Today

- Quiz 3
- Wrap up IV drug delivery and $y' = ay + b$
A drug delivered by IV accumulates at a constant rate $k_{IV}$. The body metabolizes the drug proportional to the amount of the drug.

$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$  

- What is the dose actually received by the patient?

  $$\lim_{t \to \infty} d(t) = k_{IV}/k_m$$

- When does the patient start getting that dose?

  By $t=1/k_m$ (characteristic time), the patient is getting "close" to the target dose.
Stable steady state case

Any problem of the form \( y' = c - dy \) \((d > 0)\)
with IC \( y(0) = y_0 \) has solution

\[
y(t) = c/d + (y_0 - c/d) \ e^{-dt}
\]

If you are given an alleged solution, you can always check that it solves the IVP:

- LHS: \( y'(t) = \) (on the doc cam)
- RHS: \( c - dy = \) (on the doc cam)
- \( y(0) = c/d + (y_0 - c/d) \ e^0 = y_0 \)
Stable steady state case

\[ y(t) = \frac{c}{d} + (y_0 - \frac{c}{d}) e^{-dt} \]

- With \(d>0\), as \(t\to\infty\), \(y(t)\to\frac{c}{d}\), a stable steady state.

- With \(d>0\), the characteristic time is \(1/d\).
y(t) = \frac{c}{d} + (y_0 - \frac{c}{d}) e^{-dt}

Where is \( y(t) \) going? To the steady state \( \frac{c}{d} \).

Never but at \( t = \frac{1}{d} \) it will be \( \frac{1}{e} \) of the way.

When will it get there? If the steady state condition is met.

Stable steady state case
Look different but same same.

- Newton's Law of Cooling: $T'(t) = k(E - T(t))$
  
  $c = kE$, $d = k$.

- Drug delivery: $d'(t) = k_{IV} - k_m d(t)$
  
  $c = k_{IV}$, $d = k_m$

- Terminal velocity: $v'(t) = g - \delta v(t)$
  
  $c = g$, $d = \delta$

- General form, factored: $y'(t) = d \left( \frac{c}{d} - y \right)$.

1 / characteristic time

steady state
What do you need to know?

Given a word description, write down an equation for the quantity $q(t)$ described.

- Ex. Blah is added at a constant rate and is removed proportional to how much is there...

- Ex. Blah changes proportional to the difference between blah and fixed #.

Use the shift substitution to get $z' = az$ and state that $z(t) = C_0 e^{at}$ solves it.

Substitute back to find $q(t)$.

Determine $C_0$ using the initial condition.

Answer questions about the resulting exponential $q(t)$. 
