Review for Midterm 1

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

Indeterminate form "0/0".

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

(A) 0

(B) 4

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(D) $-\infty$

(E) Does not exist

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

Indeterminate form "0/0".

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

$$\lim_{x \to 2} \frac{x - 2}{x^2 - 4x + 4}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

Indeterminate form “0/0”.

$$\lim_{x \to 2} \frac{x - 2}{x^2 - 4x + 4}$$

(A) 0

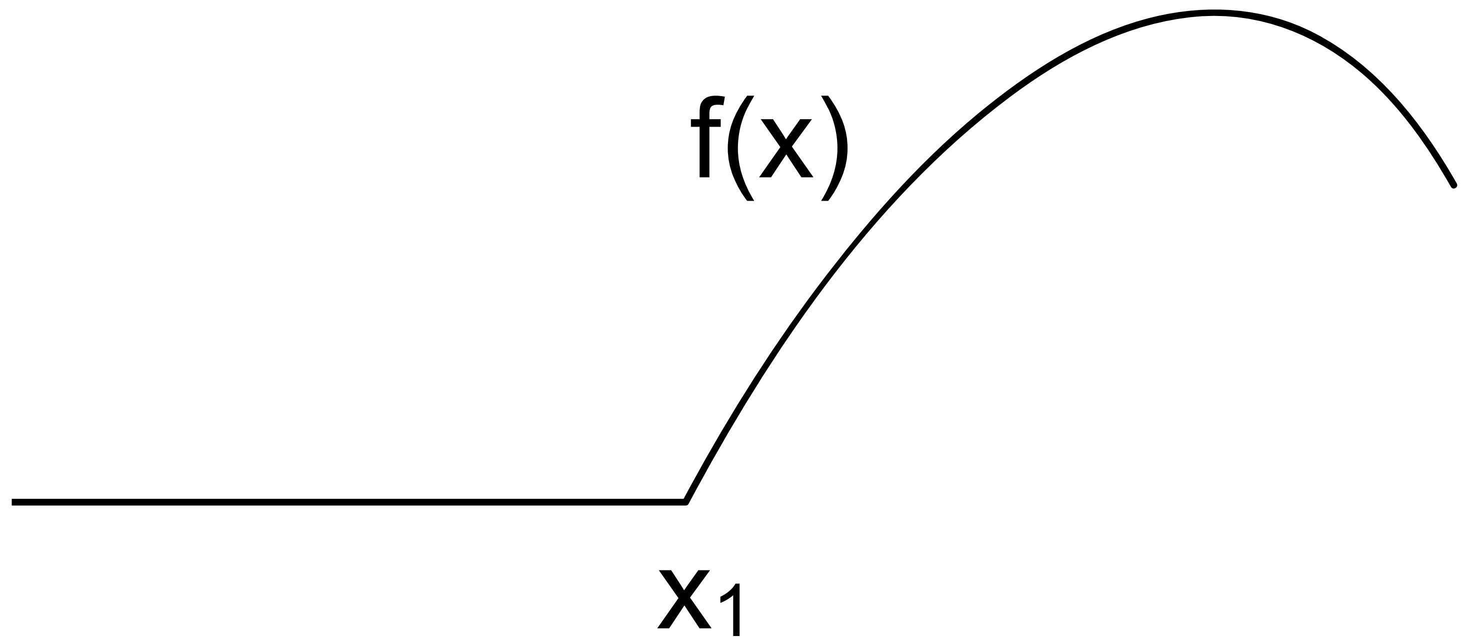
(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

Limits



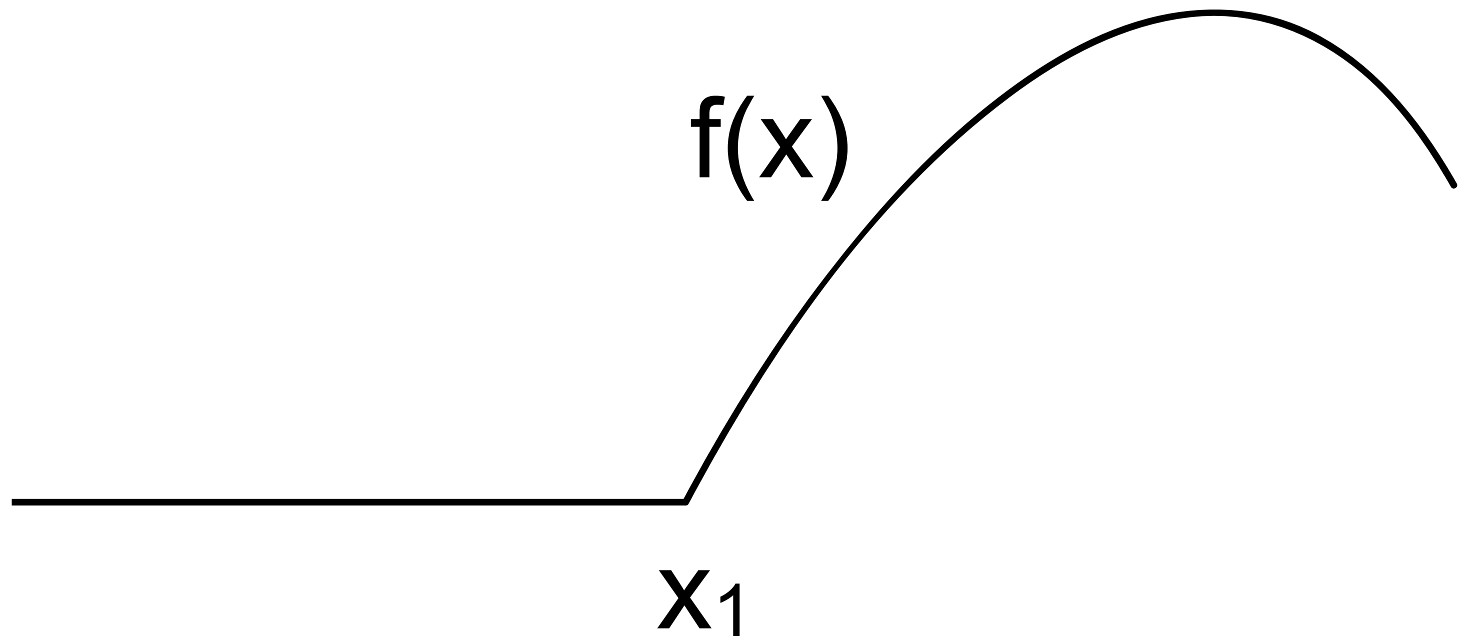
(A) $$\lim_{h\to 0} \frac{f(x_1+h)-f(x_1)}{h} = m > 0$$

(B) $$\lim_{h\to 0} \frac{f(x_1+h)-f(x_1)}{h} = 0$$

(C) Both (A) and (B)

(D) The limit does not exist.

Limits



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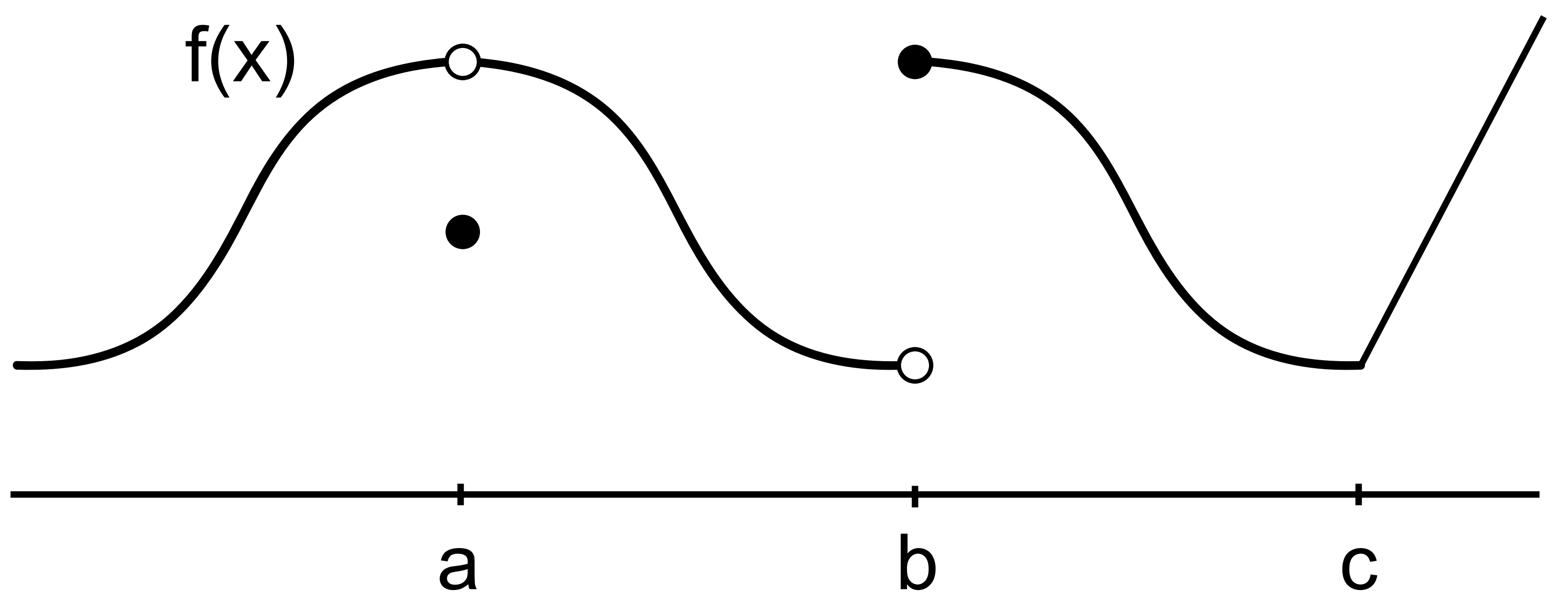


Which of the following are true?

1. $\lim_{x \to a} f(x) = f(a)$
2. $\lim_{x \to b} f(x) = f(b)$
3. $\lim_{x \to c} f(x)$ does not exist.
4. $\lim_{x \to a} f(x)$ exists.
5. $\lim_{x \to b} f(x)$ exists.

(A) 1, 4

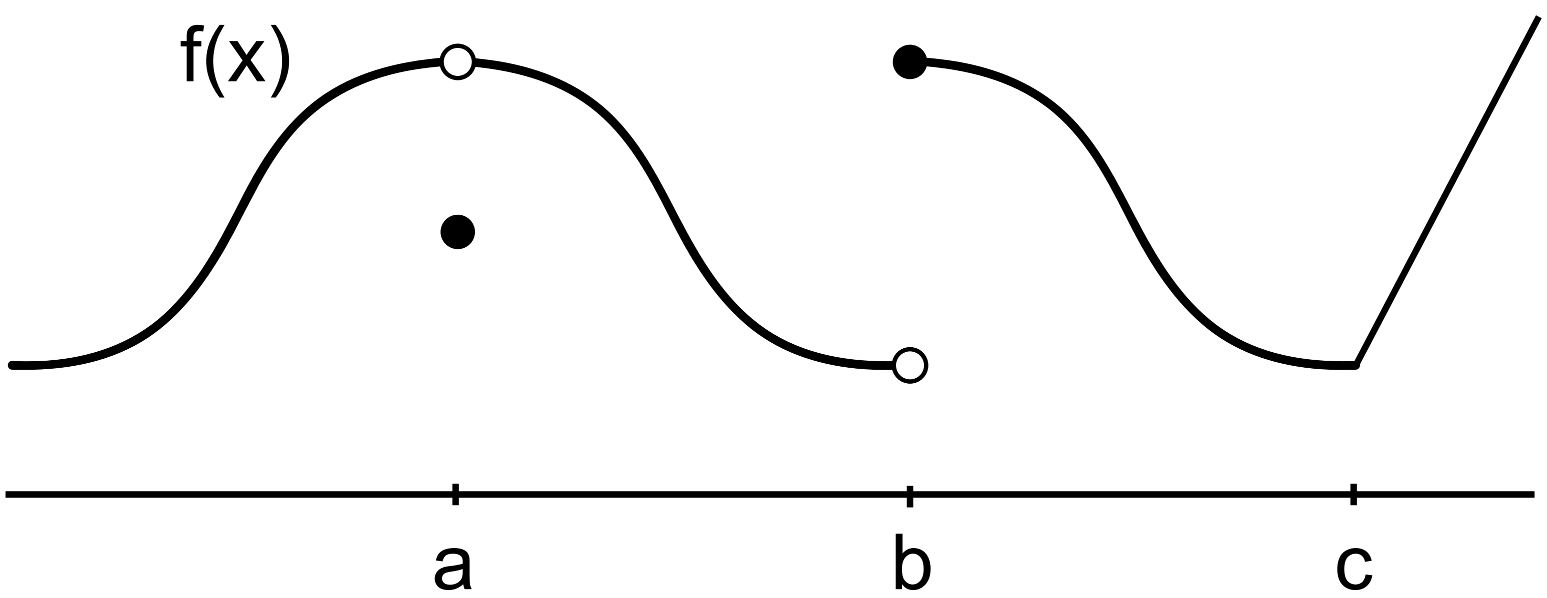
(B) 2, 5

(C) 3

(D) 4

(E) 5

Limits



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(A) 1, 4

(B) 2, 5

(C) 3

(D) 4

(E) 5

What is $\lim_{x \to -\infty} \frac{3x^n + x^2 - 1}{x^3 + 4}$?

- (A) If n=2, the limit is $-\infty$.
- (B) If n=3, the limit is ∞ .
- (C) If n>3 and even, the limit is $-\infty$.
- (D) If n>3 and odd, the limit is $-\infty$.

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- (C) If n>3 and even, the limit is $-\infty$.
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Find tangent line to $f(x)=x^2$ that goes through (1,-1).

Point of tangency is at

(A) $(1+\sqrt{2}, 3-2\sqrt{2})$

(B) $(1+\sqrt{2}, 3+2\sqrt{2})$

(C) (1, -1)

(D) $(1-\sqrt{2}, 3-2\sqrt{2})$

Find tangent line to $f(x)=x^2$ that goes through (1,-1).

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(A) $(1+\sqrt{2}, 3-2\sqrt{2})$

(B) $(1+\sqrt{2}, 3+2\sqrt{2})$

(C) (1, -1)

(D) $(1-\sqrt{2}, 3-2\sqrt{2})$

Which is true?

(A) If $f'(1/2)=0$, then $x=1/2$ must be a max or min of $f(x)$.

(B) If $f(0)=0$ and $f'(x)<0$ for $0<x<1$, then $f(1/2) <0$.

(C) If $f''(1/2) = 0$, then $x=1/2$ must be an inflection point of $f(x)$.

(D) If $f'(1/2)=0$ and $f''(1/2)<0$, then $x=1/2$ must be a min.

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Sketch $x/(1+x^2)$.

(A) $f'(x) = 1 / (2x)$

(B) $f'(x) = (1-x^2) / (1+x^2)^2$

(C) $f'(x) = (1+x^2-2x) / (1+x^2)^2$

(D) $f'(x) = 1/(1+x^2) - 2x / (1+x^2)^2$

Sketch $x/(1+x^2)$.

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Sketch $x/(1+x^2)$.

(A) $f''(x) = 2x(x^2-3) / (1+x^2)^3$

(B) $f''(x) = x(x^2-3) / (1+x^2)^2$

(C) $f''(x) = x(x^2+1) / (1+x^2)^3$

(D) $f''(x) = 2(x^2-1) / (1+x^2)^3$

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Critical points

$$f'(x) = (1-x^2) / (1+x^2)^2$$

$$f''(x) = 2x(x^2-3) / (1+x^2)^3$$

(A) x=1 is a min and x=-1 is a max.

(B) x=1 is a max and x=-1 is a min.

(C) x=1 is a max and x=-1 is neither.

(D) x=1 is neither and x=-1 is a max.

Critical points

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Inflection points

$$f''(x) = 2x(x^2-3) / (1+x^2)^3$$

(A) $x=0$, $x=\sqrt{3}$, $x=-\sqrt{3}$ are all inflection points.

(B) $x=0$ is not an inflection because f'' doesn't change sign at $x=0$.

(C) $x=\sqrt{3}$ is not an inflection point because f'' doesn't change sign at $x=\sqrt{3}$.

(D) Both $\sqrt{3}$ and $-\sqrt{3}$ are not inflection points because f'' doesn't change sign at either of them.

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Make a table

	$(-10,-\sqrt{3})$	$-\sqrt{3}$	$(-\sqrt{3},-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\sqrt{3})$	$\sqrt{3}$	$(\sqrt{3},10)$
$f(x)$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$f'(x)$	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$
$f''(x)$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$