Exponential functions

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Math 102 Section 106

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Today...

1. Check-in
2. Where are we going next?
3. Exponential functions
4. Derivative of exponential functions

Reminders:
- OSH 4 due Today
Questions? Comments? Concerns?
One of the most common type of equations that arises in scientific problems is a differential equation.

A differential equation relates one or more derivatives of a function to the function itself.

To solve a differential equation, we will look for a function that satisfies the given relationship.
Recall: Ladybugs and aphids

Predators

Prey
Ladybugs and aphids

Photo credit:

- By Gilles San Martin from Namur, Belgium - Coccinella magnifica, CC BY-SA 2.0, https://commons.wikimedia.org/w/index.php?curid=7694675
- By Shipher Wu (photograph) and Gee-way Lin (aphid provision), National Taiwan University - PLoS Biology, February 2010 direct link to the image description, CC BY 2.5, https://commons.wikimedia.org/w/index.php?curid=9590073
Ladybugs and aphids

- $x(t) =$ number of aphids at time $t$
- rate of predation by ladybugs is $P(x(t)) = \frac{Kx(t)}{a+x(t)}$
- Aphids reproduction: $G(x(t)) = rx(t)$
- The size of the aphid population satisfies the differential equation:

$$\frac{dx}{dt} = rx - \frac{Kx}{a + x}$$

- Our goals:
  1. To understand what these equations mean.
  2. To find out what $x(t)$ is (Bonus Challenge. Hint: Partial Fraction Decomposition).
Exponential Functions and their Derivatives
Exponential Functions

Q1. A bacterial culture initially contains 1 bacteria and doubles every half-hour. Find the size of the bacteria after 5 hours.

A. 32
B. 64
C. 1024
D. 2048

5 hours $\Rightarrow$ 10 doubles $\Rightarrow$ size is $2^{10} = 1024$
Q2. Which of the following is an exponential function?

A. $x^n$ power function
B. $2^x$ variable in the exponent!
C. $e^2$ this is a constant
D. $\ln(x)$ Inverse of exp. function
Q3. Exponential functions $a^x$, where $a > 1$...

A. All go through the point $(1, 1)$
B. All go through the point $(1, 0)$
C. If $a < b$, then $a^x < b^x$ for all $x > 0$ and $a^x > b^x$ for all $x < 0$
D. If $a < b$, then $a^x < b^x$ for all $x > 1$ and $a^x > b^x$ for all $x < 1$
E. None of the above
Exponential Functions

If $a < b$, then $a^x < b^x$ for all $x > 0$ and $a^x > b^x$ for all $x < 0$. 

Exponential functions

Power functions
Q4. The derivative of $f(x) = a^x$ is

A. $f'(x) = xa^{x-1}$
B. $f'(x) = ax^{a-1}$
C. $f'(x) = \frac{a^{x+h} - a^x}{h}$
D. $f'(x) = a^x$
E. $f'(x) = Ca^x$
Derivative of $f(x) = a^x$

Why $e^x$?
Differential Equations

- What real number is the same as its own square?
  - This is equivalent to asking “what $x$ satisfies $x = x^2$?”
  - This is called an algebraic equation.

- What function is equal to its own derivative?
  - Equivalent to asking “what $f(x)$ satisfies $f'(x) = f(x)$?”
  - This is called a differential equation.
Q5. Which of the following satisfies \( f'(x) = f(x) \)?

A. \( f(x) = 2^x \)
B. \( f(x) = e^x \)
C. \( f(x) = x^{-1} \)
D. \( f(x) = -x^{-1} \)
Q6. Which of the following satisfies
\[ f'(x) = (f(x))^2? \]

A. \( f(x) = 2^x \)
B. \( f(x) = e^x \)
C. \( f(x) = x^{-1} \)
D. \( f(x) = -x^{-1} \)
Summary

- Differential equations are relationships between derivatives and functions that arise in scientific problems.
- Exponential growth (e.g. bacterial cultures).
- Exponential functions are different from power functions.
- If $f(x) = a^x$, then $f'(x) = a^x \ln(a)$.
- $\frac{d}{dx} e^x = e^x$. 

Answers

1. C
2. B
3. C
4. E
5. B
6. D
1. In a river, the number of fish (in thousands), $x$, in a given year is linked to the number of salmon (in thousands), $y$, in the following year by the function

$$y = Axe^{-bx}$$

where $A, b > 0$. For what number of salmon is there no change in the number from year to the next? Find the number of salmon that would yield the largest number of salmon in the following year.
2. Sketch the graph of the function $y = x^2 e^{-x}$