# Today

- Midterm discussion
- Introduction to optimization
    - Goats
    - Wine for a wedding

# I thought the midterm was...

(A) ...easier than I expected.

(B) ...pretty much what I expected.

(C) ...harder than I expected.

# The hardest part of the midterm was...

(A) ...the multiple choice section.

(B) ...the short answer section.

(C) ...long-answer #1 (find $f'(x)$ using the def.).

(D) ...long-answer #2 (sketch $f(x)=x^4-x^2$).

(E) ...long-answer #3 (tangent line).

# The most useful thing I did to study was...

(A) ...doing/reviewing WeBWorK assignments.

(B) ...doing/reviewing OSH.

(C) ...doing the MT review problem set.

(D) ...reading the course notes.

(E) ...reviewing the lecture slides.

# I expect I'll get...

(A) A

(B) B

(C) C

(D) D

(E) F

# Optimization

- Given a scenario involving a choice of some number, use calculus to find the best value.
  - Translate scenario into a mathematical problem.
  - Solve the problem.
  - Translate back (make sure it makes sense).

I have 10 meters of fence. I want the biggest enclosure possible for my goat. I only know how to make rectangular enclosures.

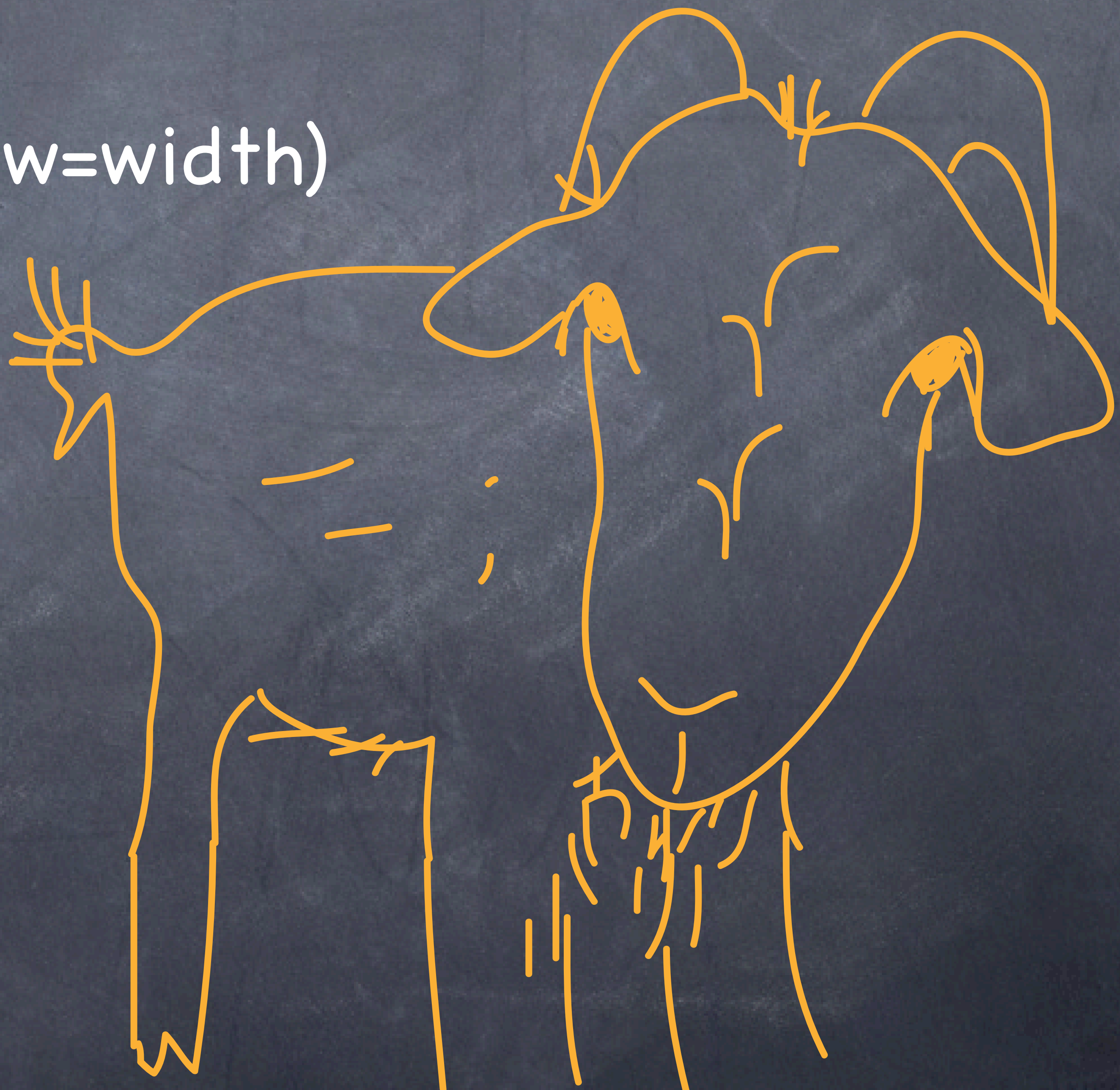
Find the max of

(A) $A(w) = lw$. ($l$=length, $w$=width)

(B) $A(w) = w(10-w)$

(C) $A(w) = w(5-2w)$

(D) $A(w) = w(5-w)$



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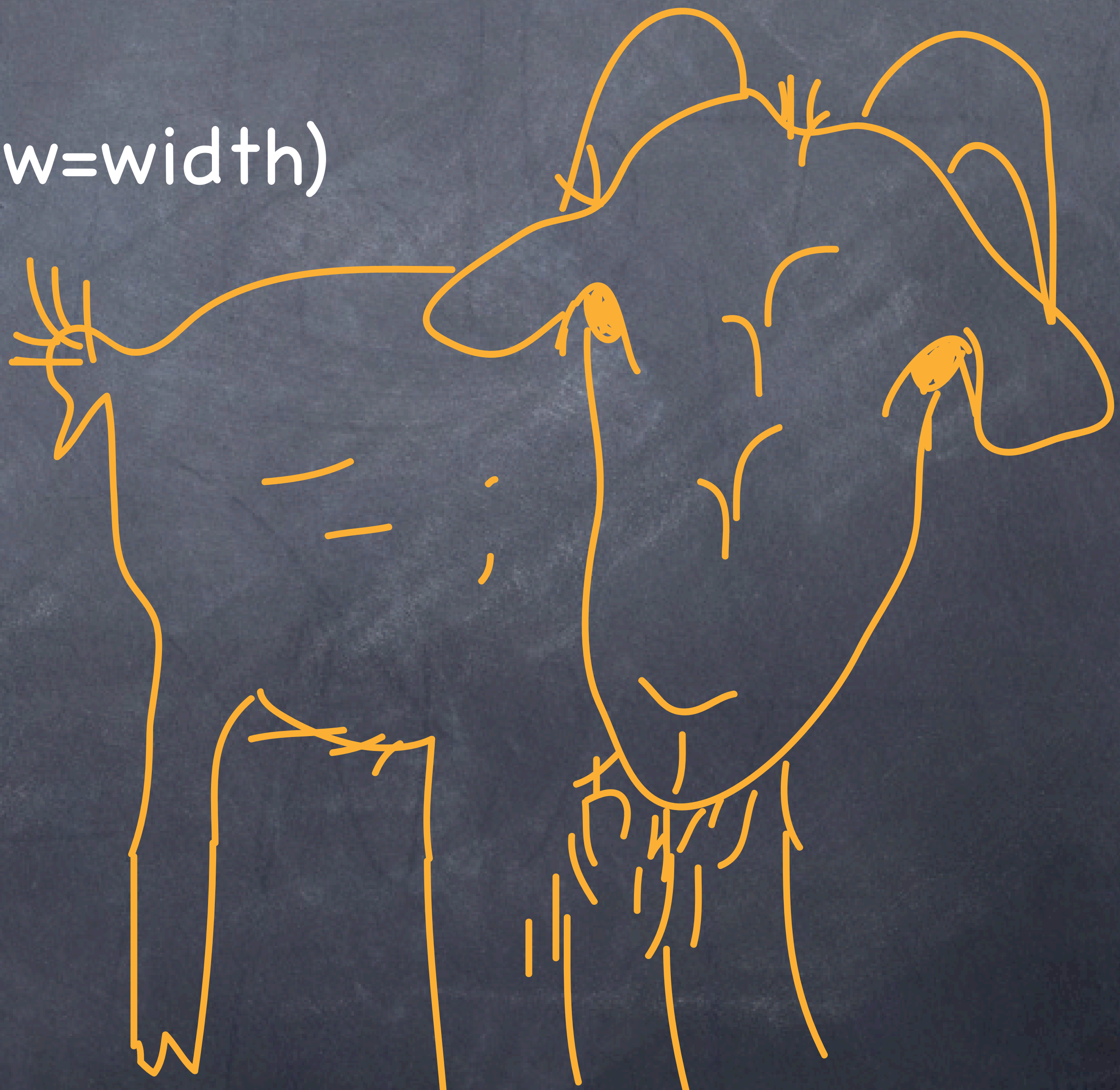


I have 10 meters of fence. I want the enclosure to be as small as possible but it can't be narrower than my goat (1/2 meter).

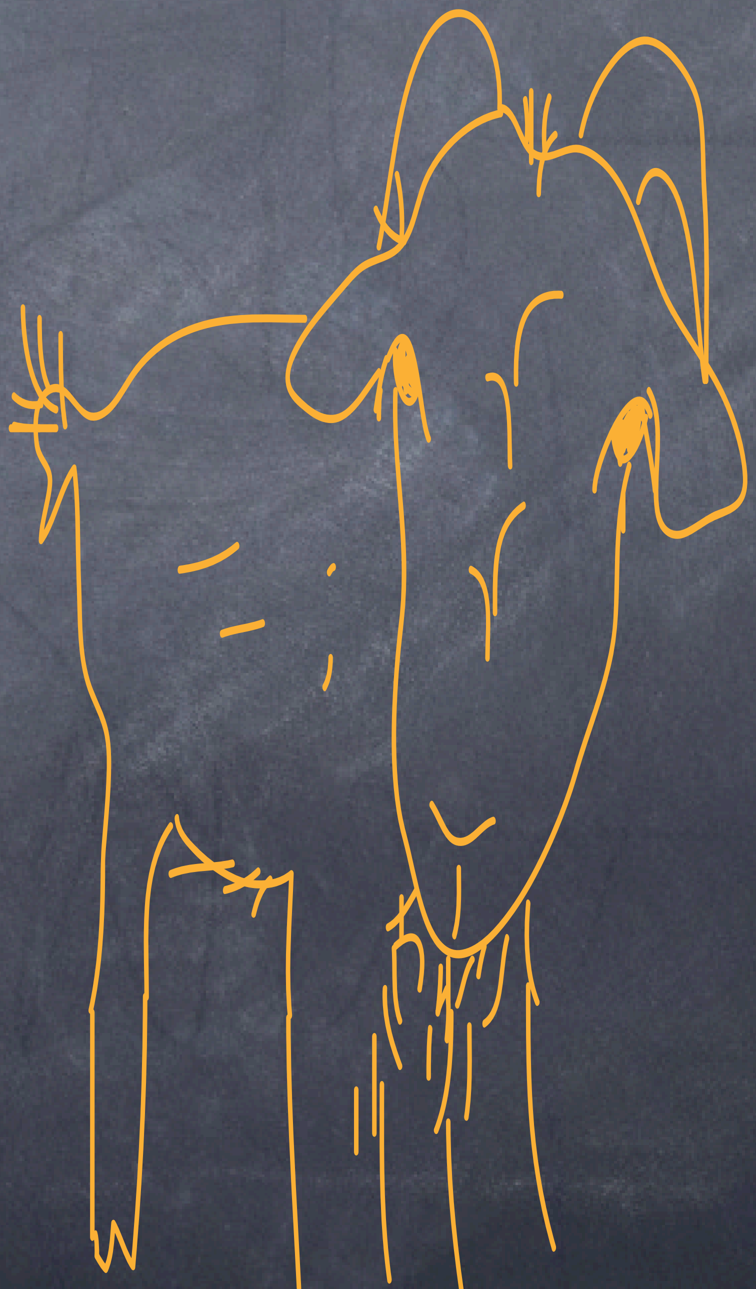
How long and how wide should I make the enclosure?

(A) l = 5/2 m, w = 5/2 m.

(B) l = 0 m, w = 5 m

(C) l = 1/2 m, w = 9/2 m

(D) l = 1/2 m, w = 19/2 m



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Find absolute min of $A(w)=w(5-w)$ on $[1/2, 9/2]$.

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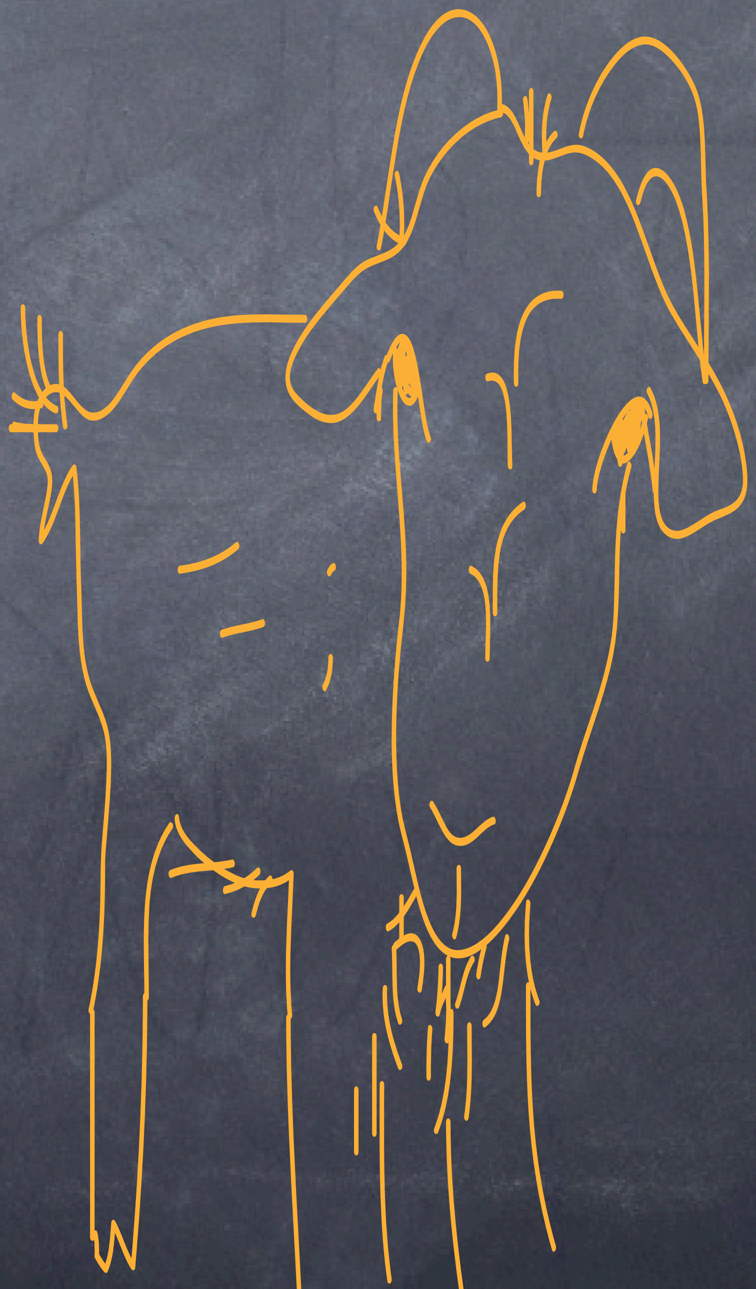

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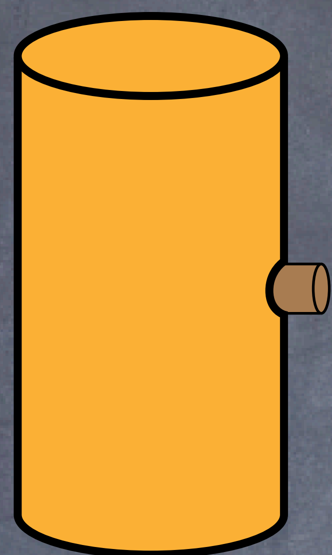
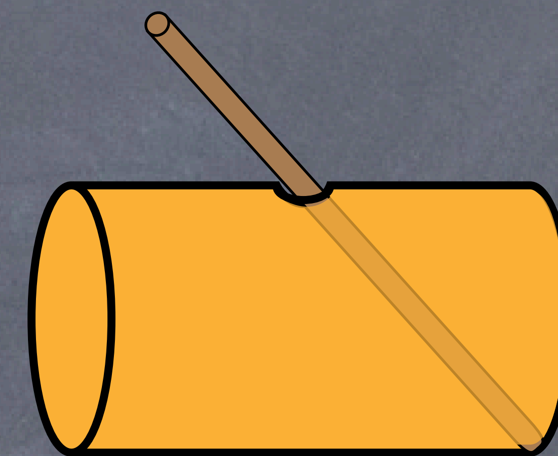
- There's an "objective function" (OF) that you want to maximize/minimize.
- The OF depends on more than one variable.
- There's a constraint relating the two variables.
- The constraint lets you simplify the OF to one variable.

$A(l,w)=lw$, $2l+2w=10$ $\to l=5-w$, $A(w)=(5-w)w$

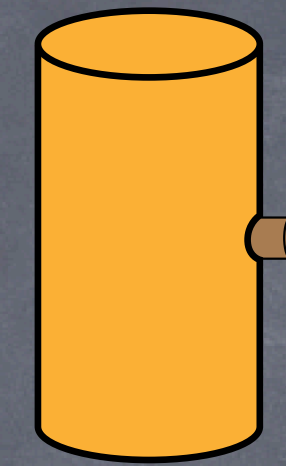
# Wine for Kepler's wedding

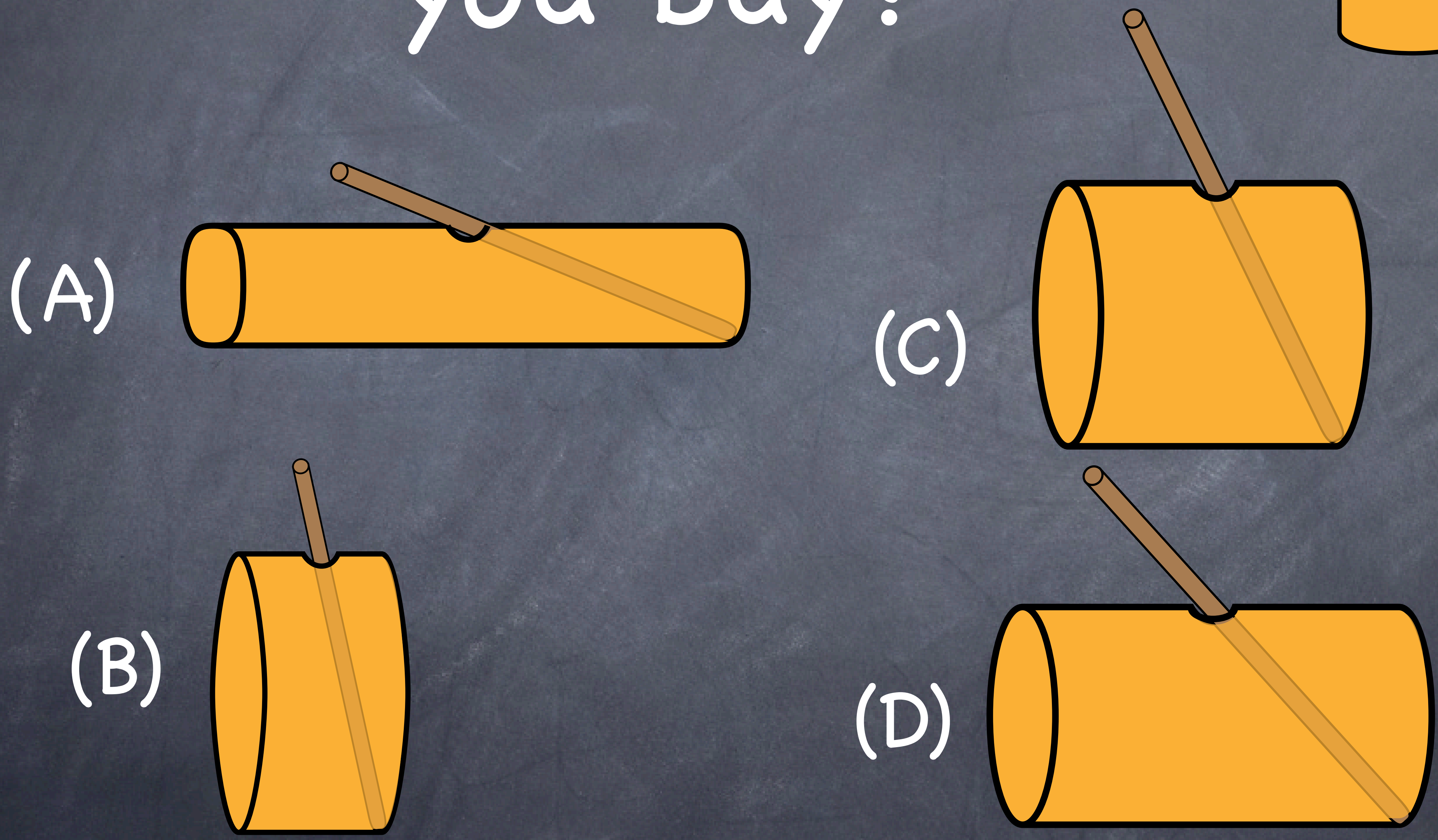




- Wine was sold by "the length of the submerged part of the rod"
- Same length of wet rod = same volume of wine?

# Which barrel would you buy?





# Kepler had enough $ for a rod-length $L_0$. How much wine can he get?

What do you expect to be the best option?

(A) Shortest possible barrel ($h=0$).

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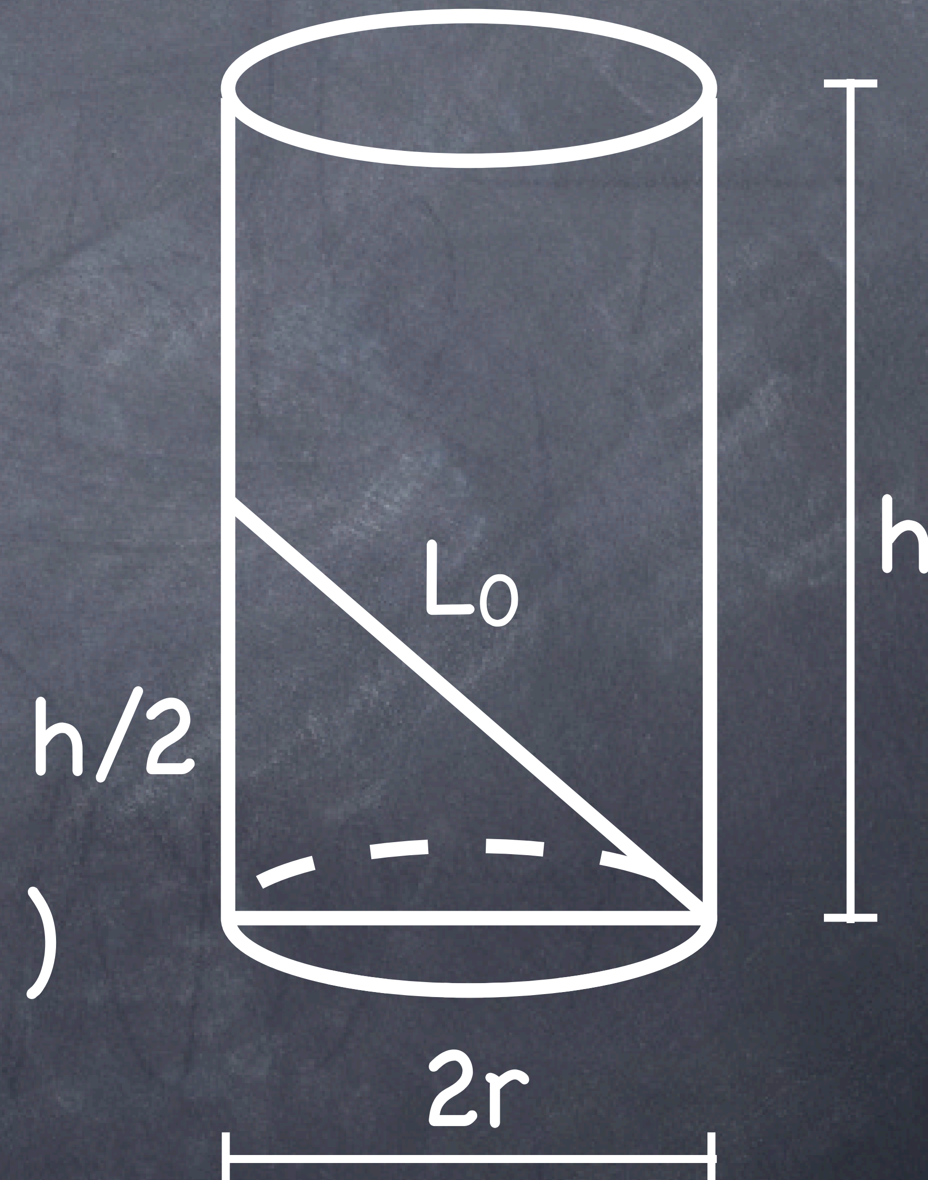
# Objective function? (to be maximized)

(A) $V = 2\pi r h$

(B) $r^2 = L_0^2/4 - h^2/16$

(C) $V = \pi r^2 h$

(D) $L_0 = \text{sqrt}( (2r)^2 + (h/2)^2 )$



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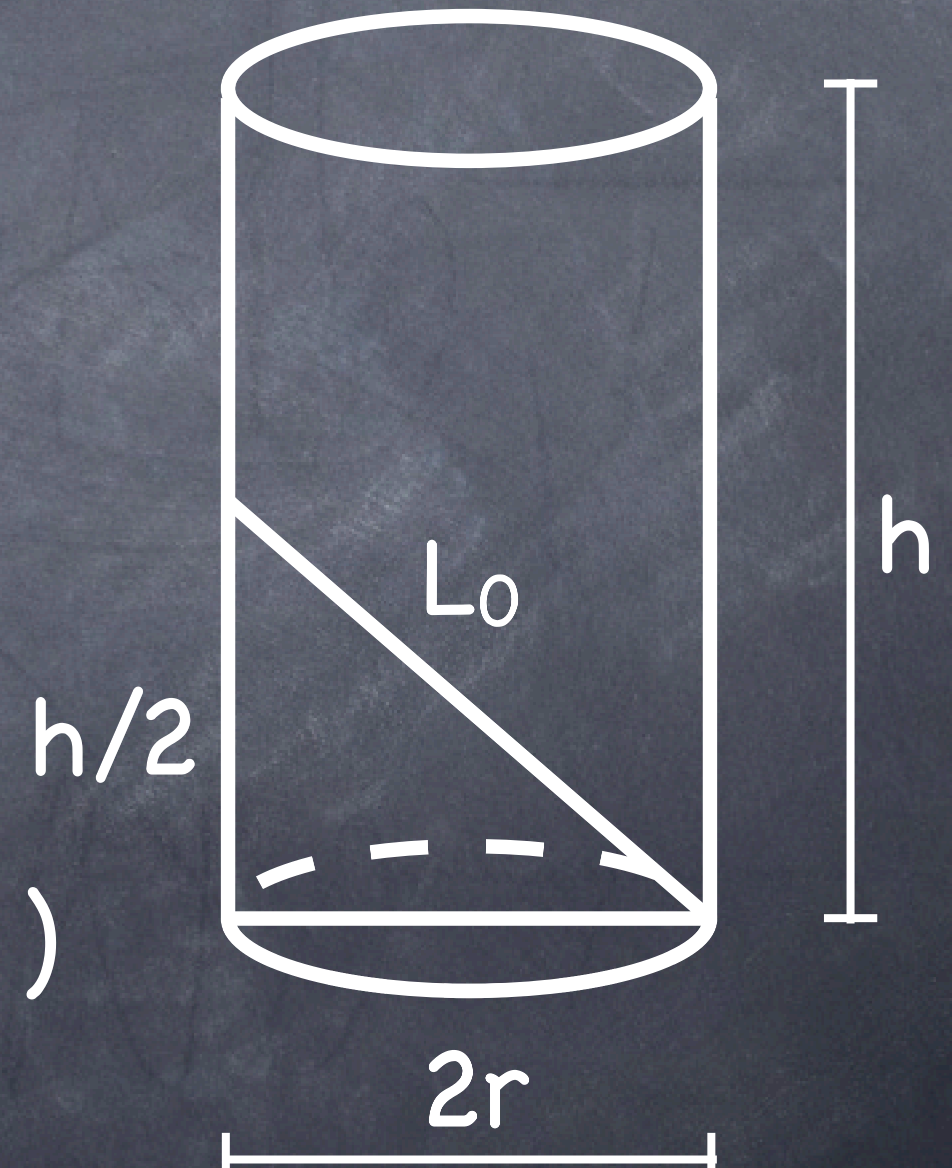

# Constraint?
# (used to simplify OF)

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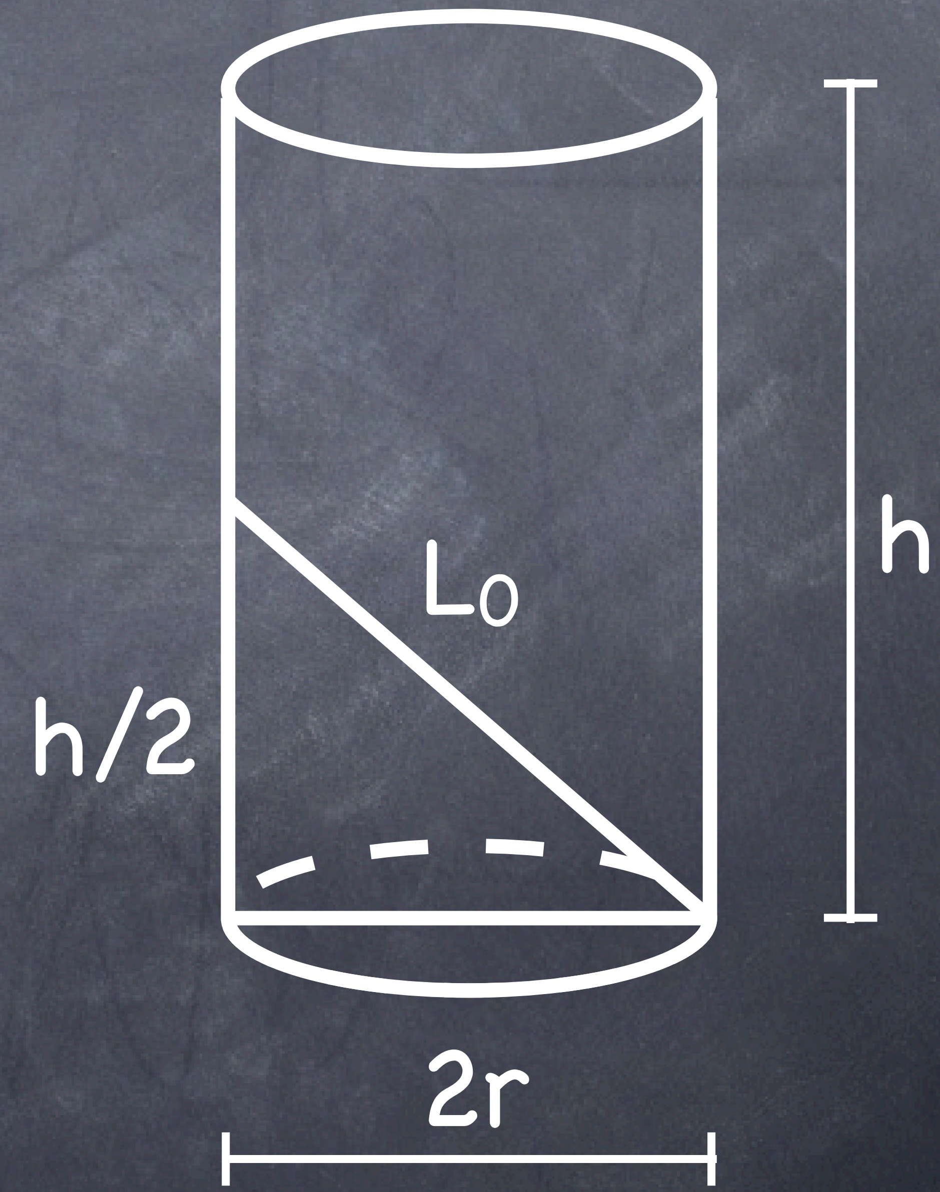

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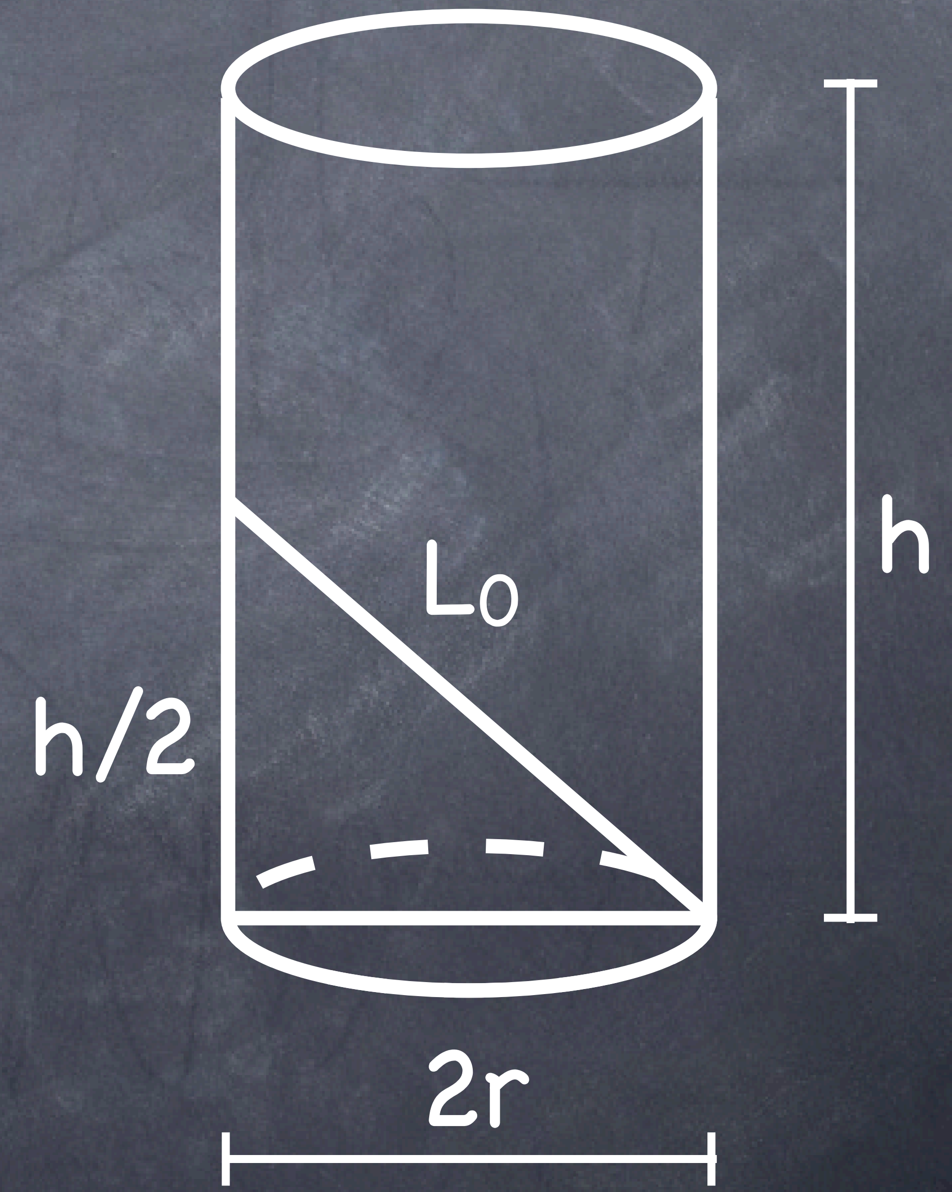

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Solve for:

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# $V = \pi h(4L_0^2 - h^2)/16$
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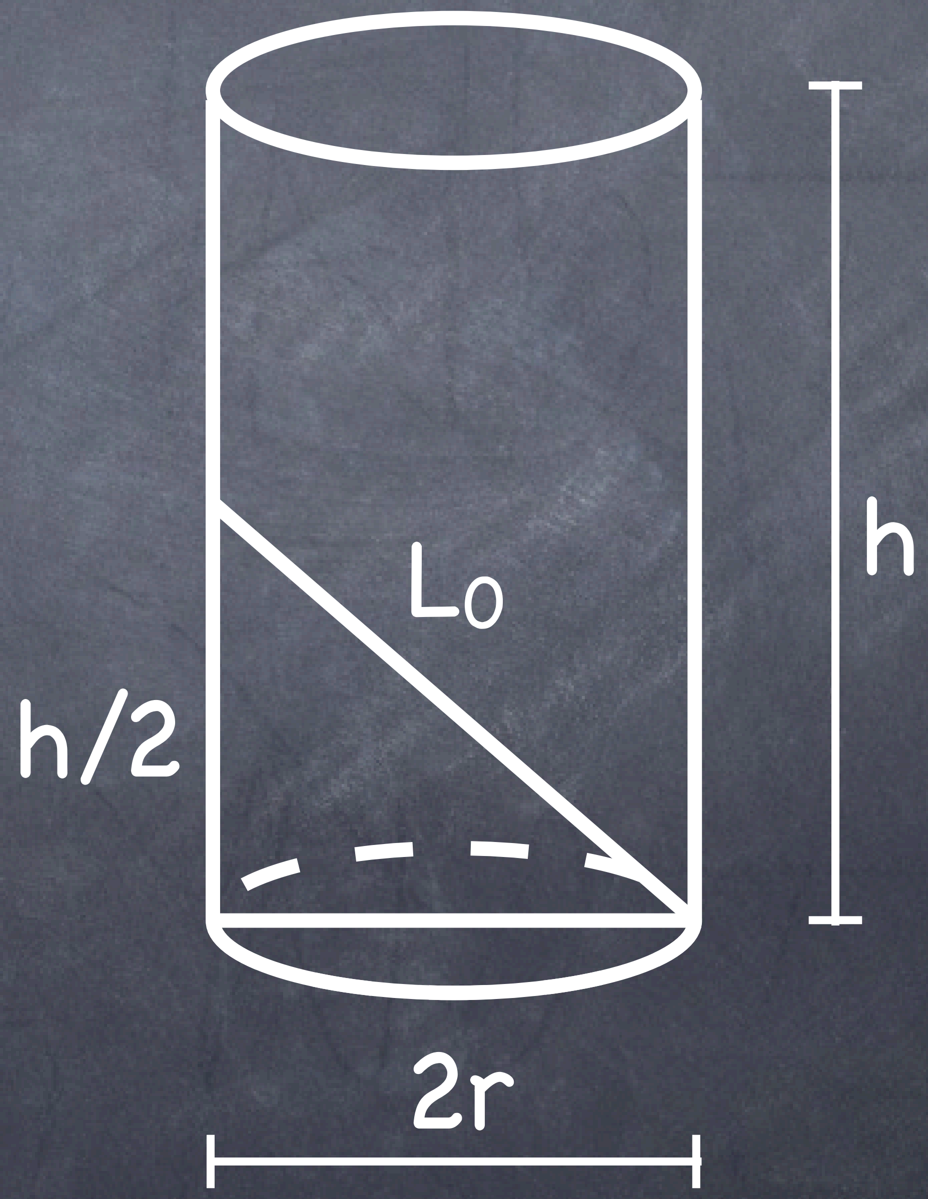

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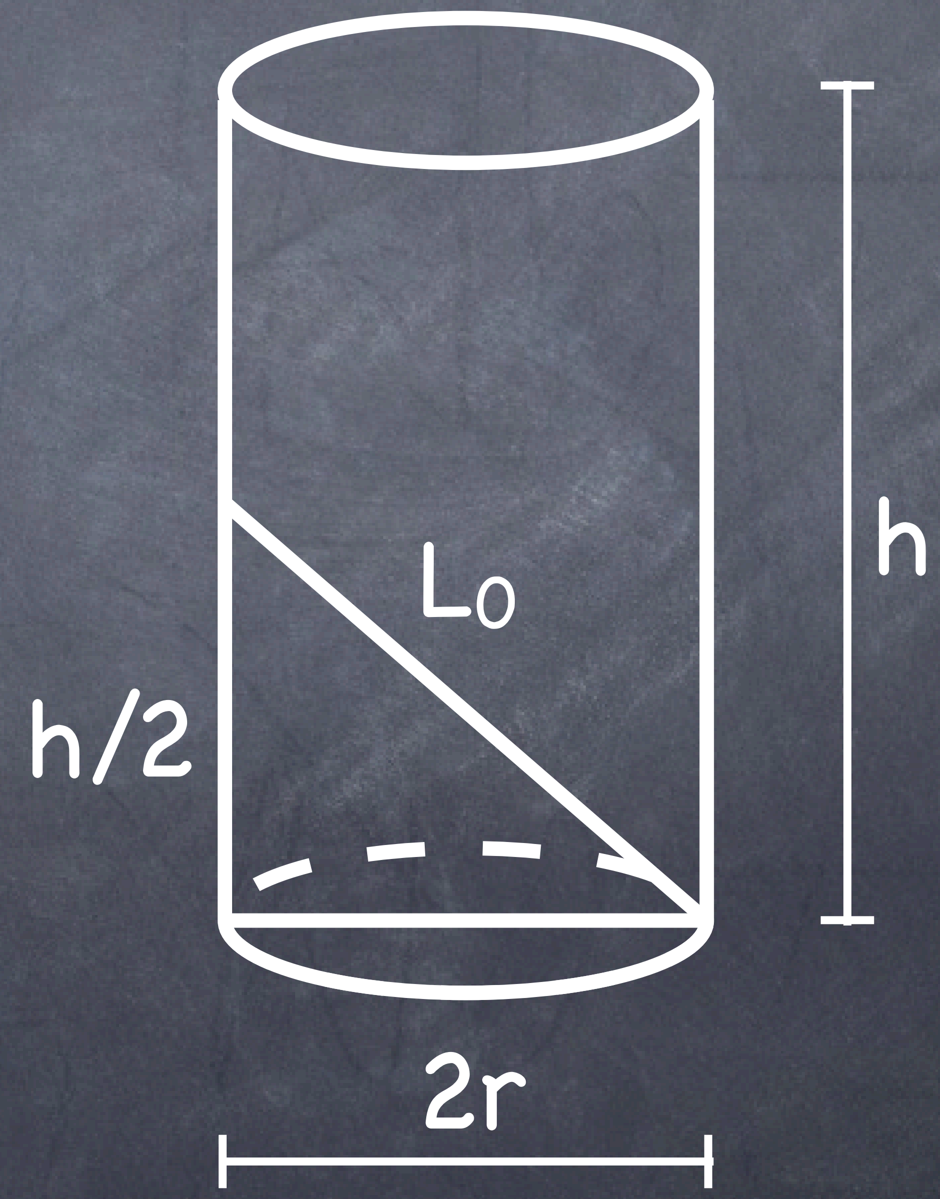


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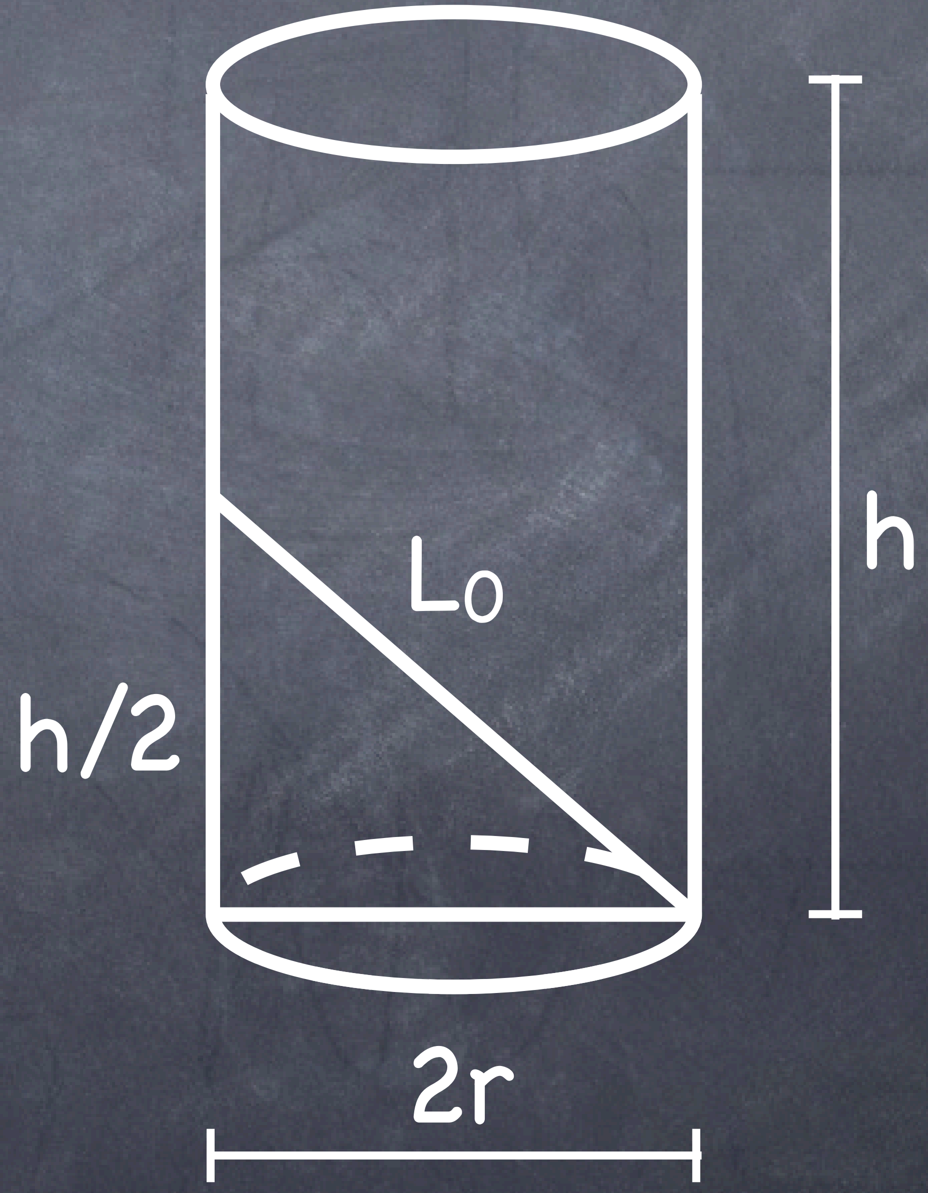

http://www.matematicasvisuales.com/english/html/history/kepler/doliometry.html

# Overall procedure

1. Determine the objective function.
2. Determine the constraint.
3. Establish an expectation (end-points or local extremum).
4. Solve constraint for one variable (make your life easy if possible).
5. Substitute it into the objective function.
6. Find the absolute extremum (check concavity!).