Today

- Requests for Friday/Monday (so far):
  - (A) Biggest cone inside a cone.
  - (B) Linear regression.
  - (C) Differential equation example.
- Solving linear differential equations.
- Requested examples (if there's time).

A drug delivered by IV accumulates at a constant rate k. The body metabolizes the drug proportional to the amount of the drug.

(A) $d'(t) = k_{IV} - k_m d(t)$

(B) $d'(t) = (k_{IV} - k_m)\, d(t)$

(C) $d'(t) = k_{IV} d(t) - k_m$

(D) $d'(t) = -k_{IV} + k_m d(t)$

A drug delivered by IV accumulates at a constant rate k. The body metabolizes the drug proportional to the amount of the drug.

(A) $d'(t) = k_{IV} - k_m d(t)$

(B) $d'(t) = (k_{IV} - k_m)\, d(t)$

(C) $d'(t) = k_{IV} d(t) - k_m$

(D) $d'(t) = -k_{IV} + k_m d(t)$

A drug delivered by IV accumulates at a constant rate k. The body metabolizes the drug proportional to the amount of the drug.

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

(A) $d(t) = k_{IV}/k_m\,(1-\exp(k_m t))$

(B) $d(t) = k_{IV}/k_m\,(1-\exp(-k_m t))$

(C) $d(t) = k_{IV}/k_m - \exp(k_m t)$

(D) $d(t) = k_{IV}/k_m - \exp(-k_m t)$

(E) $d(t) = k_{IV}/k_m - (k_{IV}-k_m)\exp(-k_m t))$

Note: $\exp(x)=e^x$.

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.
- $c(t) = k_{IV} - k_m\, d(t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.
- $c(t) = k_{IV} - k_m\, d(t)$
- $c'(t) = -k_m\, d'(t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.
- $c(t) = k_{IV} - k_m\, d(t)$
- $c'(t) = -k_m\, d'(t)$
- New equation (in $c(t)$ with no $d(t)$ appearing):

(A) $c'(t) = -k_m\, c(t)$

(B) $c'(t) = -k_{IV}\, c(t)$

(C) $c'(t) = k_m\, c(t)$

(D) $c'(t) = -k_m\, (k_{IV} - k_m\, d(t))$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.
- $c(t) = k_{IV} - k_m d(t)$
- $c'(t) = -k_m d'(t)$
- New equation (in $c(t)$ with no $d(t)$ appearing):

(A) $c'(t) = -k_m c(t)$

(B) $c'(t) = -k_{IV} c(t)$

(C) $c'(t) = k_m c(t)$

(D) $c'(t) = -k_m (k_{IV} - k_m d(t))$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.
- $c(t) = k_{IV} - k_m d(t)$
- $c'(t) = -k_m d'(t)$
- New equation (in $c(t)$ with no $d(t)$ appearing):

(A) $c'(t) = -k_m c(t)$

(B) $c'(t) = -k_{IV} c(t)$

(C) $c'(t) = k_m c(t)$

(D) $c'(t) = -k_m (k_{IV} - k_m d(t))$

What about the initial condition, $c(0) = ?$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.
- $c(t) = k_{IV} - k_m d(t)$
- $c'(t) = -k_m d'(t)$
- New equation: $c'(t) = -k_m c(t)$, $c(0)=k_{IV}$.

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.
- $c(t) = k_{IV} - k_m\, d(t)$
- $c'(t) = -k_m\, d'(t)$
- New equation: $c'(t) = -k_m\, c(t)$, $c(0) = k_{IV}$.
- Which means the solution to the $d(t)$ eq. is

(A) $d(t) = k_{IV} \exp(-k_m t)$

(B) $d(t) = k_{IV} \exp(k_m t)$

(C) $d(t) = k_{IV}/k_m\, (1 - \exp(-k_m t))$

(D) $d(t) = k_{IV}/k_m\, \exp(-k_m t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.
- $c(t) = k_{IV} - k_m\, d(t)$
- $c'(t) = -k_m\, d'(t)$
- New equation: $c'(t) = -k_m\, c(t)$, $c(0)=k_{IV}$.
- Which means the solution to the $d(t)$ eq. is

(A) $d(t) = k_{IV} \exp(-k_m t)$

(B) $d(t) = k_{IV} \exp(k_m t)$

(C) $d(t) = k_{IV}/k_m\, (1-\exp(-k_m t))$

(D) $d(t) = k_{IV}/k_m\, \exp(-k_m t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p'=kp$.

What happens to $d(t)$ as $t \to \infty$?

- New equation: $c'(t) = -k_m\, c(t)$, $c(0)=k_{IV}$.
- Which means the solution to the $d(t)$ eq. is

(A) $d(t) = k_{IV} \exp(-k_m t)$

(B) $d(t) = k_{IV} \exp(k_m t)$

(C) $d(t) = k_{IV}/k_m\, (1-\exp(-k_m t))$

(D) $d(t) = k_{IV}/k_m\, \exp(-k_m t)$

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- Make related equation that looks like $p' = kp$.

What happens to d(t) as t--> ∞?

$d(t) \to k_{IV}/k_m$

- New equation: $c'(t) = -k_m\, c(t)$, $c(0) = k_{IV}$.
- Which means the solution to the d(t) eq. is

(A) $d(t) = k_{IV} \exp(-k_m t)$

(B) $d(t) = k_{IV} \exp(k_m t)$

(C) $d(t) = k_{IV}/k_m\, (1-\exp(-k_m t))$

(D) $d(t) = k_{IV}/k_m\, \exp(-k_m t)$

General case

- Any problem of the form $y' = a - by$ with IC $y(0)=y_0$ has solution
  - $y(t) = a/b + (y_0 - a/b)\, e^{-bt}$
- Check:
  - LHS: $y'(t)$ = (on the blackboard)
  - RHS: $a - by$ = (on the blackboard)
  - $y(0) = a/b + (y_0 - a/b)\, e^0 = y_0$

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

- If b>0 then as t--> ∞, y(t) --> a/b.

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

- If b>0 then as t--> ∞, y(t) --> a/b.
- When b>0, the characteristic time is 1/b.

General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

- If b>0 then as t--> ∞, y(t) --> a/b.
- When b>0, the characteristic time is 1/b.
- Notice that if y_0=a/b then y(t) = a/b.

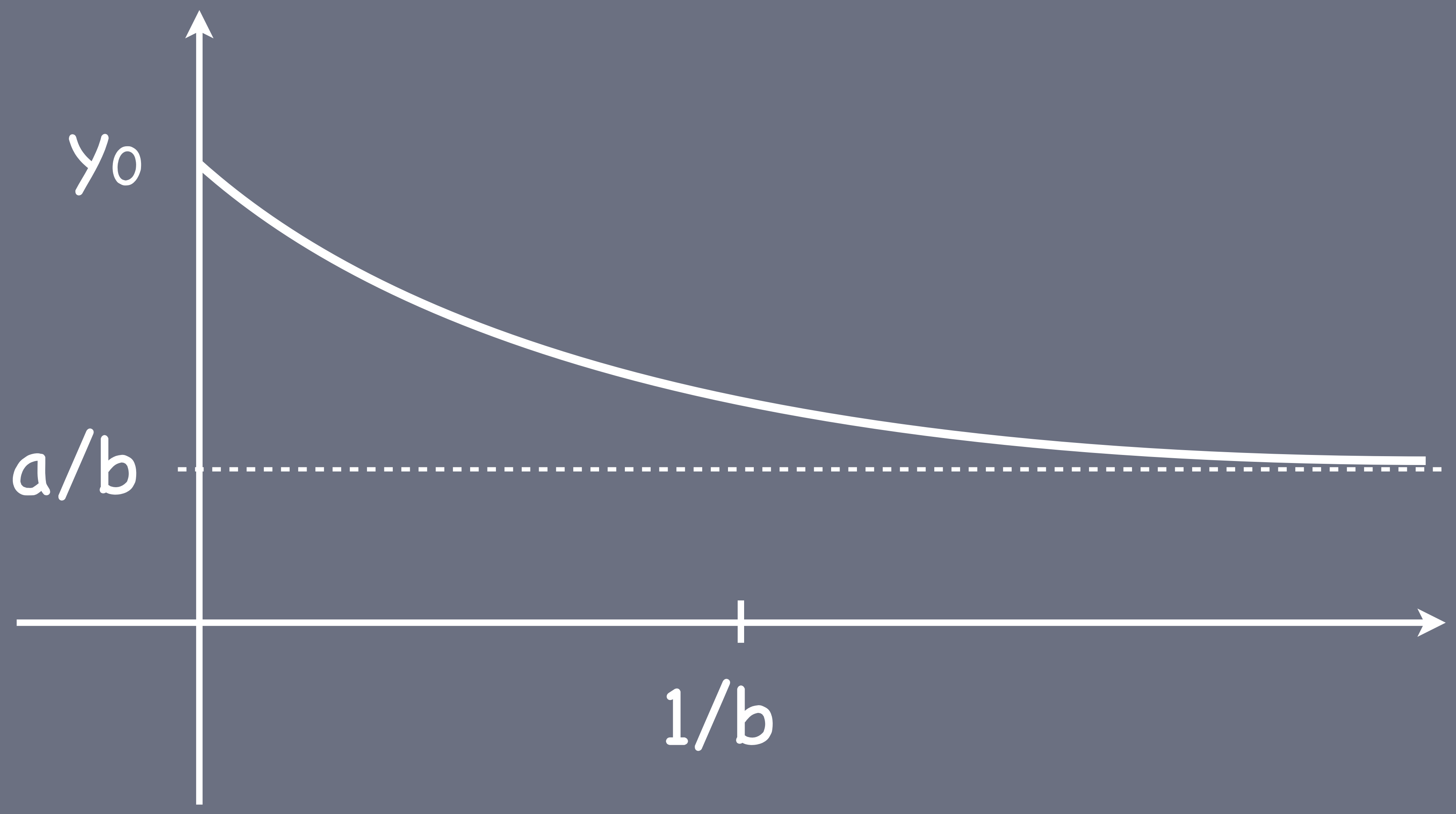
General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$

- If b>0 then as t--> ∞, y(t) --> a/b.
- When b>0, the characteristic time is 1/b.
- Notice that if y_0=a/b then y(t) = a/b.
- Constant solutions like this are called steady states.

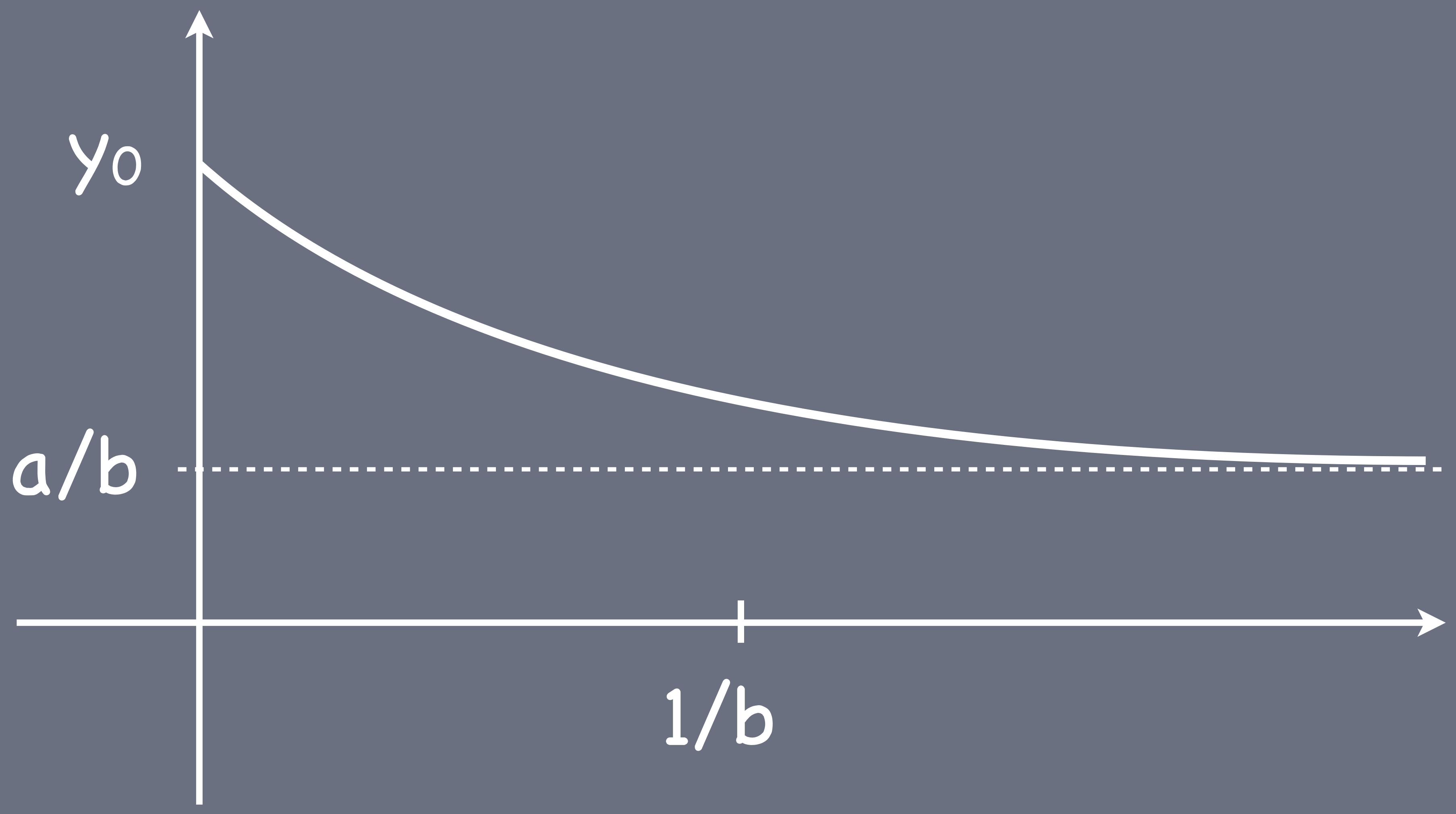
General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$



General case

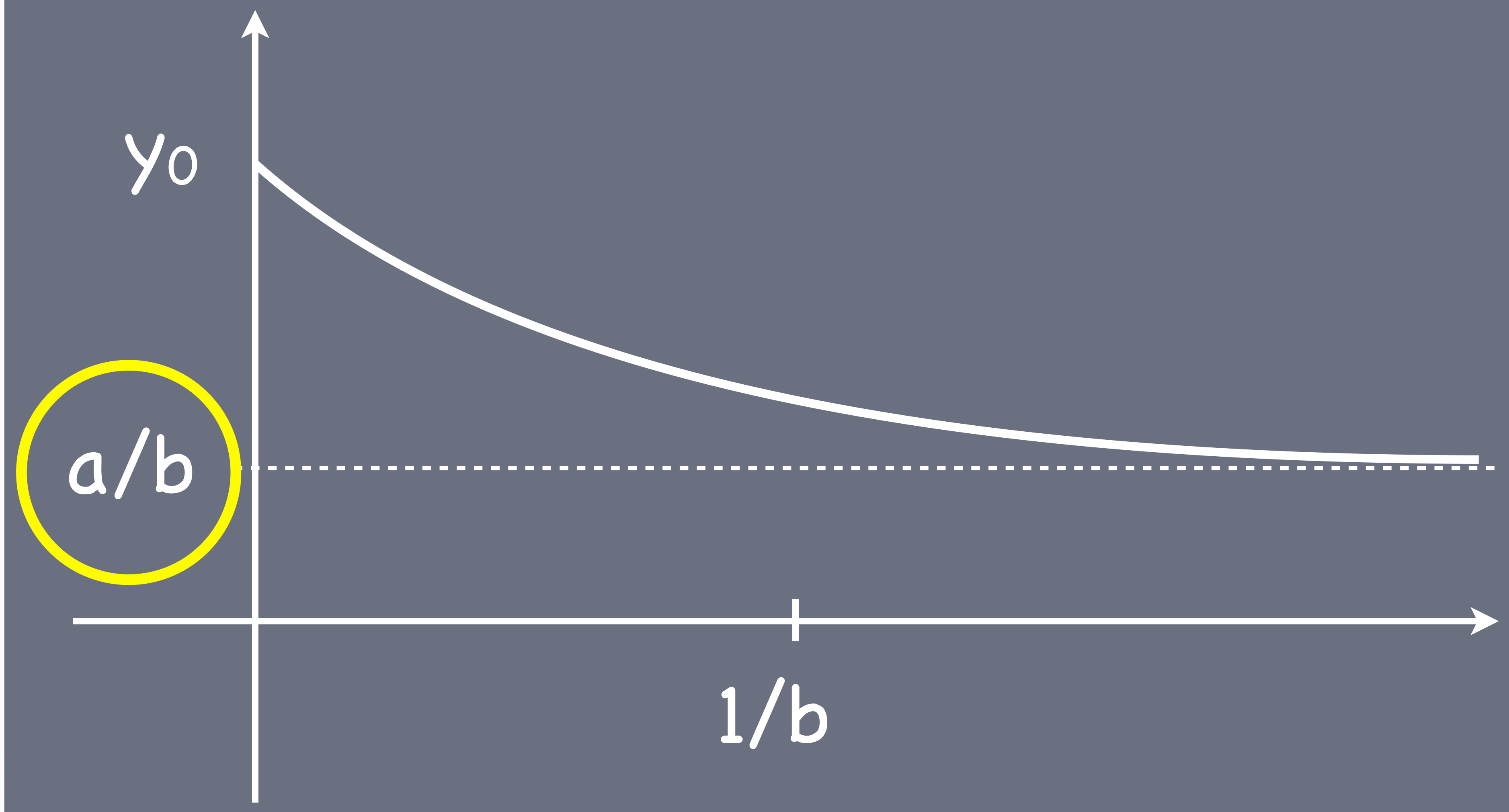
$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$



Where is y(t) going?

General case

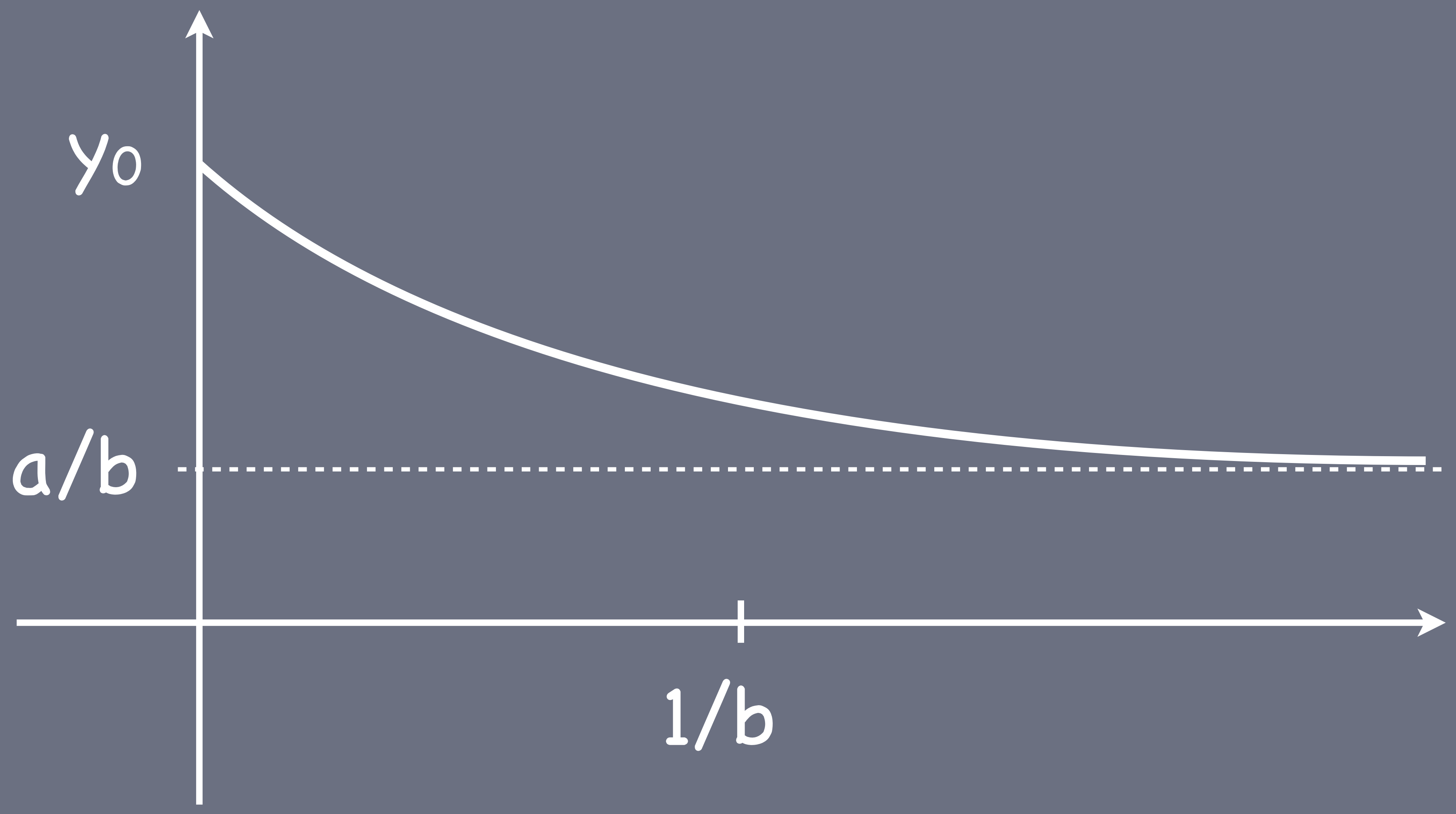
$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$



Where is y(t) going? To the steady state a/b.

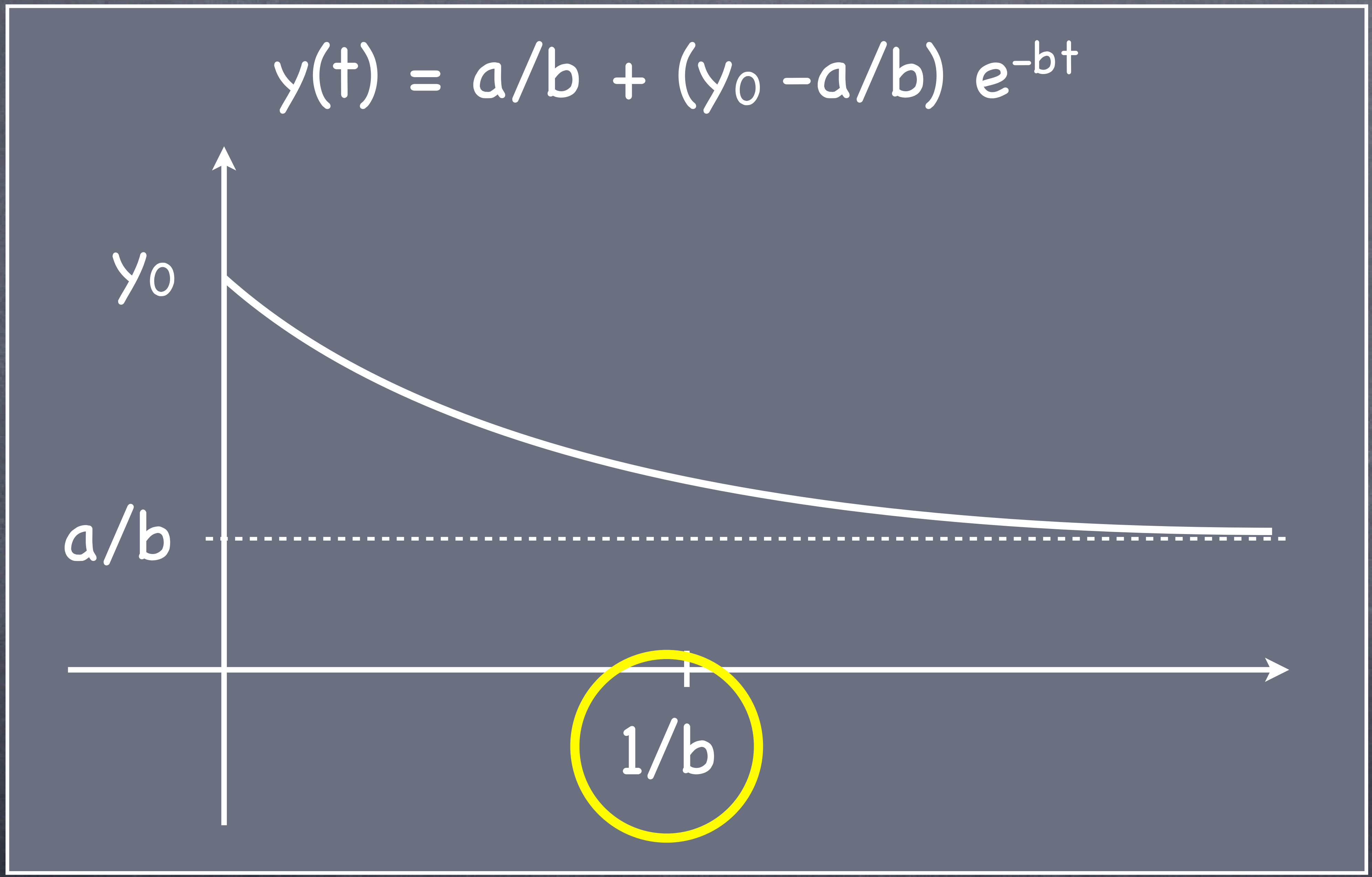
General case

$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$



When will it get there?

General case

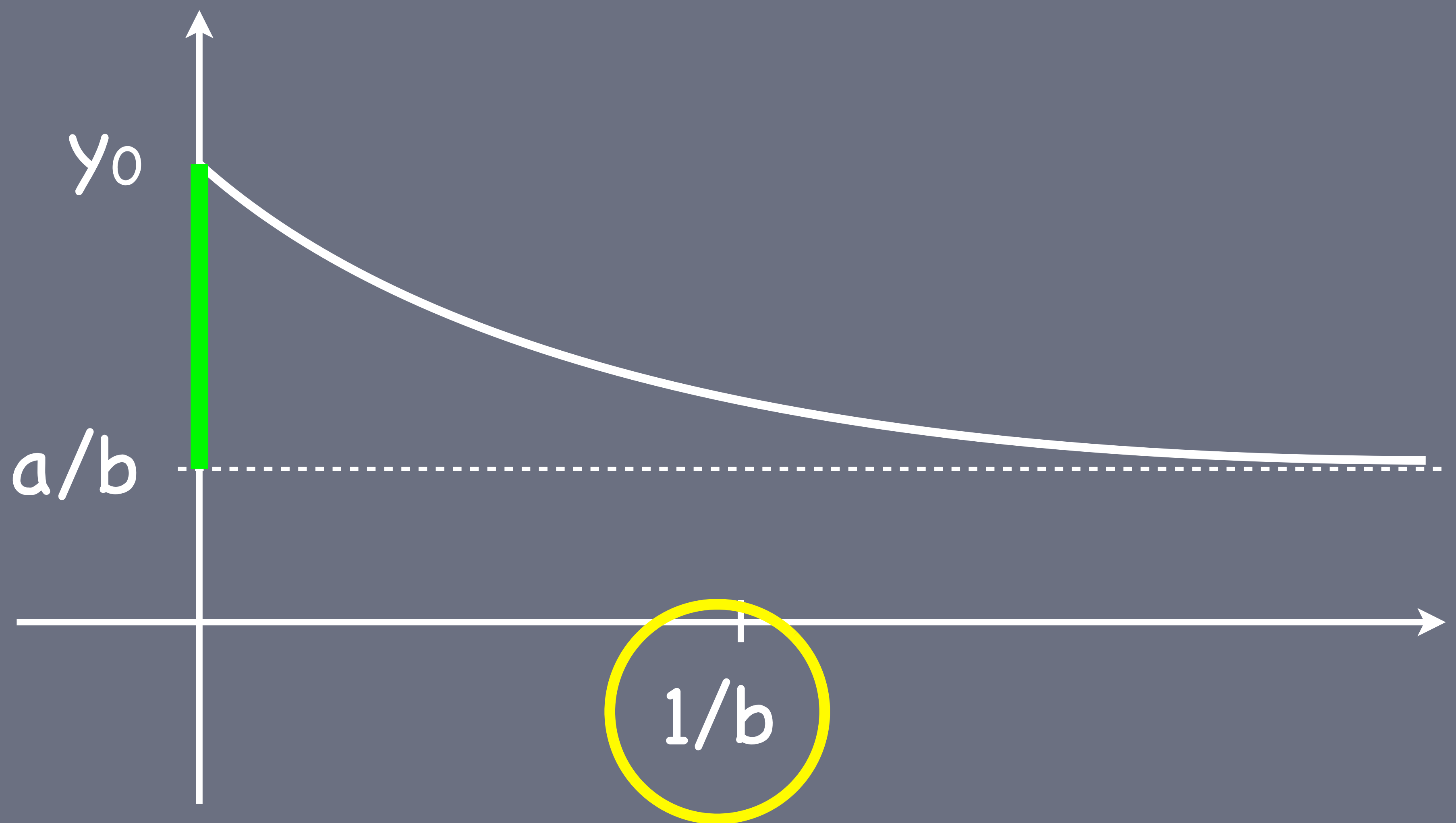


When will it get there?

Never but at t=1/b it will be 1/e of the way.

General case

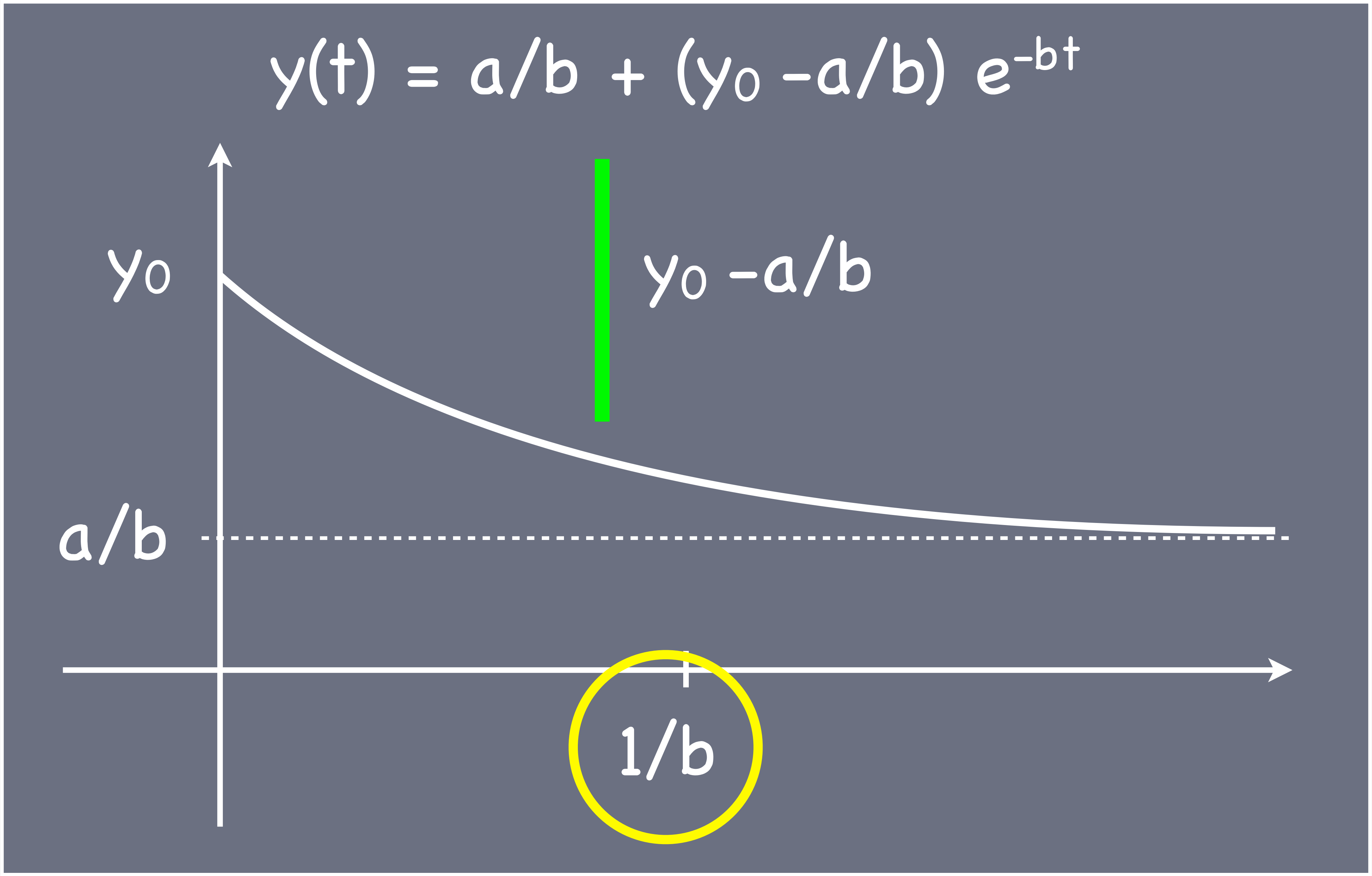
$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$



When will it get there?

Never but at t=1/b it will be 1/e of the way.

General case

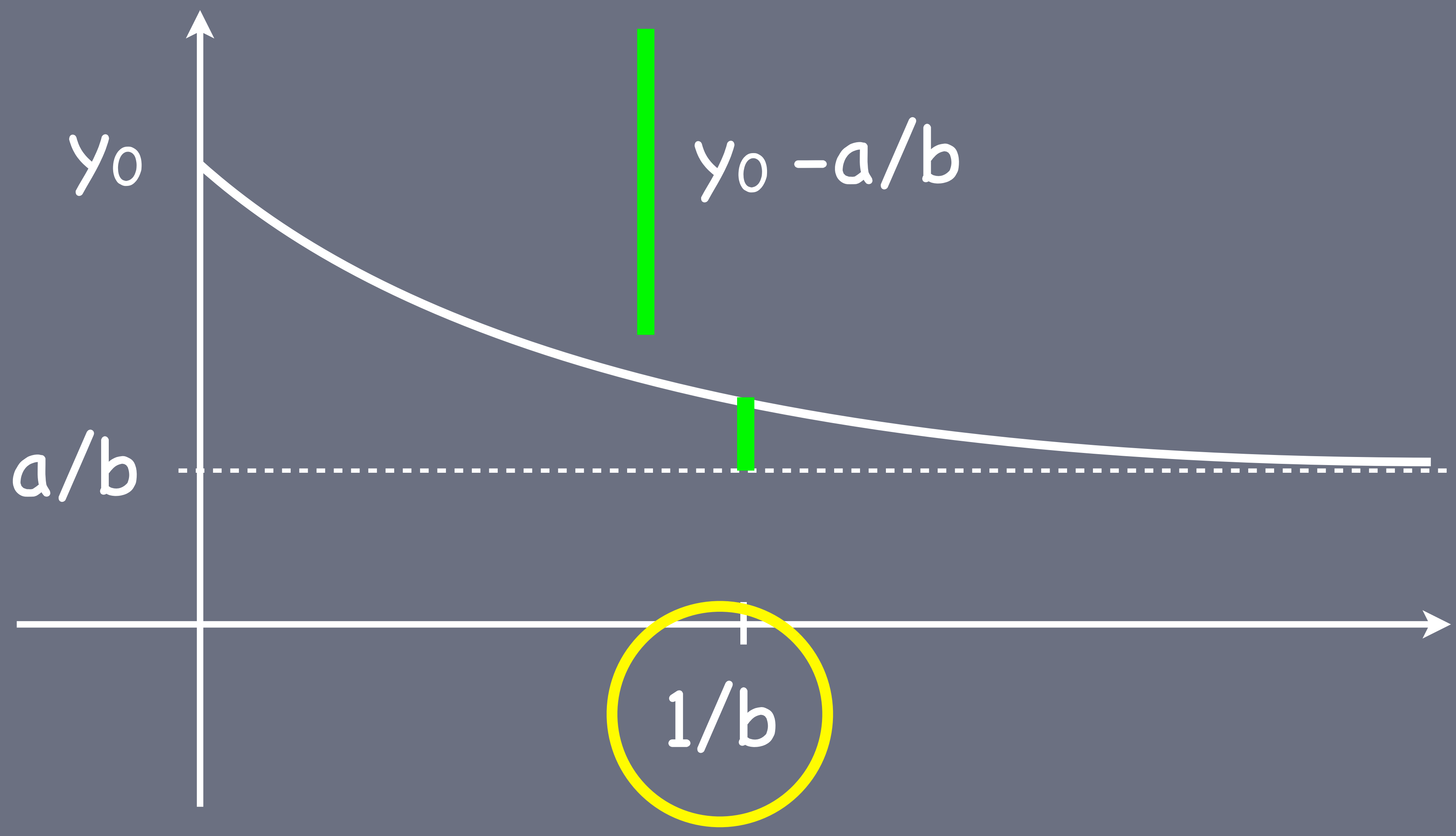


When will it get there?

Never but at $t=1/b$ it will be $1/e$ of the way.

General case

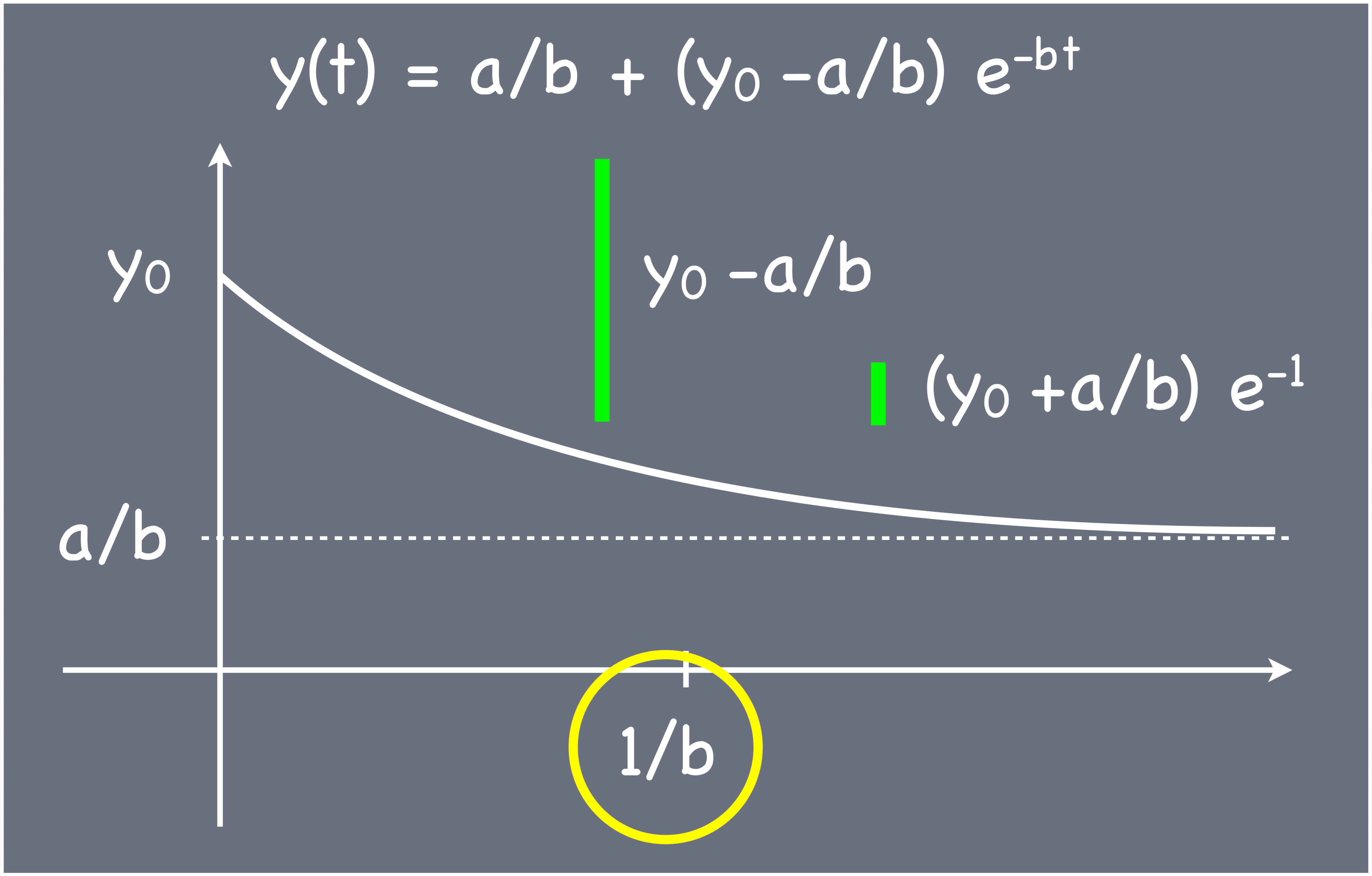
$$y(t) = a/b + (y_0 - a/b)\, e^{-bt}$$



When will it get there?

Never but at t=1/b it will be 1/e of the way.

General case



When will it get there?

Never but at $t=1/b$ it will be $1/e$ of the way.

Look different, same same.

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E - T(t))$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$
    - $a=kE$, $b=k$.

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$
    - $a=kE$, $b=k$.
- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$
    - $a=kE$, $b=k$.
- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$
    - $a=k_{IV}$, $b=k_m$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$
    - $a=kE$, $b=k$.
- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$
    - $a=k_{IV}$, $b=k_m$
- Terminal velocity: $mv'(t) = f - \mu v(t)$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$
  - $a=kE$, $b=k$.
- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$
  - $a=k_{IV}$, $b=k_m$
- Terminal velocity: $mv'(t) = f - \mu v(t)$
  - $a=f/m$, $b=\mu/m$

Look different, same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$
  - $a=kE$, $b=k$.
- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$
  - $a=k_{IV}$, $b=k_m$
- Terminal velocity: $mv'(t) = f - \mu v(t)$
  - $a=f/m$, $b=\mu/m$
- General form, factored: $y'(t) = b\ (a/b - y)$

What do you need to know?

What do you need to know?

- Given a word description, write down an equation for the quantity $q(t)$ described.

What do you need to know?

- Given a word description, write down an equation for the quantity $q(t)$ described.
  - Ex. Blah is added at a constant rate and is removed proportional to how much is there...

What do you need to know?

- Given a word description, write down an equation for the quantity $q(t)$ described.
  - Ex. Blah is added at a constant rate and is removed proportional to how much is there...
  - Ex. Blah changes proportional to the difference between blah and fixed #.

What do you need to know?

- Given a word description, write down an equation for the quantity q(t) described.
  - Ex. Blah is added at a constant rate and is removed proportional to how much is there...
  - Ex. Blah changes proportional to the difference between blah and fixed #.
- Substitute as in the drug problem to get $y'=ky$ and state that $y(t)=Ce^{kt}$ solves it.

What do you need to know?

- Given a word description, write down an equation for the quantity q(t) described.
  - Ex. Blah is added at a constant rate and is removed proportional to how much is there...
  - Ex. Blah changes proportional to the difference between blah and fixed #.
- Substitute as in the drug problem to get $y'=ky$ and state that $y(t)=Ce^{kt}$ solves it.
- Substitute back to find q(t).

What do you need to know?

- Given a word description, write down an equation for the quantity q(t) described.
  - Ex. Blah is added at a constant rate and is removed proportional to how much is there...
  - Ex. Blah changes proportional to the difference between blah and fixed #.
- Substitute as in the drug problem to get $y'=ky$ and state that $y(t)=Ce^{kt}$ solves it.
- Substitute back to find q(t).
- Determine C using the initial condition.

What do you need to know?

- Given a word description, write down an equation for the quantity q(t) described.
  - Ex. Blah is added at a constant rate and is removed proportional to how much is there...
  - Ex. Blah changes proportional to the difference between blah and fixed #.
- Substitute as in the drug problem to get $y'=ky$ and state that $y(t)=Ce^{kt}$ solves it.
- Substitute back to find q(t).
- Determine C using the initial condition.
- Answer questions about the resulting q(t).