Lecture 10 (Sept. 27, 2013)

- Learning Goals: ① Find local extrema of a given function
  ② Sketch a given function

Example 1: Find the local extrema of $f(x) = x^4$

① Find critical points: $f'(x) = 4x^3 = 0 \Rightarrow x = 0$

② Use the test

#1 $f'(x) < 0$ for $x < 0$ ↘
$f'(x) > 0$ for $x > 0$ ↗ $\Rightarrow f(0) = 0$ is a local minimum

#2 $f''(x) = 12x^2$

* only need to check the sign of $f''(x)$ at the critical points

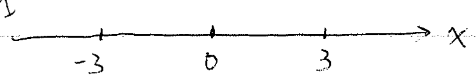
$f''(0) = 0$, inconclusive

Example 2: Find the local extrema of $f(x) = -x^5 + 15x^3$

① $f'(x) = -5x^4 + 45x^2 = -5x^2(x^2 - 9) = 0 \Rightarrow x = 0,\ x = -3,\ x = 3$

② Need to consider the sign of $f'(x)$ in all four intervals

#1



	$(-\infty, -3)$	-3	$(-3, 0)$	0	$(0, 3)$	3	$(3, +\infty)$
$f'(x)$	$-$	0	$+$	0	$+$	0	$-$
$f(x)$	↘	$f(-3)$	↗	0	↗	$f(3)$	↘

$f(-3)$ is a local minimum　$f(0)$ is not local extrema　$f(3)$ is a local maximum

#2 $f''(x) = -20x^3 + 90x = -10x(2x^2 - 9)$

$f''(-3) = 30 \cdot (18 - 9) > 0 \Rightarrow$ concave up $\cup$ $\Rightarrow f(-3)$ is a local minimum

$f''(0) = 0$, inconclusive, need to get back to first derivative test

$f''(3) = -30 \cdot (18 - 9) < 0 \Rightarrow$ concave down $\cap$ $\Rightarrow f(3)$ is a local maximum

Notice: different statement "$f(x)$ has local extrema at $x = ?$"
"find the local maximum/minimum $f(x) = ?$"

• Sketch a given function $f(x)$

Procedure: ① domain, vertical asymptotes (usually where $f(x)$ DNE)

② behaviour of $f(x)$ when $x \to \infty$; horizontal asymptotes

③ first derivative: critical points, local extrema; decreasing/increasing

④ second derivative: inflection points, concavity

⑤ Other properties: symmetry, x, y-intercepts

⑥ Sketch: mark all the points we know

mark the asymptotes (make sure the curve never crosses those lines)

connect the points smoothly, considering increasing/decreasing, concavity ...

Example 3: Sketch $f(x) = -2x^3 + 6x^2 - 3$

① domain: $(-\infty, +\infty)$

② $x \to +\infty$, $f(x) \to -\infty$; $x \to -\infty$, $f(x) \to +\infty$

③ $f'(x) = -6x^2 + 12x = -6x \cdot (x-2)$ $\Rightarrow$ critical points $x=0$ and $x=2$

	$(-\infty, 0)$	0	$(0, 2)$	2	$(2, +\infty)$
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	$\searrow$	-3	$\nearrow$	5	$\searrow$

$f(0)$ is a local minimum $f(2)$ is a local maximum

④ $f''(x) = -12x + 12 = -12 \cdot (x-1)$ $\Rightarrow$ possible inflection point $x=1$

	$(-\infty, 1)$	1	$(1, +\infty)$
$f''(x)$	$+$	0	$-$
$f(x)$	$\cup$	$\vert$	$\cap$

$(1, f(1))$ is an inflection point

⑤ Skip

⑥