Today

- Related rates problem continued.
- Implicit differentiation
    - Geometric example (tangent line to circle)
    - Power rule for fractional powers

Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.

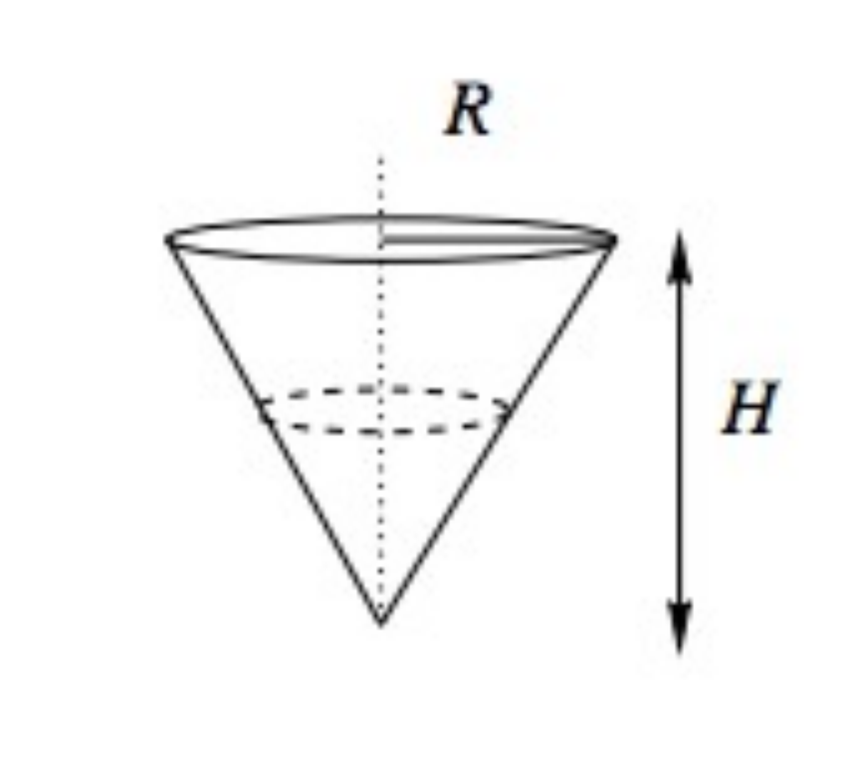
Which is the relevant equation relating the quantities (not rates of change yet)?

(A) $V = 1/3\ \pi\ R^2 H$

(B) $V = 1/3\ \pi\ (R^2/H^2)\ h$

(C) $V = 1/3\ \pi\ (R^2/H^2)\ h^3$

(D) $V = 1/3\ \pi\ r^2 h$



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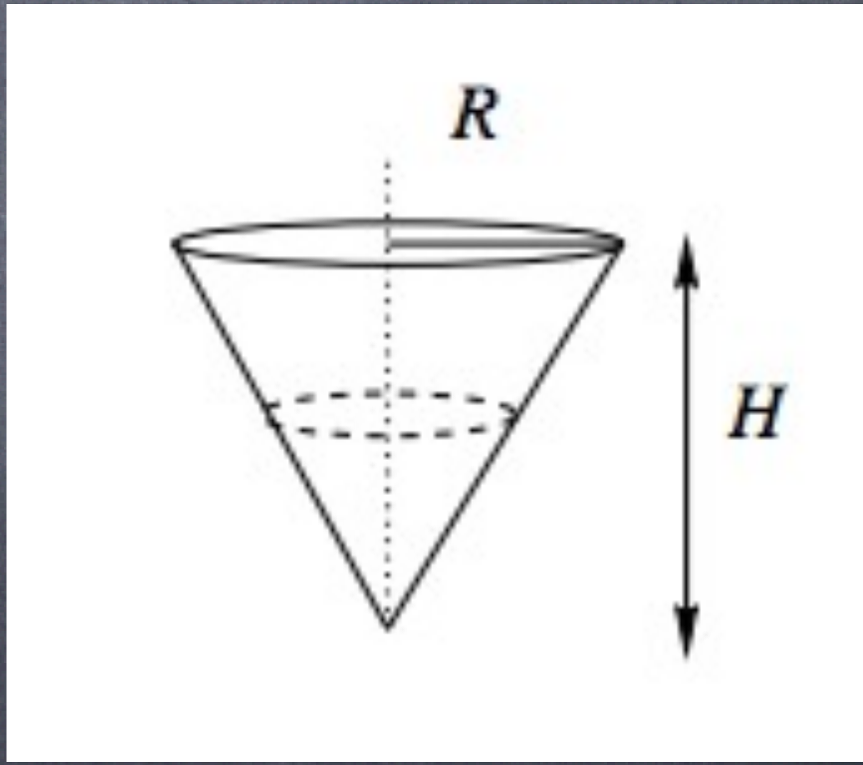
Which is the relevant equation relating the rates of change?

(A) $-k = 1/3\ \pi\ (R^2/H^2)\ h'$

(B) $V' = \pi\ (R^2/H^2)\ h^2\ k$

(C) $-k = \pi\ (R^2/H^2)\ h^2\ h'$

(D) $V' = 1/3\ \pi\ (2rr'\ h + r^2\ h')$



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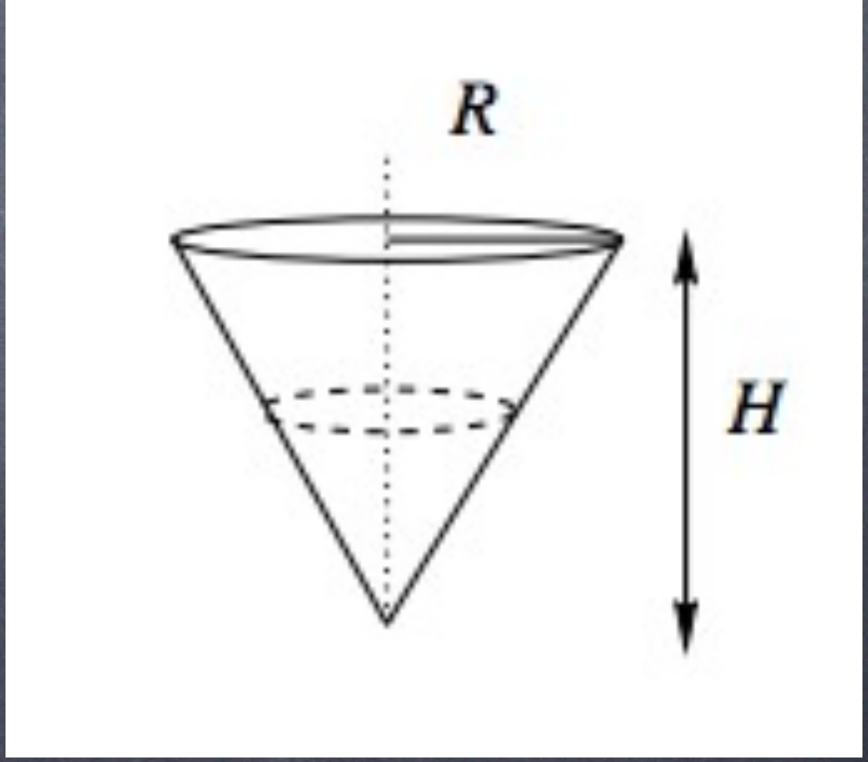

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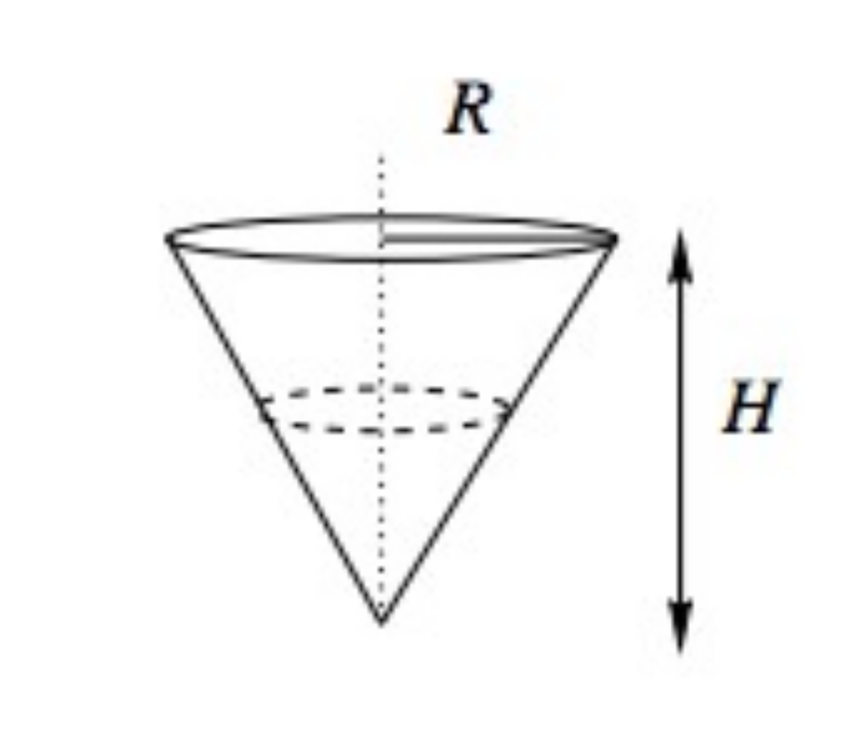

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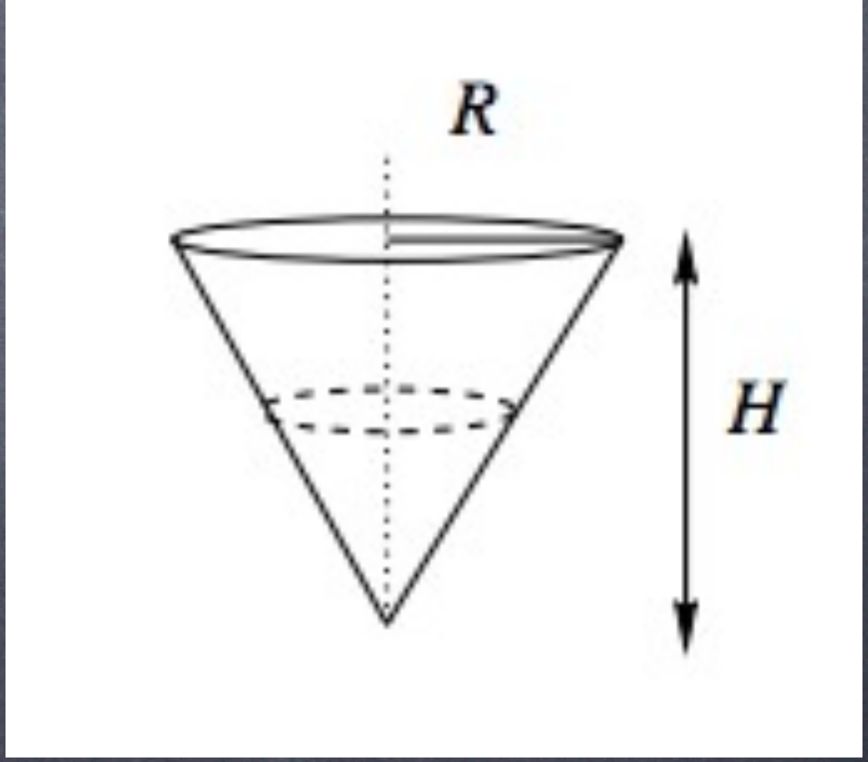

Procedure

- Establish expectation based on sketch or otherwise.
- Find equation relating Q_1 and Q_2.
- Take derivatives on both sides.
- Finally, plug in specific values.
- Reality check - compare answer against expectation.

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- Let $y=y(x)$ and take "implicit derivative" of e.g. $x^2+y(x)^2=25$ ------>

Find the tangent line to the curve defined by $x^2+y^2=25$ at (3,-4).

What can you predict about the answer without calculus?

(A) The slope of the tangent line will be positive.

(B) The slope of the tangent line will be negative.

(C) The slope of the tangent line will be 4/3.

(D) The slope of the tangent line will be 3/4.

(E) The slope of the tangent line will be -3/4.

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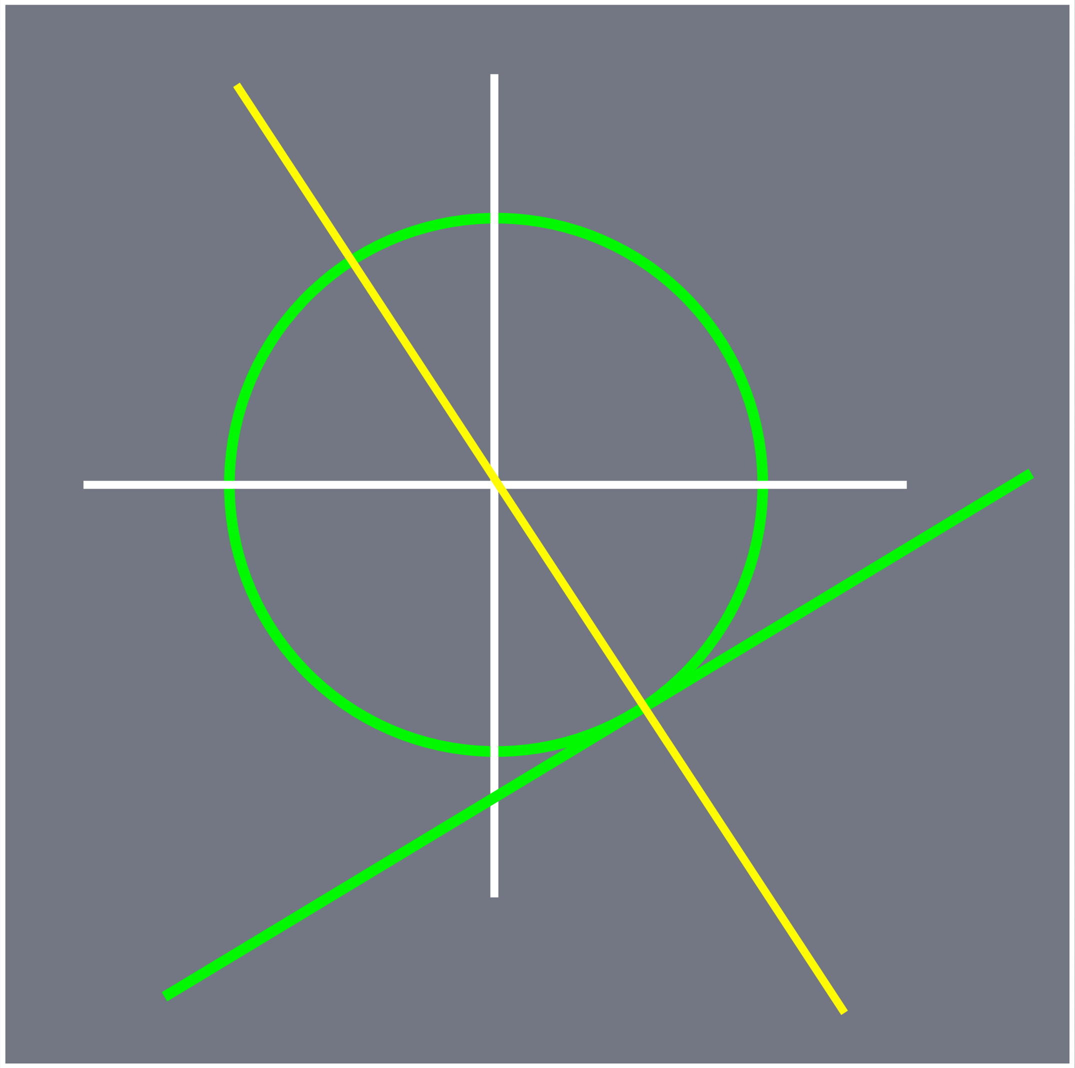

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The derivative of each side of this equation must also be equal. That means...

(A) $2xx' + 2yy' = 25.$

(B) $2xx' + 2y = 25.$

(C) $2xx' + 2yy' = 0.$

(D) $2x + 2yy' = 0.$

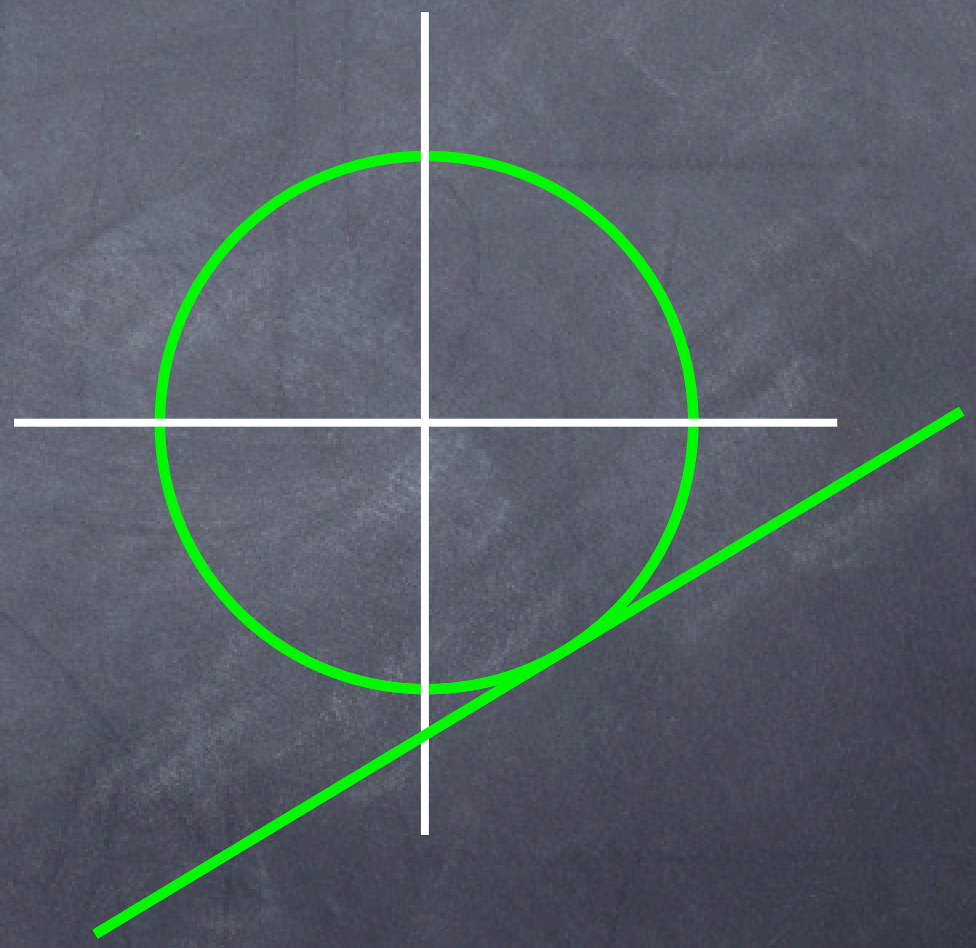
Find the tangent line to the curve defined by $x^2+y^2=25$ at (3,-4).

(A) $y' = -2x / 2y$

(B) $y = 3/4 (x-3) - 4$

(C) $2x + 2yy' = 0$

(D) $y' = 3/4$



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