Today

- Optimal foraging
- Intro to least squares
Foraging
Foraging time includes
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- a commute ($t_0 \rightarrow \text{constant}$),
Foraging time includes
- a commute ($t_0 \rightarrow$ constant),
- a visit to each patch ($t_p$)
Foraging success is characterized by $f(t_p) = \text{resource collected from a single patch after a time } t_p \text{ spent in the patch.}$

Remember the definition of $f(t_p)$ for an upcoming clicker Q.
Which of the following graphs matches the given description of \( f(t) \)?

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Foraging

What to maximize?

food
Foraging

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- Total amount collected?
Foraging

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- Stay in a patch forever.
Foraging

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  - Stay in a patch forever.
- If you can move to a new patch, move when the returns diminish enough to make the new patch look better.
Foraging

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  • Stay in a patch forever.
• If you can move to a new patch, move when the returns diminish enough to make the new patch look better.
  • Leave right away!
Foraging

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Don’t forget travel time!
Foraging

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What to maximize?

- Total amount collected?
  - Stay in a patch forever.
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  - Leave right away!
- Maximize average rate of collection.

food

Don’t forget travel time!
Foraging

food(t)
Foraging

I waited

(A) not long enough.
(B) just the right amount of time.
(C) too long.
Foraging

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\[ R(t) = \frac{\text{food}(t) - \text{food}(0)}{t - 0} \]
Foraging

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Choose \( t \) to maximize \( R(t) \).
Foraging
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Foraging

\[ \text{food} \]

\[ t \]

\[ \text{food} \]
Foraging
Foraging

food(t)
food(t) = \begin{cases} 
0 & \text{for } 0 \leq t \leq t_0 \\
\food(t-t_0) & \text{for } t > t_0
\end{cases}
Foraging

$$\text{food}(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq t_0 \\ f(t-t_0) & \text{for } t > t_0 \end{cases}$$

Average Rate of Collection

$$R(t) = \frac{\text{food}(t) - \text{food}(0)}{t - 0}$$
Foraging

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Average Rate of Collection

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$R(t)$ maximal at $t_{max}$. 
Foraging

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R(t) maximal at \( t_{\text{max}} \).

Optimal \( t_p = t_{\text{max}} - t_0 \).
Foraging

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Average Rate of Collection

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\( R(t) \) maximal at \( t_{\text{max}} \).

Optimal \( t_p = t_{\text{max}} - t_0 \).

Could have maximized \( R(t_p + t_0) = \frac{\text{f}(t_p)}{t_p + t_0} \) to get best \( t_p \).
Least squares model fitting
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How do we find the best line to fit through the data?
Least squares model fitting

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$y = ax$

$(x^n, y^n)$

$i = 1, 2, 3, \ldots, n$
Least squares model fitting

\[ y = ax \]

i = 1, 2, 3, ..., n

\((x_i, y_i)\)

\((x_1, y_1)\)

\((x_n, y_n)\)
Least squares model fitting

Each red bar is called a residual. We want all the residuals to be as small as possible.

\[ y = ax \]

\( (x_i, y_i) \quad \text{for} \quad i = 1, 2, 3, \ldots, n \)
The residuals are...

(A) \( r_i = y_i^2 + x_i^2 \)

(B) \( r_i = a^2 (y_i^2 + x_i^2) \)

(C) \( r_i = y_i - ax_i \)

(D) \( r_i = y_i - x_i \)

(E) \( r_i = x_i - y_i \)
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