Today

- Qualitative analysis of DEs continued.
  - Drawing the phase line.
  - Determining long term behaviour.
  - Sketching solutions from the phase line.

Qualitative analysis

Qualitative analysis

- Finding a formula for a solution to a DE is ideal but what if you can't?

Qualitative analysis

- Finding a formula for a solution to a DE is ideal but what if you can't?
- Qualitative analysis - extract information about the general solution without solving.

Qualitative analysis

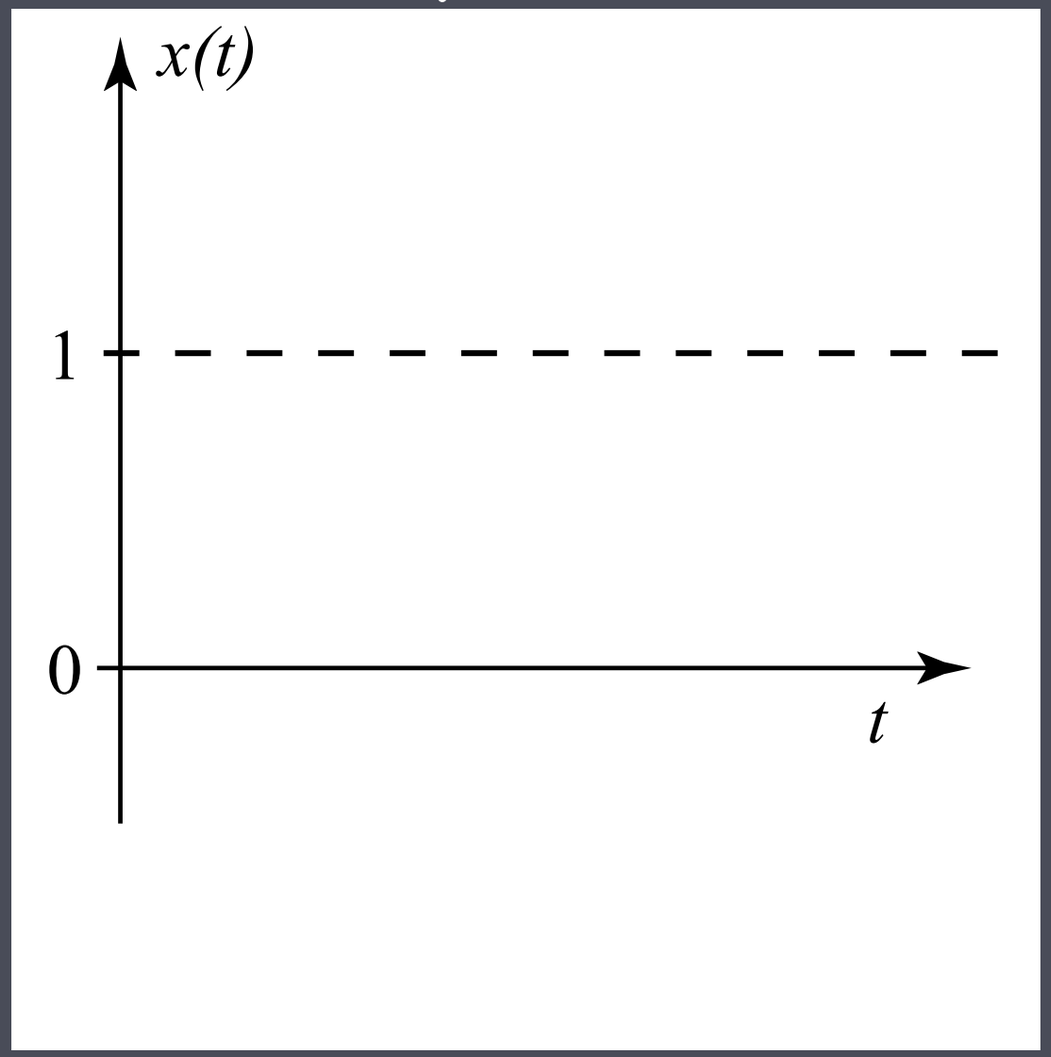
- Finding a formula for a solution to a DE is ideal but what if you can't?
- Qualitative analysis - extract information about the general solution without solving.
    - Steady states
    - Slope fields
    - Stability of steady states
    - Plotting y' versus y (state space/phase line)

$$x' = x(1 - x)$$

velocity

position

Slope field



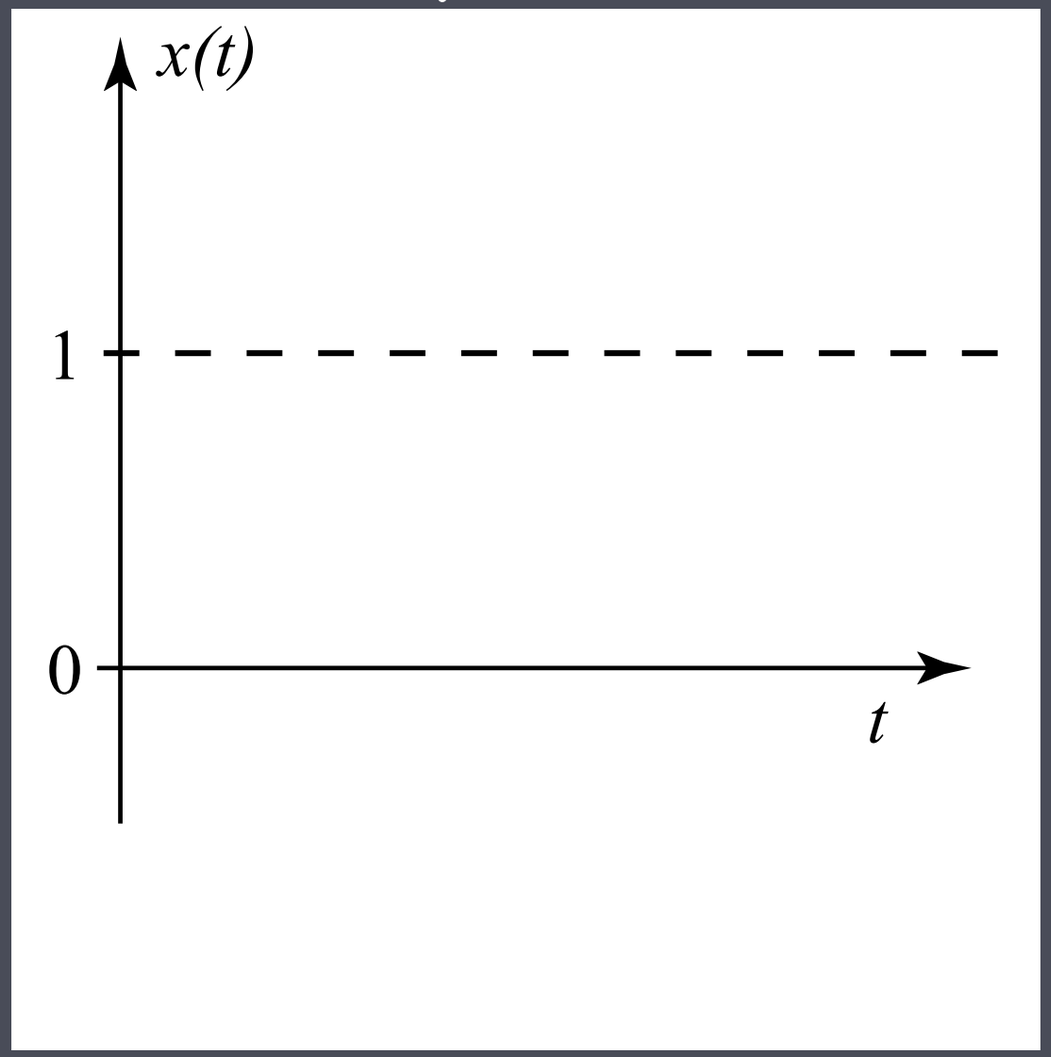
$$x' = x(1 - x)$$

velocity

position

Slope field

- Slope field.



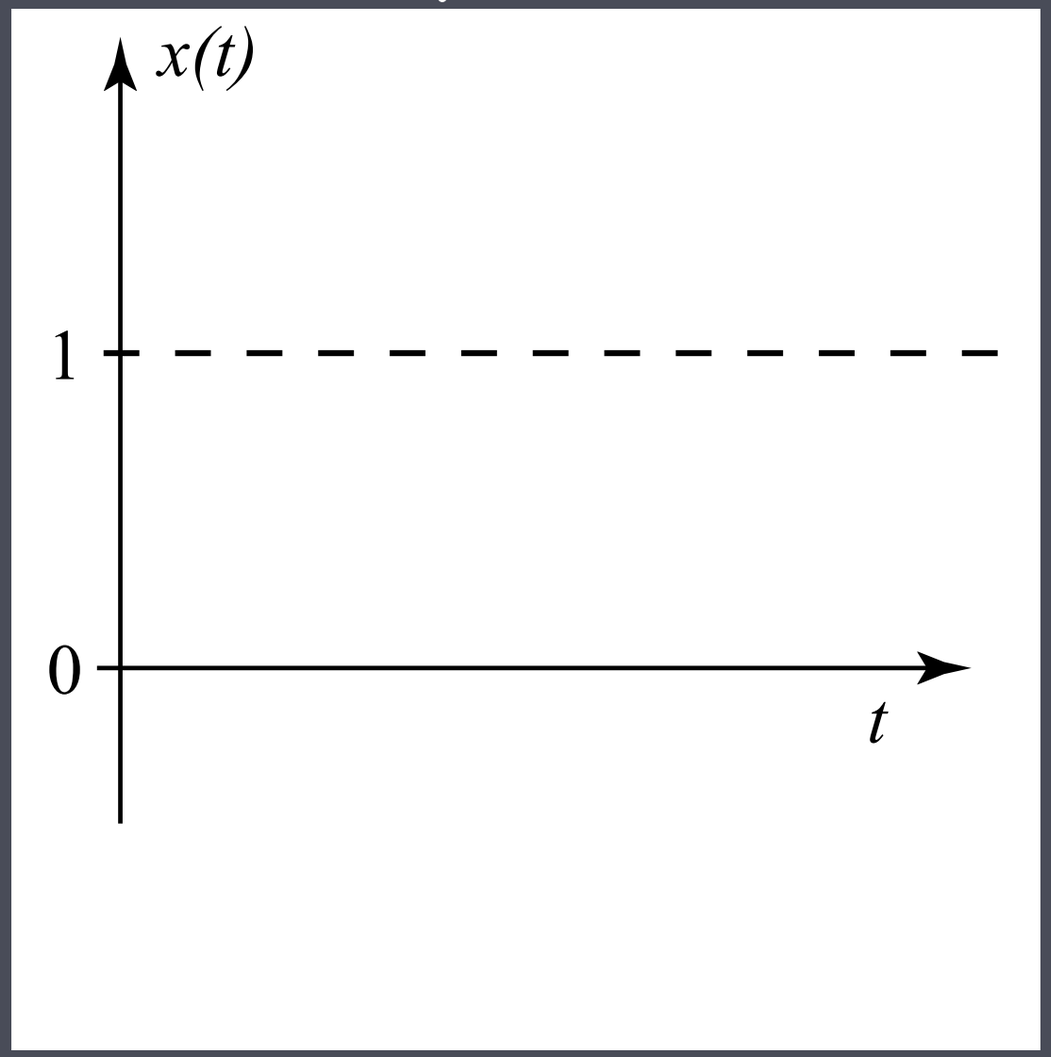
$$x' = x(1 - x)$$

velocity

position

Slope field

- Slope field.
- At any t, don't know x yet so plot all possible x' values

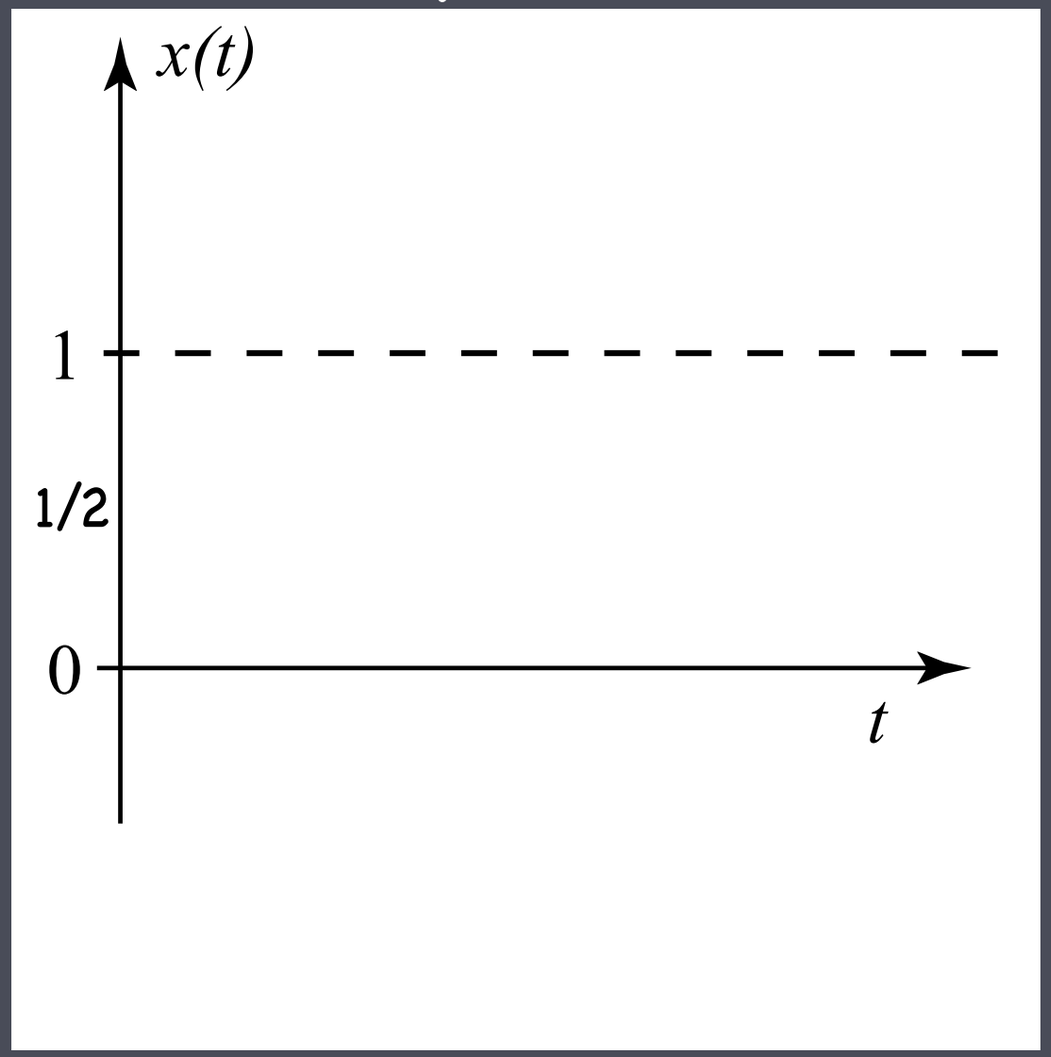


$$x' = x(1 - x)$$

velocity

position

Slope field



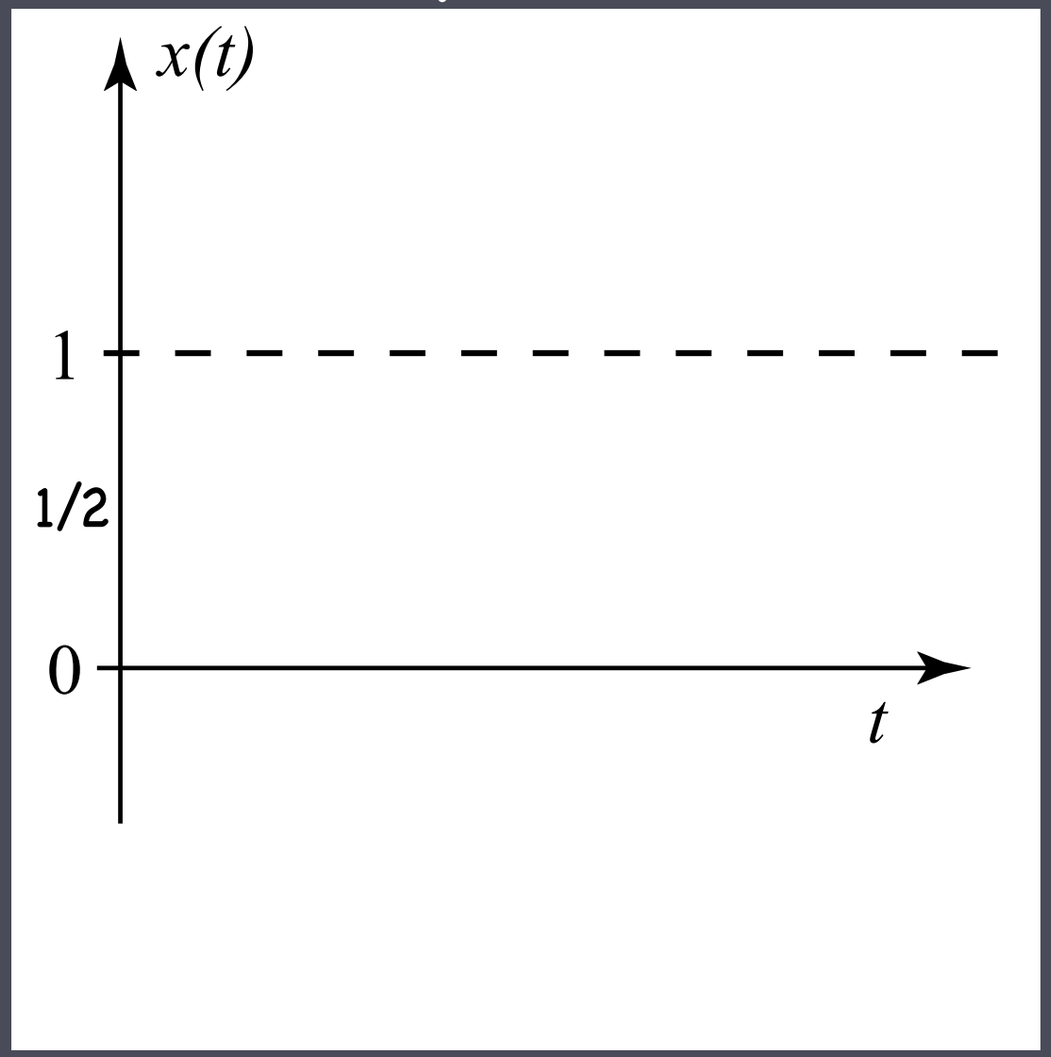
- Slope field.
- At any t, don't know x yet so plot all possible x′ values
  - When x(t)=1/2 what is x′?
    (A) 0
    (B) 1/4
    (C) 1/2
    (D) 1

$$x' = x(1 - x)$$

velocity

position

Slope field



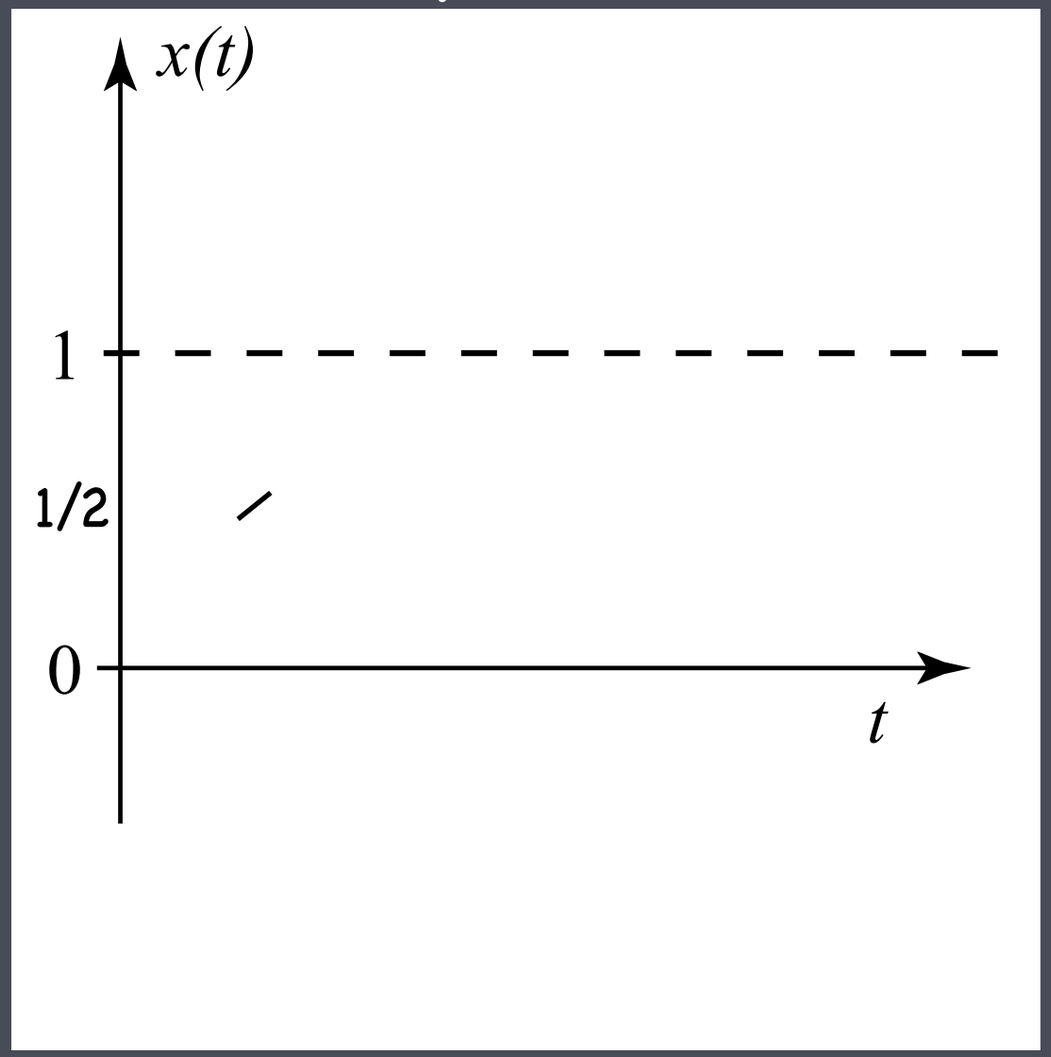
- Slope field.
- At any t, don't know x yet so plot all possible x′ values
  - When x(t)=1/2 what is x′?
    (A) 0
    (B) 1/4
    (C) 1/2
    (D) 1

$$x' = x(1 - x)$$

velocity

position

Slope field



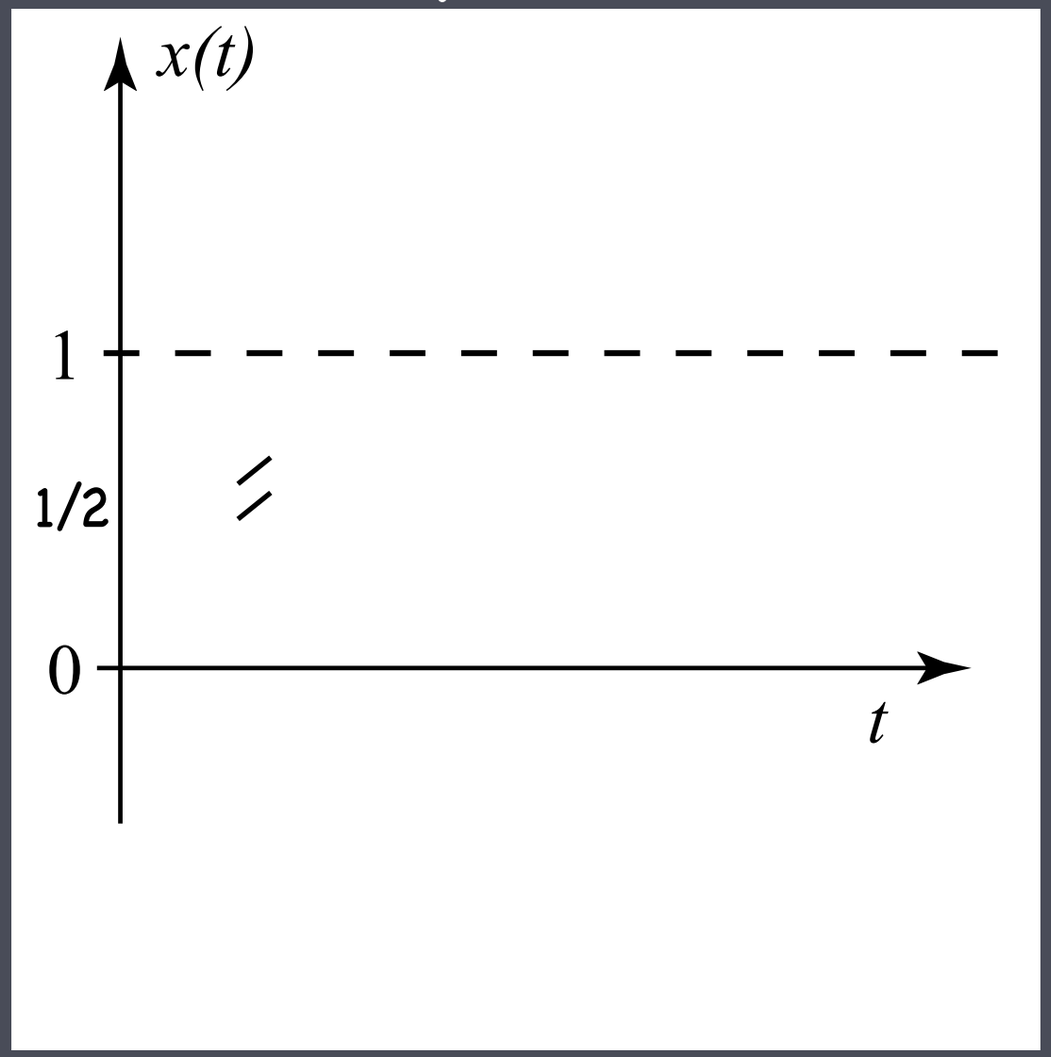
- Slope field.
- At any t, don't know x yet so plot all possible x′ values

$$x' = x(1 - x)$$

velocity

position

Slope field



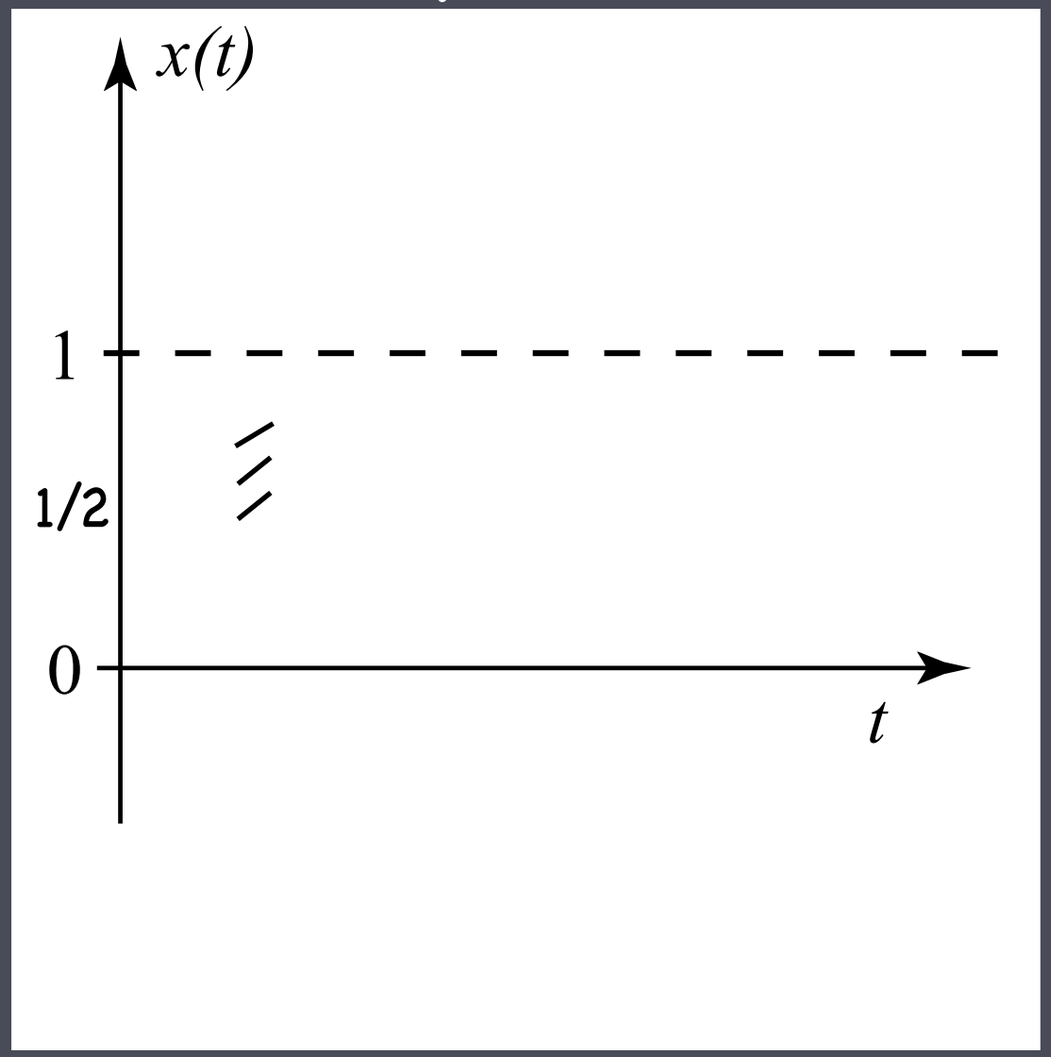
- Slope field.
- At any t, don't know x yet so plot all possible x' values

$$x' = x(1 - x)$$

velocity

position

Slope field



- Slope field.
- At any t, don't know x yet so plot all possible x' values

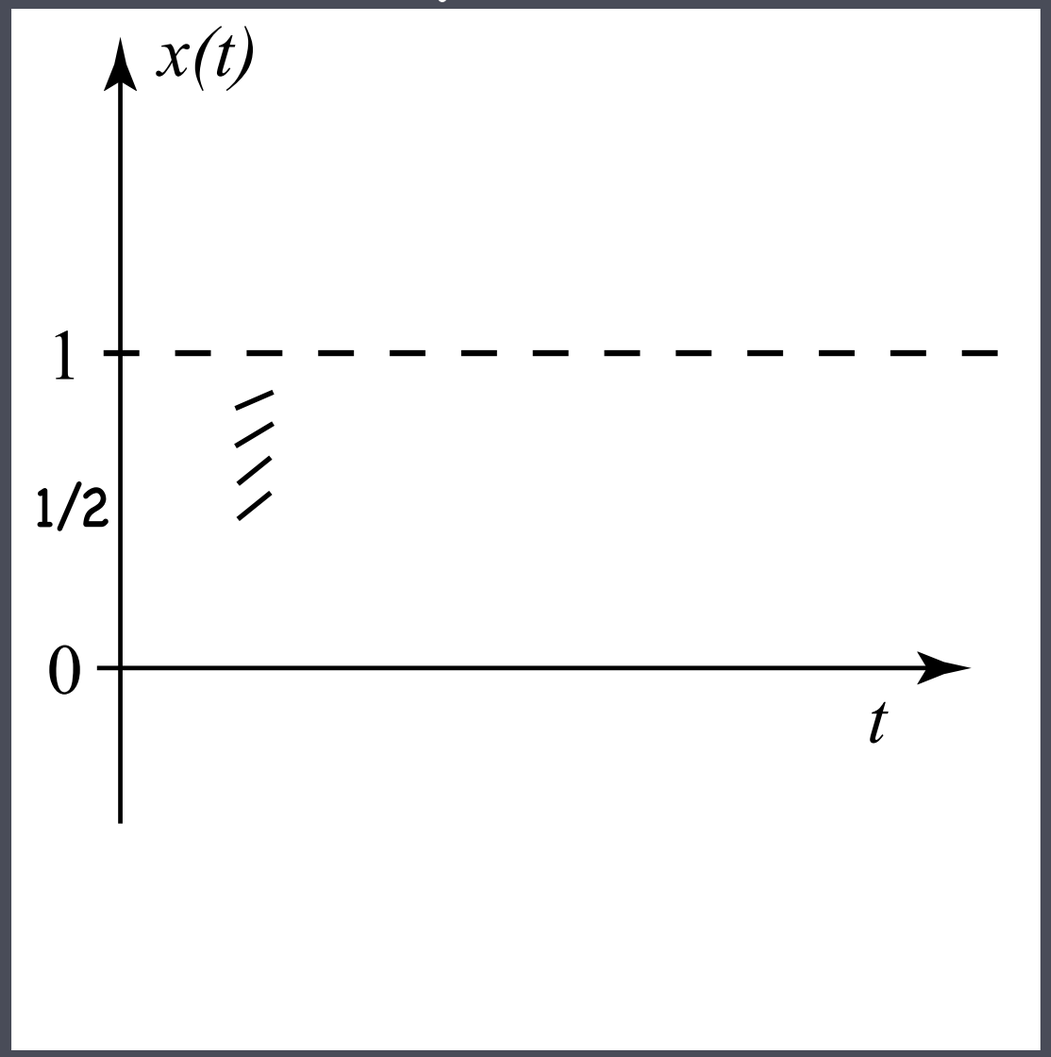
$$x' = x(1 - x)$$

velocity

position

Slope field

- Slope field.
- At any t, don't know x yet so plot all possible x' values

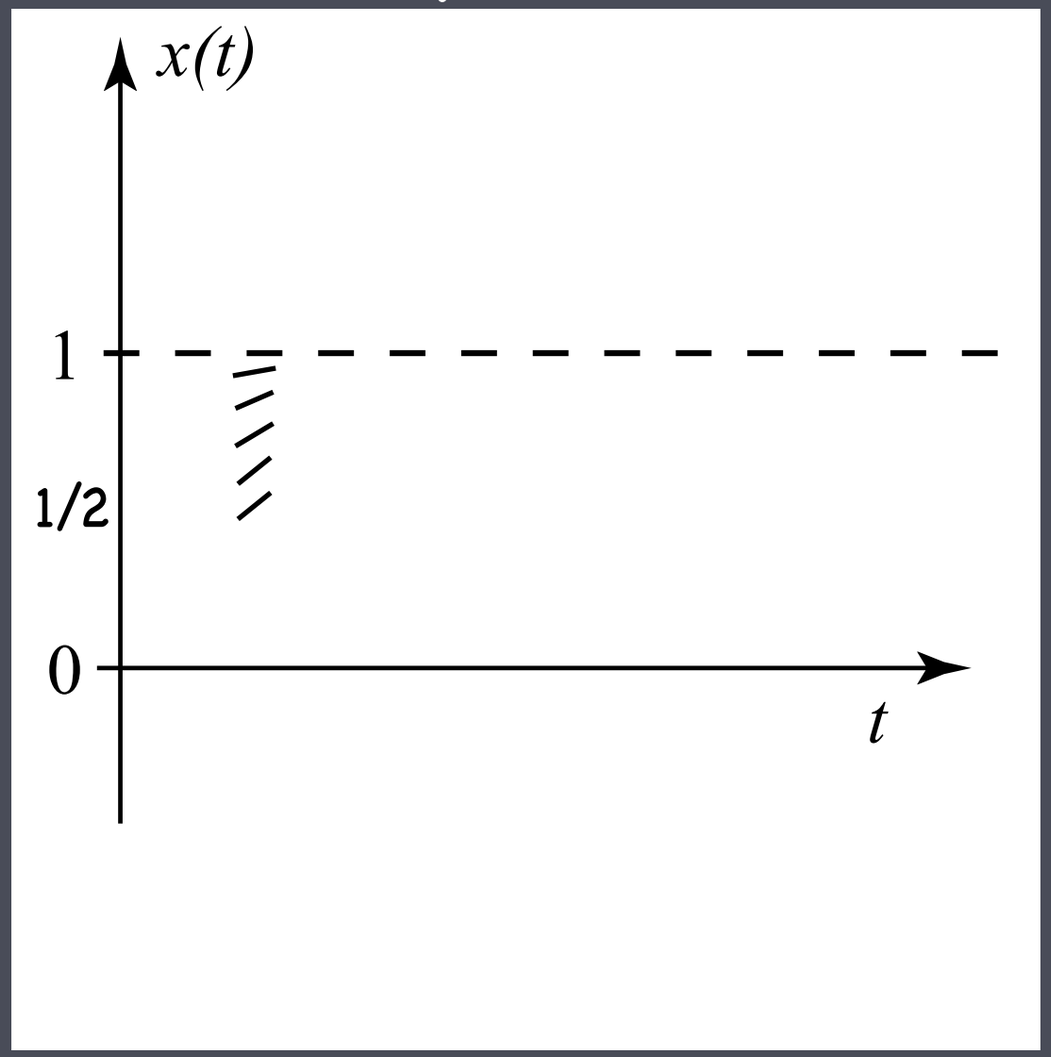


$$x' = x(1 - x)$$

velocity

position

Slope field



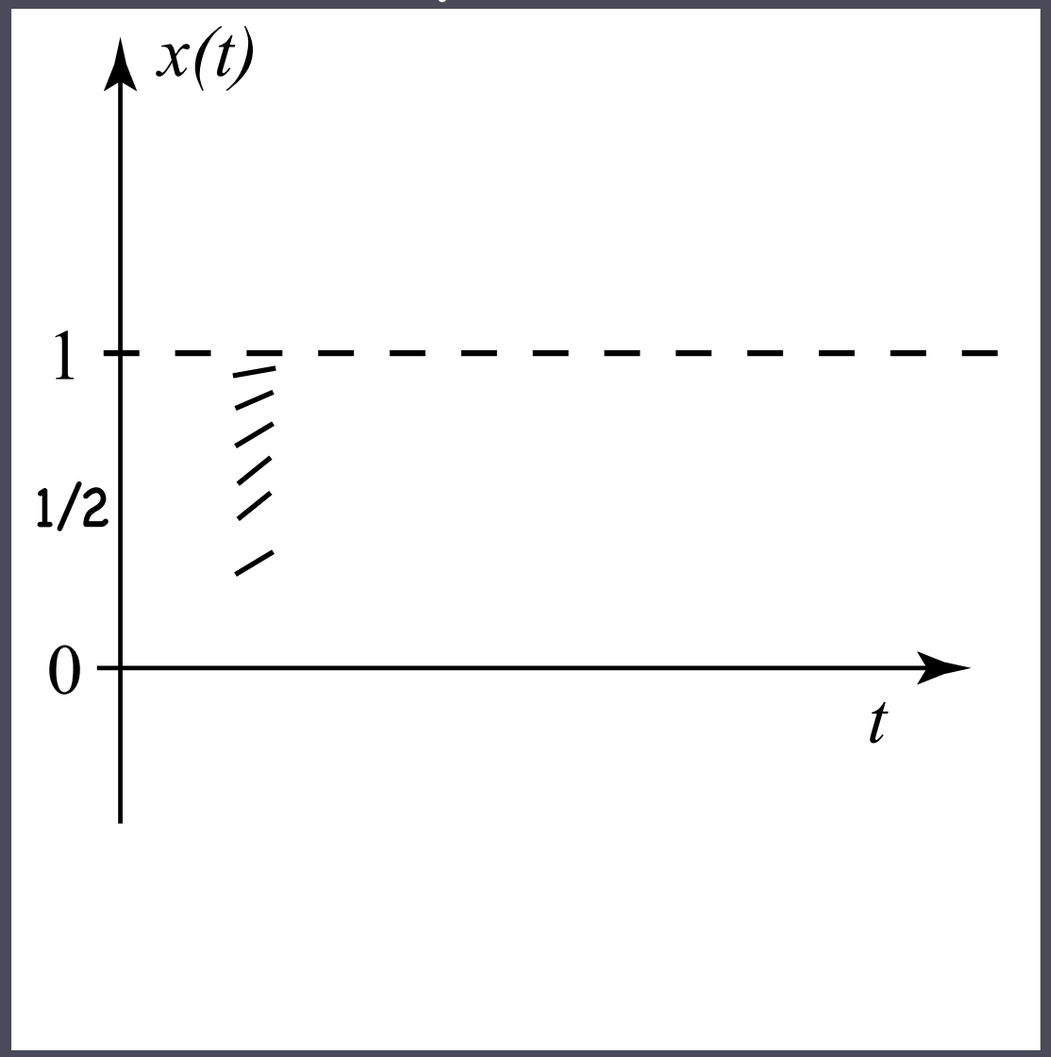
- Slope field.
- At any t, don't know x yet so plot all possible x' values

$$x' = x(1 - x)$$

velocity

position

Slope field



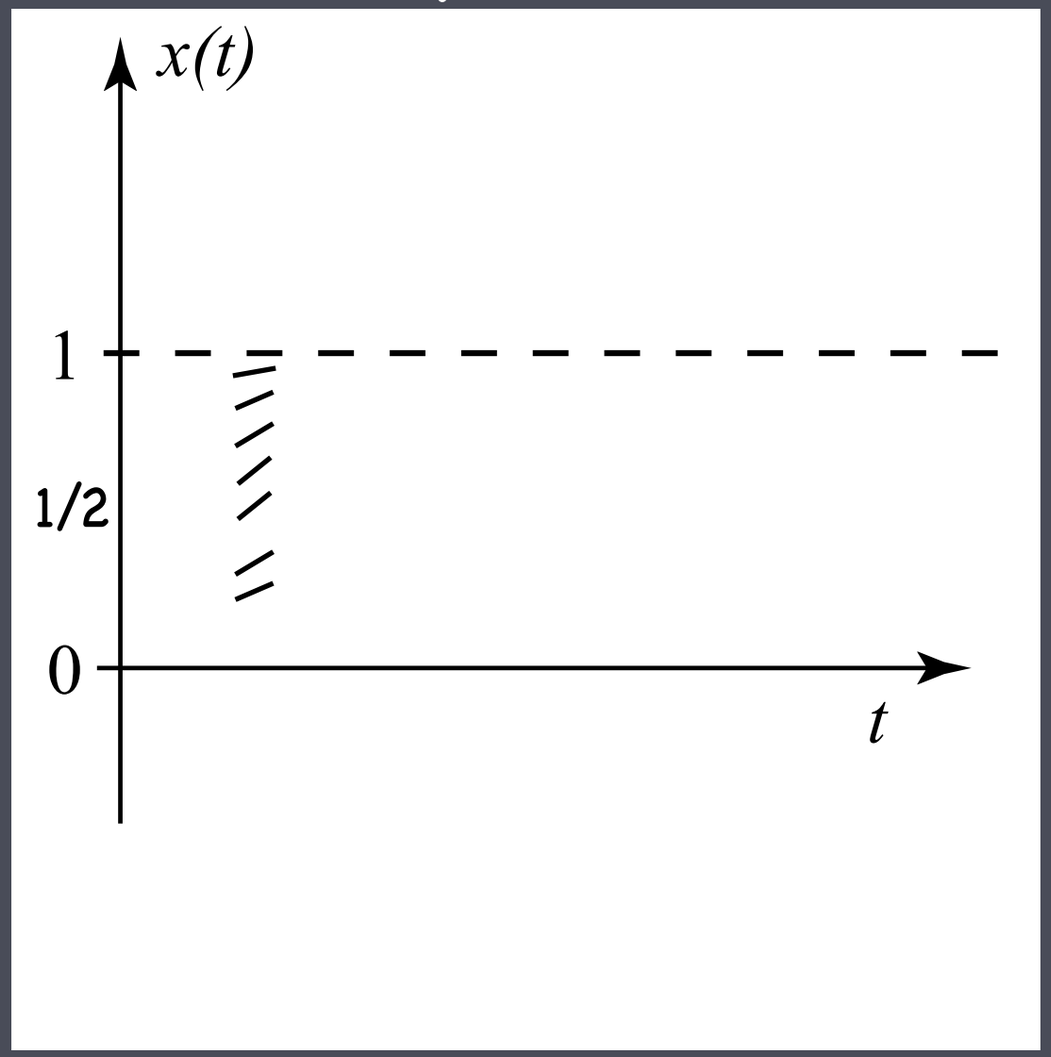
- Slope field.
- At any t, don't know x yet so plot all possible x' values

$$x' = x(1 - x)$$

velocity

position

Slope field



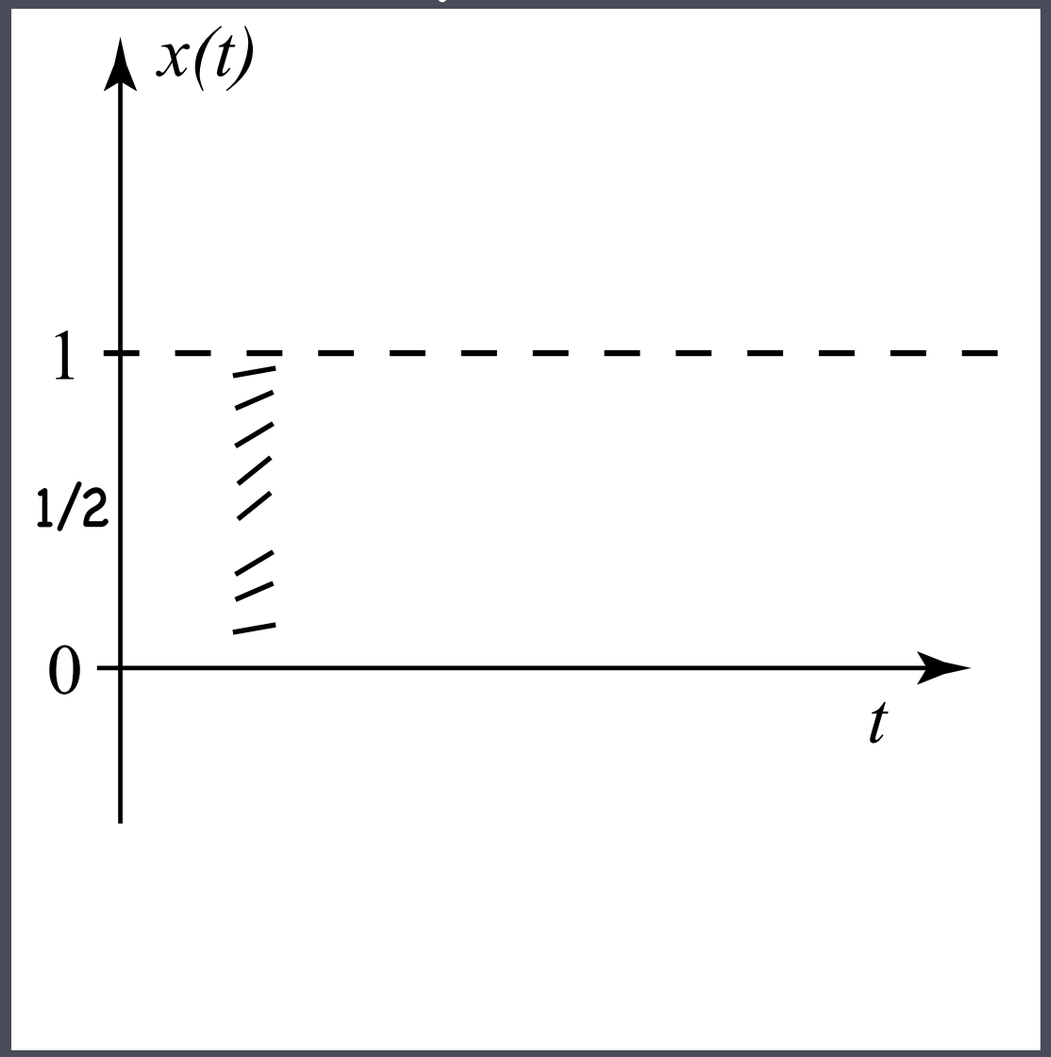
- Slope field.
- At any t, don't know x yet so plot all possible x' values

$$x' = x(1 - x)$$

velocity

position

Slope field



- Slope field.
- At any t, don't know x yet so plot all possible x' values

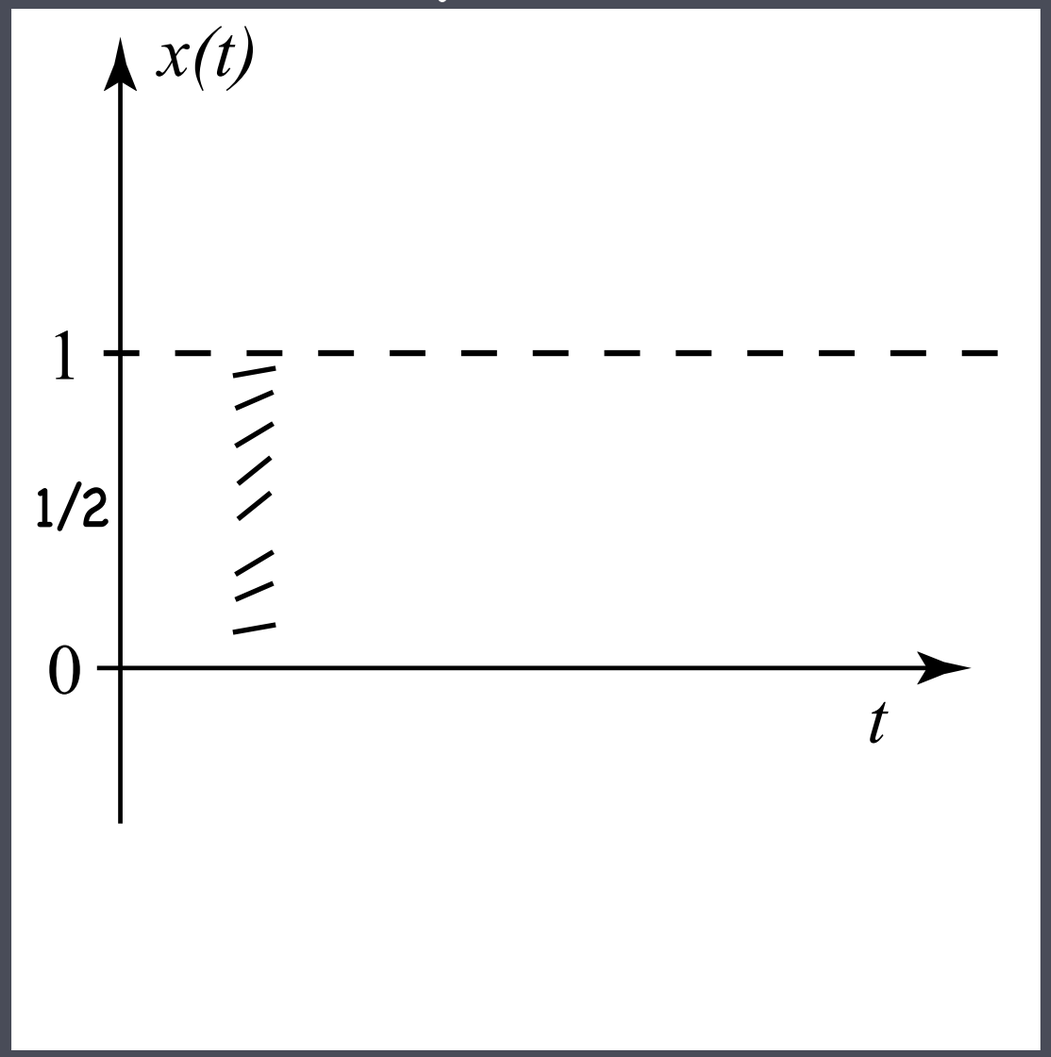
$$x' = x(1 - x)$$

velocity

position

Slope field

- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.

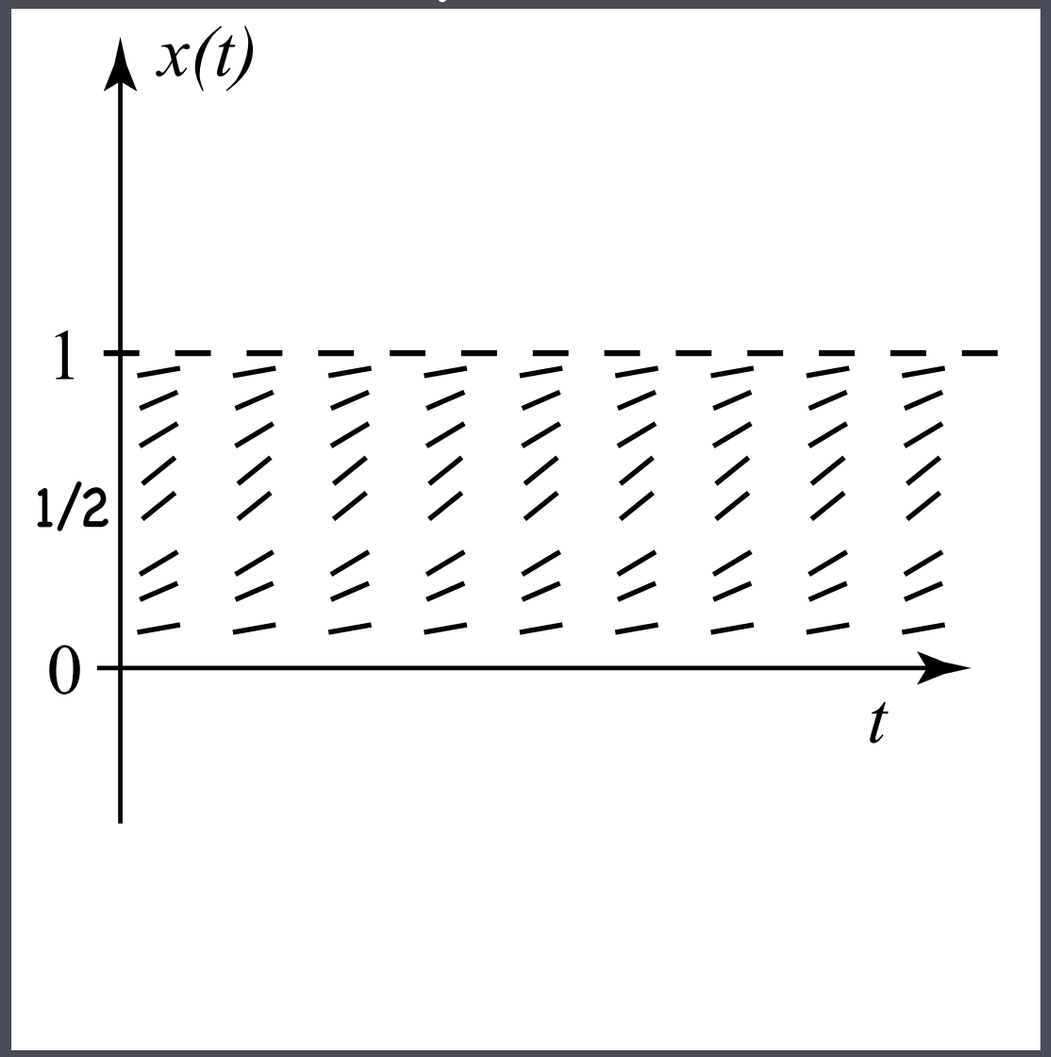


$$x' = x(1 - x)$$

velocity

position

Slope field



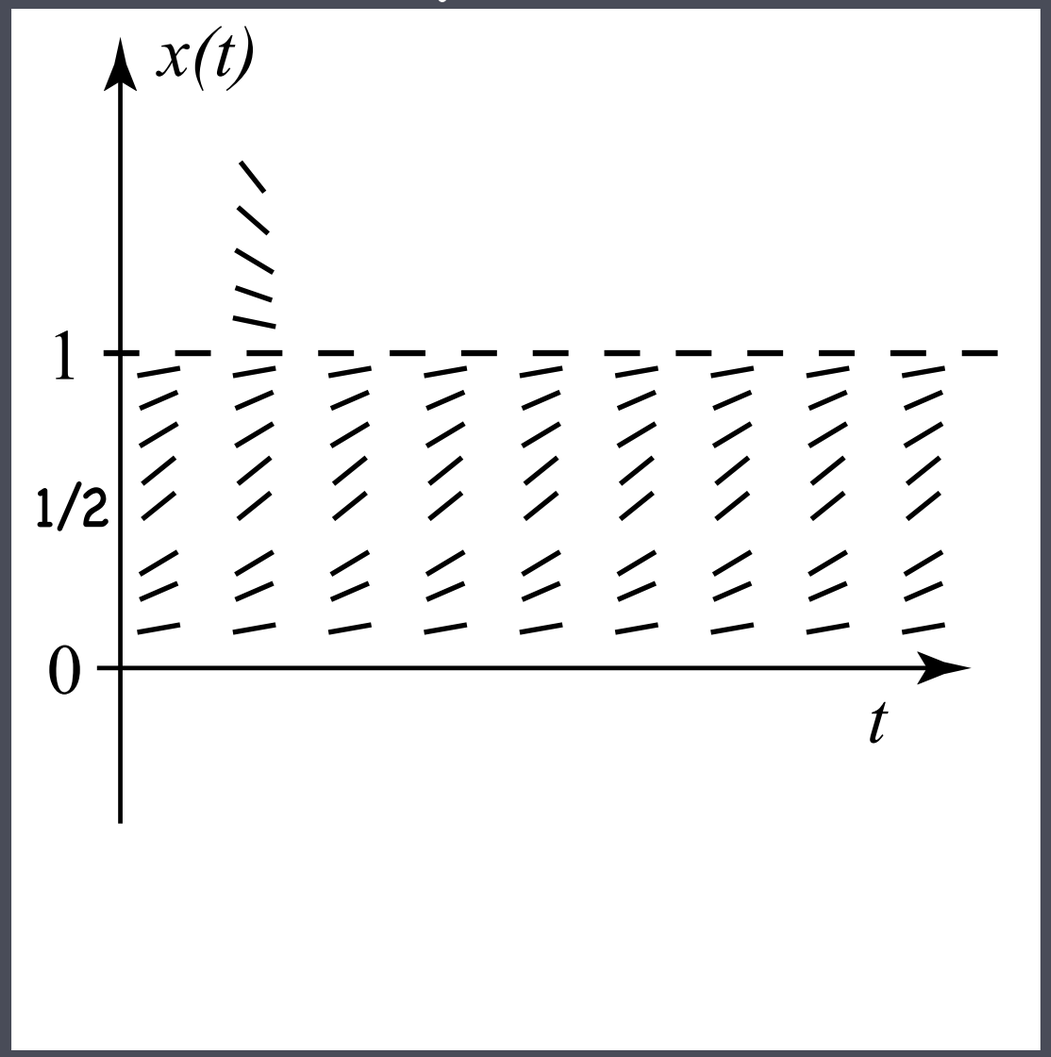
- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.

$$x' = x(1 - x)$$

velocity

position

Slope field



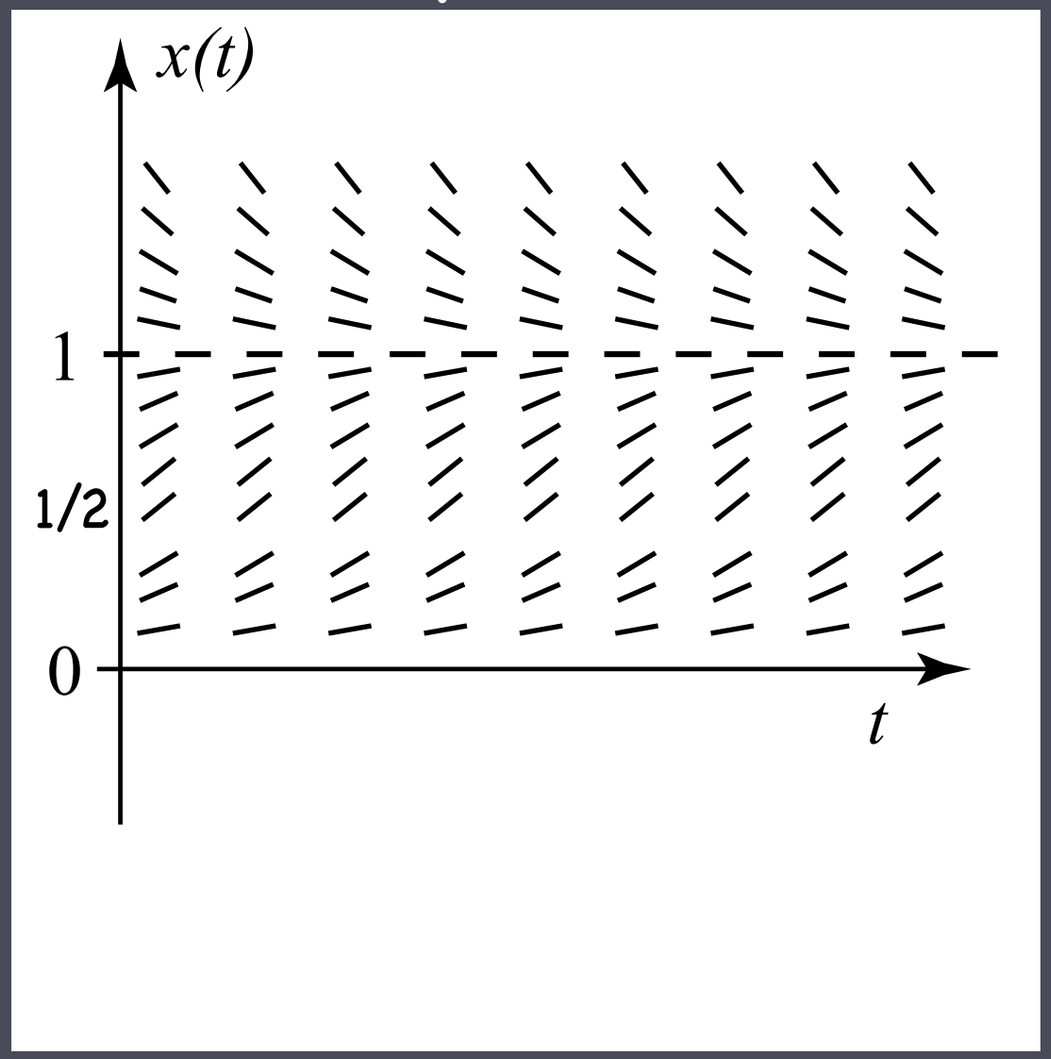
- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.

$$x' = x(1 - x)$$

velocity

position

Slope field

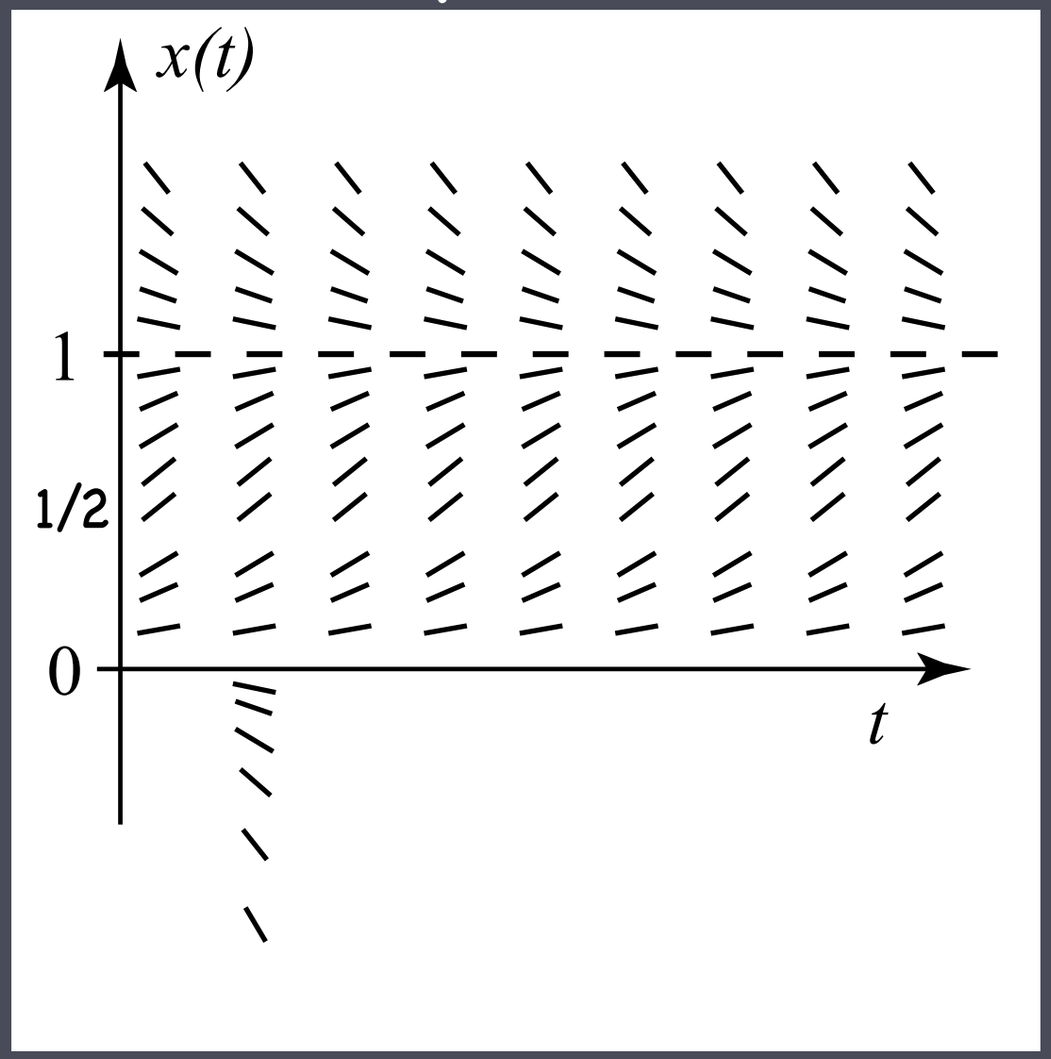


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.

$$x' = x(1 - x)$$

velocity position

Slope field

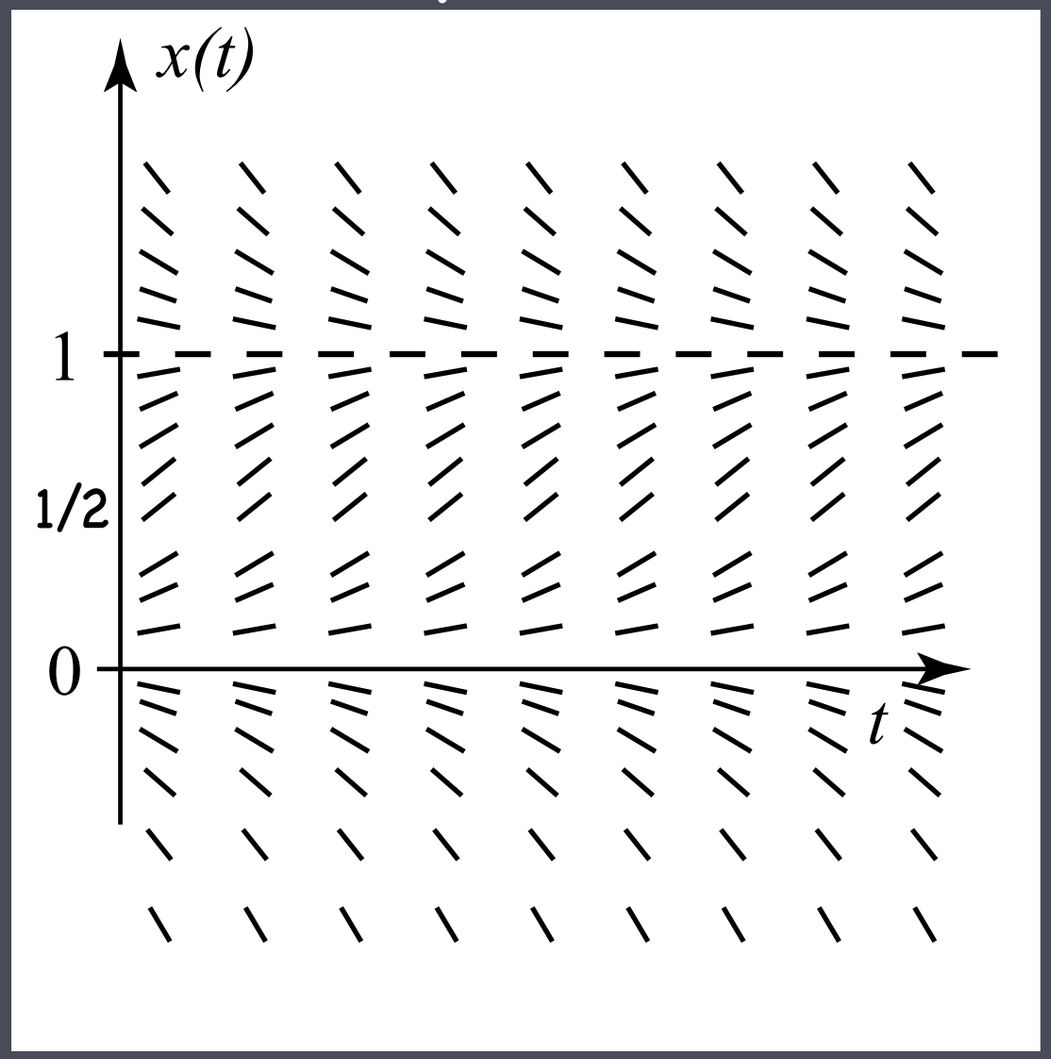


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.

$$x' = x(1 - x)$$

velocity position

Slope field

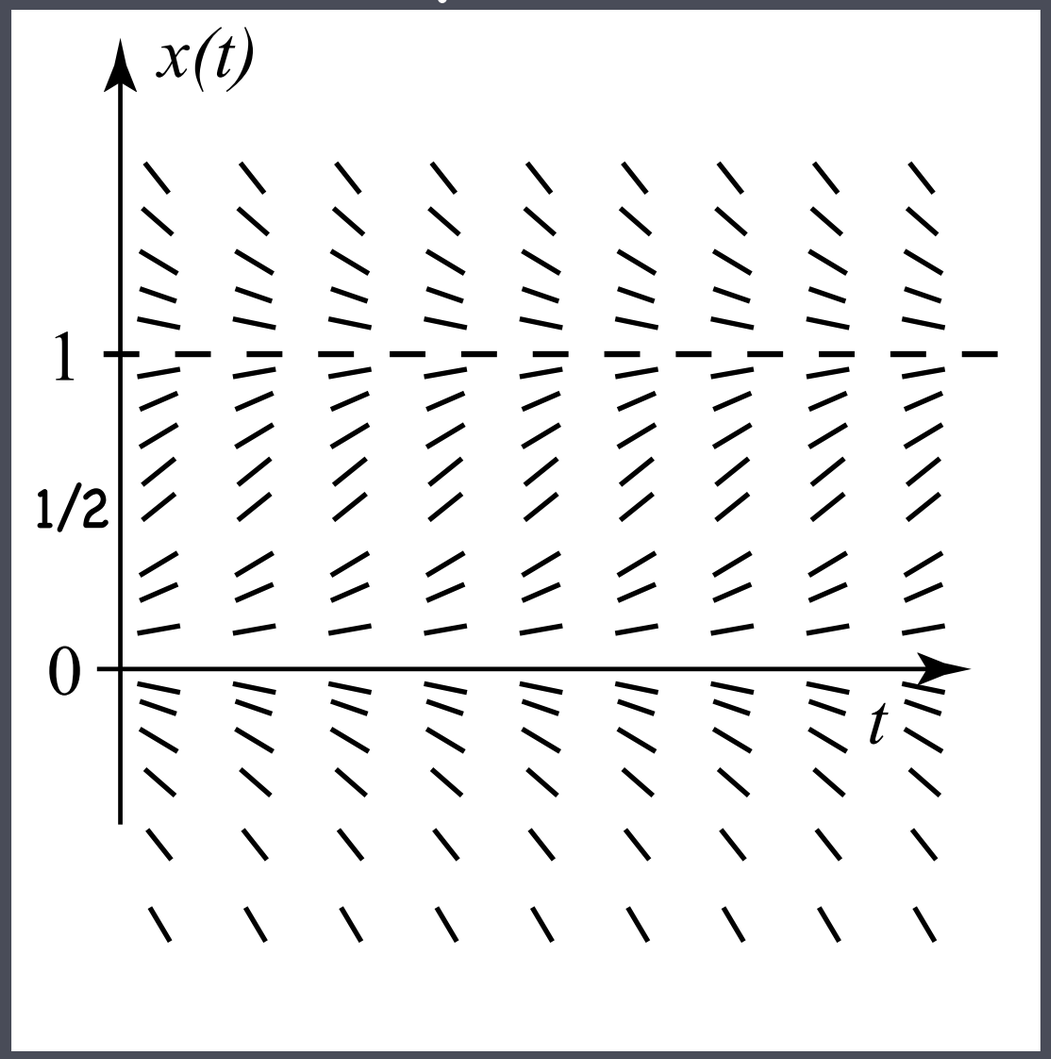


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.

$$x' = x(1 - x)$$

velocity position

Slope field

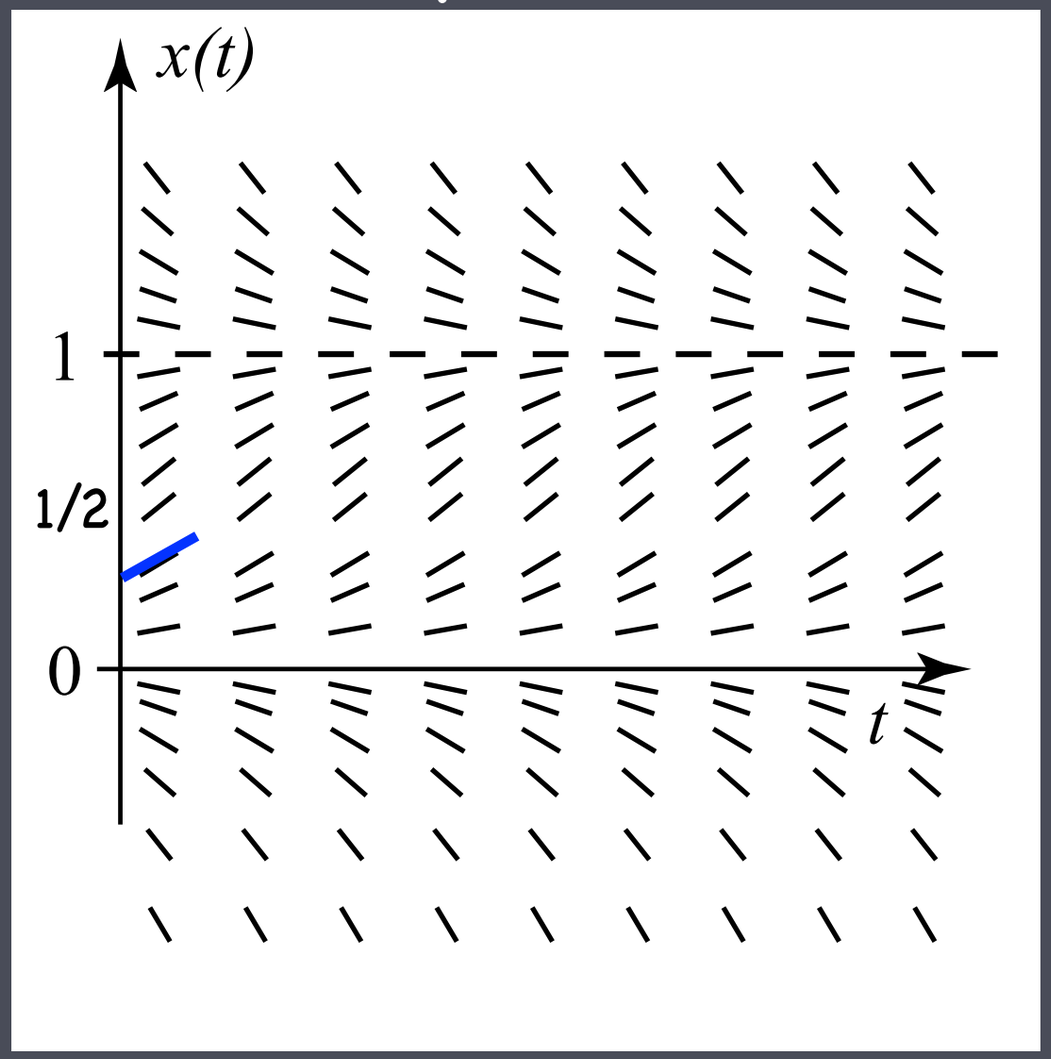


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.
- Solution curves must be tangent to slope field everywhere.

$$x' = x(1 - x)$$

velocity position

Slope field

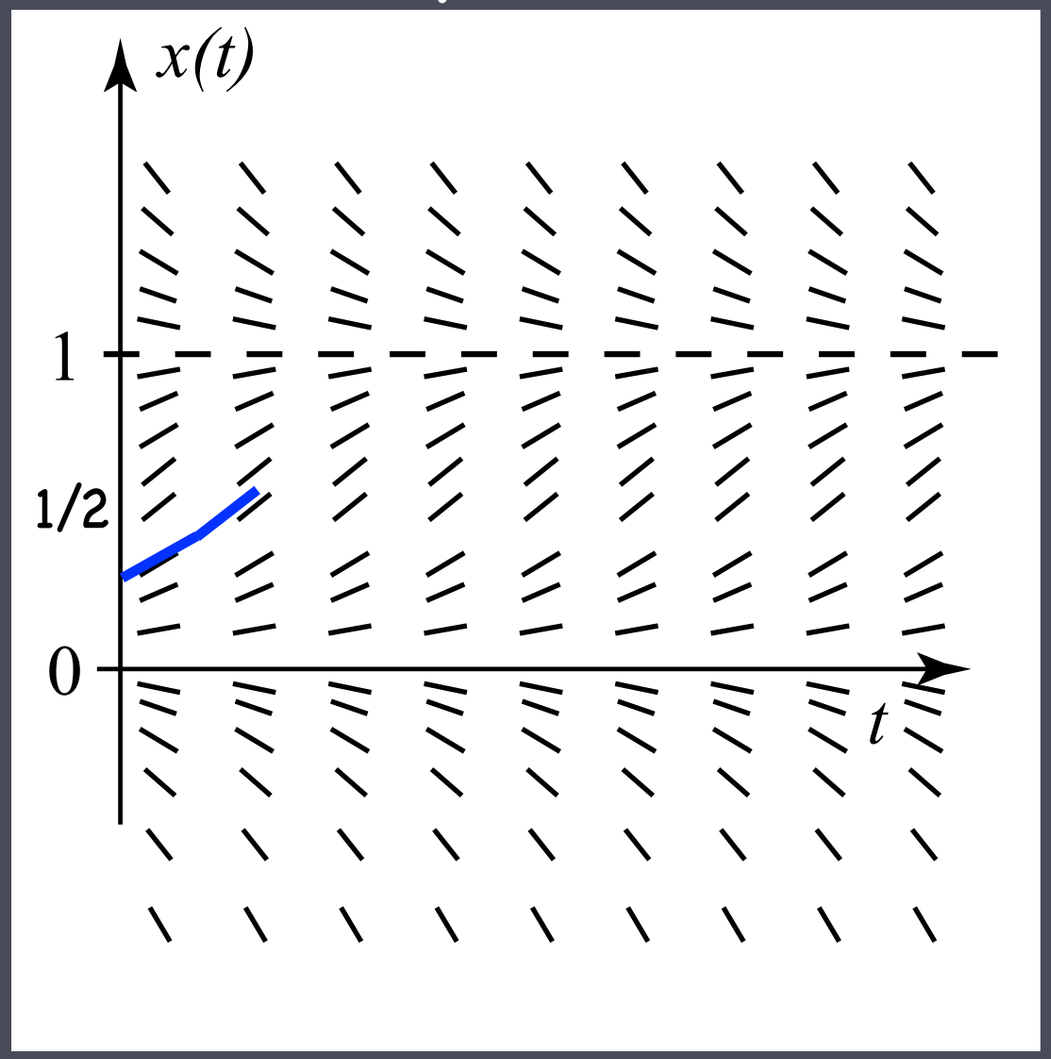


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.
- Solution curves must be tangent to slope field everywhere.

$$x' = x(1 - x)$$

velocity position

Slope field

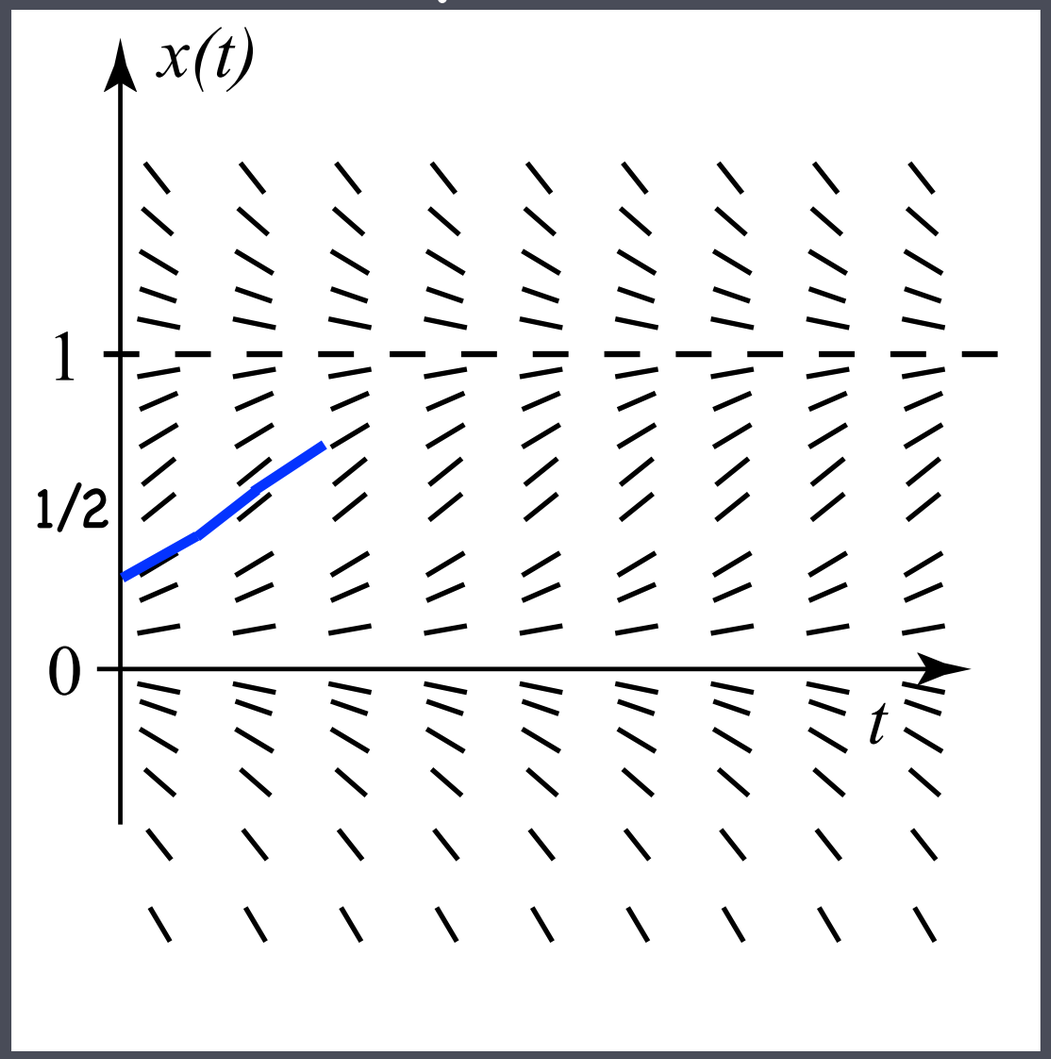


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.
- Solution curves must be tangent to slope field everywhere.

$$x' = x(1 - x)$$

velocity position

Slope field

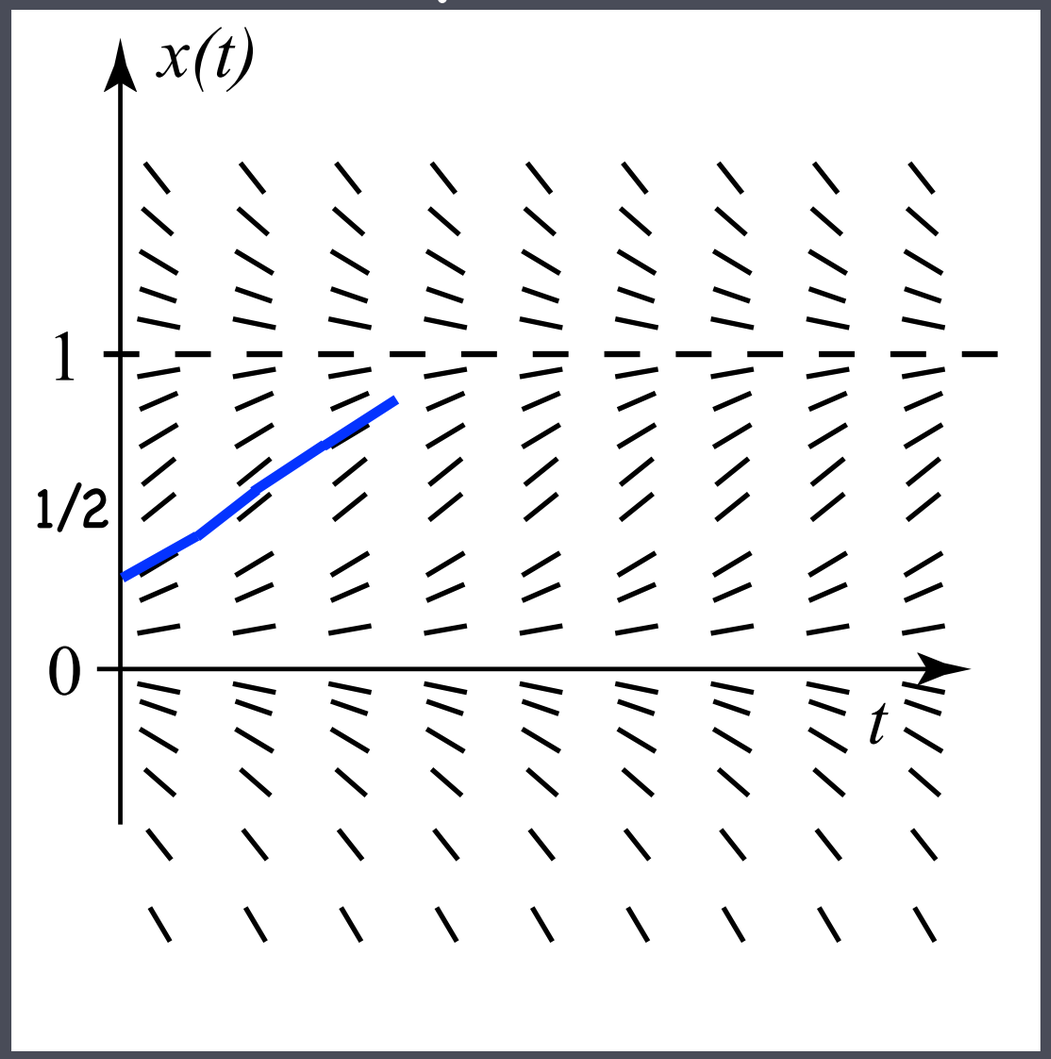


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.
- Solution curves must be tangent to slope field everywhere.

$$x' = x(1 - x)$$

velocity position

Slope field

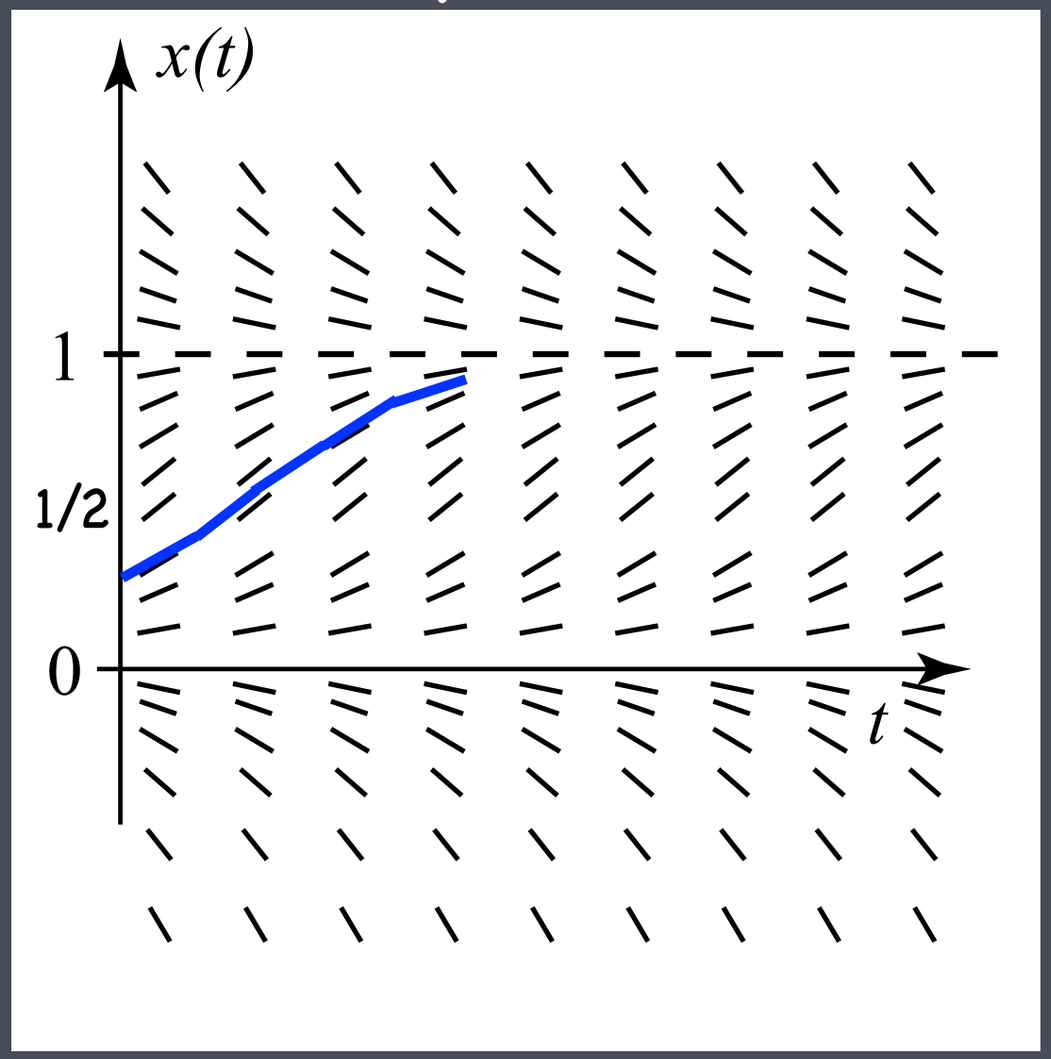


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.
- Solution curves must be tangent to slope field everywhere.

$$x' = x(1 - x)$$

velocity position

Slope field

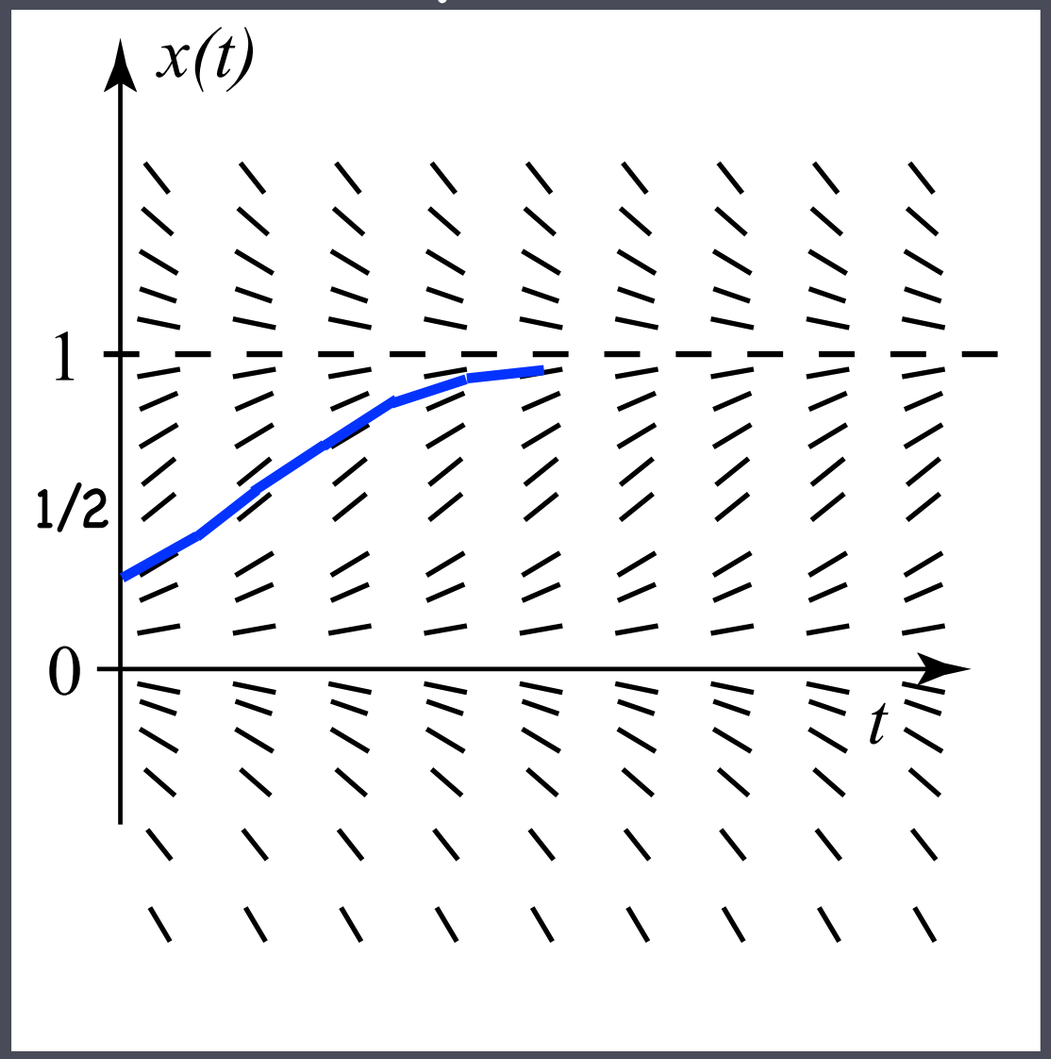


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.
- Solution curves must be tangent to slope field everywhere.

$$x' = x(1 - x)$$

velocity position

Slope field

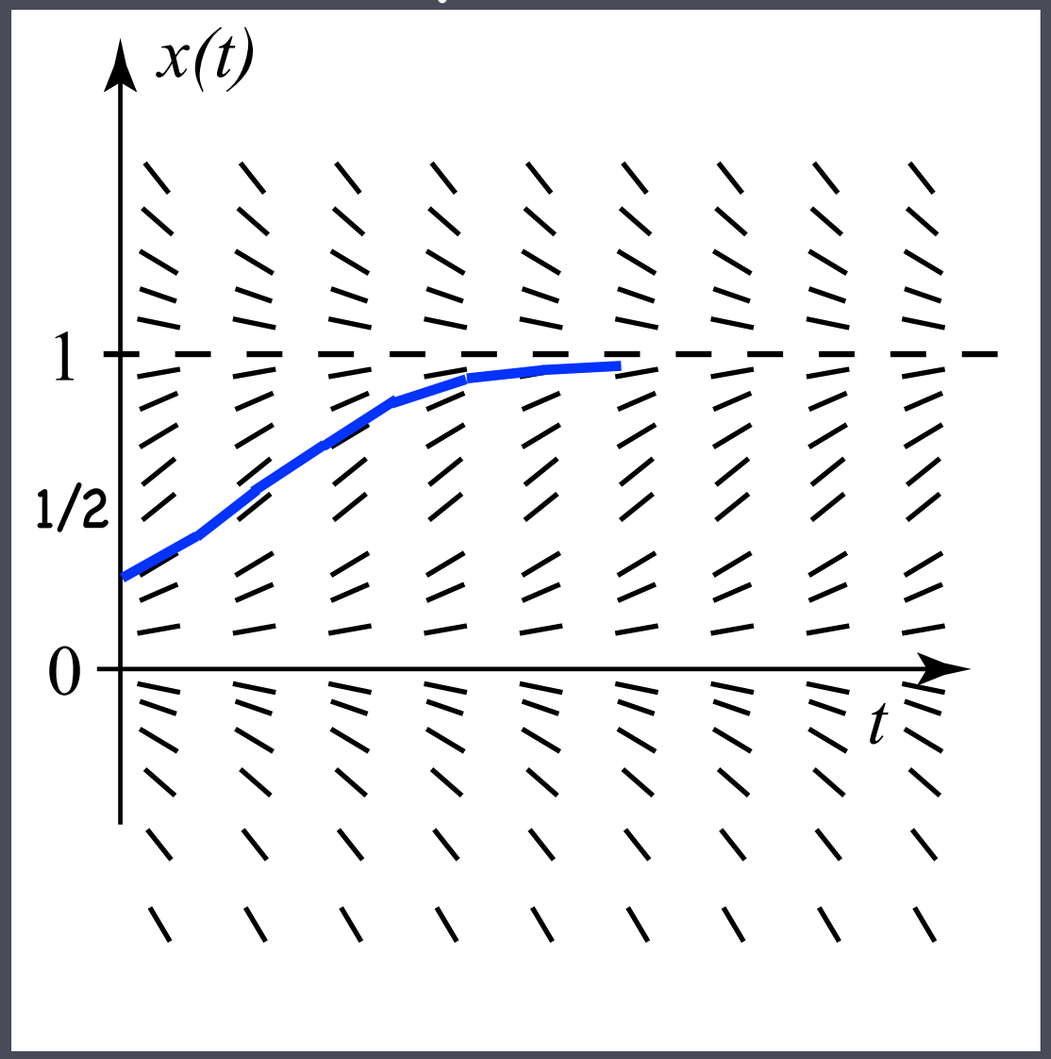


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.
- Solution curves must be tangent to slope field everywhere.

$$x' = x(1 - x)$$

velocity position

Slope field

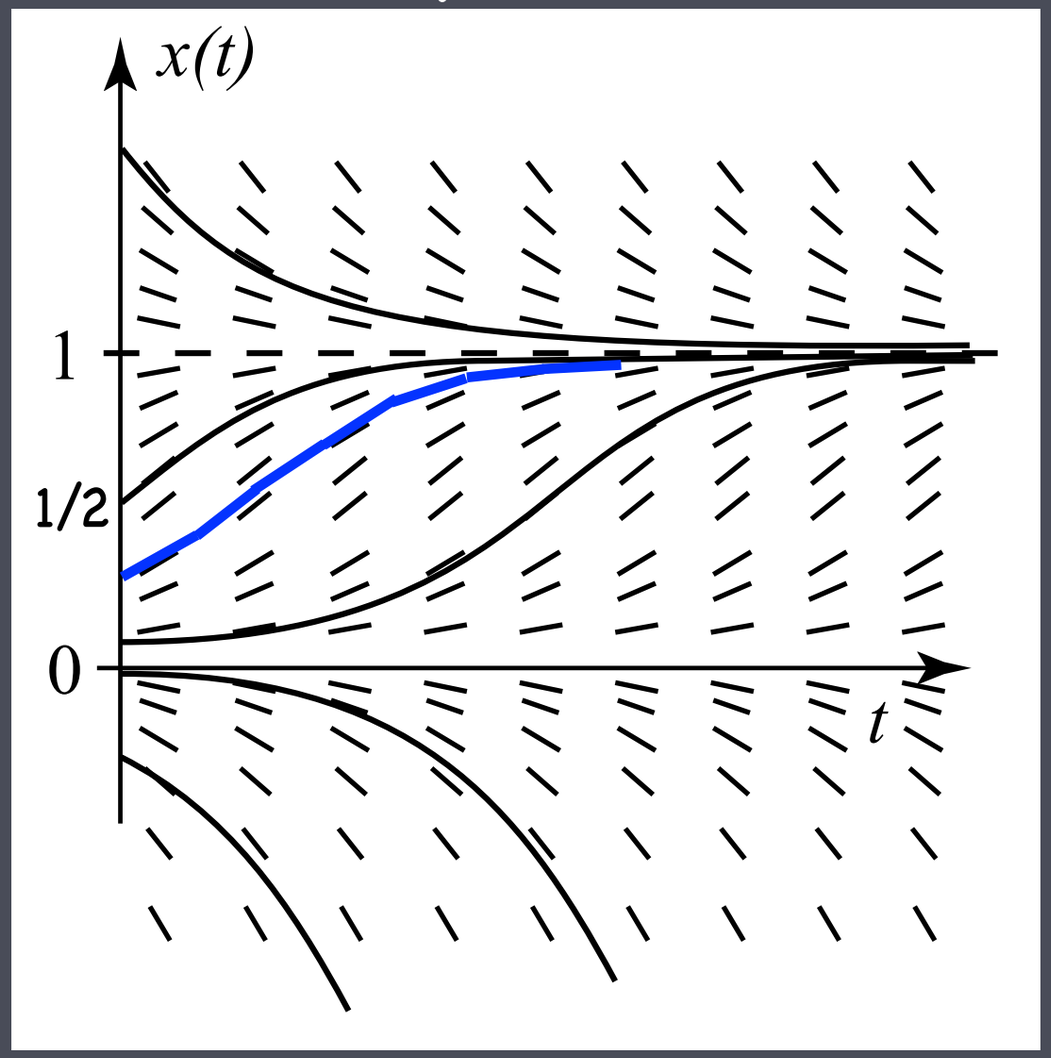


- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.
- Solution curves must be tangent to slope field everywhere.

$$x' = x(1 - x)$$

velocity position

Slope field



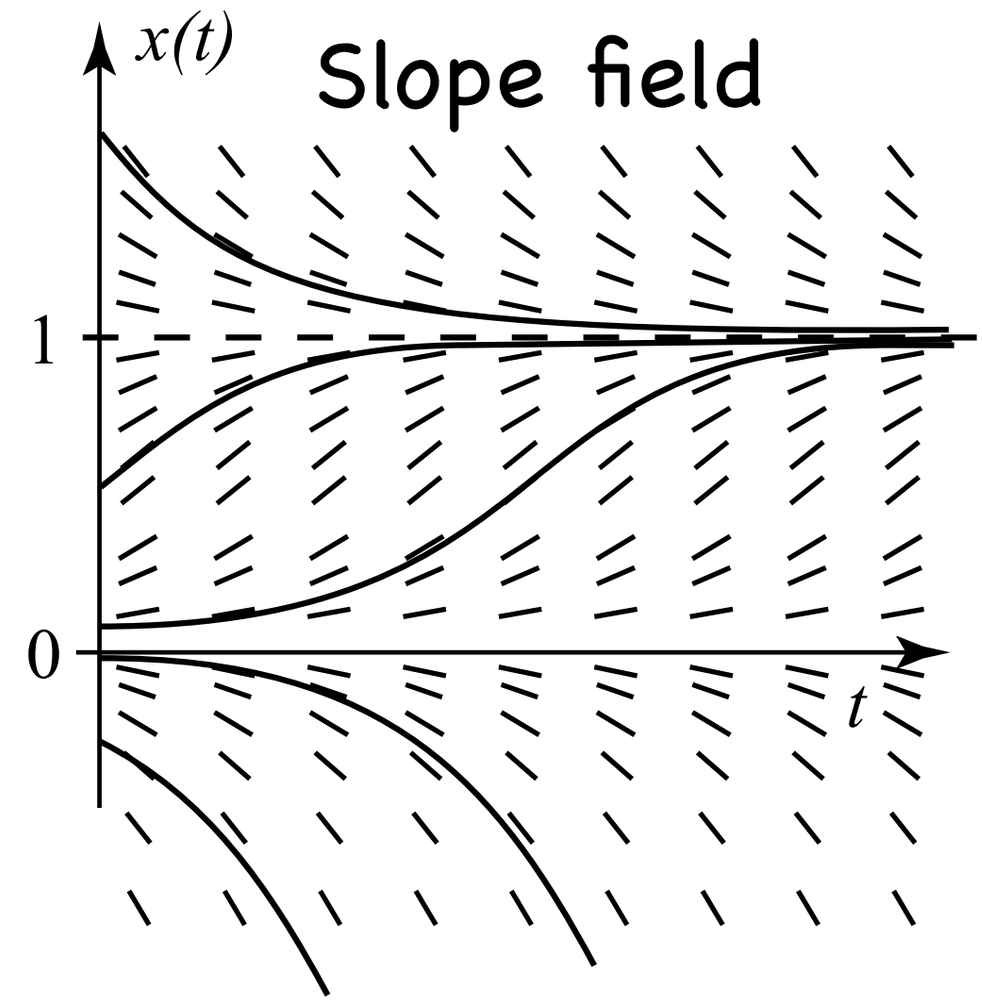
- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw them for all t.
- Solution curves must be tangent to slope field everywhere.

$$y' = -y(y-1)(y+1)$$

- What are the steady states of this equation?
- Draw the slope field for this equation.
    - Include the steepest slope element in each interval between steady states and two others (roughly).

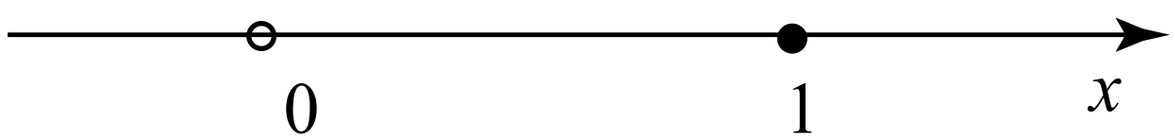
Velocity versus position

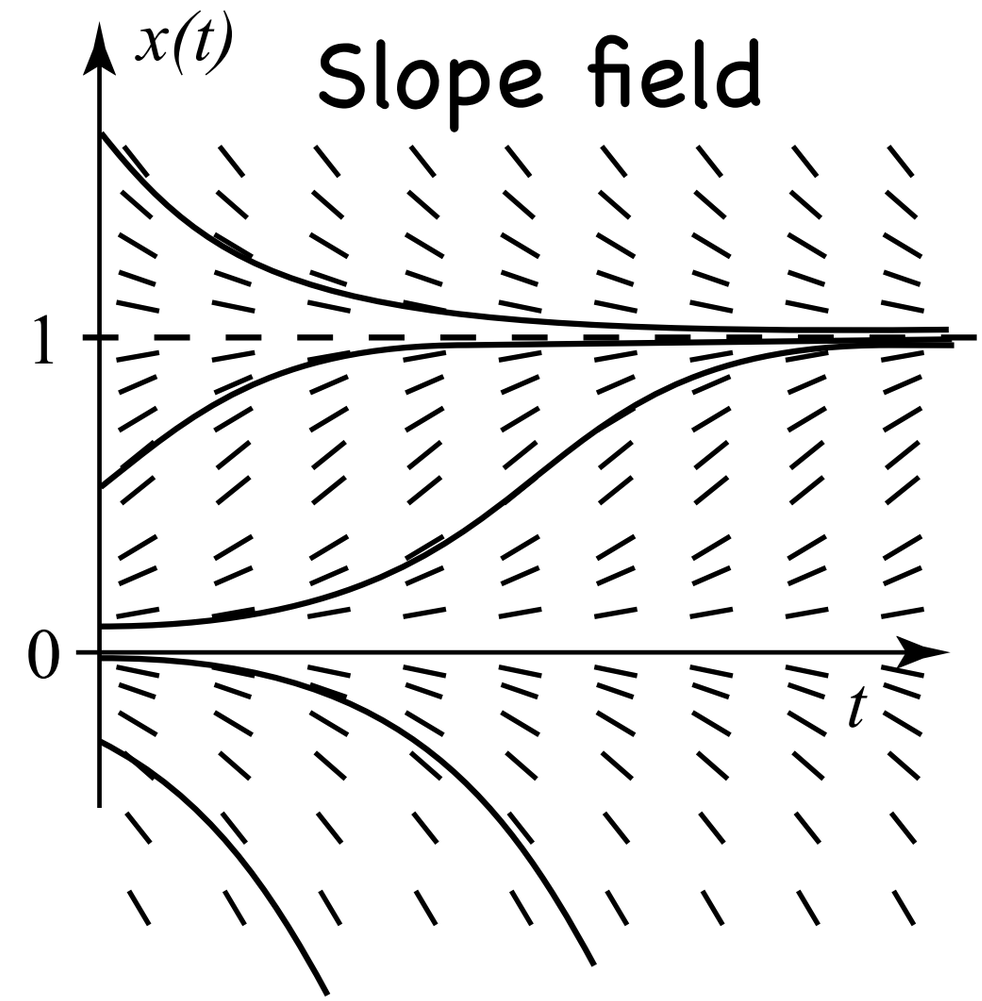
Velocity (x′) vs. position (x)



Velocity versus position

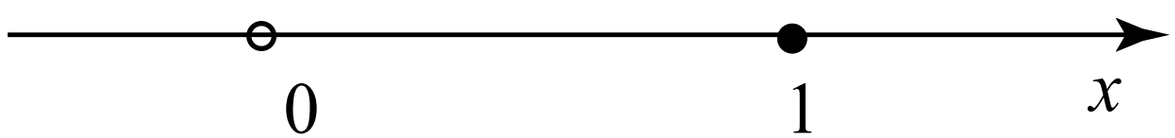
Velocity (x′) vs. position (x)



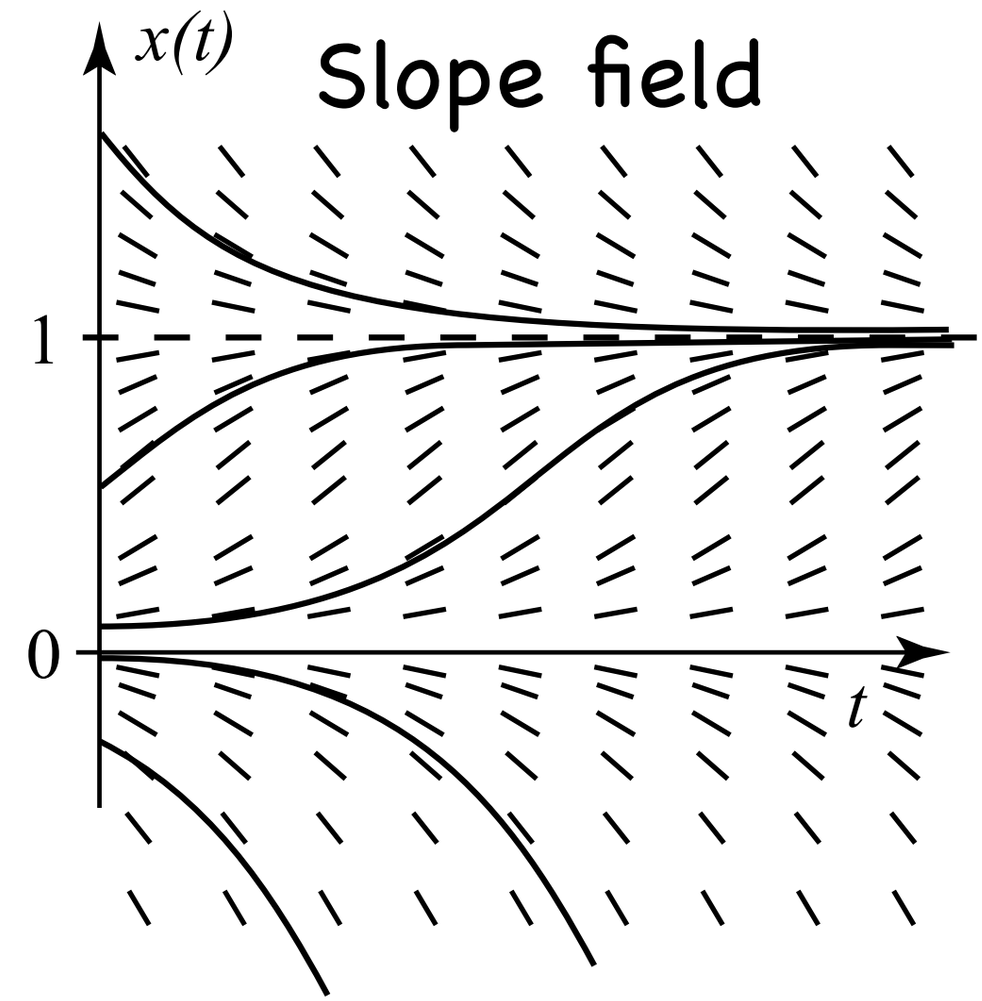


Velocity versus position

Velocity (x′) vs. position (x)

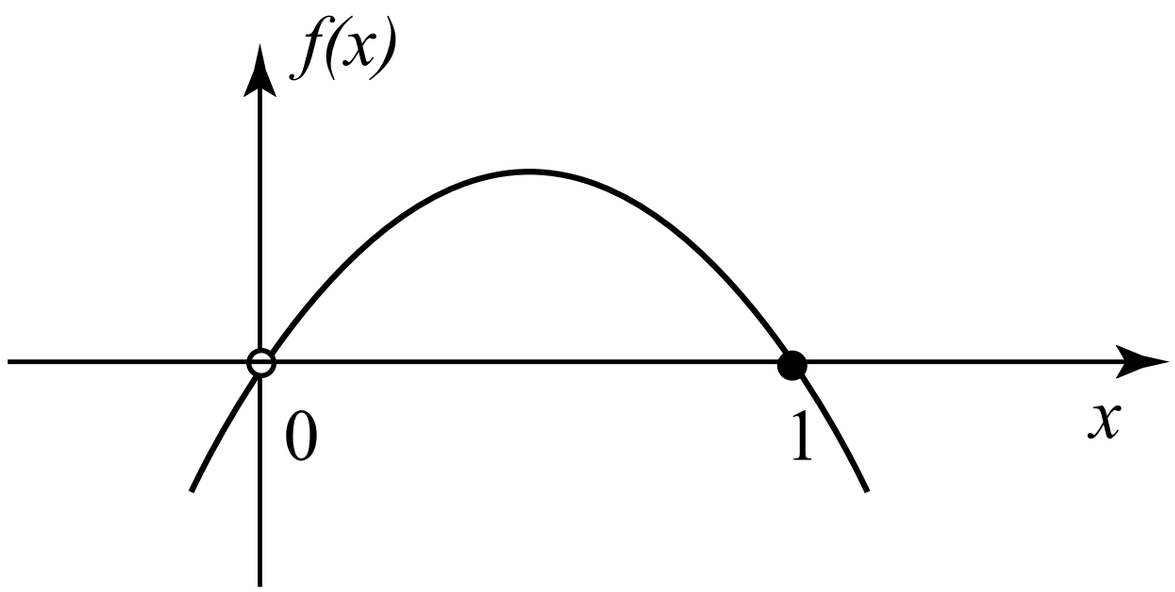


$x' = f(x) = x(1-x)$

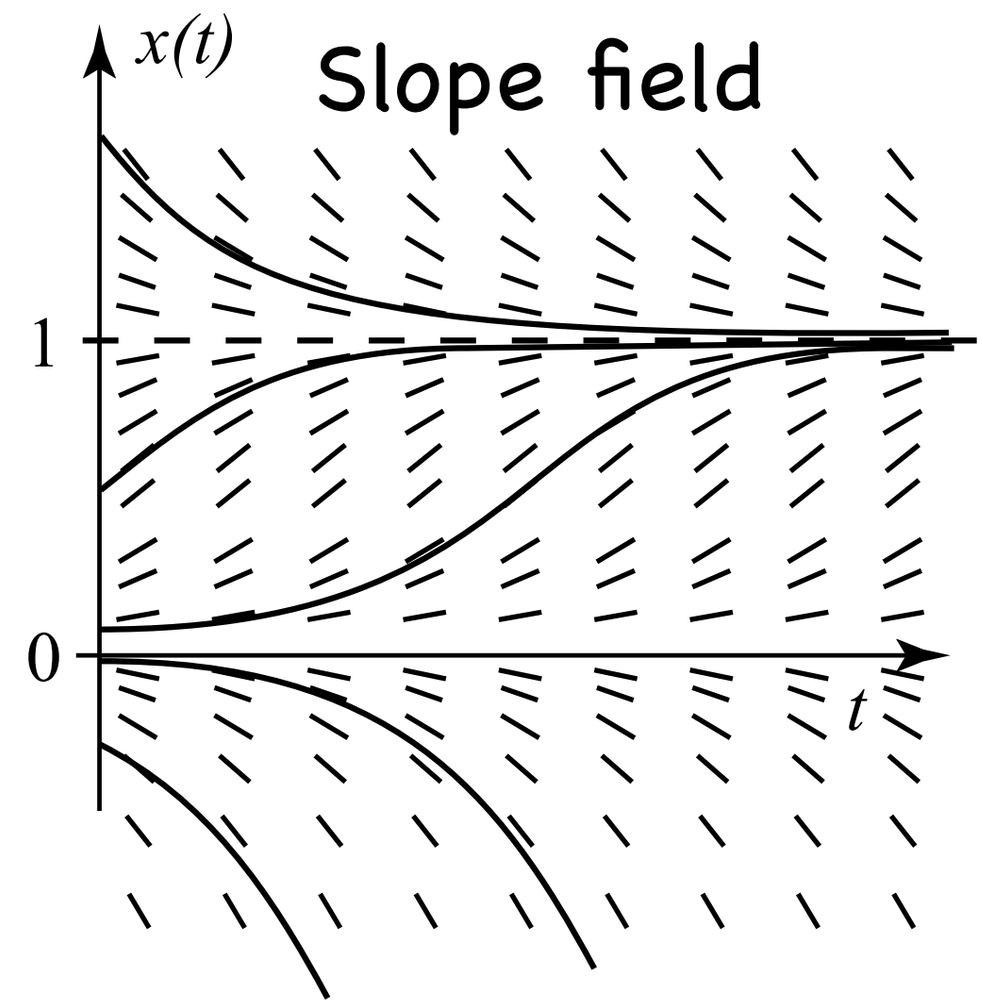


Velocity versus position

Velocity (x′) vs. position (x)

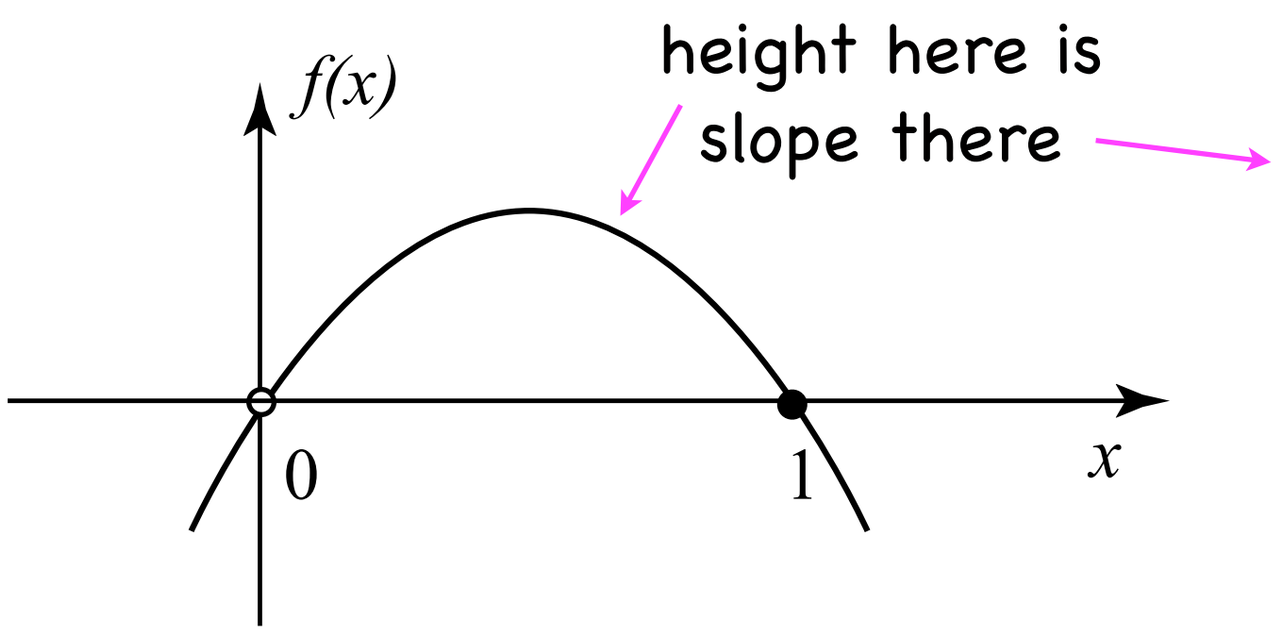


$x' = f(x) = x(1-x)$

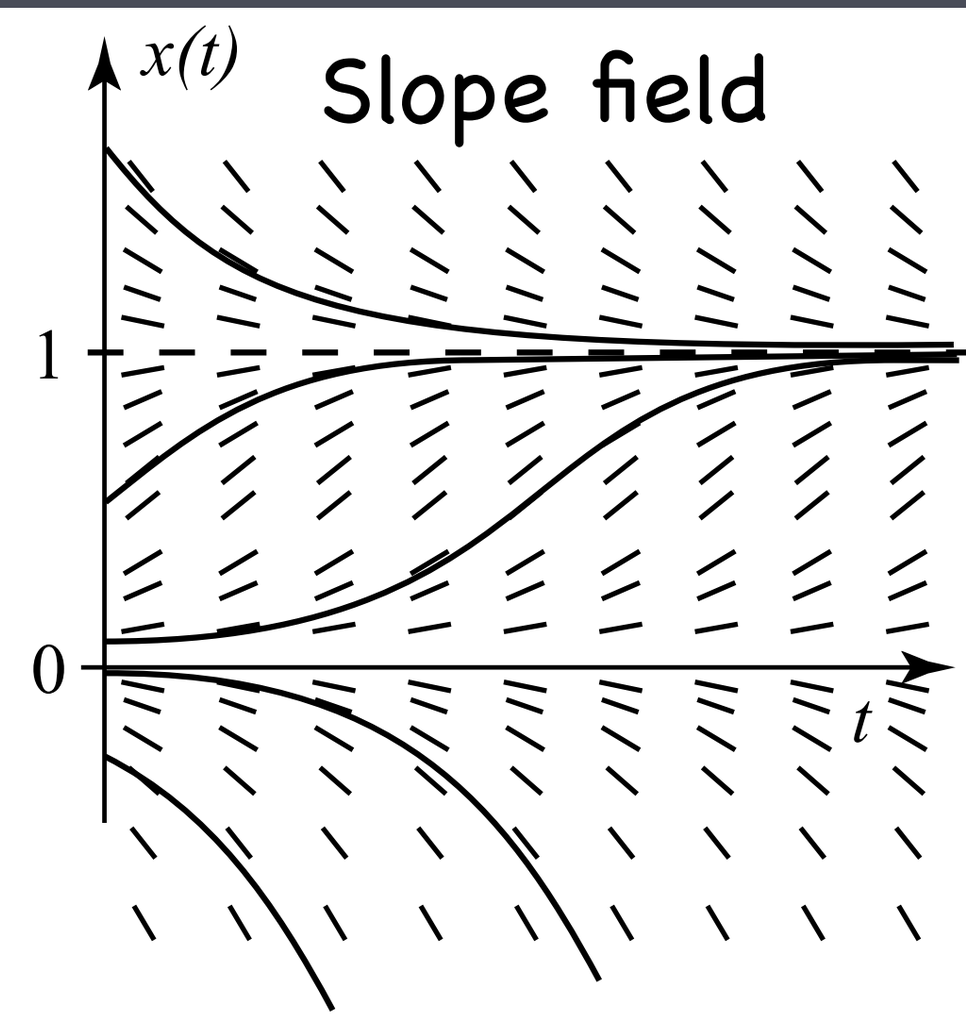


Velocity versus position

Velocity (x′) vs. position (x)

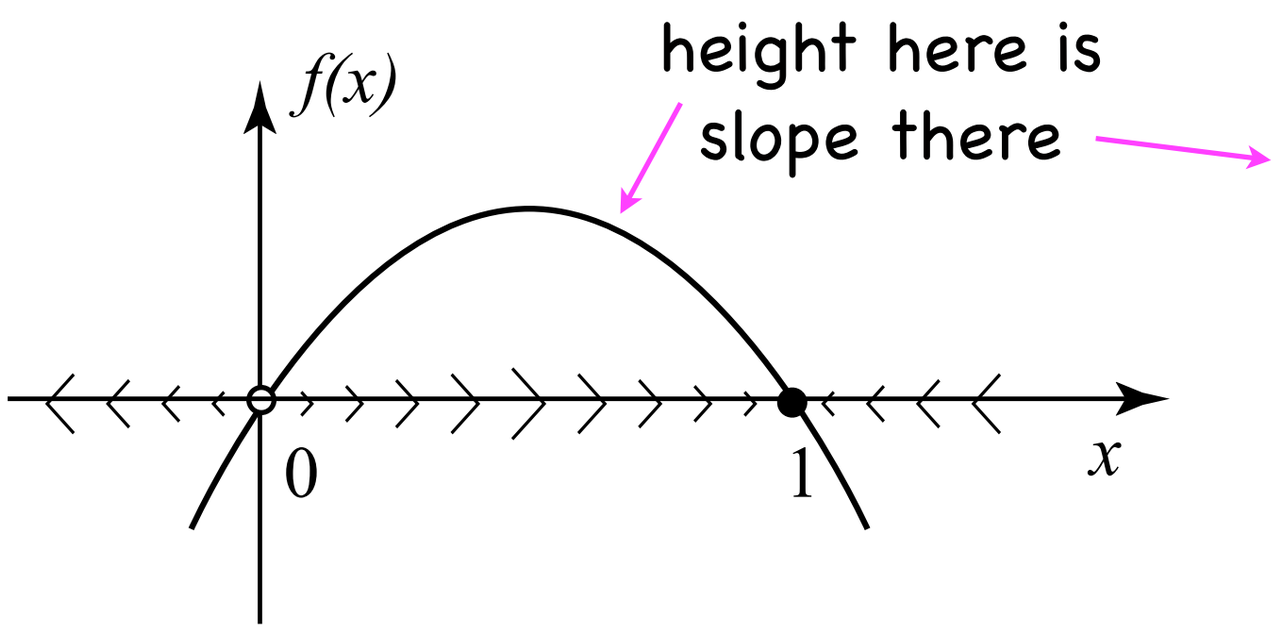


$x' = f(x) = x(1-x)$

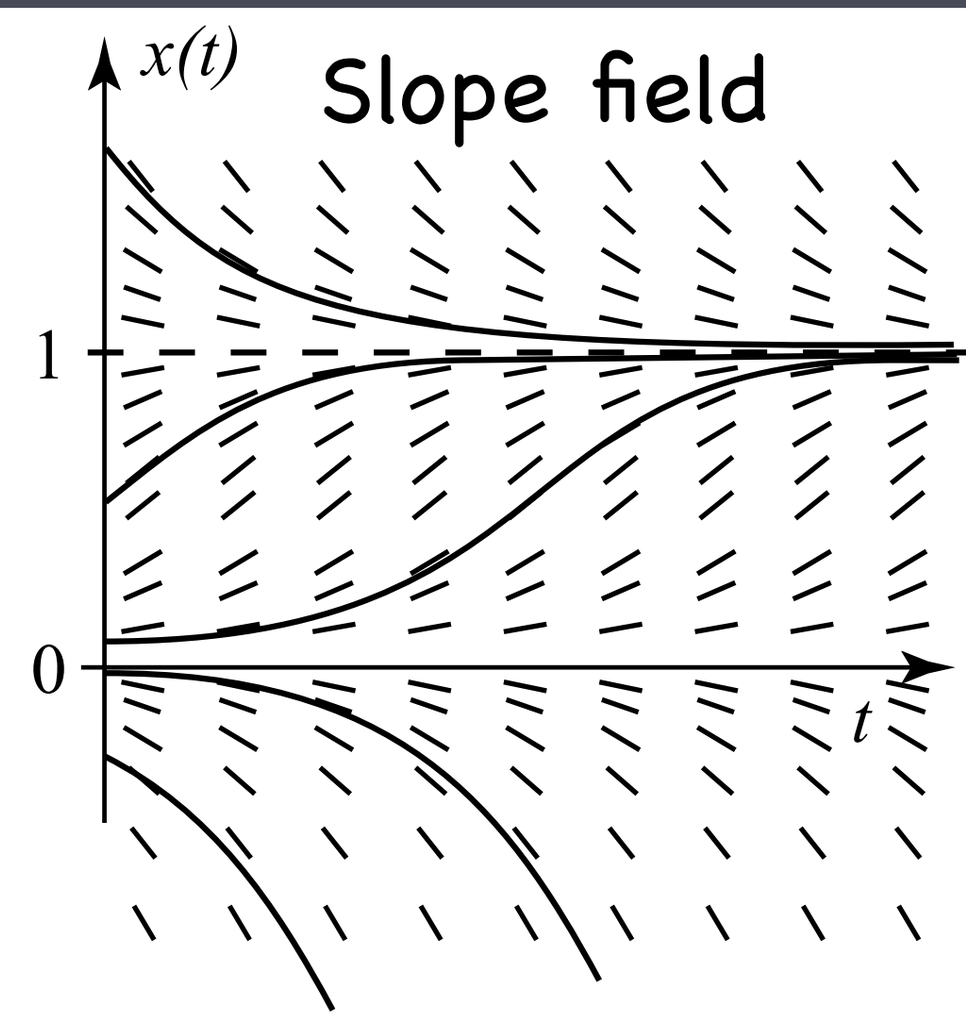


Velocity versus position

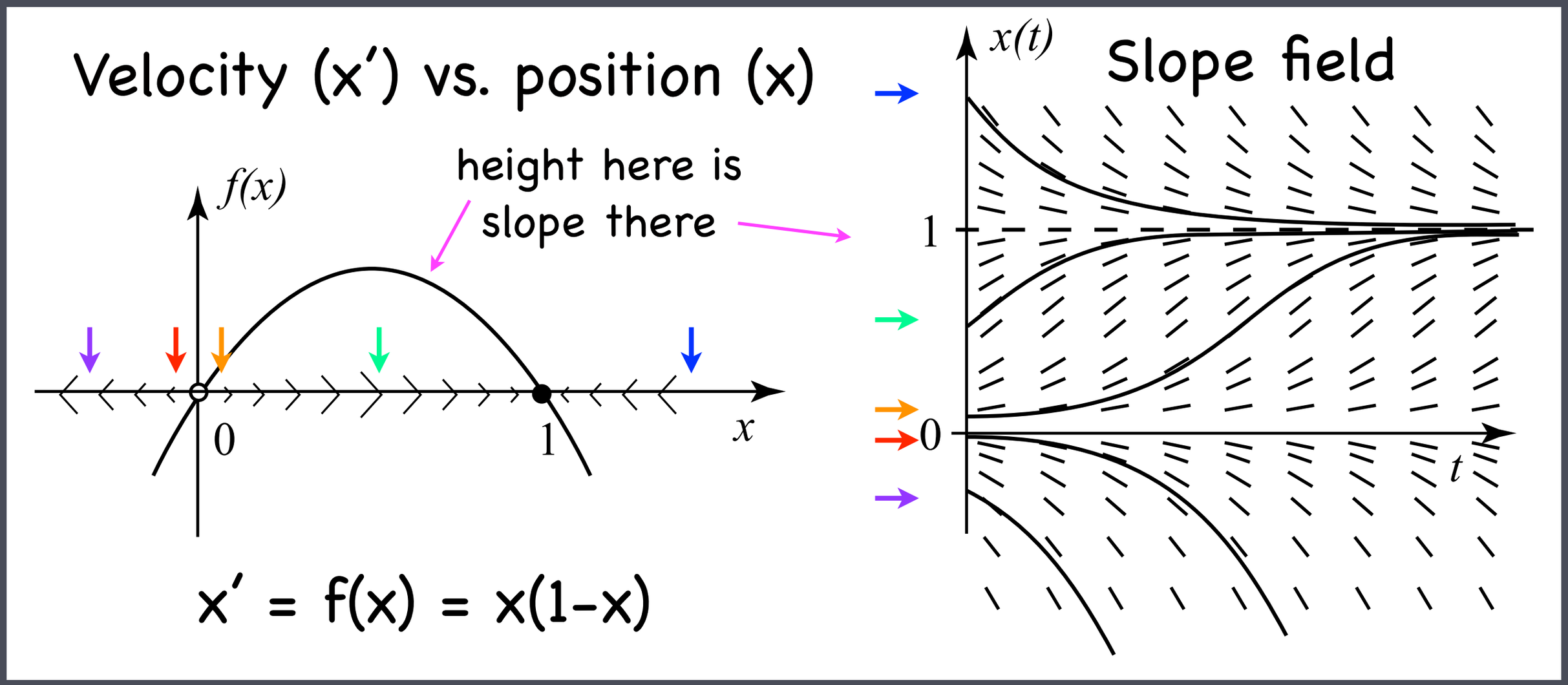
Velocity (x′) vs. position (x)



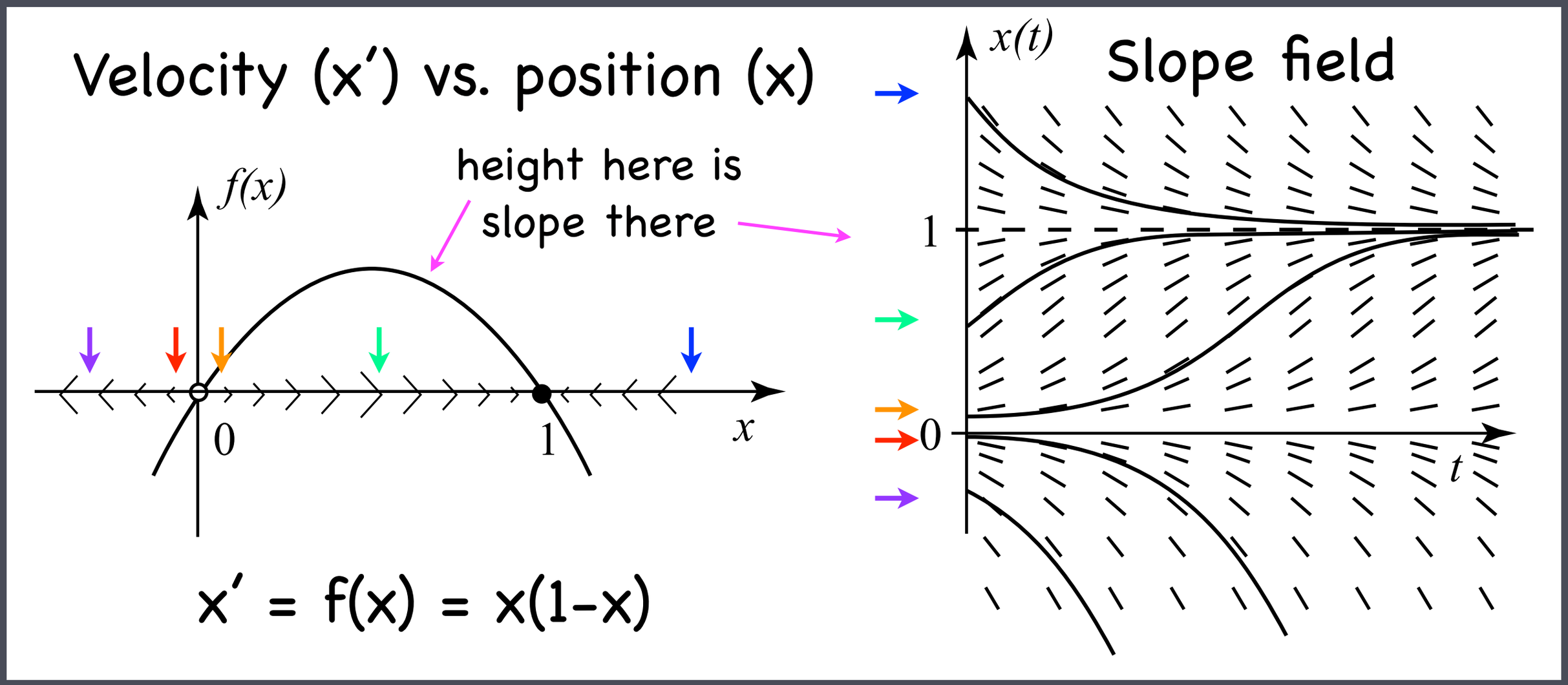
$x' = f(x) = x(1-x)$



Velocity versus position



Velocity versus position

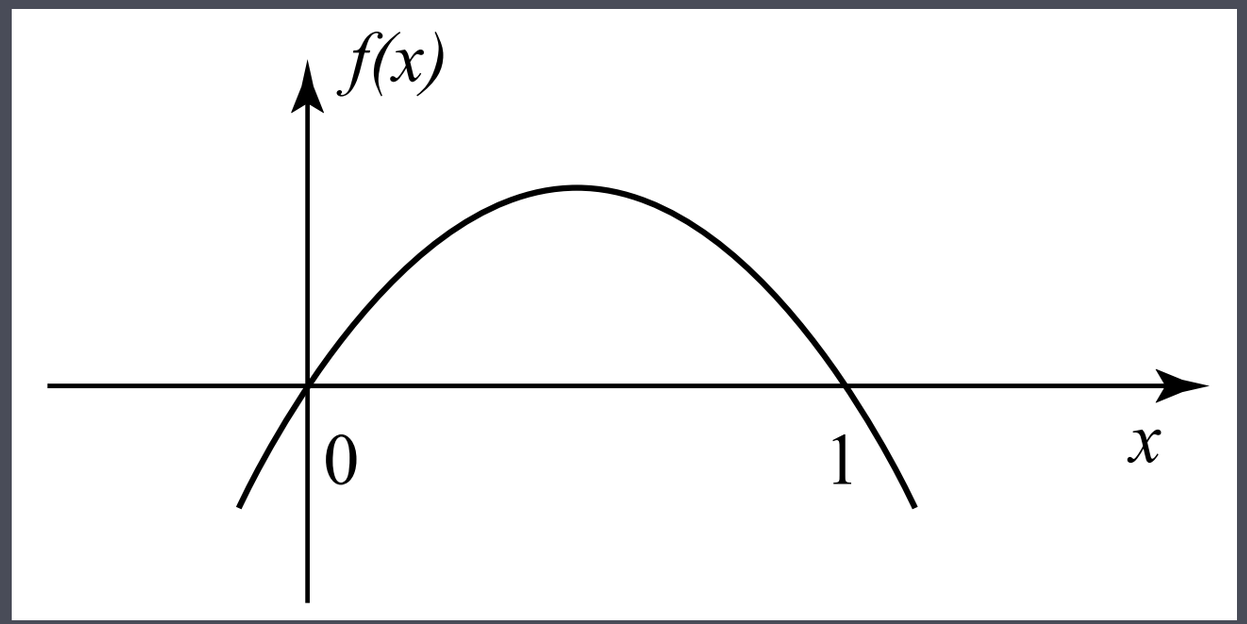


Stable steady state- all nearby solutions approach

Unstable steady state - not stable

Determine stability

$$x' = x(1-x)$$



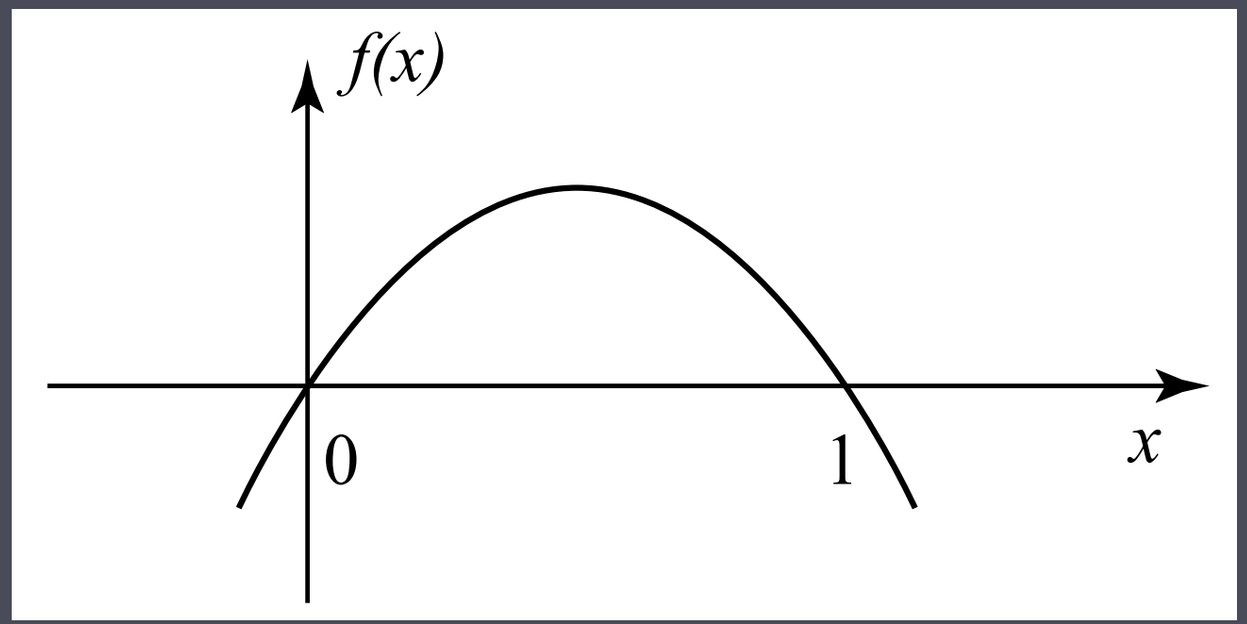
- If you start at x(0) = -0.01, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



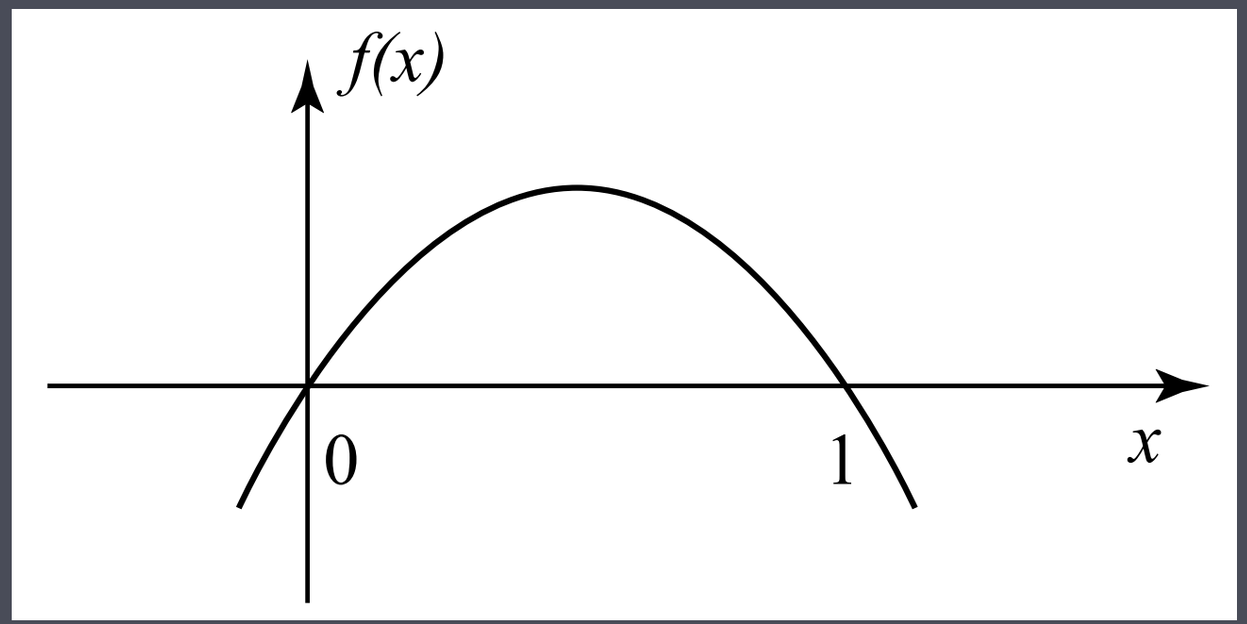
- If you start at x(0) = 0.01, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



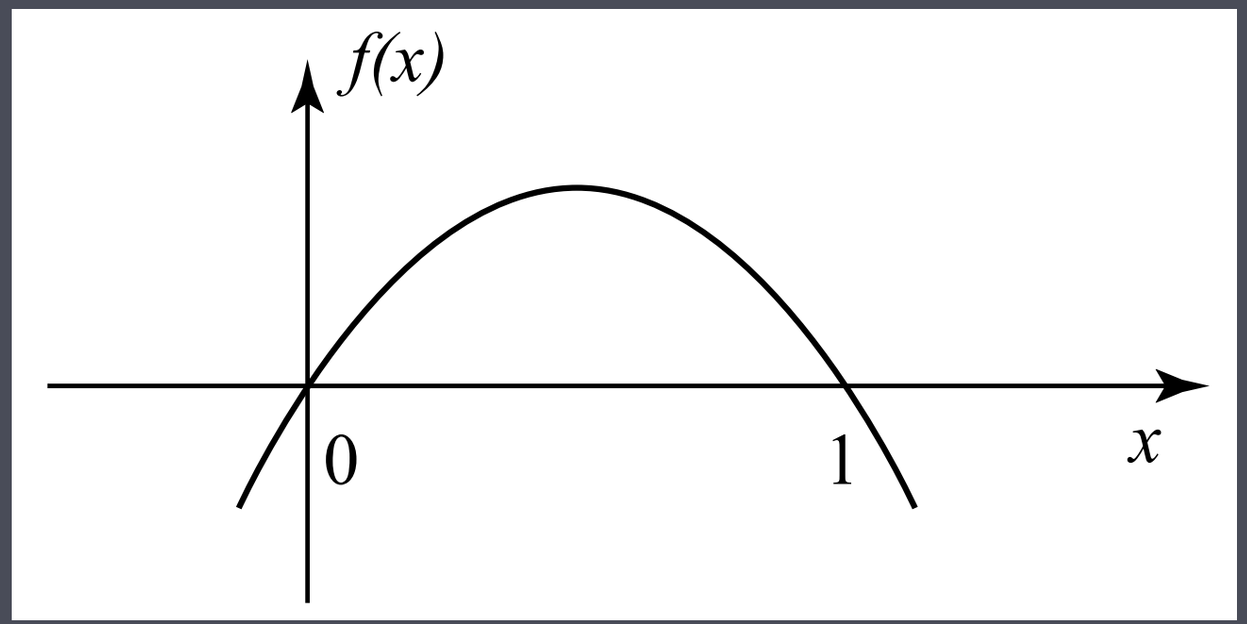
- If you start at x(0) = 0.99, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



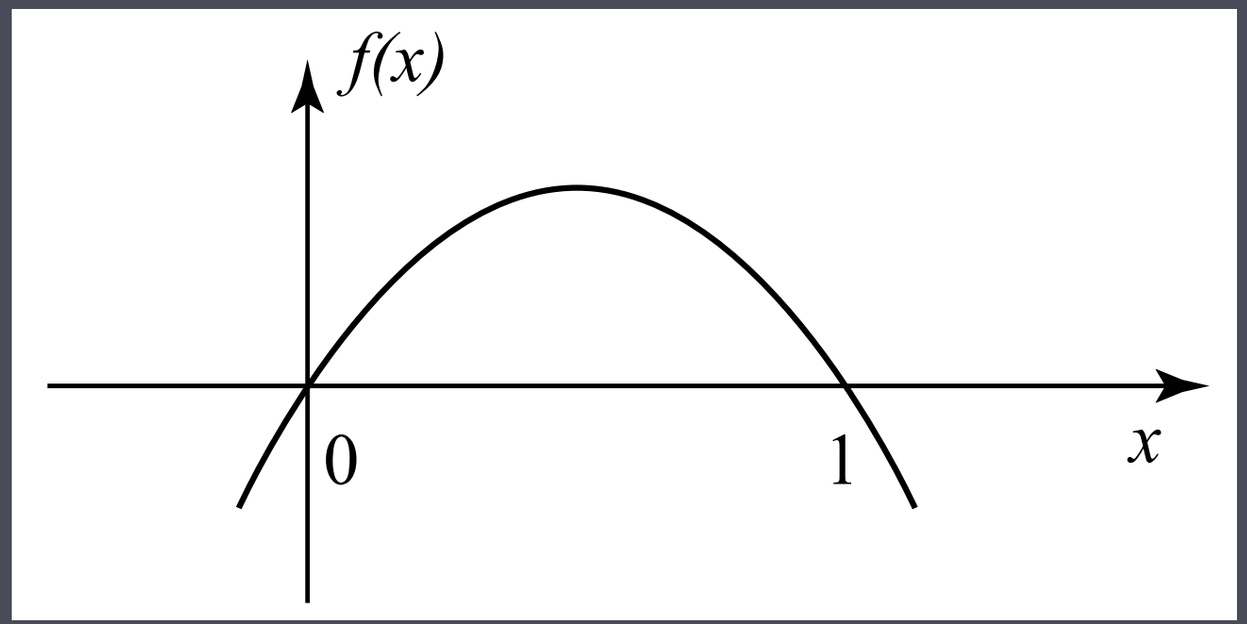
- If you start at x(0) = 1.01, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



(A) Both x(t)=0 and x(t)=1 are stable steady states.

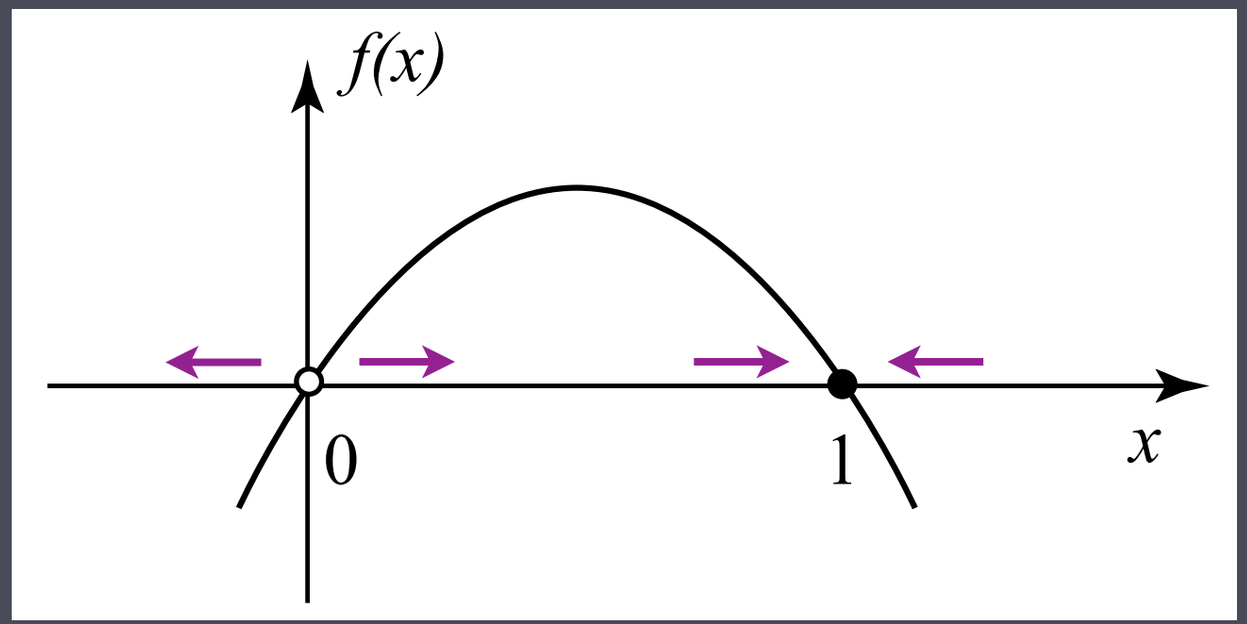
(B) x(t)=0 is stable and x(t)=1 is unstable.

(C) x(t)=0 is unstable and x(t)=1 is stable.

(D) Both x(t)=0 and x(t)=1 are unstable steady states.

Determine stability

$$x' = x(1-x)$$



(A) Both x(t)=0 and x(t)=1 are stable steady states.

(B) x(t)=0 is stable and x(t)=1 is unstable.

(C) x(t)=0 is unstable and x(t)=1 is stable.

(D) Both x(t)=0 and x(t)=1 are unstable steady states.

Stable - solid dot. Unstable - empty dot.