Today

- Trig related rate problems:
  - Triangle with height-angle relation (fish).
  - Triangle with angle--opposite-side relation (clock).
- Linear approximation (3 examples).

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

Relate the two changing quantities (h and θ):

(A) $\sin(\theta) = 2/h$

(B) $\sin(\theta/2) = 1/h$

(C) $\sin(\theta/2) = 1/\mathrm{sqrt}(1+h^2)$

(D) $\tan(\theta) = 2/h$

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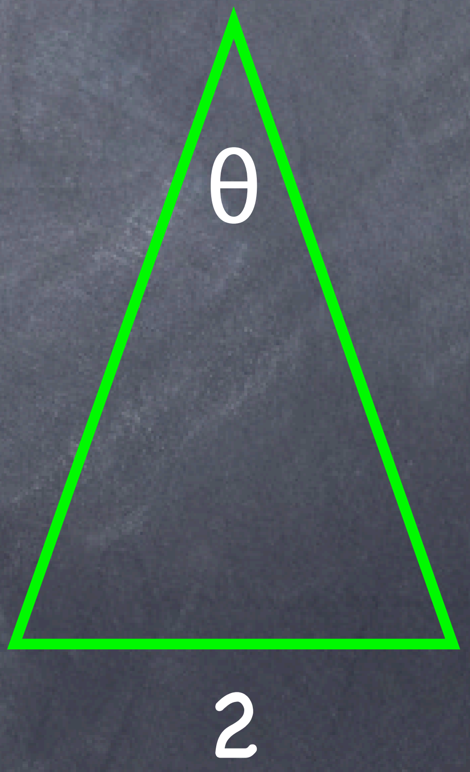

If the height of an isosceles triangle with base 2m changes at a rate h′=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

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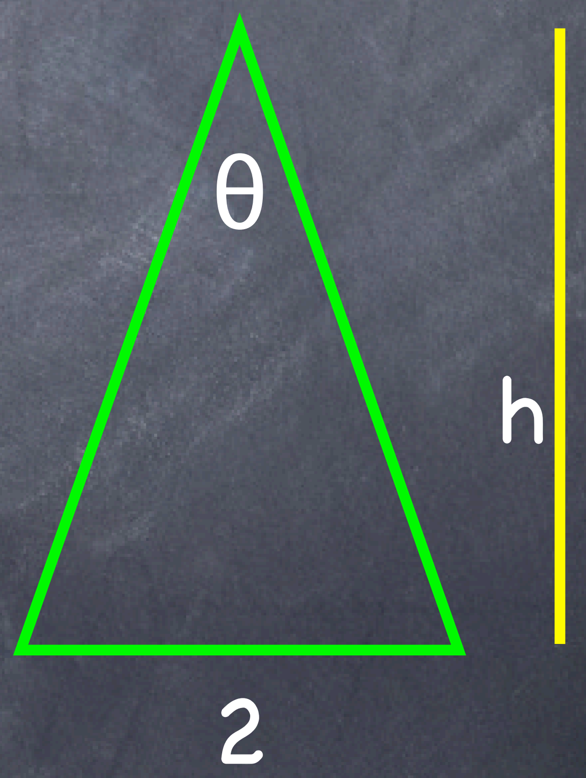

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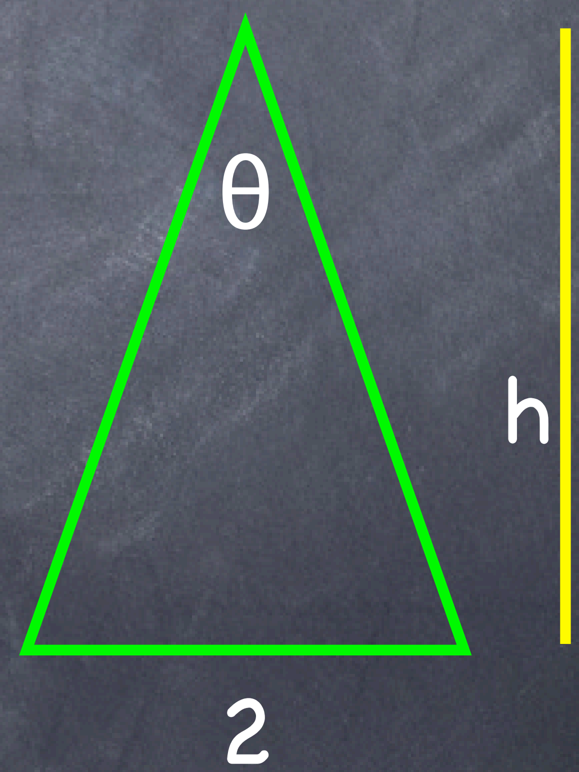

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$\theta = \ldots$ (A) $\pi/6$ (B) $\pi/4$ (C) $\pi/3$ (D) $2\pi/3$ (E) π.

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    $= -2 \cos^2(\theta/2) \quad = -3/2$ radians/s

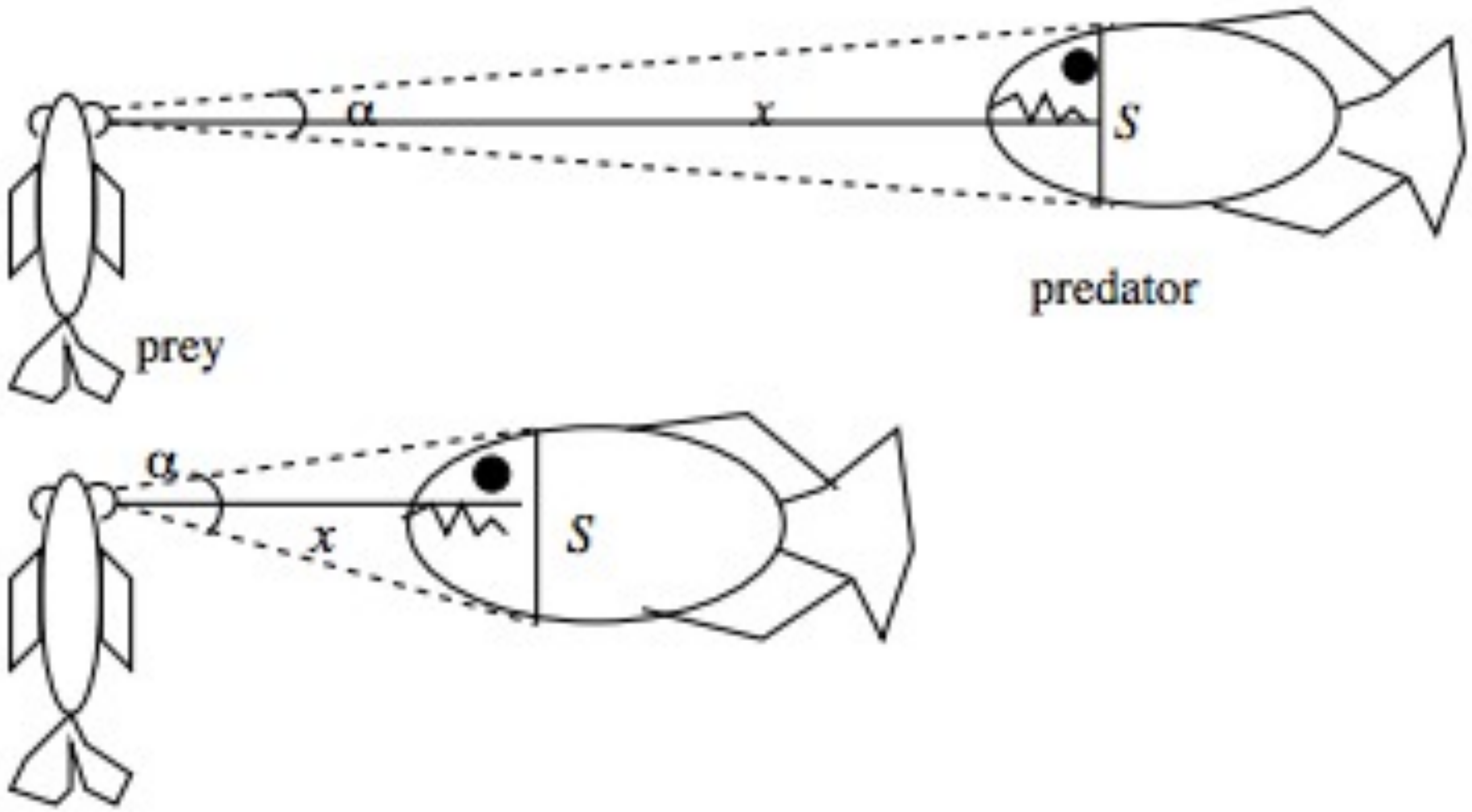
θ = ... (A) π/6 (B) π/4 (C) π/3 (D) 2π/3 (E) π.

Zebra Danio escape response

http://en.wikipedia.org/wiki/File:Zebrafisch.jpg

Zebra Danio escape response

10.9.1 The Zebra danio and its escape response



ZD tries to escape when α′ is above a threshold value.

Zebra Danio escape response

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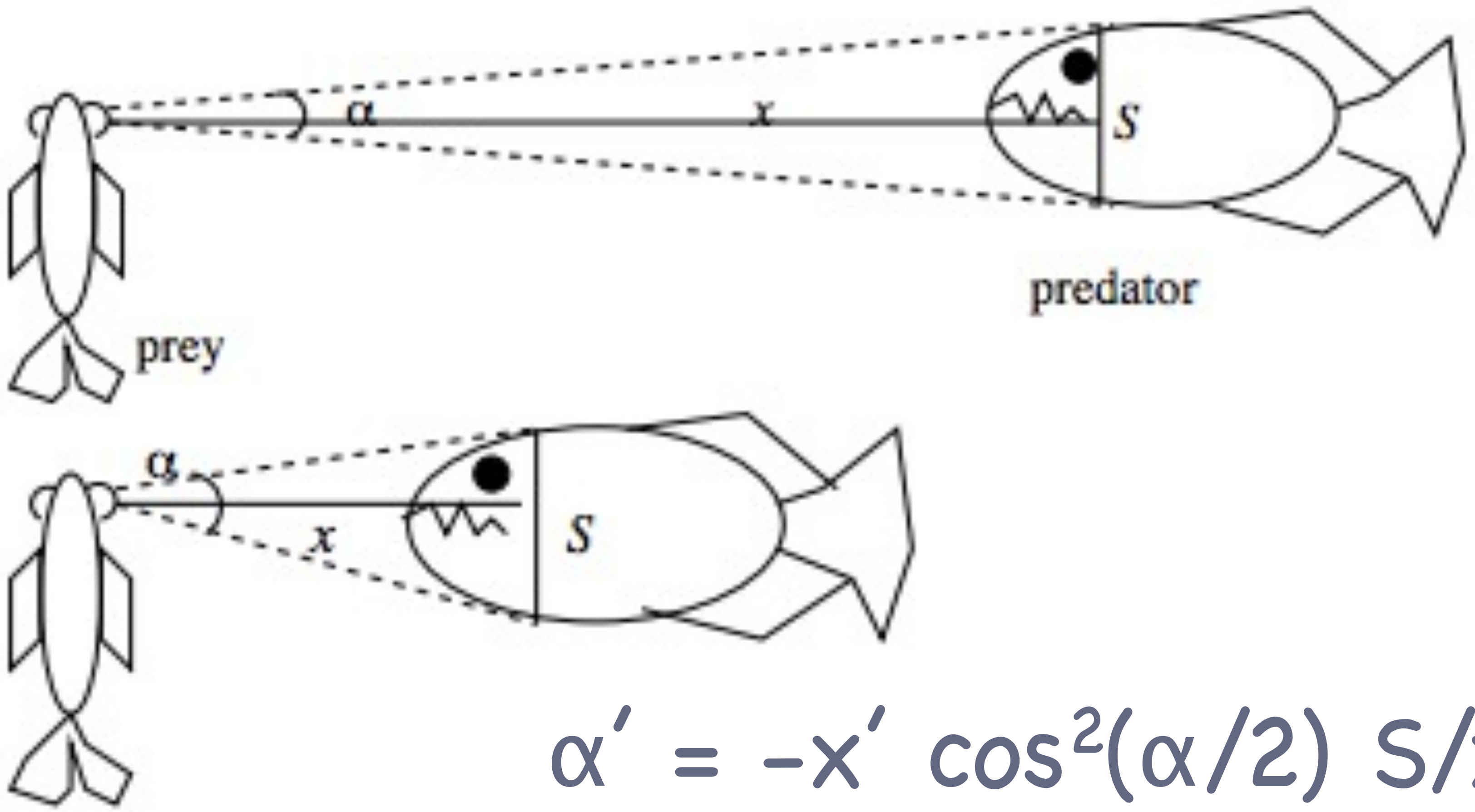

$$\alpha' = -x' \cos^2(\alpha/2)\, S/x^2$$

ZD tries to escape when α' is above a threshold value.

What is $\cos^2(a)$ when $\tan(a)=p/q$?

(A) $(p^2+q^2) / q^2$

(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) p^2/q^2

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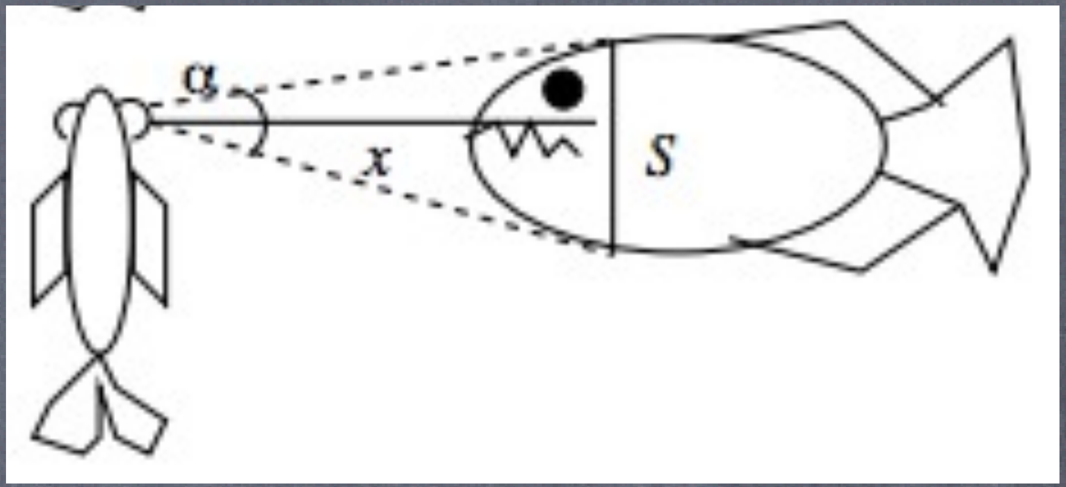

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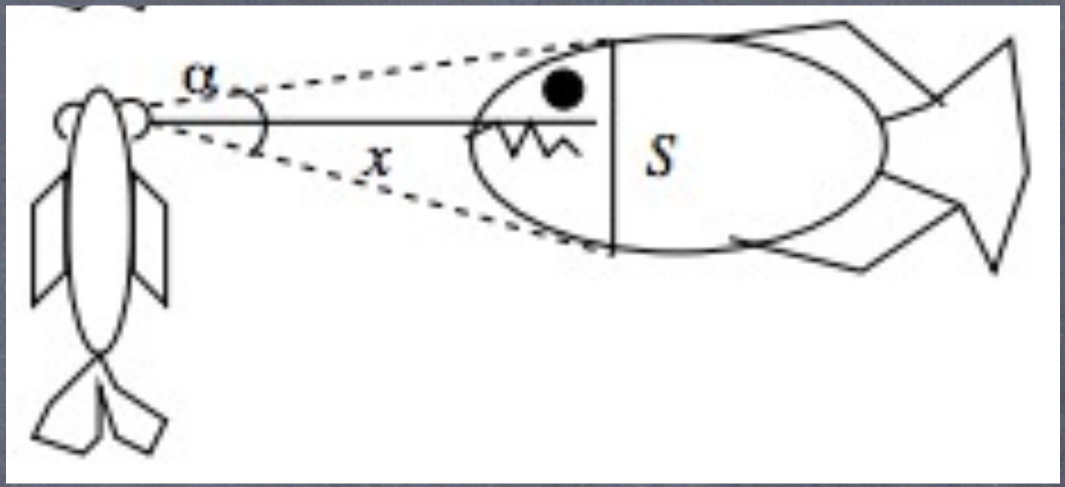

$$\alpha' = -x' \cos^2(\alpha/2)\, S/x^2$$

$$= -x'\, x^2/(x^2+S^2/4)\, S/x^2$$

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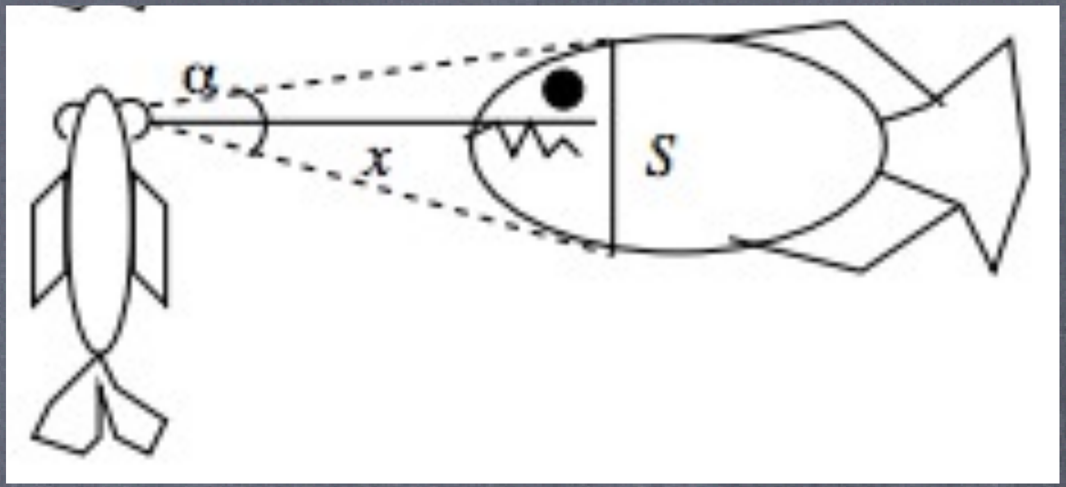

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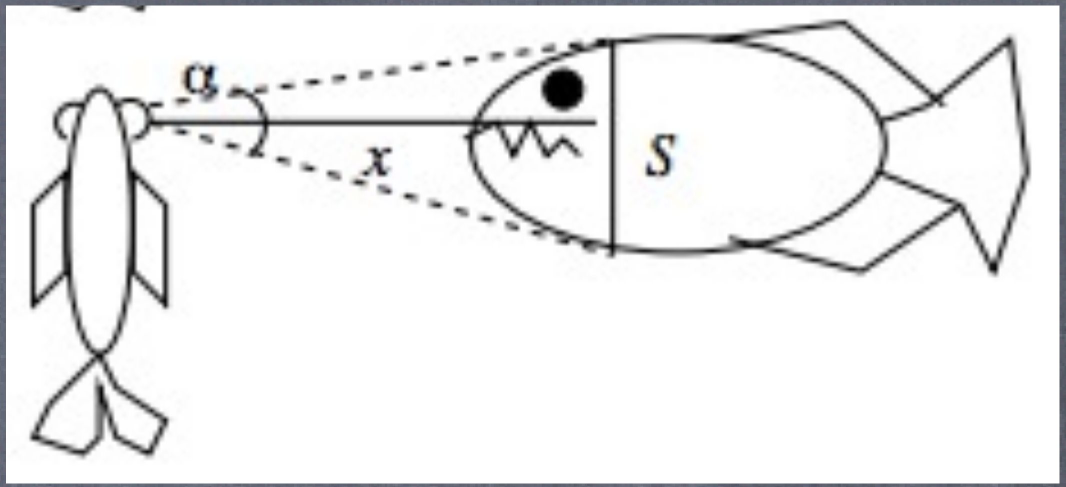

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$$= -x'\ S/(x^2+S^2/4)$$

$$= v\ S/(x^2+S^2/4)$$

Assuming the Zebra Danio reacts to a rapidly changing optical angle α, it will try to escape from...

(A) ...a very large predator.

(B) ...a very small predator.

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If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator distance x constant, plot $\alpha' = v\, S/(x^2+S^2/4)$ as function of S.

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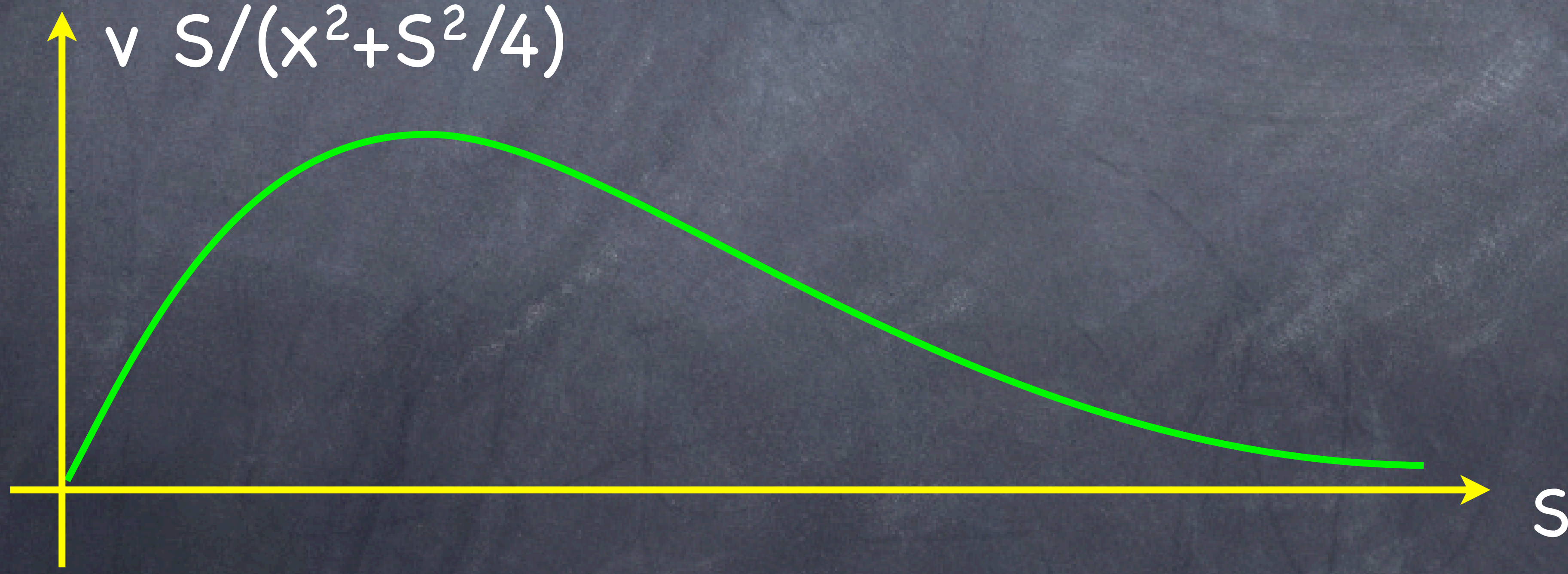

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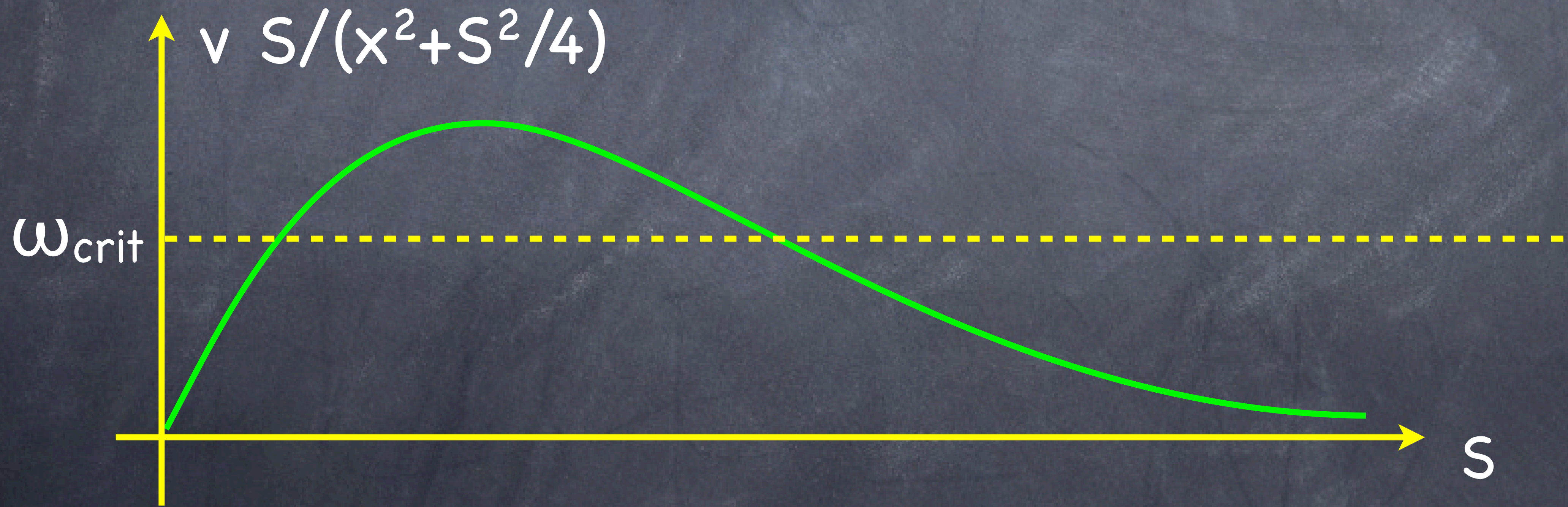

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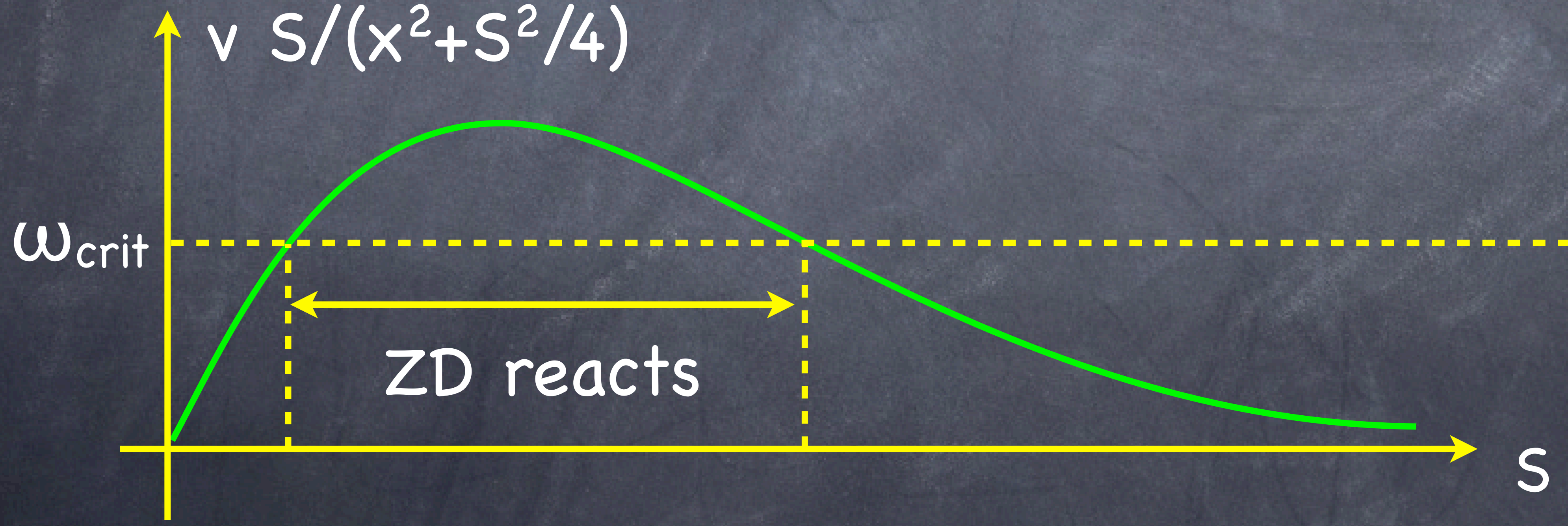

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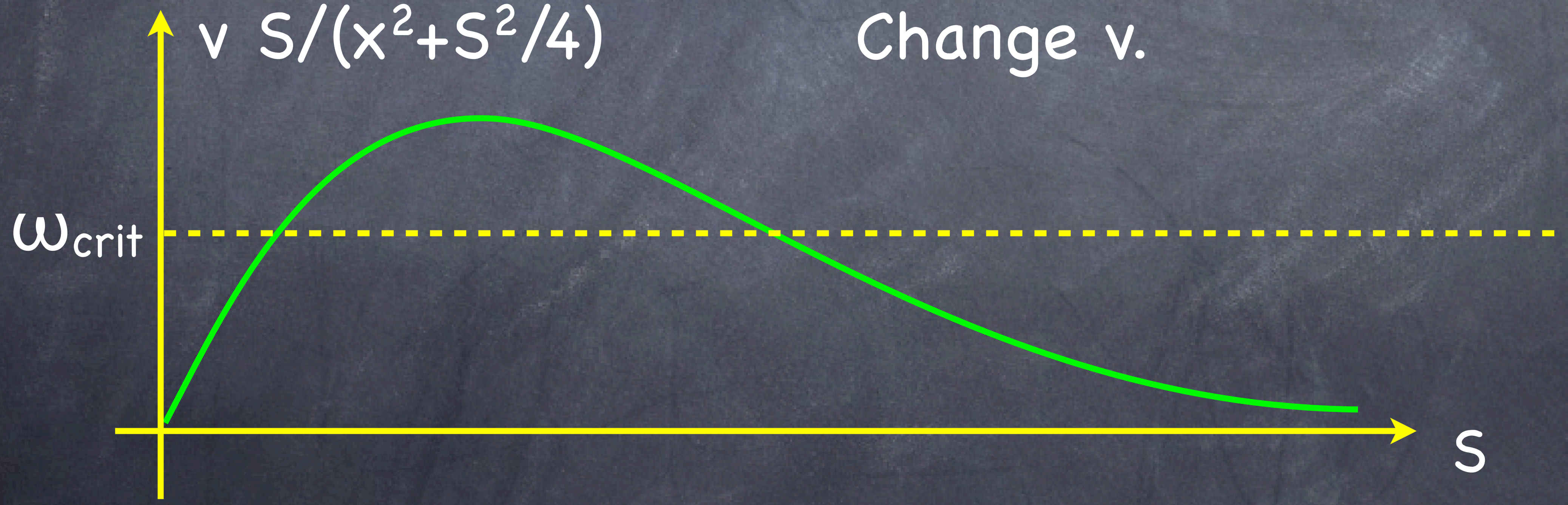
Change v.



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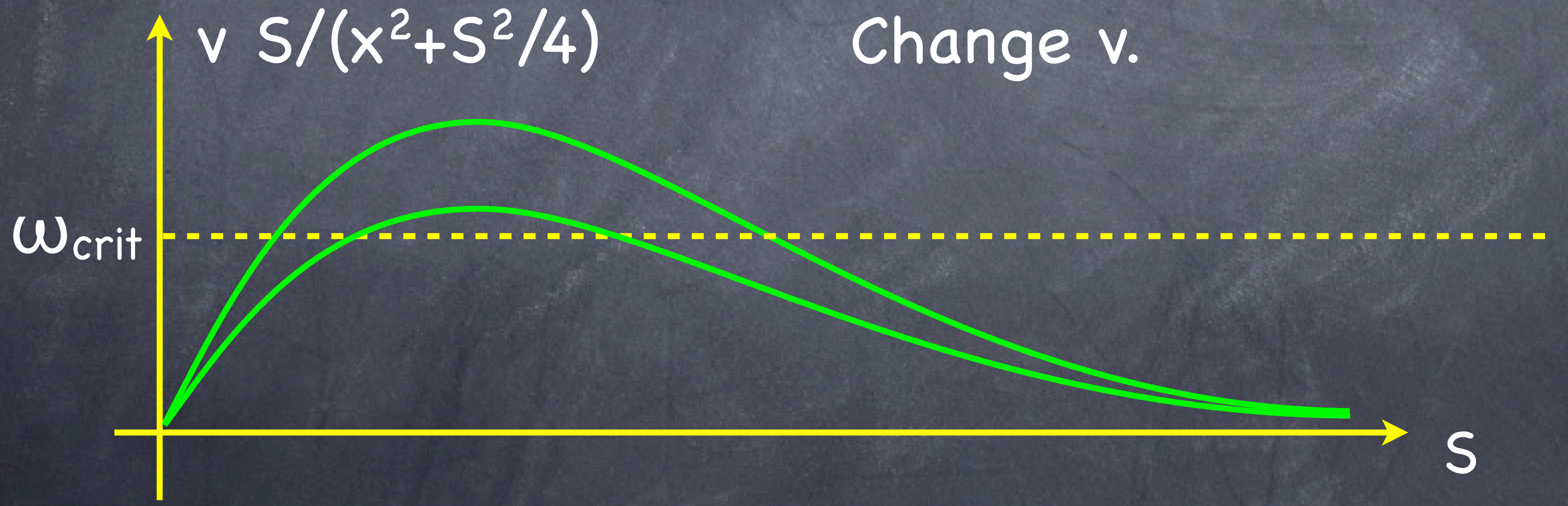

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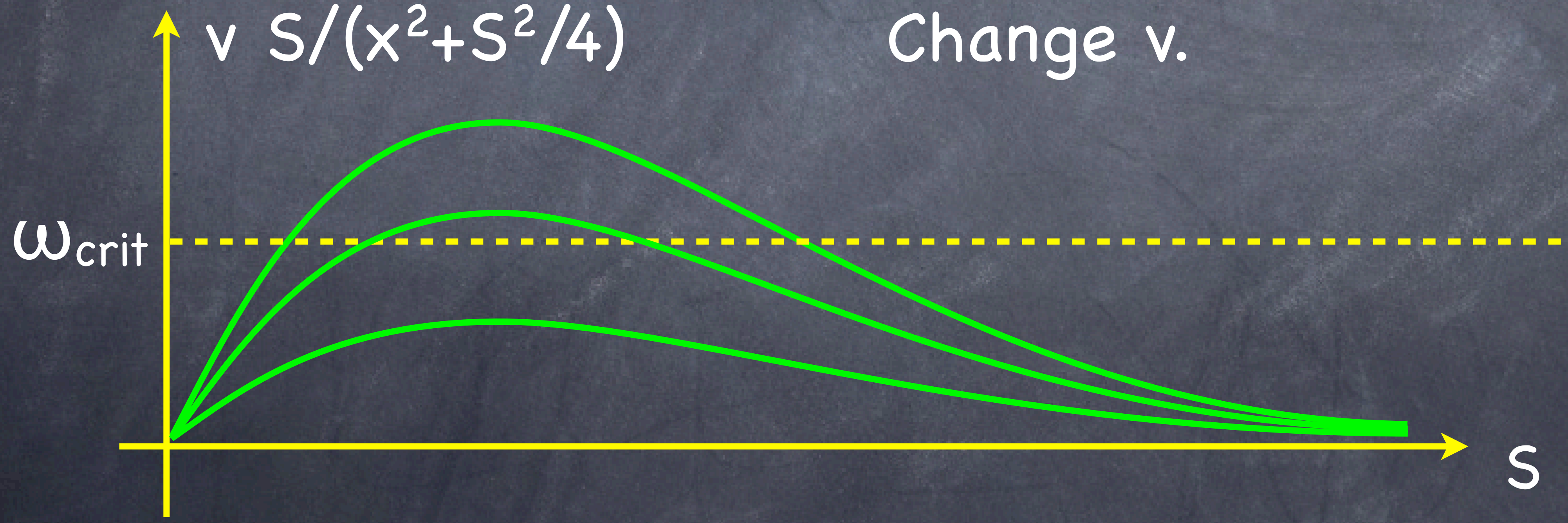

Change v.

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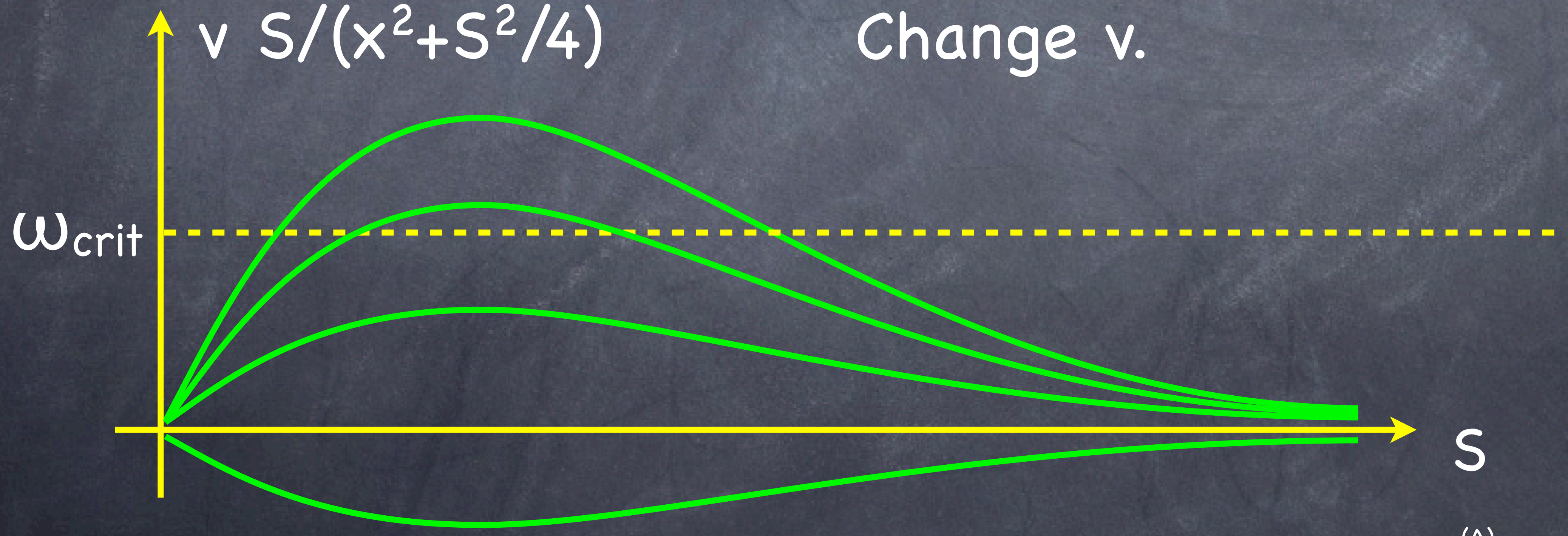

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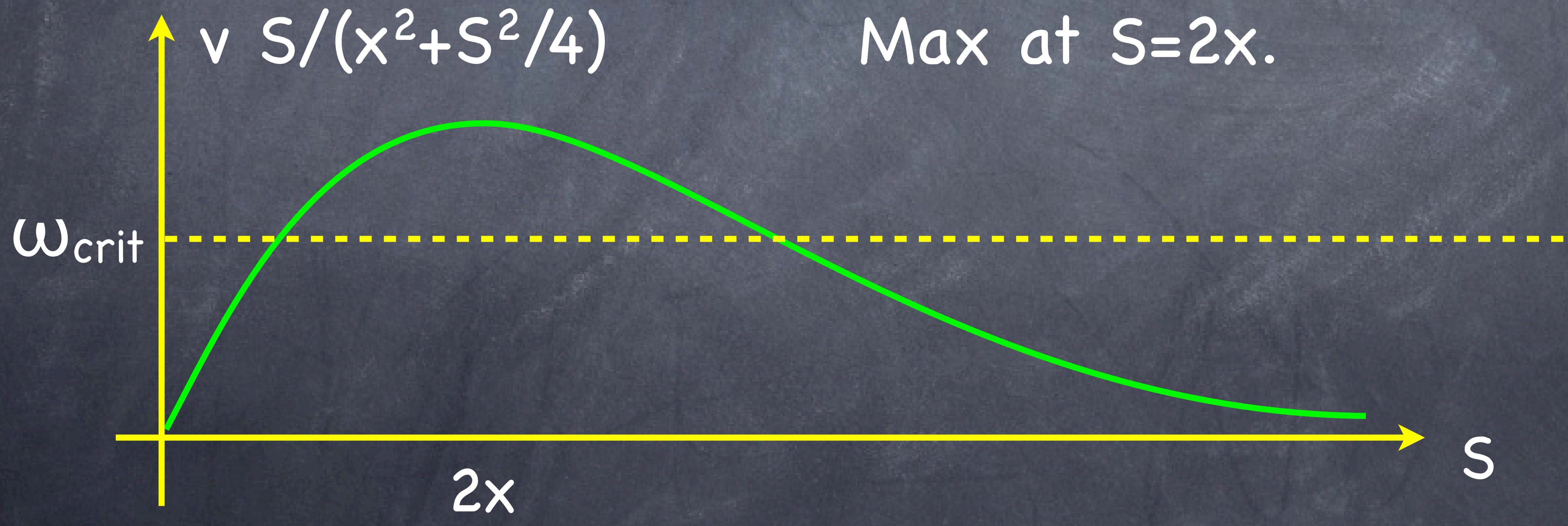

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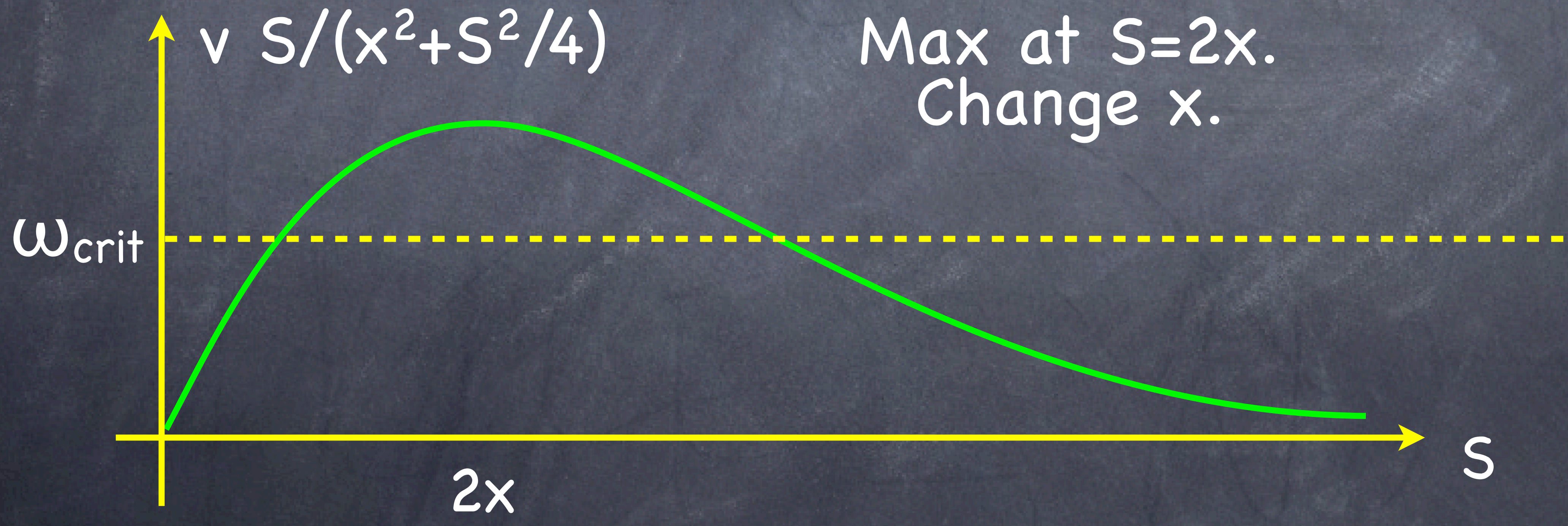

Max at S=2x.
Change x.

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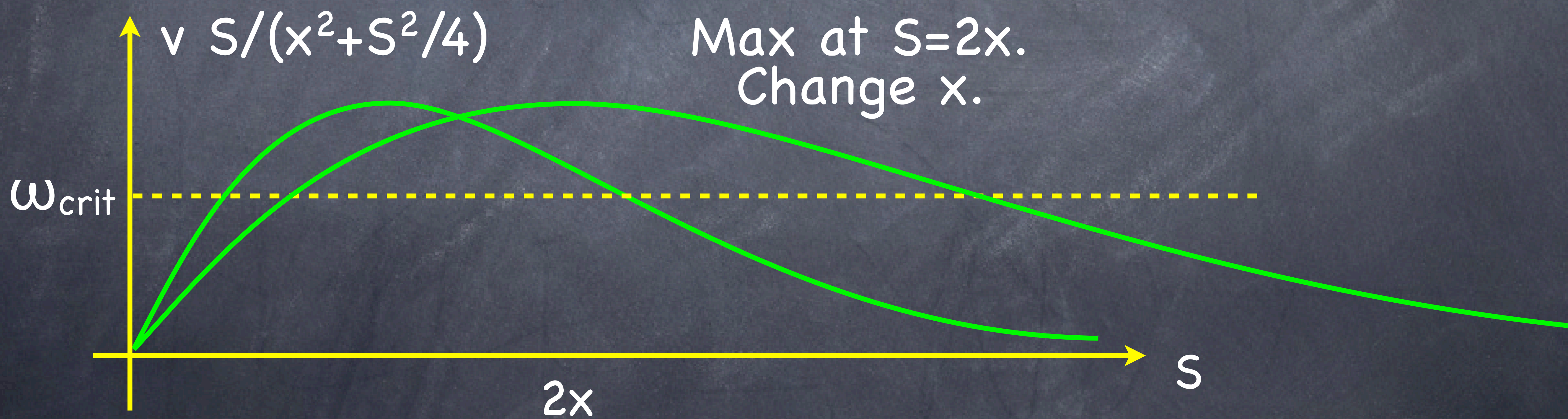

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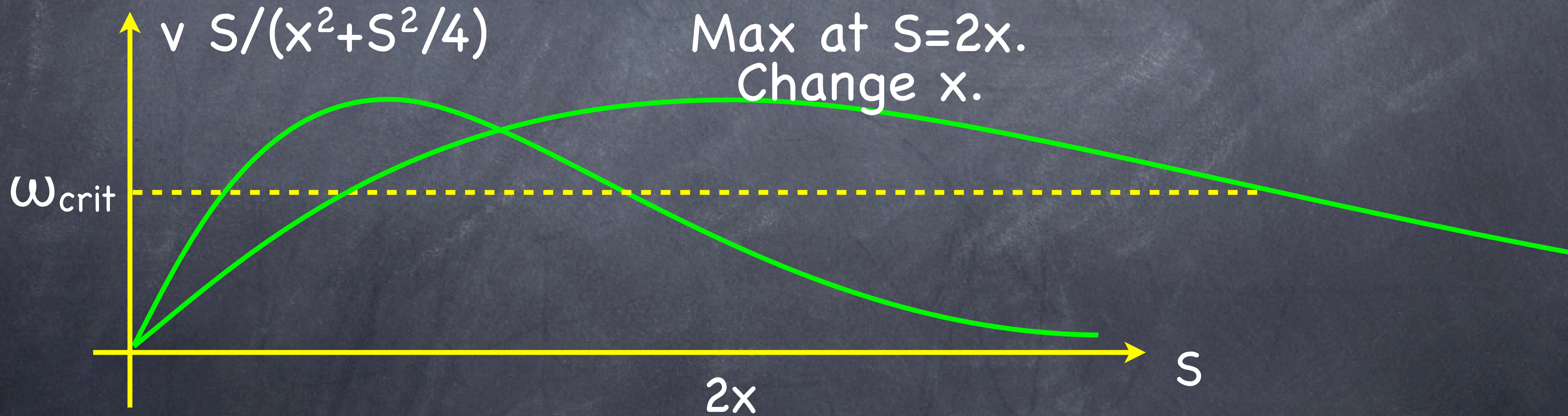

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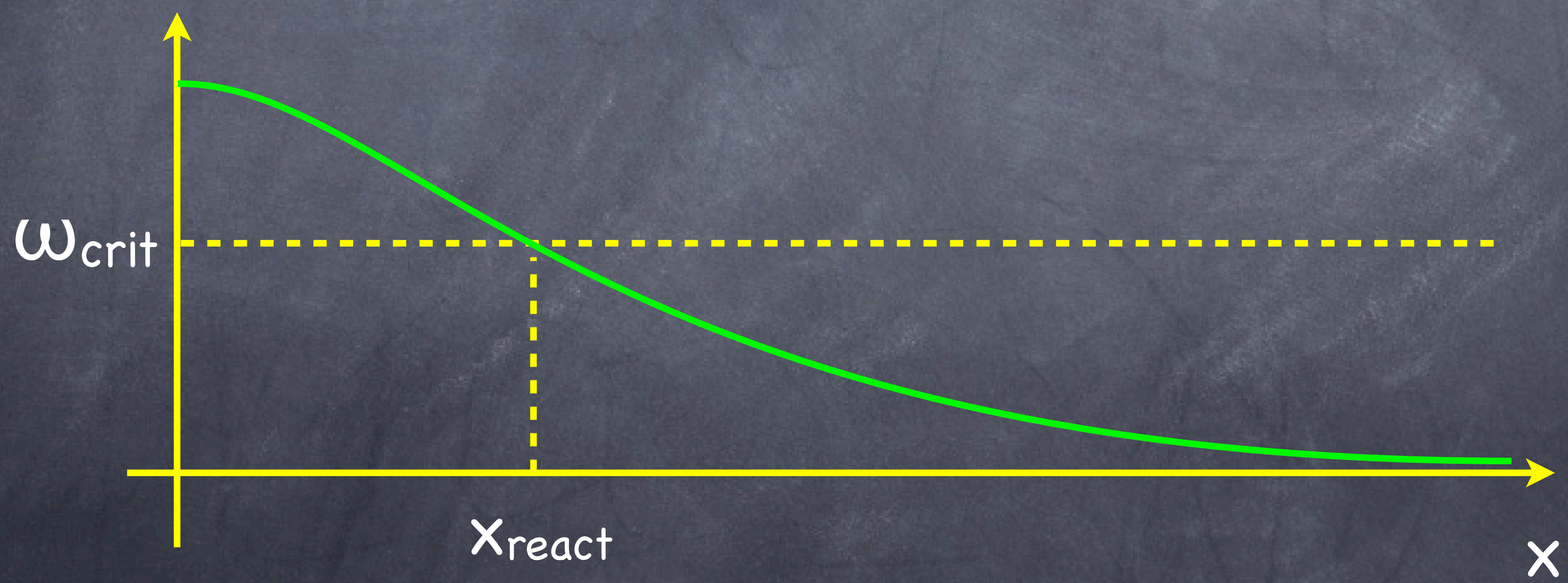

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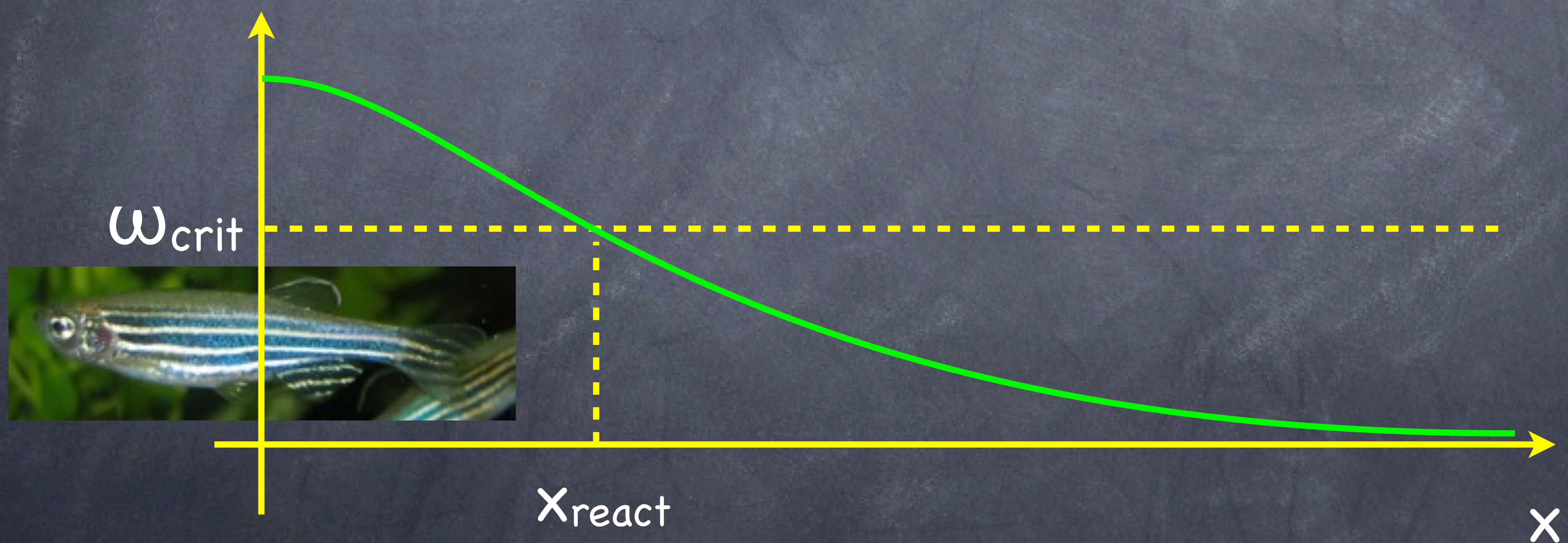

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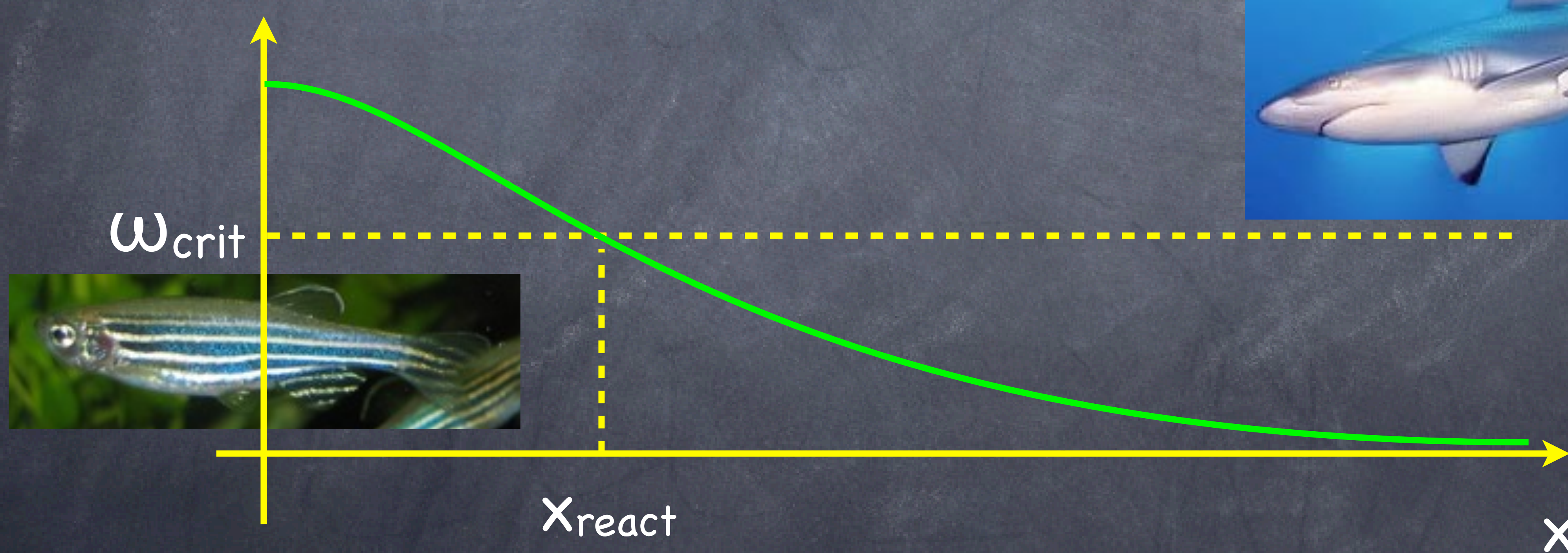

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Image - http://en.wikipedia.org/wiki/File:Tibur%C3%B3n.jpg

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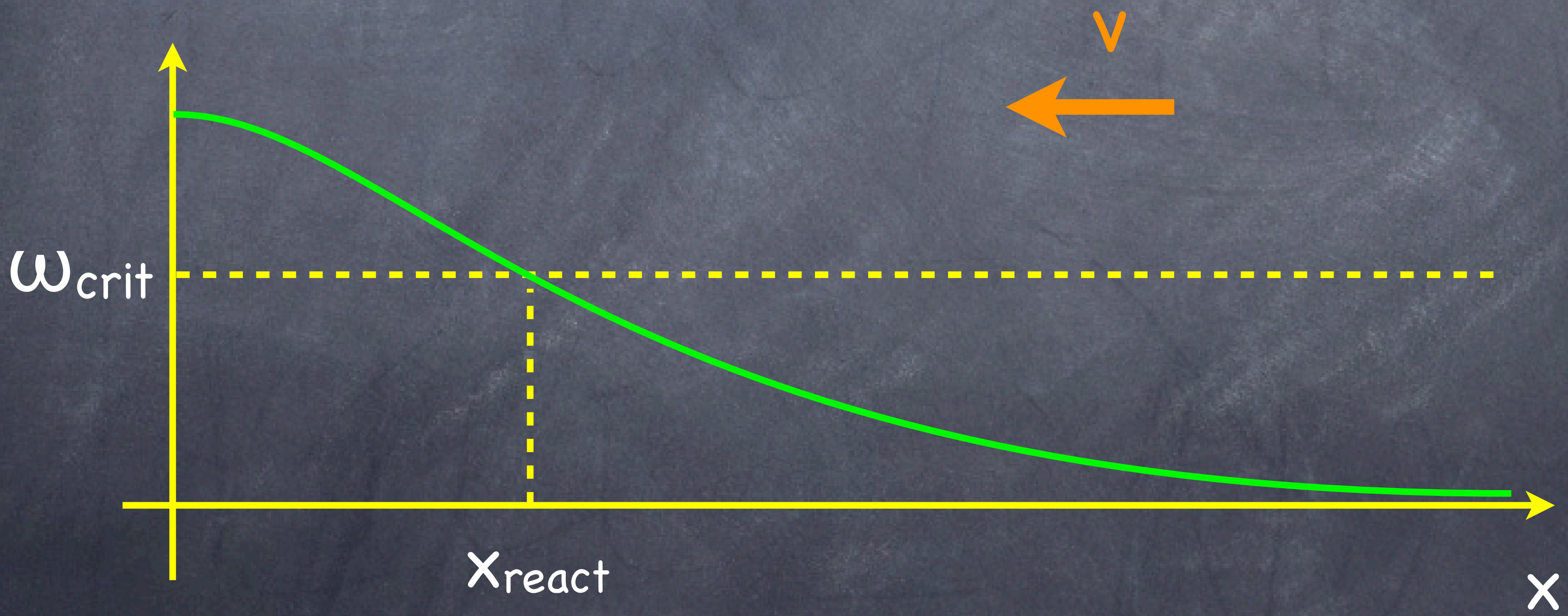

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Triangle with two sides of fixed length, angle between them changes.

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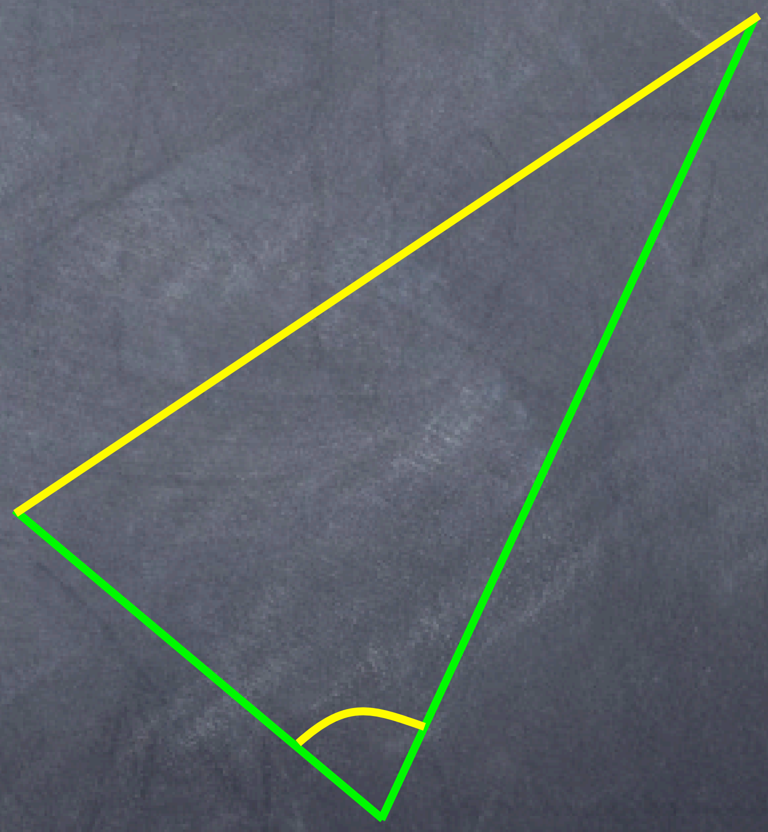

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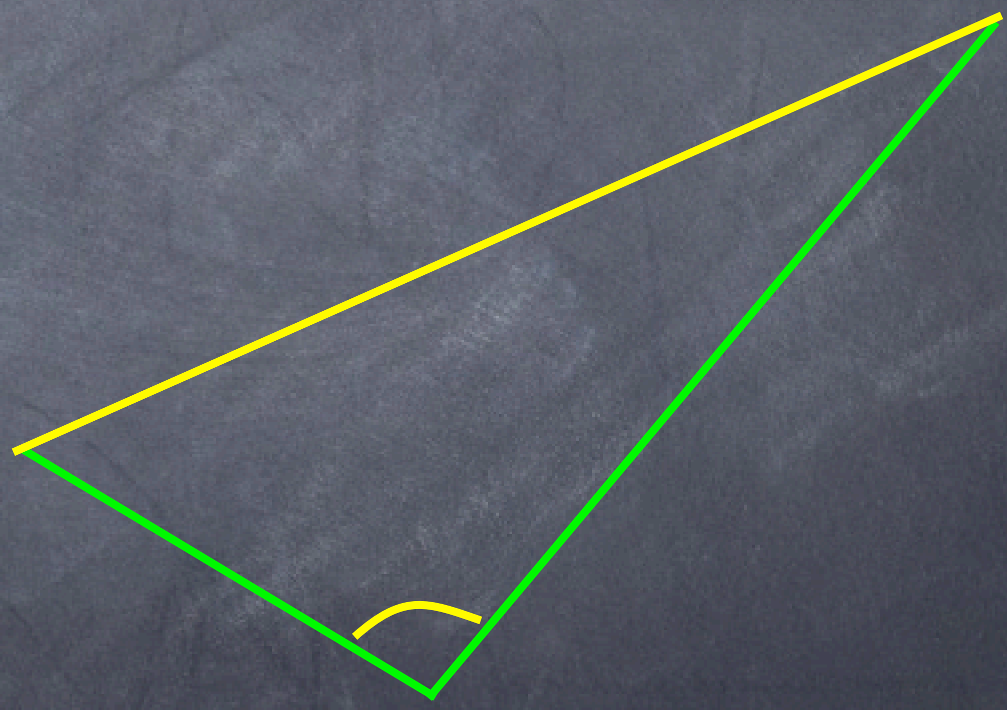


Triangle with two sides of fixed length, angle between them changes.

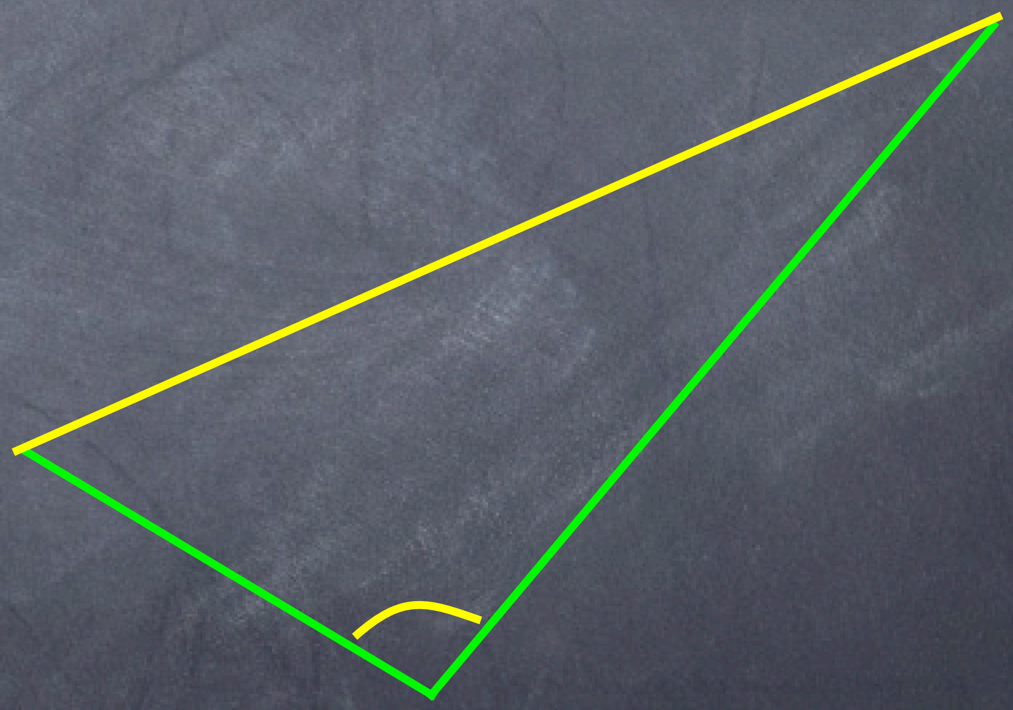
Relate the two changing quantities:

(A) $a^2 = b^2+c^2$

(B) $a^2 = b^2+c^2 - 2bc\cos(\theta)$

(C) $a/\sin(A) = b/\sin(B)$

(D) $\sin(\theta) = a/b$



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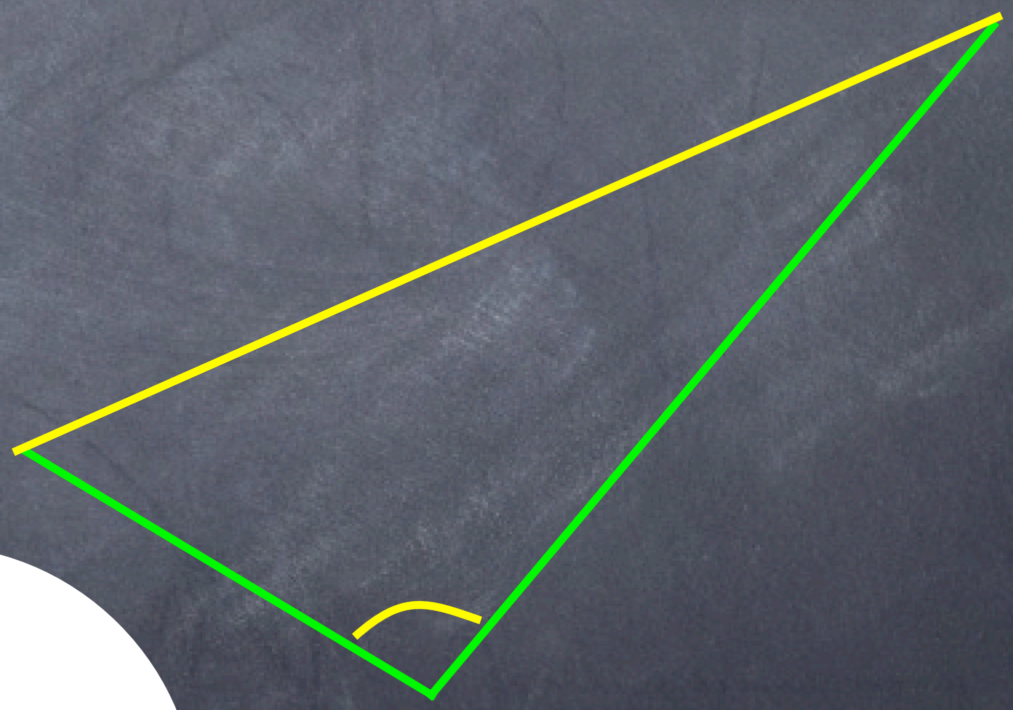
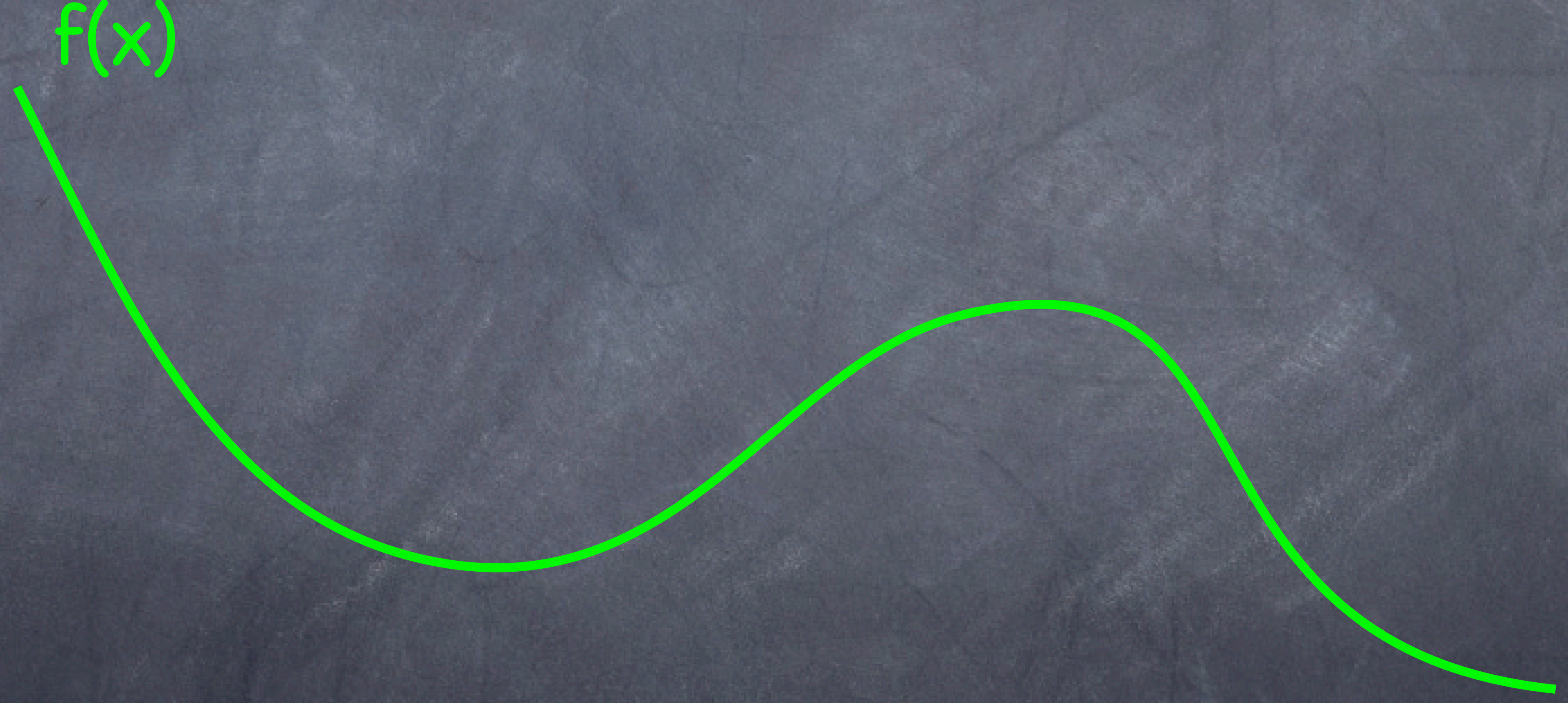
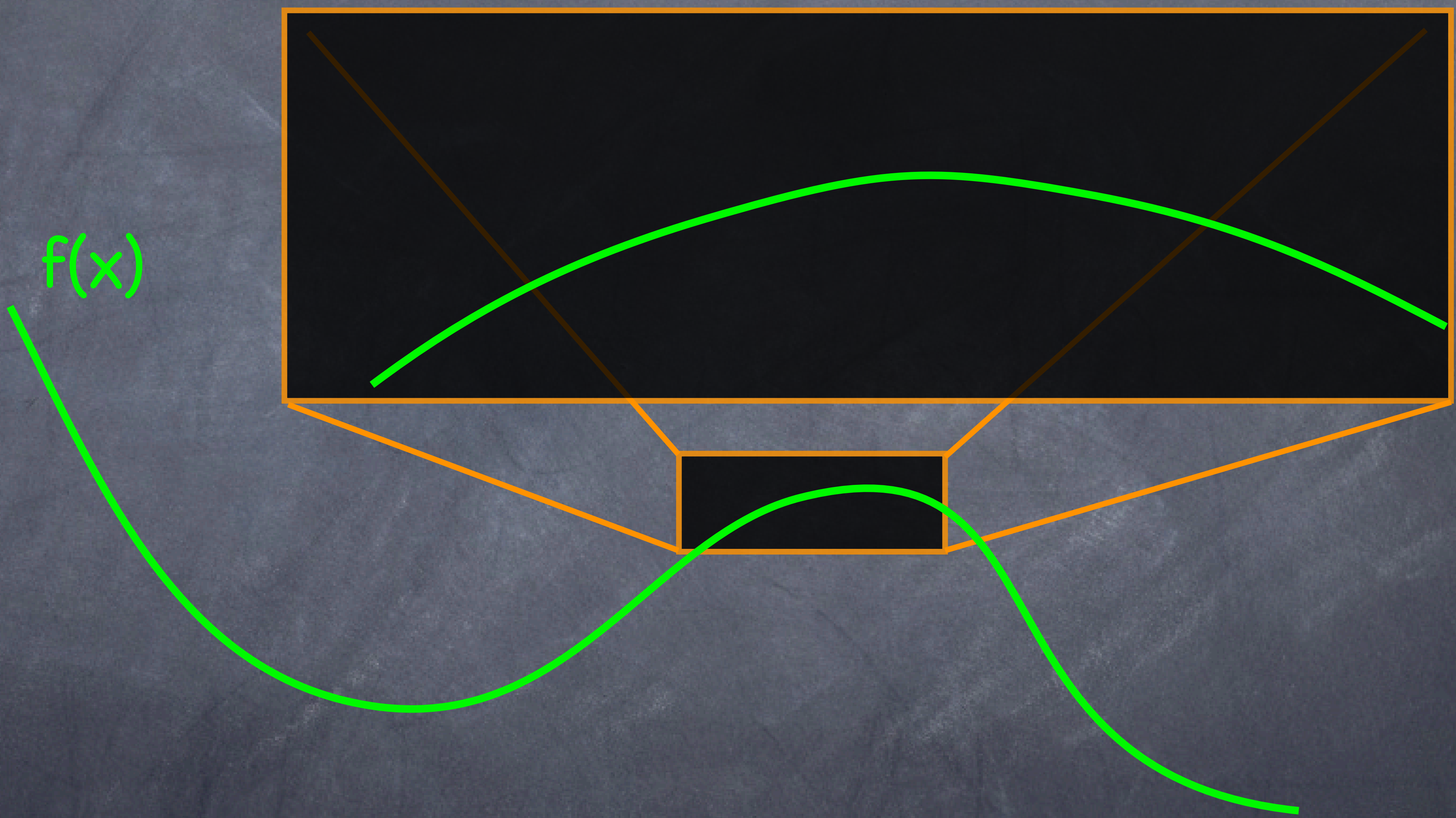


Linear approximation



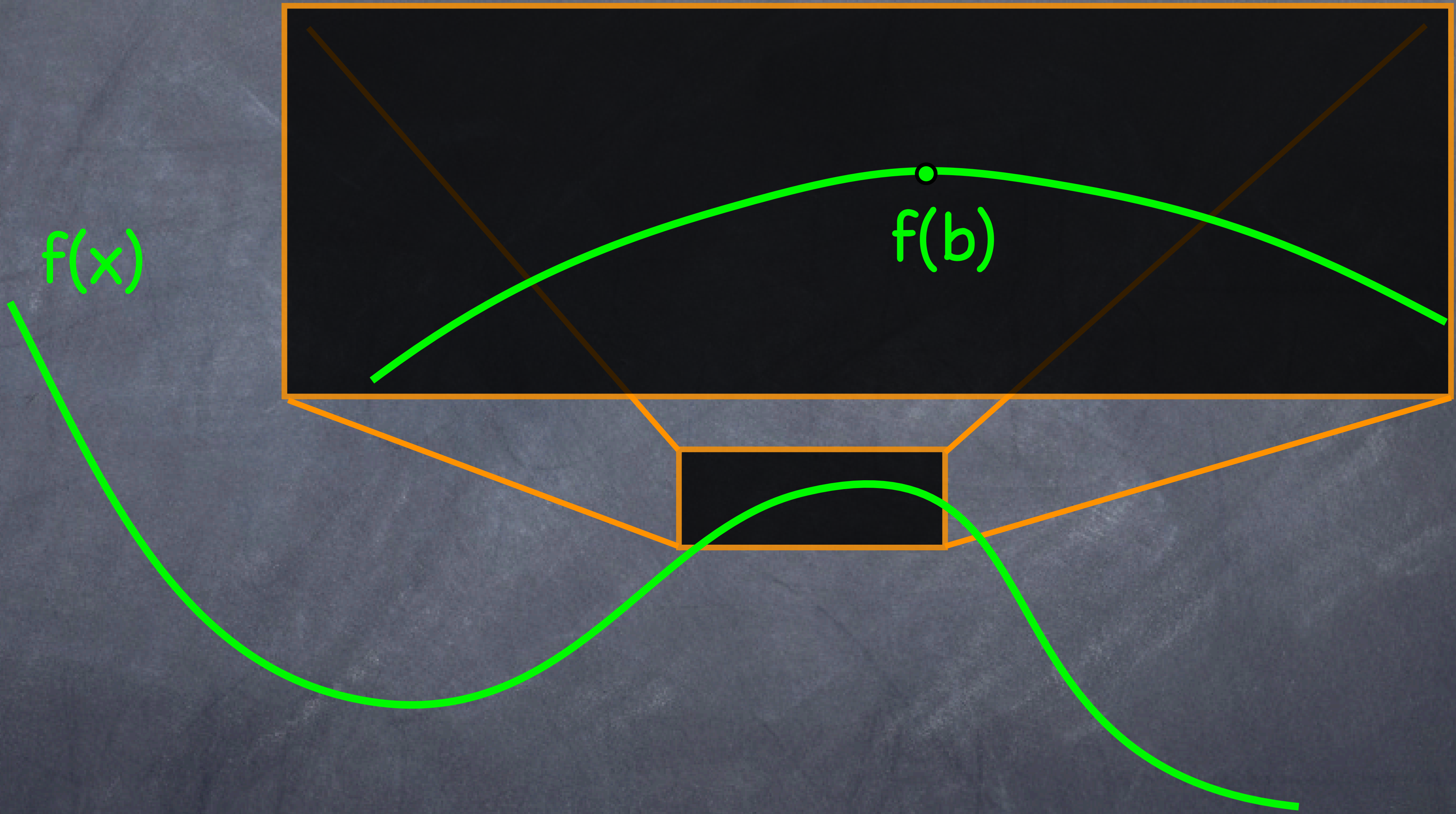
Suppose you want to know f(b) but it's hard to calculate.
If a is near b and both f(a) and f'(a) are easy to calculate, use tangent line to approximate f(b).

Linear approximation



Suppose you want to know f(b) but it's hard to calculate.
If a is near b and both f(a) and f′(a) are easy to calculate, use tangent line to approximate f(b).

Linear approximation



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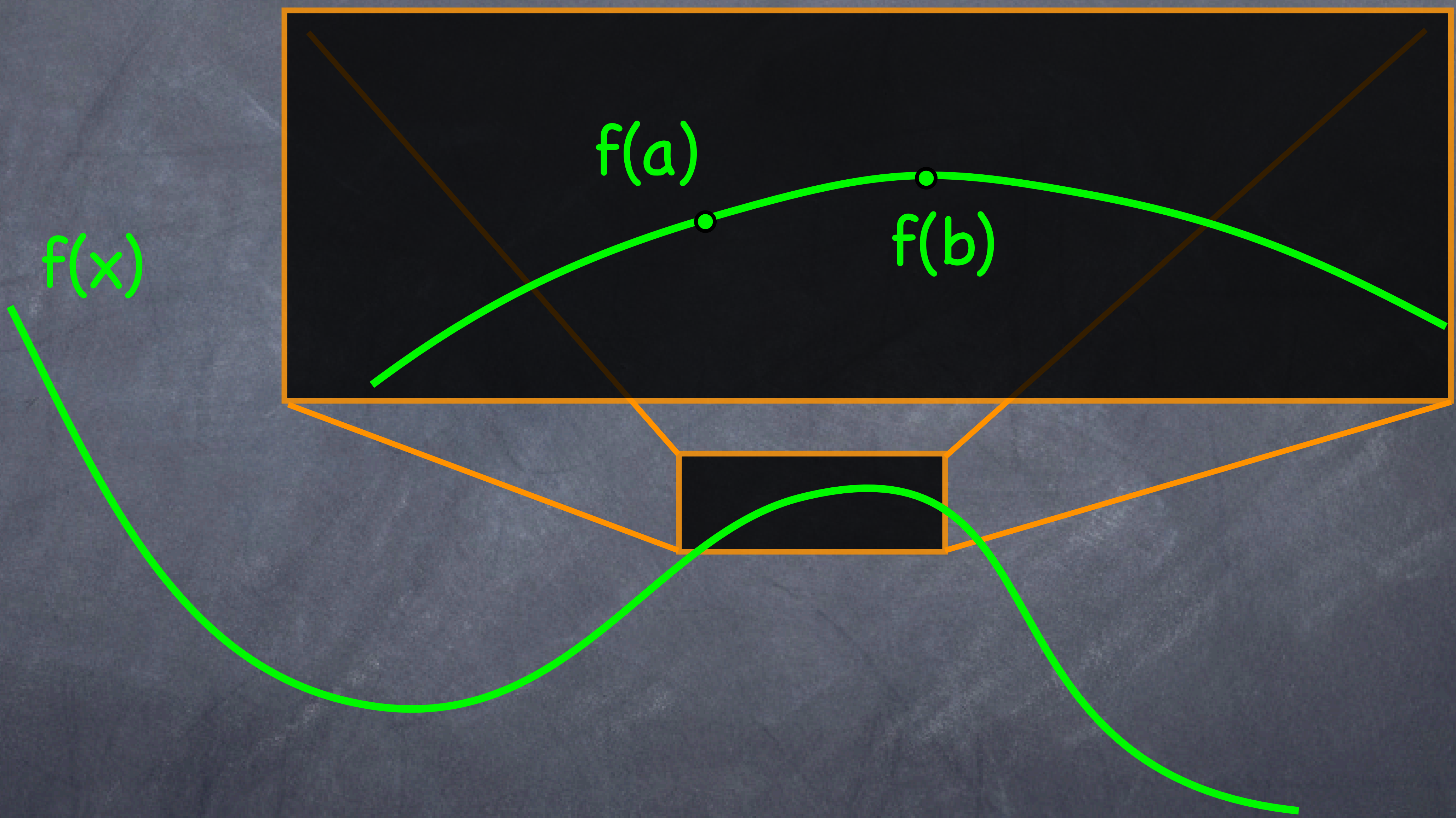

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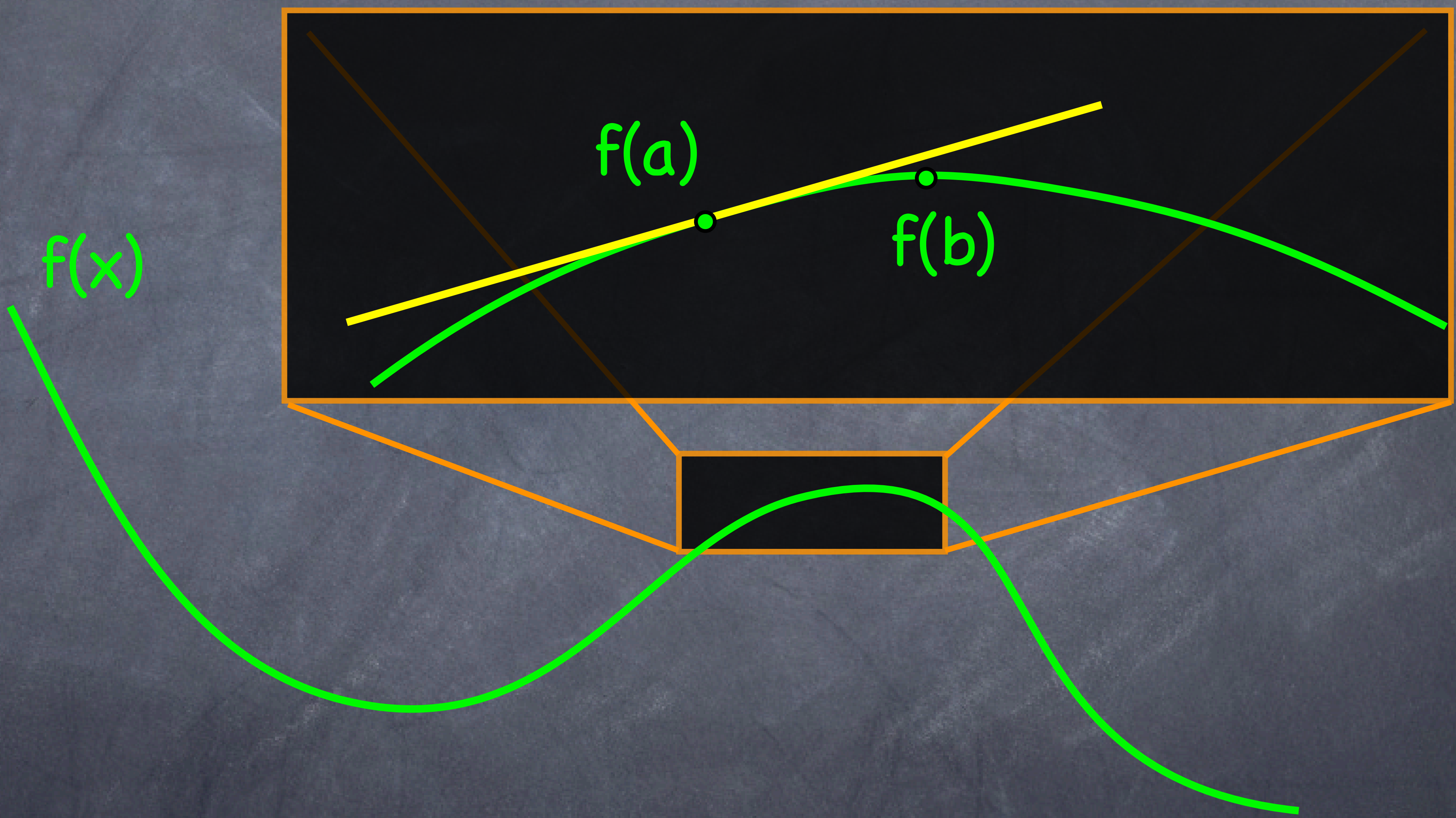

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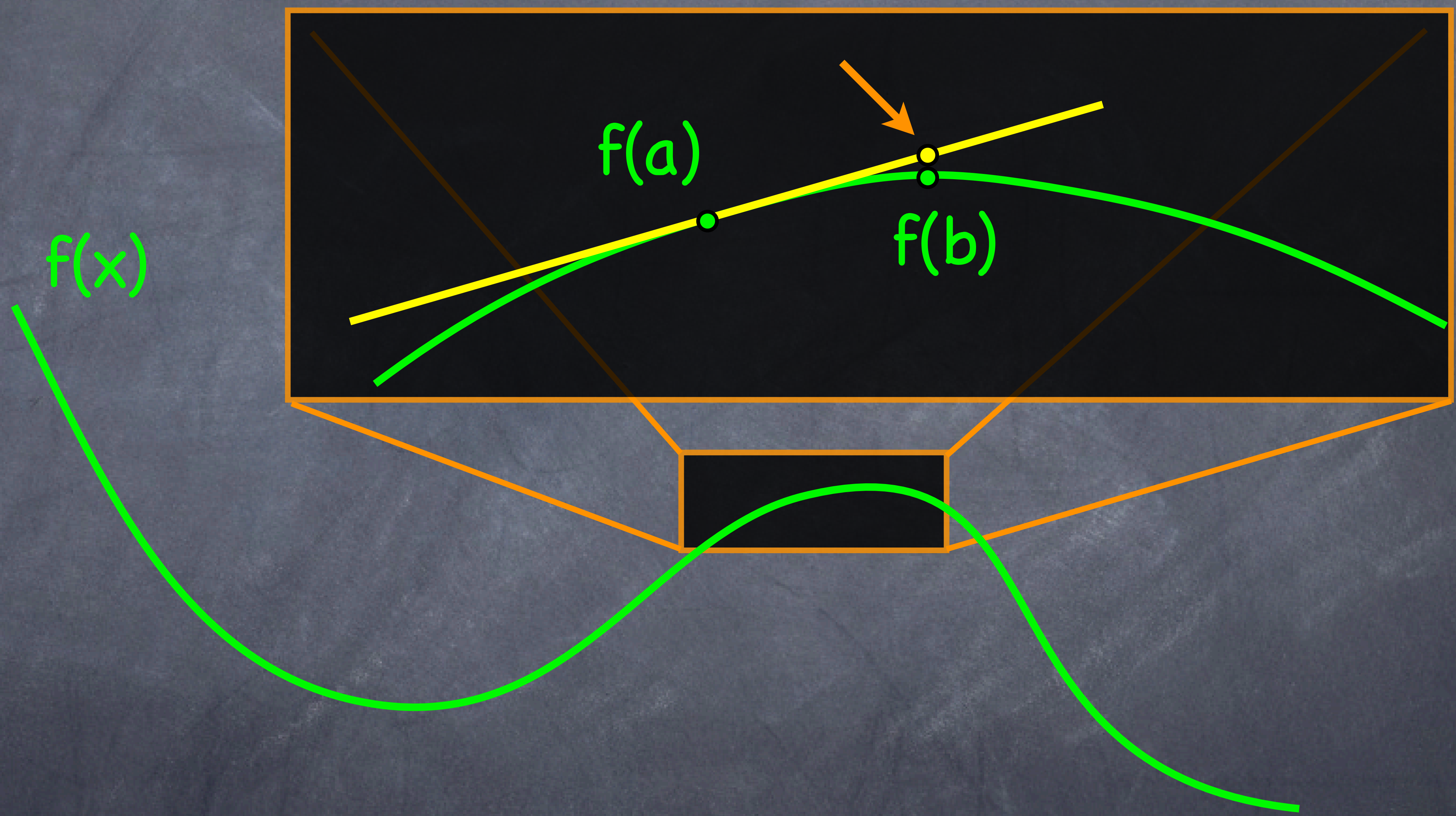

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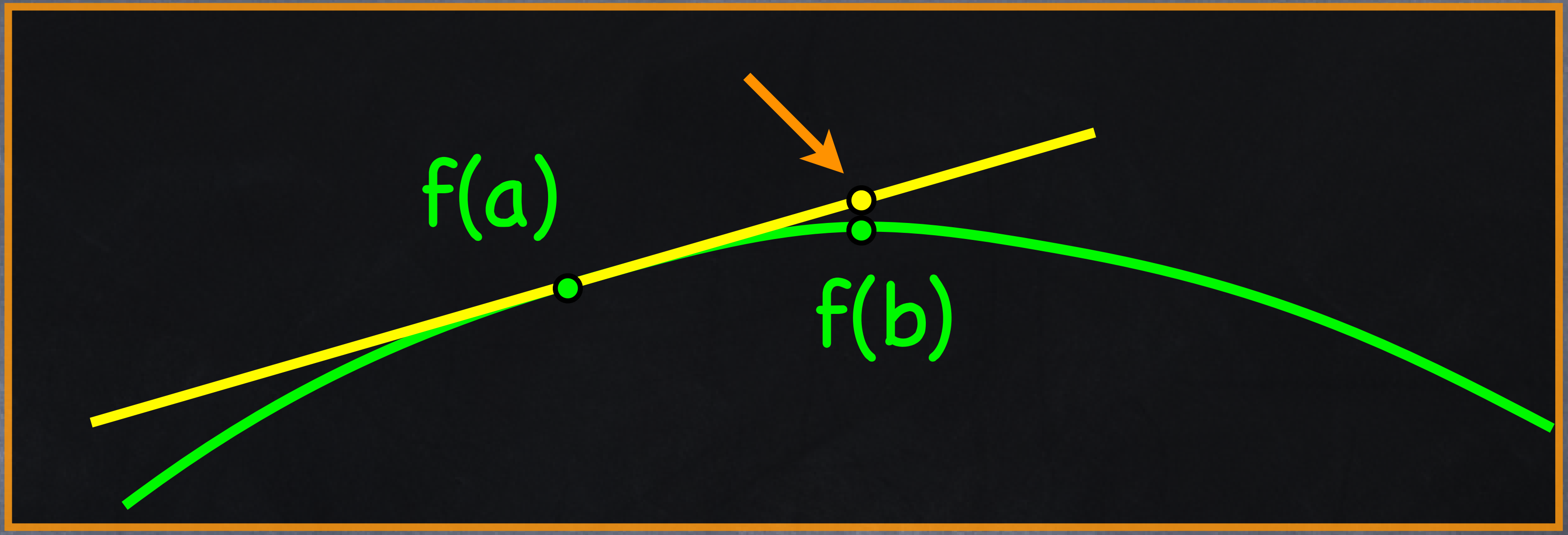

(A) $f(b) \approx f(b)+f'(b)(x-b)$

(B) $f(b) \approx f(a)+f'(a)(x-a)$

(C) $f(b) \approx f(a)+f'(a)(b-a)$

(D) $f(a) \approx f(b)+f'(b)(a-b)$

(E) $f(a) \approx f(b)+f'(b)(x-b)$