Absolute extrema

- A continuous function on a closed interval [a,b] takes on its highest (lowest) value either at a local maximum (minimum) or at an end point (x=a or x=b). Call this an absolute maximum (minimum).

- When looking for absolute extrema, check critical points AND end points!

Where does $f(x)=x^3-x^2$ take on its absolute minimum on the interval [-1,2]?

(A) $x=-1$

(B) $x=0$

(C) $x=2/3$

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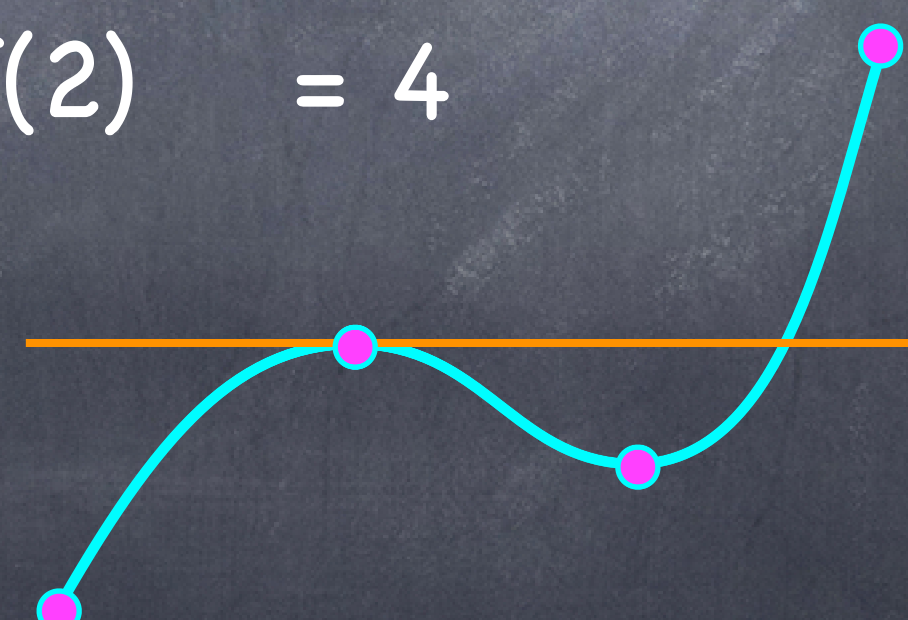
$f(0) = 0$

$f(2/3) = -4/27$

$f(2) = 4$



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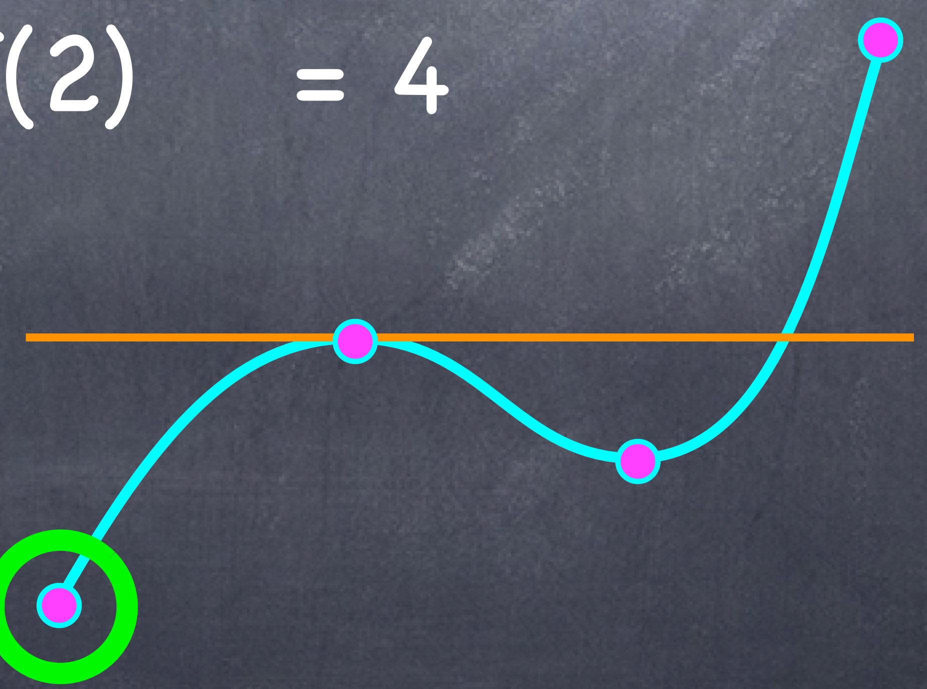

Optimization

- Given a scenario involving a choice of some number, use calculus to find the best value.
  - Translate scenario into a mathematical problem.
  - Solve the problem.
  - Translate back (make sure it makes sense).

I have 10 meters of fence. I want the biggest enclosure possible for my goat. I only know how to make rectangular enclosures.

Find the max of

(A) $A(w) = lw$. (l=length, w=width)

(B) $A(w) = w(10-w)$

(C) $A(w) = w(5-2w)$

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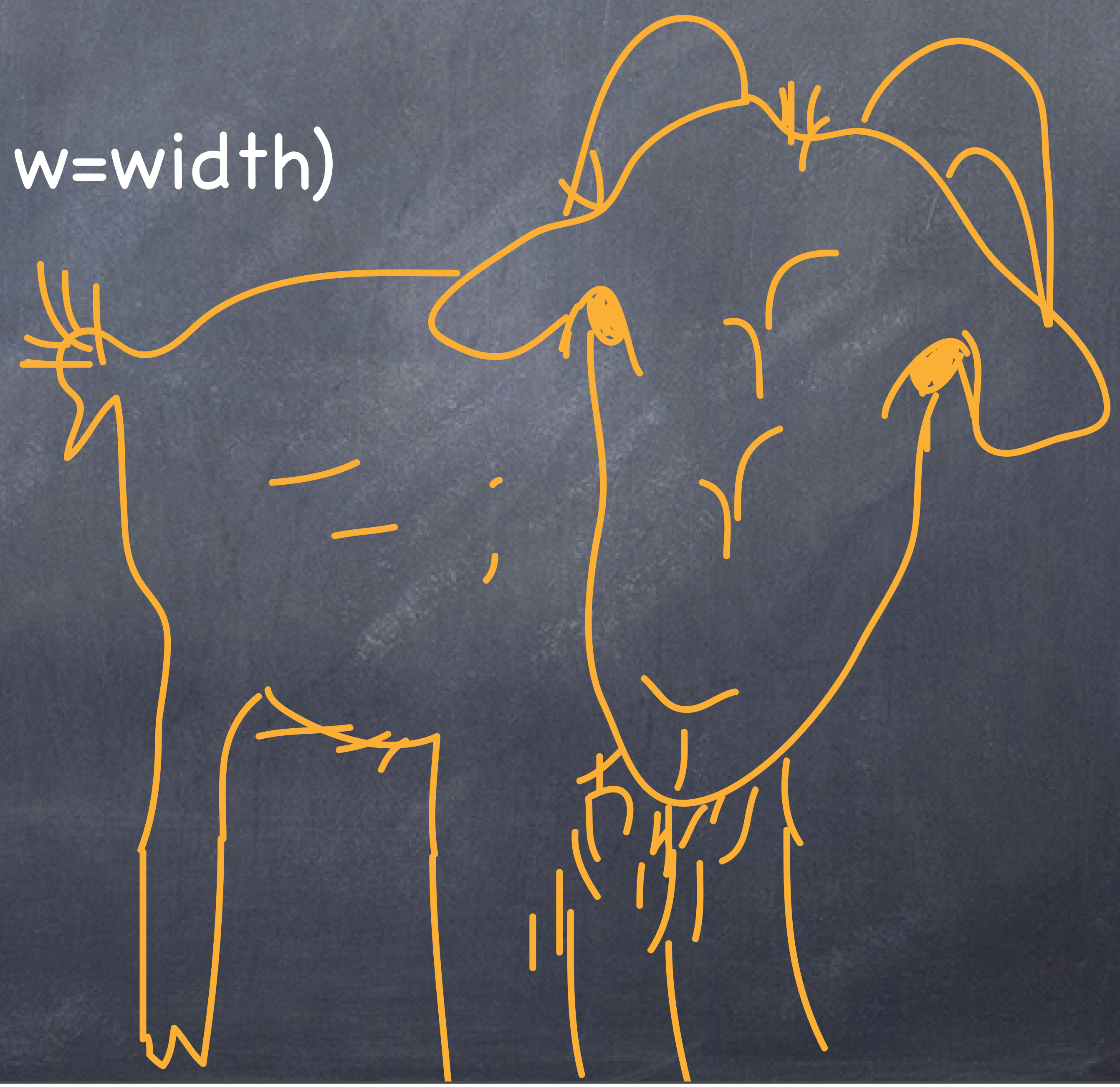

I have 10 meters of fence. I want the enclosure to be as small as possible but it can't be narrower than my goat (1/2 meter).

How long and how wide should I make the enclosure?

(A) l = 5/2 m, w = 5/2 m.

(B) l = 0 m, w = 5 m

(C) l = 1/2 m, w = 9/2 m

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Find absolute min of $A(w)=w(5-w)$ on $[1/2, 9/2]$.

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- The OF depends on more than one variable.
- There's a constraint relating the two variables.
- The constraint lets you simplify the OF to one variable.

$A(l,w)=lw$, $2l+2w=10$ --> $l=5-w$, $A(w)=(5-w)w$