Today

- Qualitative analysis of DEs continued.
- Drawing the phase line.
- Determining long term behaviour.
- Sketching solutions from the phase line.
Qualitative analysis
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Finding a formula for a solution to a DE is ideal but what if you can't?
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Qualitative analysis - extract information about the general solution without solving.
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Finding a formula for a solution to a DE is ideal but what if you can’t?

Qualitative analysis – extract information about the general solution without solving.

- Steady states
- Slope fields
- Stability of steady states
- Plotting y’ versus y (state space/phase line)
\[ x' = x(1 - x) \]

velocity

position

Slope field

\[ x(t) \]

1

0
\[ x' = x(1 - x) \]

Slope field.
Slope field.

At any $t$, don't know $x$ yet so plot all possible $x'$ values.
\[ x' = x(1 - x) \]

**Slope field.**

At any \( t \), don’t know \( x \) yet so plot all possible \( x' \) values

When \( x(t) = 1/2 \) what is \( x' \)?

- (A) 0
- (B) 1/4
- (C) 1/2
- (D) 1
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At any \( t \), don’t know \( x \) yet so plot all possible \( x' \) values

Now draw them for all \( t \).
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Slope field.

At any \( t \), don’t know \( x \) yet so plot all possible \( x' \) values

Now draw them for all \( t \).

Solution curves must be tangent to slope field everywhere.
\[ x' = x(1 - x) \]

\[ \begin{array}{c}
\text{velocity} \\
\text{position}
\end{array} \]

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\( x(t) \)  
\( t \)

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- Now draw them for all \( t \).
- Solution curves must be tangent to slope field everywhere.
\[ y' = -y(y-1)(y+1) \]

What are the steady states of this equation?

Draw the slope field for this equation.

Include the steepest slope element in each interval between steady states and two others (roughly).
Velocity (x’) vs. position (x)

Slope field
Velocity versus position

Velocity ($x'$) vs. position ($x$)

Slope field

$0 \quad 1 \quad x$

$0 \quad 1 \quad x(t)$
Velocity versus position

Velocity (x') vs. position (x)

\[ x' = f(x) = x(1-x) \]
Velocity versus position

Velocity \((x')\) vs. position \((x)\)

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Velocity versus position

Velocity (x') vs. position (x)

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height here is slope there

Slope field
Velocity versus position

Velocity (x’) vs. position (x)

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height here is slope there
Velocity versus position

**Velocity (x’) vs. position (x)**

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**Slope field**

Height here is slope there
Velocity versus position

**Velocity (x’) vs. position (x)**

\[ x' = f(x) = x(1-x) \]

- **Stable steady state** - all nearby solutions approach
- **Unstable steady state** - not stable
Determine stability

\[ x' = x(1 - x) \]

If you start at \( x(0) = -0.01 \), the solution

(A) increases

(B) decreases
Determine stability

\[
x' = x(1 - x)
\]

If you start at \(x(0) = 0.01\), the solution

(A) increases

(B) decreases
Determine stability

If you start at $x(0) = 0.99$, the solution

(A) increases

(B) decreases
Determine stability

If you start at \( x(0) = 1.01 \), the solution

(A) increases
(B) decreases
Determine stability

\[ x' = x(1 - x) \]

(A) Both \( x(t) = 0 \) and \( x(t) = 1 \) are stable steady states.

(B) \( x(t) = 0 \) is stable and \( x(t) = 1 \) is unstable.

(C) \( x(t) = 0 \) is unstable and \( x(t) = 1 \) is stable.

(D) Both \( x(t) = 0 \) and \( x(t) = 1 \) are unstable steady states.
Determine stability

\[ x' = x(1 - x) \]

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(C) \( x(t)=0 \) is unstable and \( x(t)=1 \) is stable.

(D) Both \( x(t)=0 \) and \( x(t)=1 \) are unstable steady states.