\[ y' = \sin(y) \]

Sketch a few solutions \( y(t) \).
\[ y' = \sin(y) \]
$y' = \sin(y)$
What you should be able to do:

- Identify steady states for a DE.
- Draw/interpret the phase line for a DE.
- Draw/interpret a slope field for a DE.
- Determine stability of steady states.
- Determine long-term behaviour of solutions.
- Sketch the graphs of solutions using phase line and/or slope fields (slopes, concavity, IPs, h-asymptotes).
Some biological examples

Allee effect: \( P' = rP (1-P/K) (P/T-1) \) where \( T<K \)

Lac operon: \( c' = c^2/(k^2+c^2) - ac \)
The rate of change of an object’s temperature is proportional to the difference between the object’s temperature and the surrounding environment.

(A) \( T'(t) = k ( T(t) - E ) \)

(B) \( T'(t) = k ( E - T(t) ) \)

(C) \( T'(t) = E - kT(t) \)

(D) \( T'(t) = kT(t) - E \)

Assume \( k>0 \) and \( E \) is the temperature of the environment.
The rate of change of an object’s temperature is proportional to the difference between the object’s temperature and the surrounding environment.

(A) $T'(t) = k \left( T(t) - E \right)$

(B) $T'(t) = k \left( E - T(t) \right)$

(C) $T'(t) = E - kT(t)$

(D) $T'(t) = kT(t) - E$

Assume $k > 0$ and $E$ is the temperature of the environment.
The rate of change of an object’s temperature is proportional to the difference between the object’s temperature and the surrounding environment.

\((A)\ T'(t) = k \ (T(t) - E)\)

\((B)\ T'(t) = k \ (E - T(t))\)

\((C)\ T'(t) = E - kT(t)\)

\((D)\ T'(t) = kT(t) - E\)

Assume \(k > 0\) and \(E\) is the temperature of the environment.
The rate of change of an object’s temperature is proportional to the difference between the object’s temperature and the surrounding environment.

\[ (A) \ T'(t) = k \ ( T(t) - E ) \]
\[ (B) \ T'(t) = k \ ( E - T(t) ) \]
\[ (C) \ T'(t) = E - kT(t) \]
\[ (D) \ T'(t) = kT(t) - E \]

Make sure eq matches physical intuition. If the coefficient on T(t) is +ive, the solution \( \longrightarrow \infty \).

Assume k>0 and E is the temperature of the environment.
The rate of change of an object’s temperature is proportional to the difference between the object’s temperature and the surrounding environment.

\( (A) \ T'(t) = k \ (T(t) - E) \)

\( (B) \ T'(t) = k \ (E - T(t)) \)

\( (C) \ T'(t) = E - kT(t) \)

\( (D) \ T'(t) = kT(t) - E \)

Assume \( k > 0 \) and \( E \) is the temperature of the environment.

Make sure eq matches physical intuition.

If the coefficient on \( T(t) \) is +ive, the solution \( \longrightarrow \infty \).

Units have to match!
Solving $T' = k(E-T)$
Solving $T' = k(E-T)$

Suppose $T$ is measured in Kelvin and $E = 273K$ is the freezing point of H$_2$O.
Solving $T' = k(E-T)$

Suppose $T$ is measured in Kelvin and $E = 273K$ is the freezing point of $H_2O$.

Change to centigrade.
Solving $T' = k(E-T)$

Suppose $T$ is measured in Kelvin and $E = 273K$ is the freezing point of $H_2O$.

Change to centigrade.

Define $S(t) = T(t) - E$. 
Solving $T' = k(E-T)$

Suppose $T$ is measured in Kelvin and $E = 273K$ is the freezing point of $H_2O$.

Change to centigrade.

Define $S(t) = T(t) - E$.

$S'(t) = T'(t)$
Solving $T' = k(E-T)$

- Suppose $T$ is measured in Kelvin and $E = 273K$ is the freezing point of $H_2O$.
- Change to centigrade.
  - Define $S(t) = T(t) - E$.
  - $S'(t) = T'(t)$
  - $S' = T' = k(E-T) = k(-S) = -kS$
Solving $T' = k(E-T)$

Suppose $T$ is measured in Kelvin and $E = 273K$ is the freezing point of $H_2O$.

Change to centigrade.

Define $S(t) = T(t) - E$.

$S'(t) = T'(t)$

$S' = T' = k(E-T) = k(-S) = -kS$

Solution: $S(t) = S_0e^{-kt}$. Change back to $T(t)$:
Solving $T' = k(E-T)$

Suppose $T$ is measured in Kelvin and $E = 273K$ is the freezing point of $H_2O$.

Change to centigrade.

Define $S(t) = T(t) - E$.

$S'(t) = T'(t)$

$S' = T' = k(E-T) = k(-S) = -kS$

Solution: $S(t) = S_0e^{-kt}$. Change back to $T(t)$:

$T(t) = S(t) + E = S_0e^{-kt} + E$
Solving $T' = k(E-T)$

Suppose $T$ is measured in Kelvin and $E = 273K$ is the freezing point of $H_2O$.

Change to centigrade.

Define $S(t) = T(t) - E$.

$S'(t) = T'(t)$

$S' = T' = k(E-T) = k(-S) = -kS$

Solution: $S(t) = S_0 e^{-kt}$. Change back to $T(t)$:

$T(t) = S(t) + E = S_0 e^{-kt} + E$

With $T(0)=T_0$, $T(0) = S_0 + E = T_0$ so $S_0 = T_0 - E$. 
Solving $T' = k(E-T)$

Suppose $T$ is measured in Kelvin and $E = 273K$ is the freezing point of $H_2O$.

Change to centigrade.

Define $S(t) = T(t) - E$.

$S'(t) = T'(t)$

$S' = T' = k( E-T ) = k ( -S ) = - kS$

Solution: $S(t) = S_0 e^{-kt}$. Change back to $T(t)$:

$T(t) = S(t) + E = S_0 e^{-kt} + E = (T_0-E)e^{-kt} + E$

With $T(0)=T_0$, $T(0) = S_0 + E = T_0$ so $S_0 = T_0 - E$. 
Phase line for NLC:
\[
\frac{dT}{dt} = k(E - T)
\]

Notice that the arrows are always the same for any \(E\), just shifted left or right.
Phase line for NLC:

\[
\frac{dT}{dt} = k(E - T)
\]

Notice that the arrows are always the same for any \( E \), just shifted left or right.
Phase line for NLC:

\[
\frac{dT}{dt} = k(E - T)
\]

Notice that the arrows are always the same for any E, just shifted left or right.
Phase line for NLC:

\[
\frac{dT}{dt} = k(E - T)
\]

Notice that the arrows are always the same for any E, just shifted left or right.
What does the phase line tell us without even solving the equation?

\[ \frac{dT}{dt} = k(E - T) \]

What influence does \( k \) have on this diagram?
What does the phase line tell us without even solving the equation?

\[
\frac{dT}{dt} = k(E - T)
\]

What influence does \( k \) have on this diagram?
What does the phase line tell us without even solving the equation?

\[
\frac{dT}{dt} = k(E - T)
\]

What influence does \( k \) have on this diagram?

What influence does \( k \) have on this diagram?
What does the phase line tell us without even solving the equation?

\[
\frac{dT}{dt} = k(E - T)
\]

What influence does \( k \) have on this diagram?
What does the phase line tell us without even solving the equation?

\[
\frac{dT}{dt} = k(E - T)
\]

What influence does \( k \) have on this diagram?
What does the phase line tell us without even solving the equation?

\[
\frac{dT}{dt} = k(E - T)
\]

What influence does \( k \) have on this diagram?
What does the phase line tell us without even solving the equation?

\[
\frac{dT}{dt} = k(E - T)
\]

What influence does \( k \) have on this diagram?
What does the phase line tell us without even solving the equation?\[
\frac{dT}{dt} = k\left(E - T\right)
\]

What influence does \( k \) have on this diagram?
What does the phase line tell us without even solving the equation?

\[ \frac{dT}{dt} = k(E - T) \]

What influence does \( k \) have on this diagram?