Today

- Quiz 3
- Wrap up IV drug delivery and $y'=ay+b$

A drug delivered by IV accumulates at a constant rate k_{IV}. The body metabolizes the drug proportional to the amount of the drug.

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

- What is the dose actually received by the patient?

$$\lim_{t \to \infty} d(t) = k_{IV}/k_m$$

- When does the patient start getting that dose?

By $t=1/k_m$ (characteristic time), the patient is getting "close" to the target dose.

Stable steady state case

- Any problem of the form $y' = c - dy$ $(d > 0)$ with IC $y(0)=y_0$ has solution

$$y(t) = c/d + (y_0 - c/d)\, e^{-dt}$$

- If you are given an alleged solution, you can always check that it solves the IVP:
  - LHS: $y'(t)$ = (on the doc cam)
  - RHS: $c - dy$ = (on the doc cam)
  - $y(0) = c/d + (y_0 - c/d)\, e^0 = y_0$

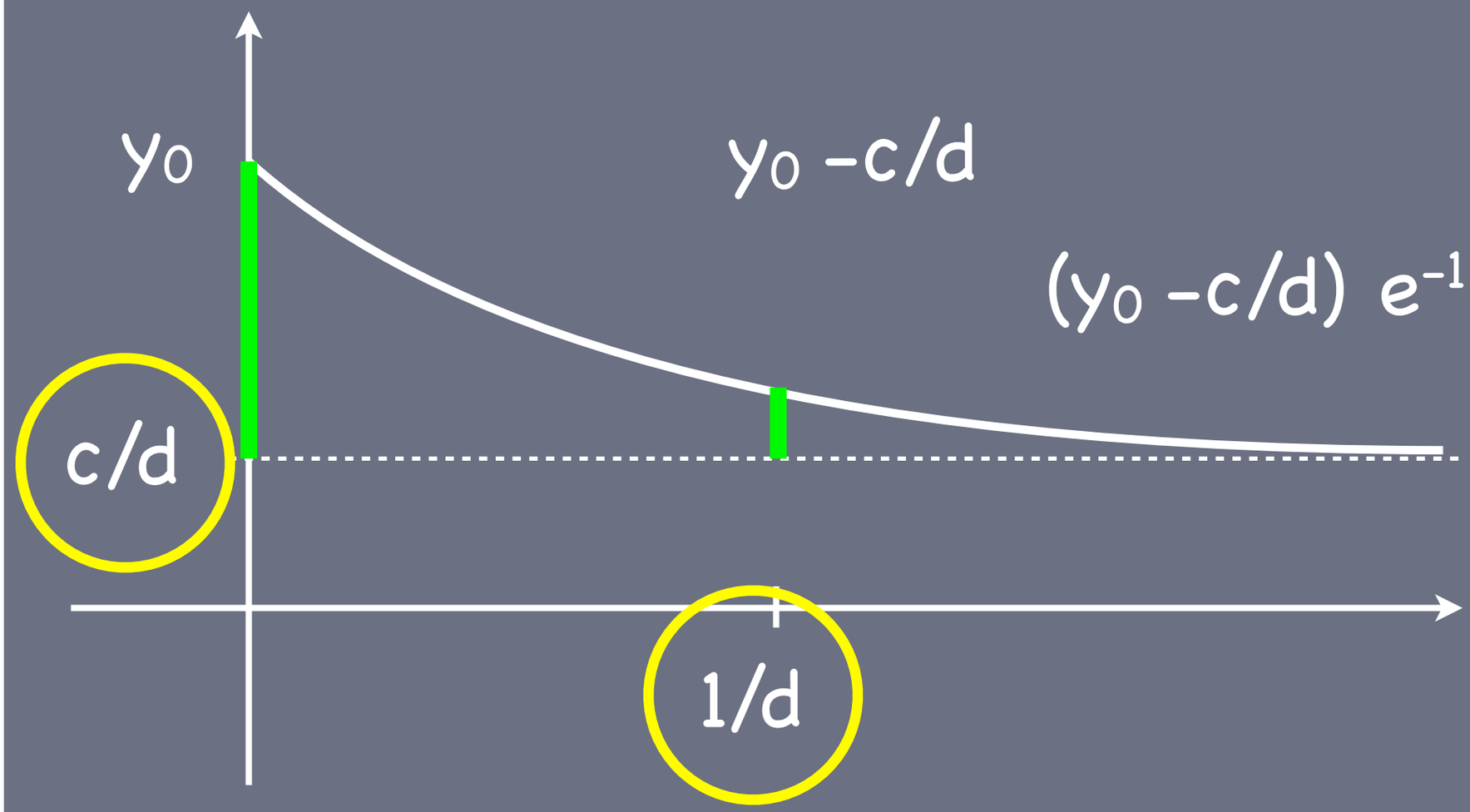
Stable steady state case

$$y(t) = c/d + (y_0 - c/d)\, e^{-dt}$$

- With d>0, as t--> ∞, y(t) --> c/d, a stable steady state.
- With d>0, the characteristic time is 1/d.

Stable steady state case

$$y(t) = c/d + (y_0 - c/d)\, e^{-dt}$$



When will it get there?

Where is y(t) going? To the steady state c/d.

Never but at t=1/d it will be 1/e of the way.

Look different but same same.

- Newton's Law of Cooling: $T'(t) = k(E-T(t))$

  $c = kE$, $d = k$.

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$

  $c = k_{IV}$, $d = k_m$

- Terminal velocity: $v'(t) = g - \delta v(t)$

  $c = g$, $d = \delta$

- General form, factored: $y'(t) = d\,(c/d - y)$.

1 / characteristic time

steady state

What do you need to know?

- Given a word description, write down an equation for the quantity q(t) described.
  - Ex. Blah is added at a constant rate and is removed proportional to how much is there...
  - Ex. Blah changes proportional to the difference between blah and fixed #.
- Use the shift substitution to get $z'=az$ and state that $z(t) = C_0e^{at}$ solves it.
- Substitute back to find q(t).
- Determine C_0 using the initial condition.
- Answer questions about the resulting exponential q(t).