The derivative:
computational aspects

Math 102 Section 106
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Math 102: Announcements

- **Quiz Friday**: Individual and Group Work
- **Office Hours**:
  - Today: 3-4 pm Math Annex 1118
  - Thursday: 3-4 pm LSK 300B (next to MLC)
The derivative of a function $y = f(x)$ at $x_0$ is

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$ 

You can also write $\frac{dy}{dx} \bigg|_{x_0}$ to denote $f'(x_0)$.

You can also write $f'(x)$ or $\frac{dy}{dx}$ to denote the derivative as a function of $x$. 

Derivative
Derivative

Example

Derivative of $f(x) = \frac{1}{x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left( \frac{1}{x + h} - \frac{1}{x} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \left( \frac{x - (x + h)}{(x + h)x} \right)$$

$$= \lim_{h \to 0} \frac{-1}{(x + h)x}$$

$$= \frac{1}{x^2}$$
Last time: geometric view

- The tangent line to a point on the graph is of a function is the line we see when we zoom into the graph at that point.
Example \((f(x) = x^3)\)
Sketch the first and second derivatives of \(f(x) = x^3\) together on the document camera/board.
Example \((f(x) = e^{-x^2} \sin(x))\)

Sketch the first derivative of \(f(x) = e^{-x^2} \sin(x)\) together on the document camera/board.
Going backwards...

Q1. Given $f'(x)$, sketch the original function.
Going backwards...

Q1. Given \( f'(x) \), sketch the original function.
Q2. Given $f'(x)$, sketch the second derivative, $f''(x)$. 
Second derivative?

Q2. Given $f'(x)$, sketch the second derivative, $f''(x)$.

(A) 

(B) 

(C) 

(D) 

(E)
Position, velocity, & acceleration

- \( x(t) = \) position
- \( v(t) = x'(t) = \) velocity
- \( a(t) = v'(t) = x''(t) \) acceleration

**Acceleration** is the second derivative of position.
Q3. Which is \(x, v, a\)?

(A) \(x, v, a\)

(B) \(x, v, a\)

(C) \(x, v, a\)

(D) \(x, v, a\)

Check max/mins --> zeros, check inc/dec --> +/-.
Q3.

Which is $x, v, a$?

(A) $x, v, a$

(B) $x, v, a$

(C) $x, v, a$

(D) $x, v, a$

Check max/mins $\rightarrow$ zeros, check inc/dec $\rightarrow$ +/-.
Derivative: computational aspects

- Spreadsheets?
  - Basic computer literacy
  - Useful common tool with many applications:
    - Budgets
    - Manipulate data for labs

- Familiarity with common software used by business, industry, accounting

- Introduction to scientific computing

**BONUS CALCULUS UNDERSTANDING:**
  - Approximations
  - Accuracy and refining a result
  - Iteration methods
Approximating the derivative

Q4. The derivative of a function $f(t)$ can be approximated by

A. $f'(t) \approx \frac{f(t+h)-f(t)}{h}$ for a large value of $h$

B. $f'(t) \approx \frac{f(t+h)-f(t)}{h}$ for a small value of $h$

C. $f'(t) \approx \frac{f(t+h)-f(t)}{t}$ for a large value of $h$

D. $f'(t) \approx \frac{f(t+h)-f(t)}{t}$ for a small value of $h$
Approximating the derivative

Our goals:

- Use a spreadsheet to compute an approximation of $f(x) = x^3$ over the interval $0 \leq x \leq 1$ for $\Delta x = 0.25$ and compare to the true derivative.

- Comment on the comparison.

- Recompute the approximation to the derivative with $\Delta x = 0.05$.

For notes: See Section 3.3 of the course notes and example spreadsheet on wiki.
Today...

- More sketching examples: think about slopes and think about how zeros of the derivative correspond to maxima/minima.
- Going backwards: given $f'(x)$, sketch $f(x)$.
- Position, velocity and acceleration are related...
- Approximating the derivative computationally (an introduction to scientific computing)
Answers

1. A
2. B
3. C
4. B
1. When we approximate the derivative of a function by $f'(t) \approx \frac{f(t+h)-f(t)}{h}$ then we are
   A. Approximating the slope of the secant line by the slope of a tangent line.
   B. Approximating the slope of the tangent line by the slope of a secant line.
   C. Approximating the slope of the tangent line by the value of the function.
   D. Trying to avoid having to calculate a limit.
   E. None of the above.
Related exam problems

(a) Microtubules (MTs) are biological polymers important in cell structure, cell division, and transport of material inside cells. The length of microtubules (MT length) grows and shrinks as shown in the following figure from Janulevicius et al. (2009) Biophys. J. 90:788-798. Use this figure to draw a sketch of MT growth rate (i.e. rate of change of microtubule length per unit time) over the same time interval.

(b) Which has a greater magnitude: the rate of shrinkage (per unit time) or the rate of growth (per unit time)?