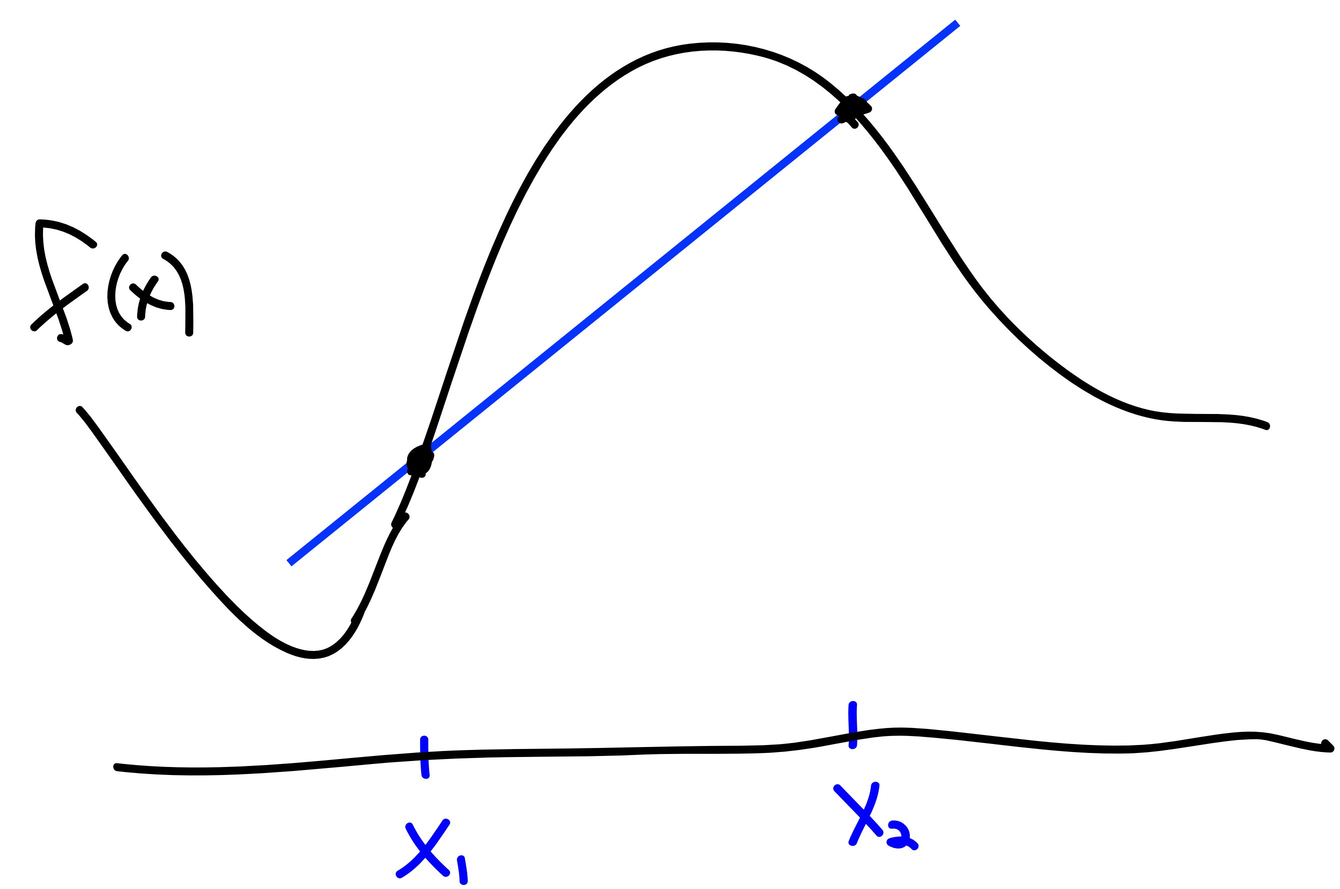
# Today...

- Demo WeBWorK tricks
- From secant line to tangent line.
- The Definition of the Derivative.
- More about limits

# What if you want the rate of change AT $x_1$?
## (instantaneous instead of average)



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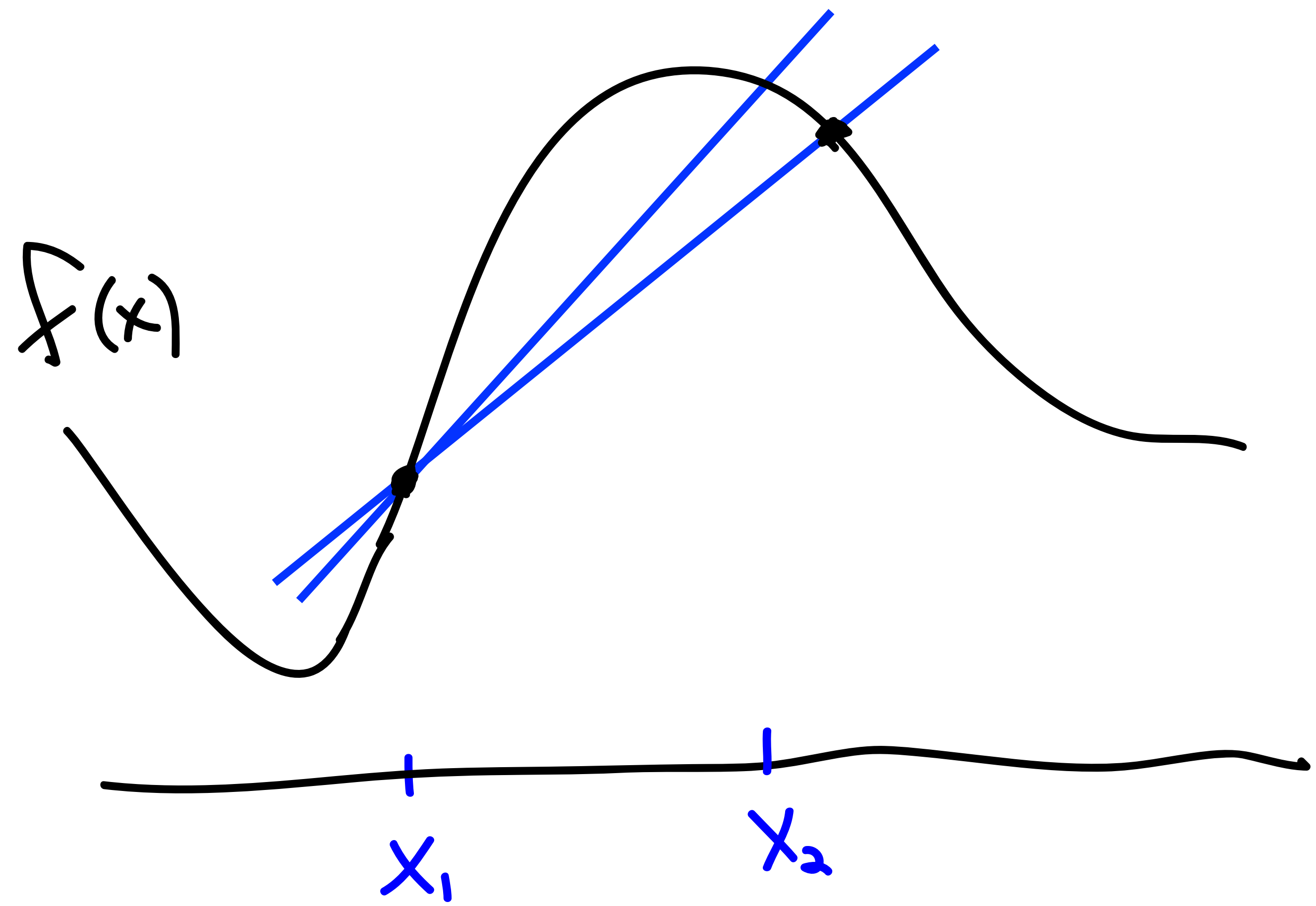

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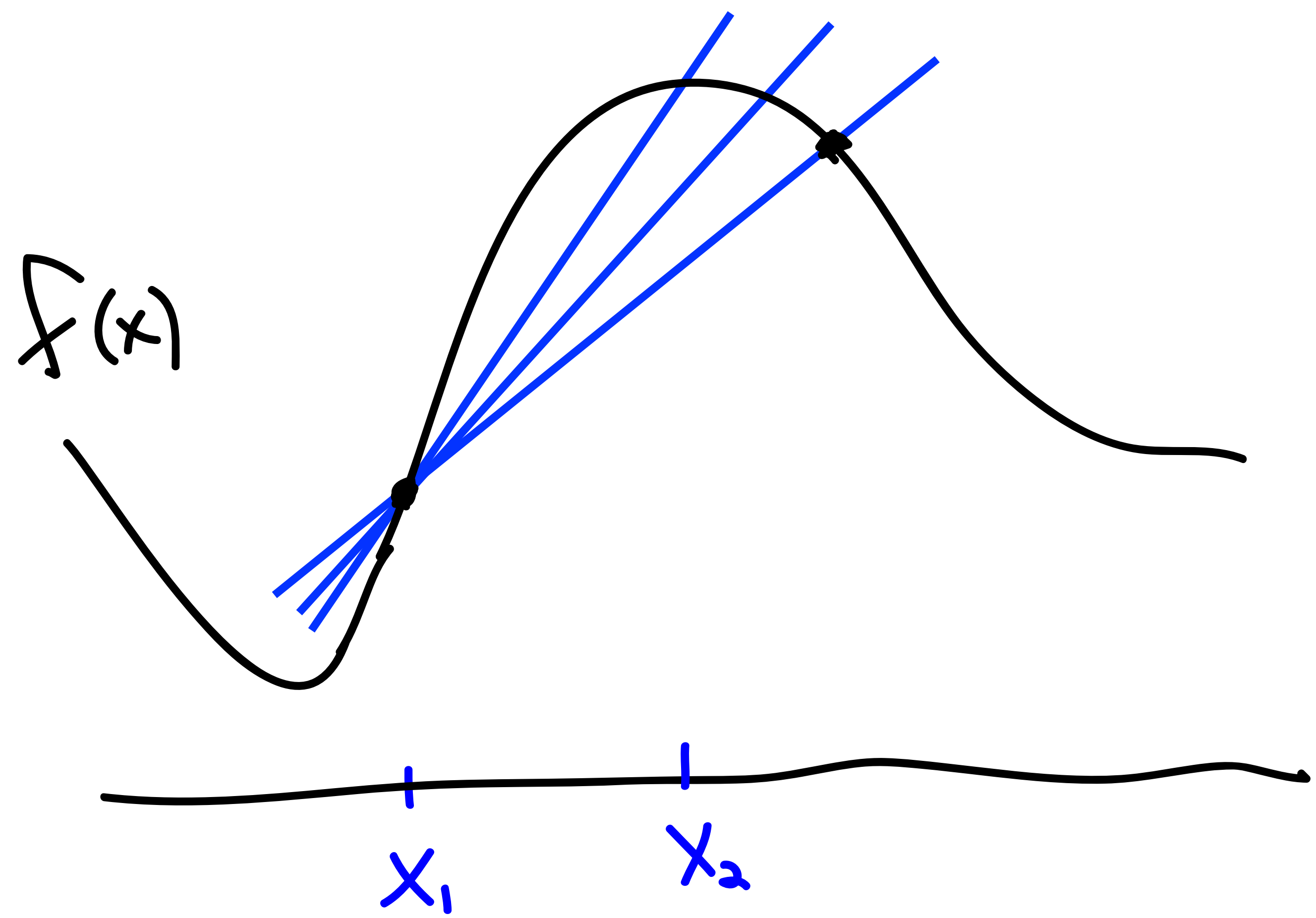

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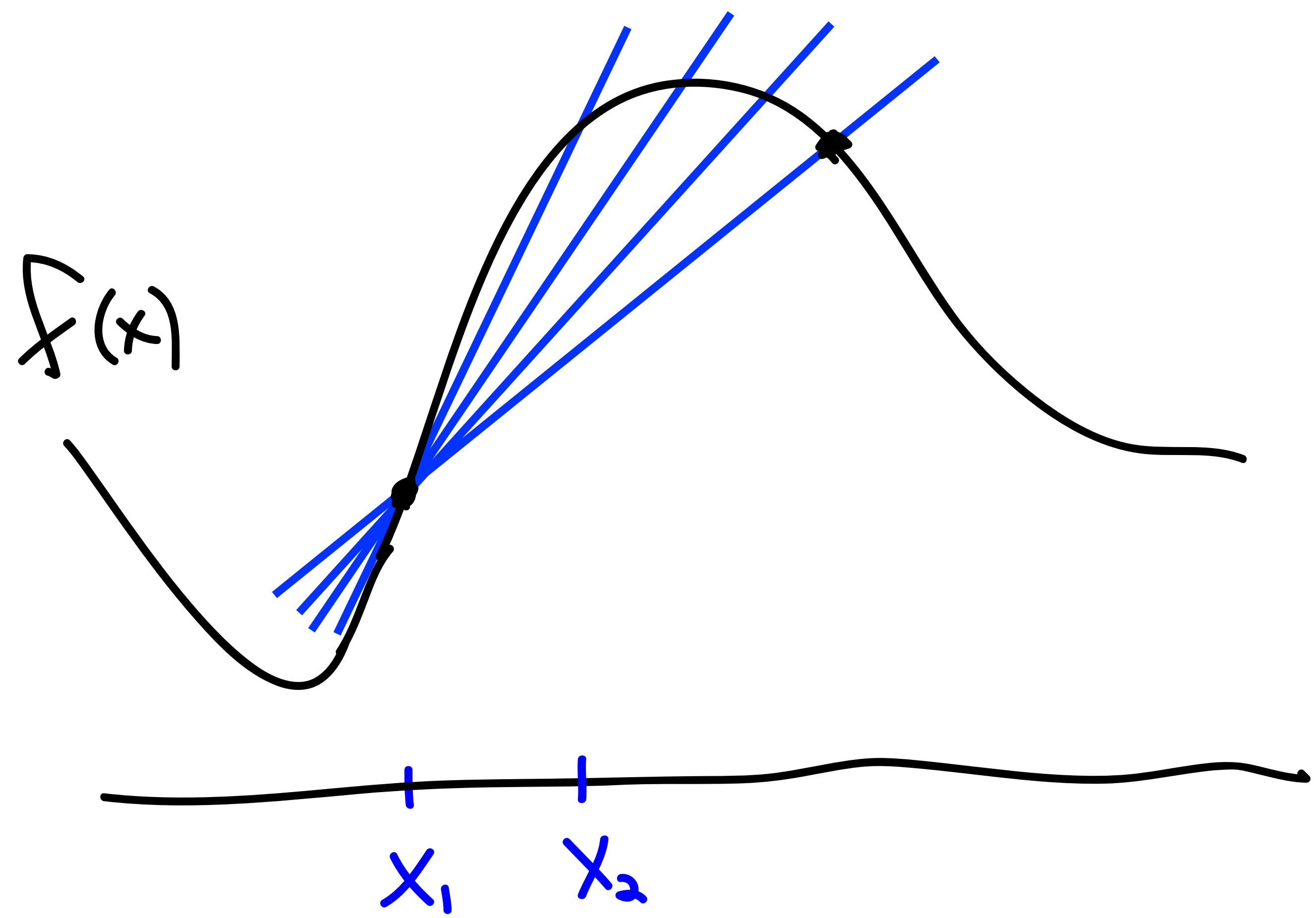

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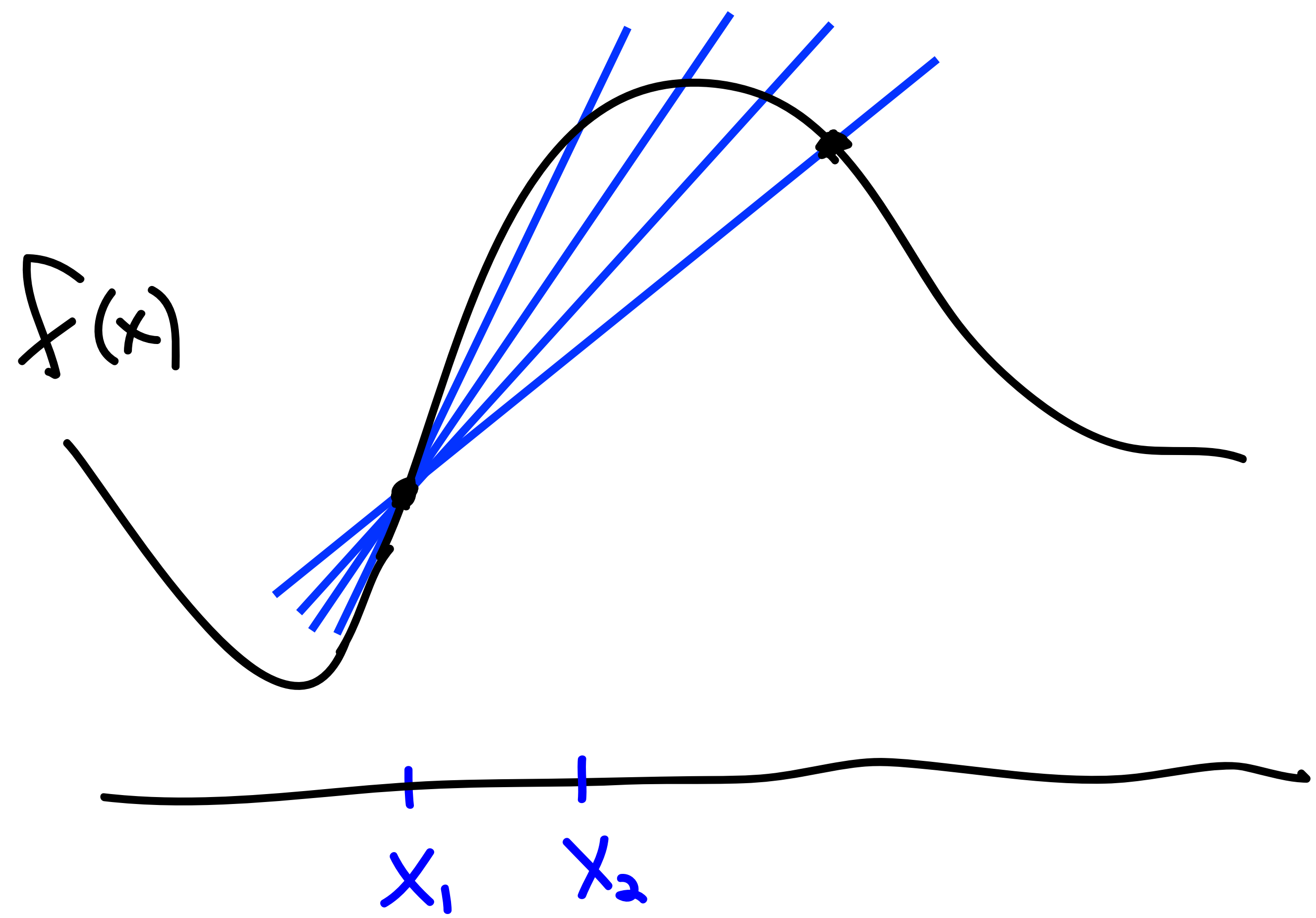


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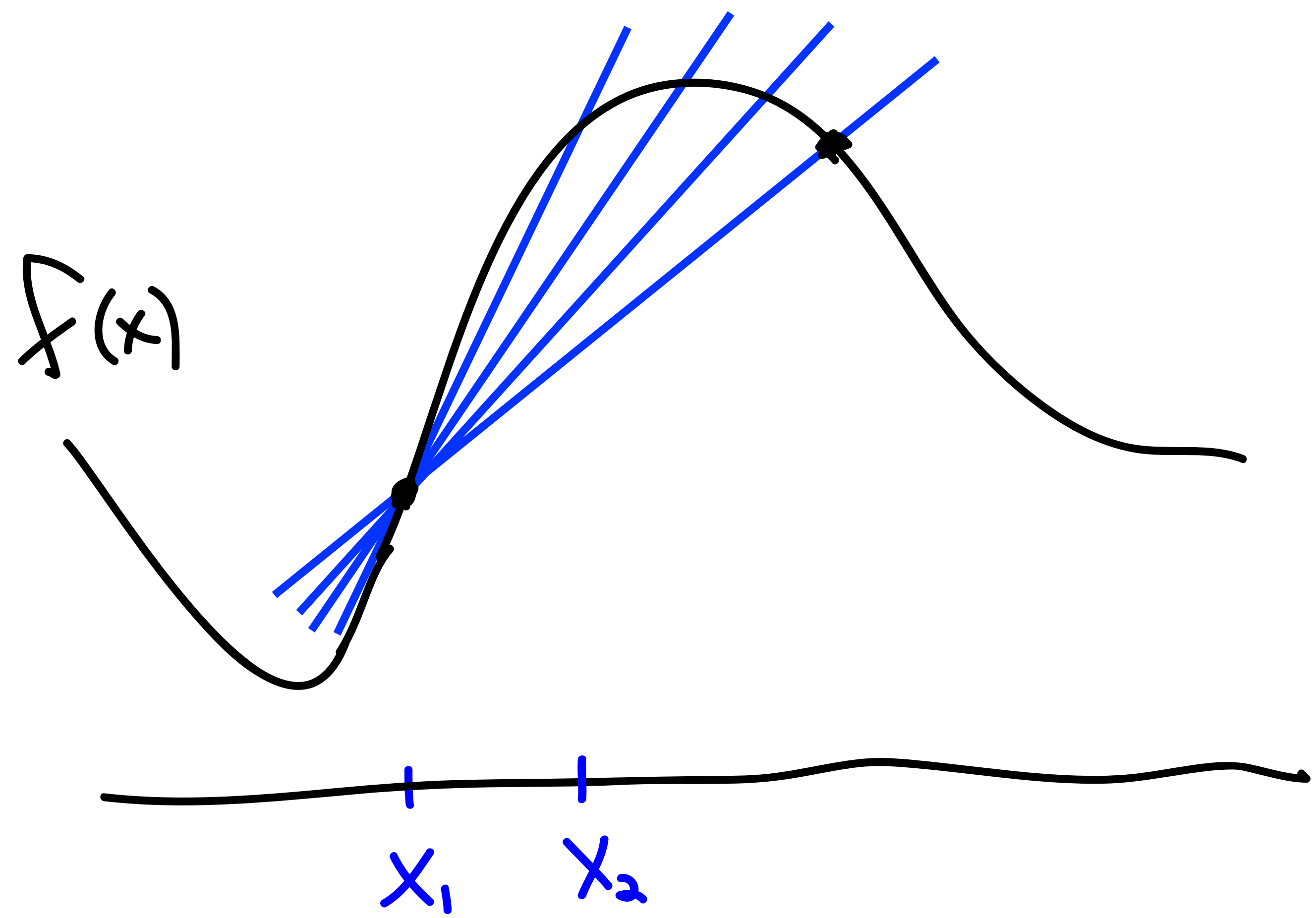


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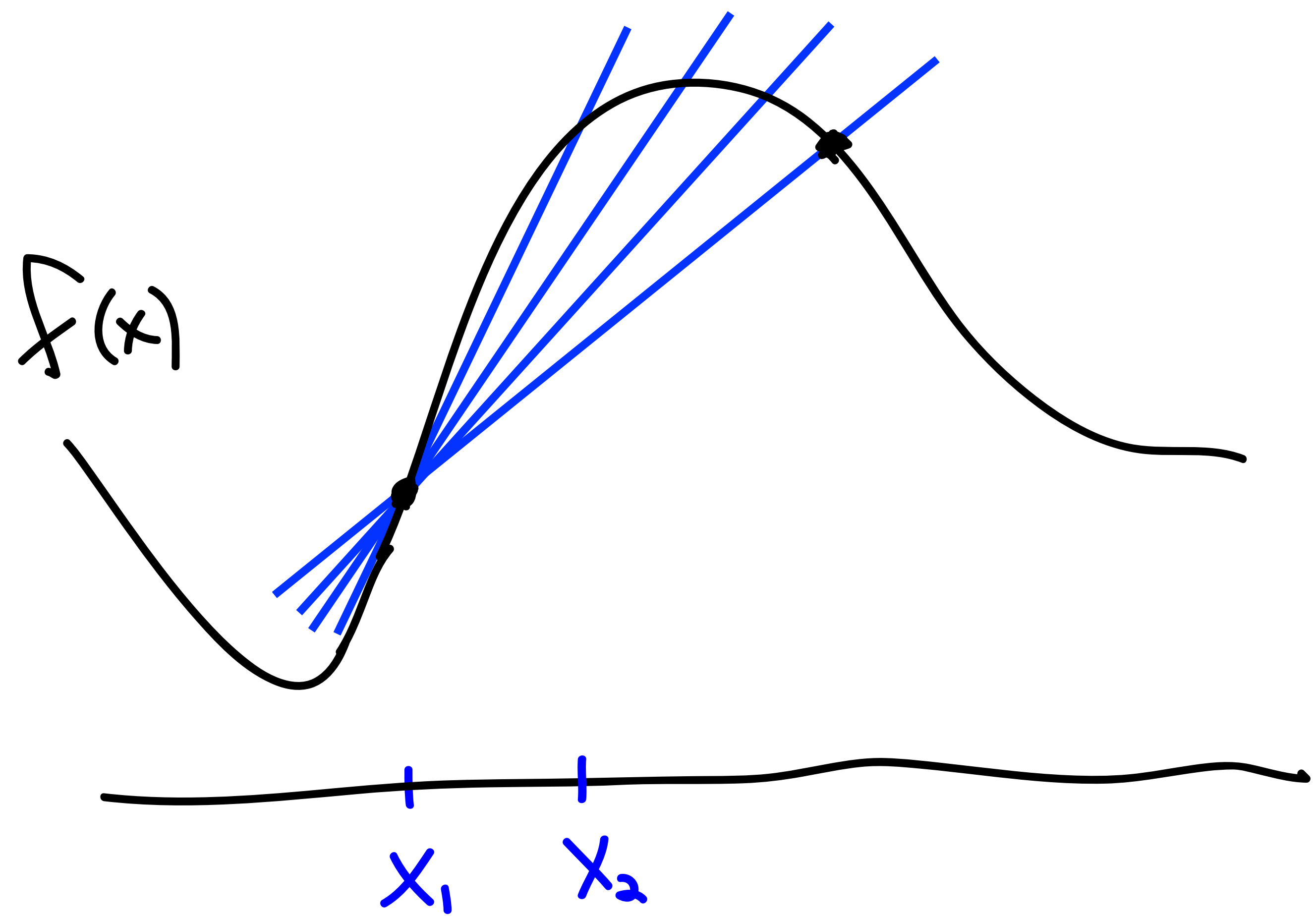


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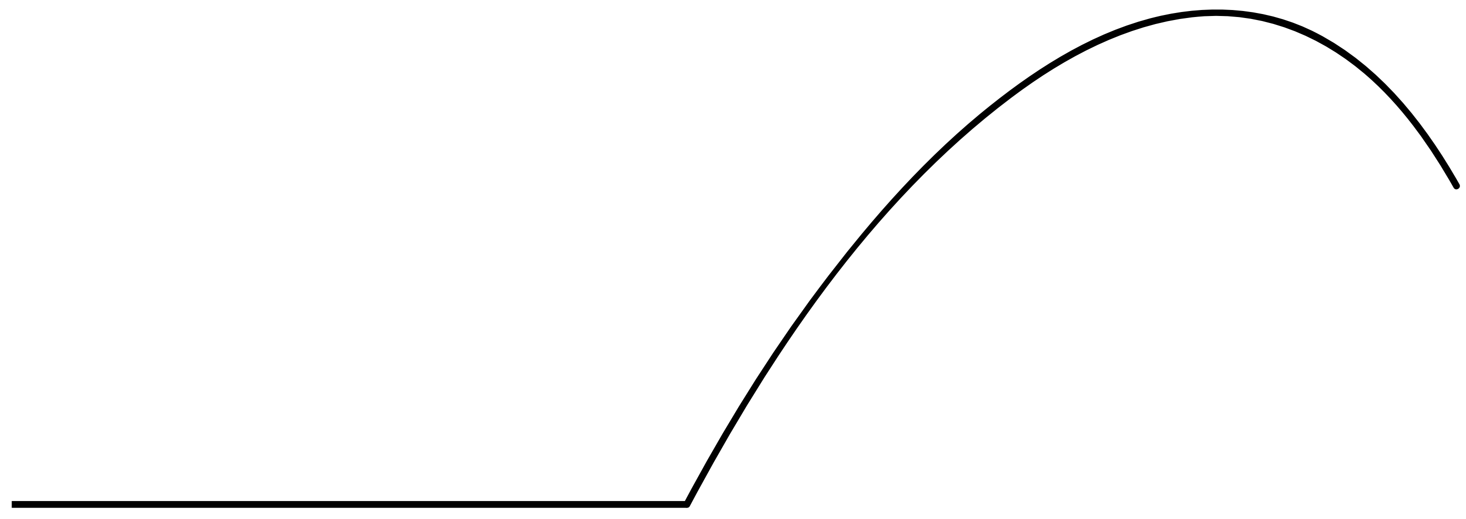
$$\text{slope at } x_1 = f'(x_1) = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$

# Another example



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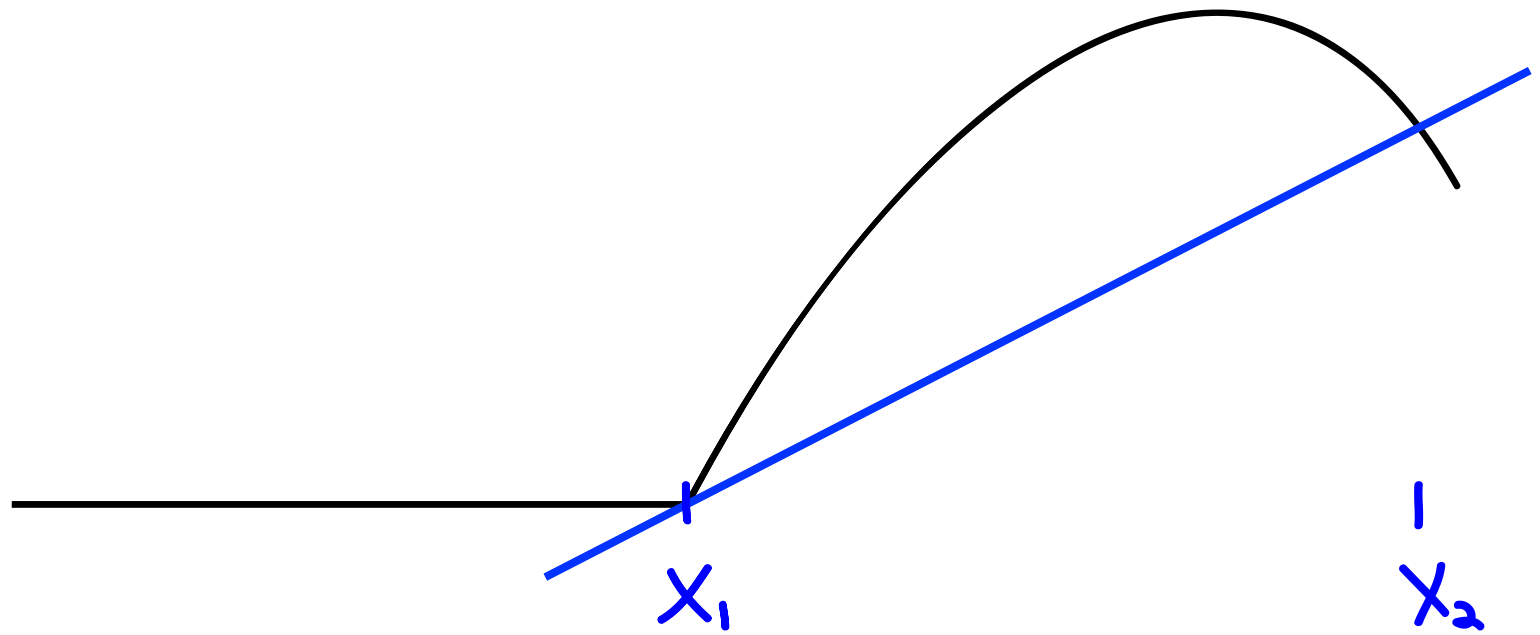

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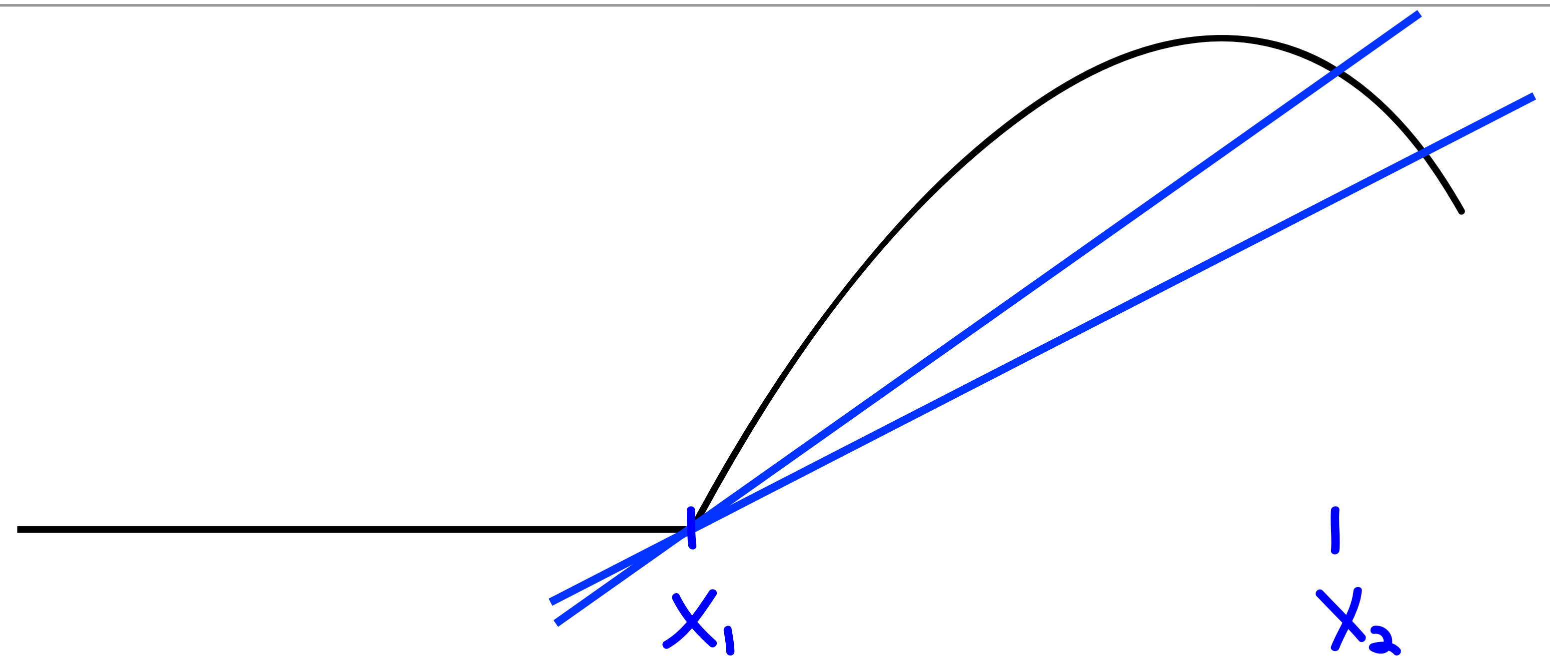

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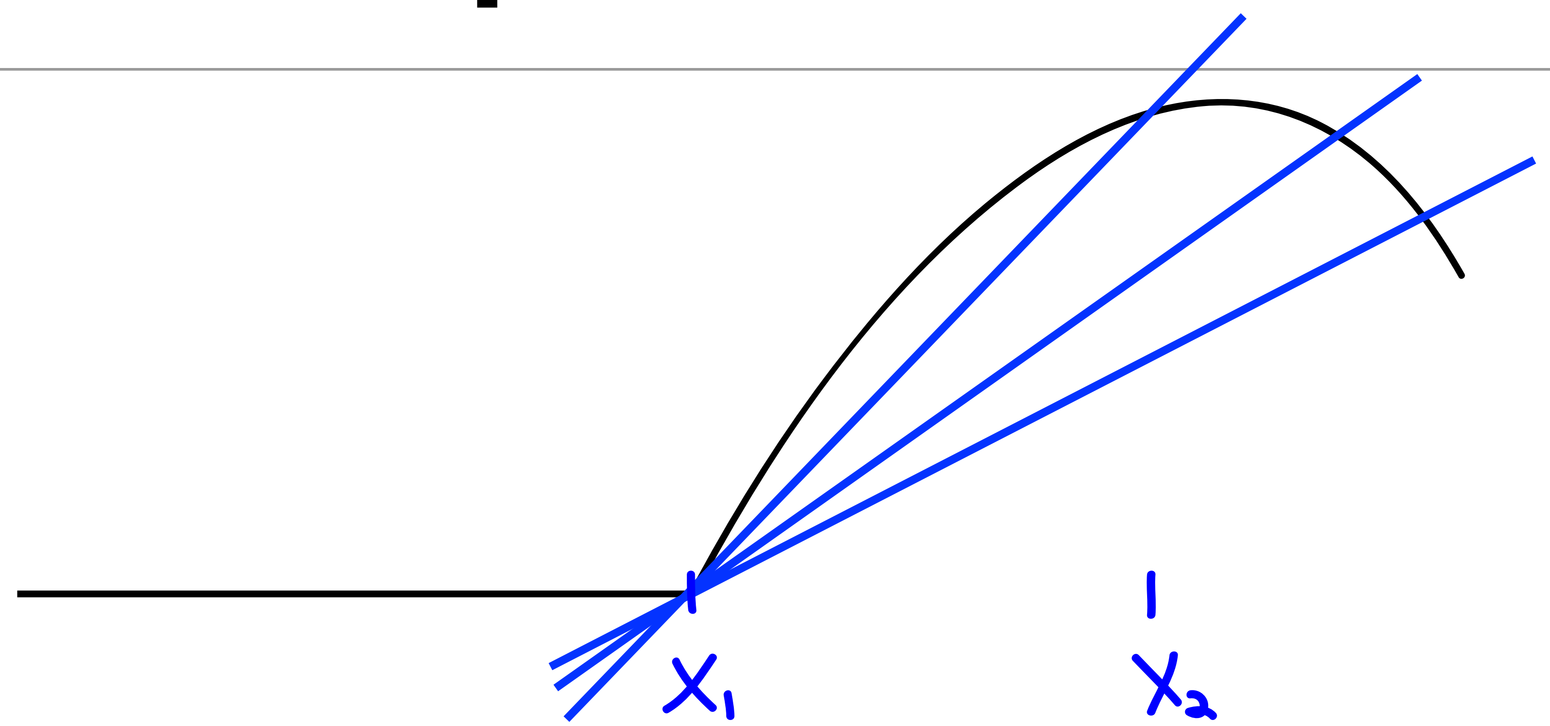

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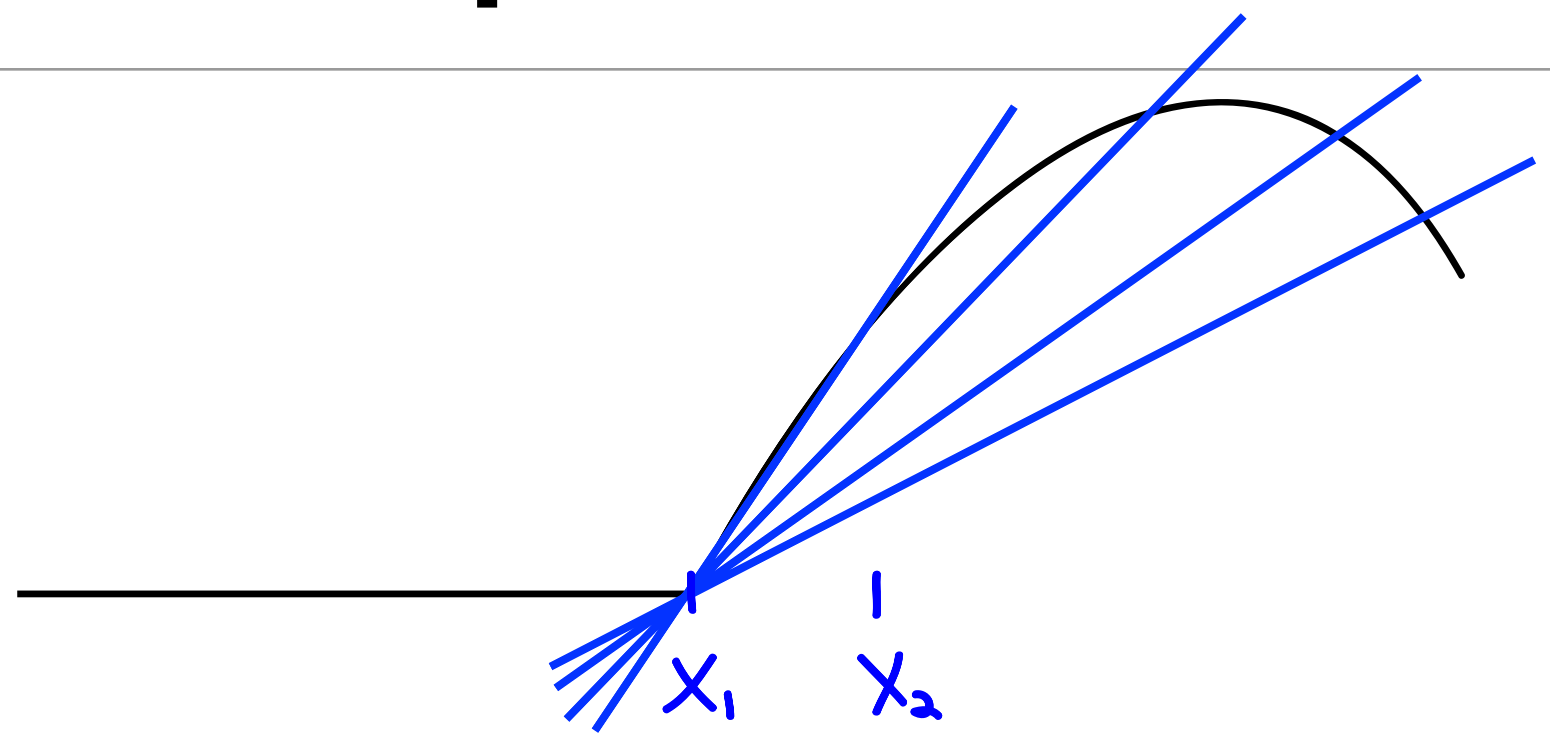

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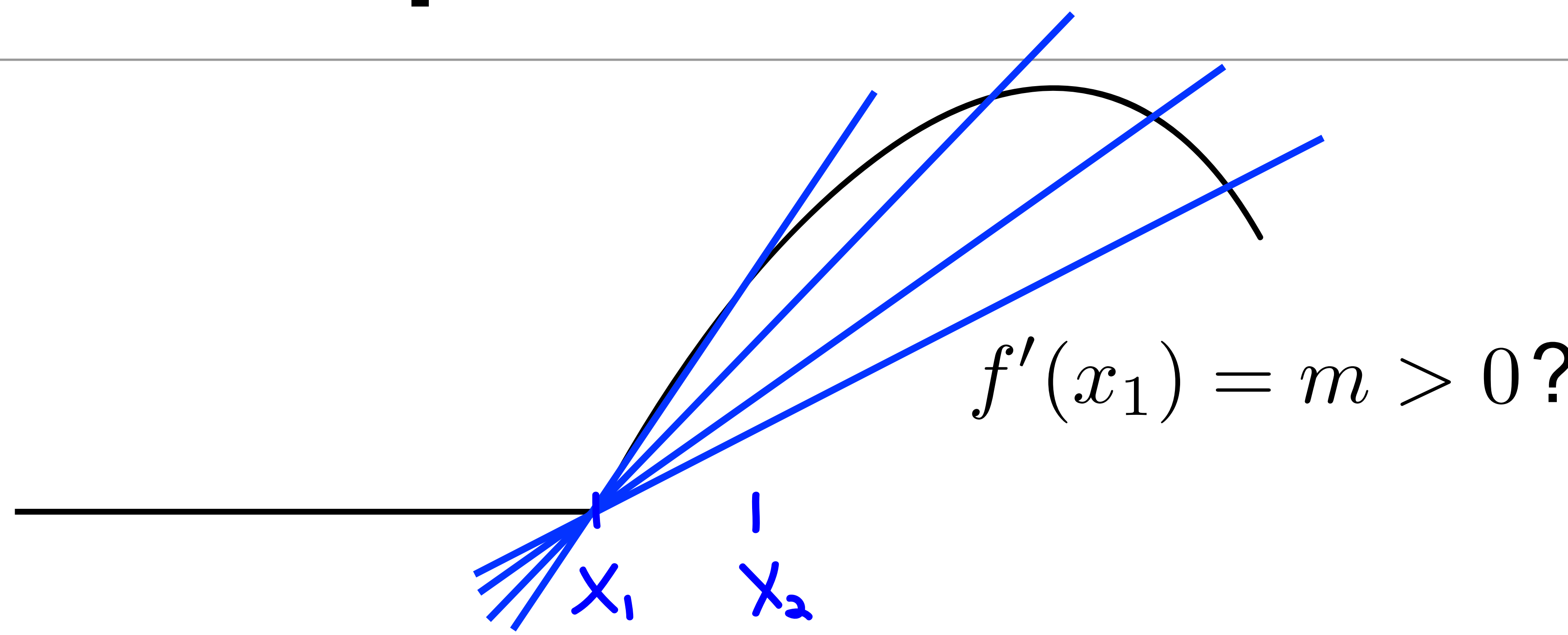

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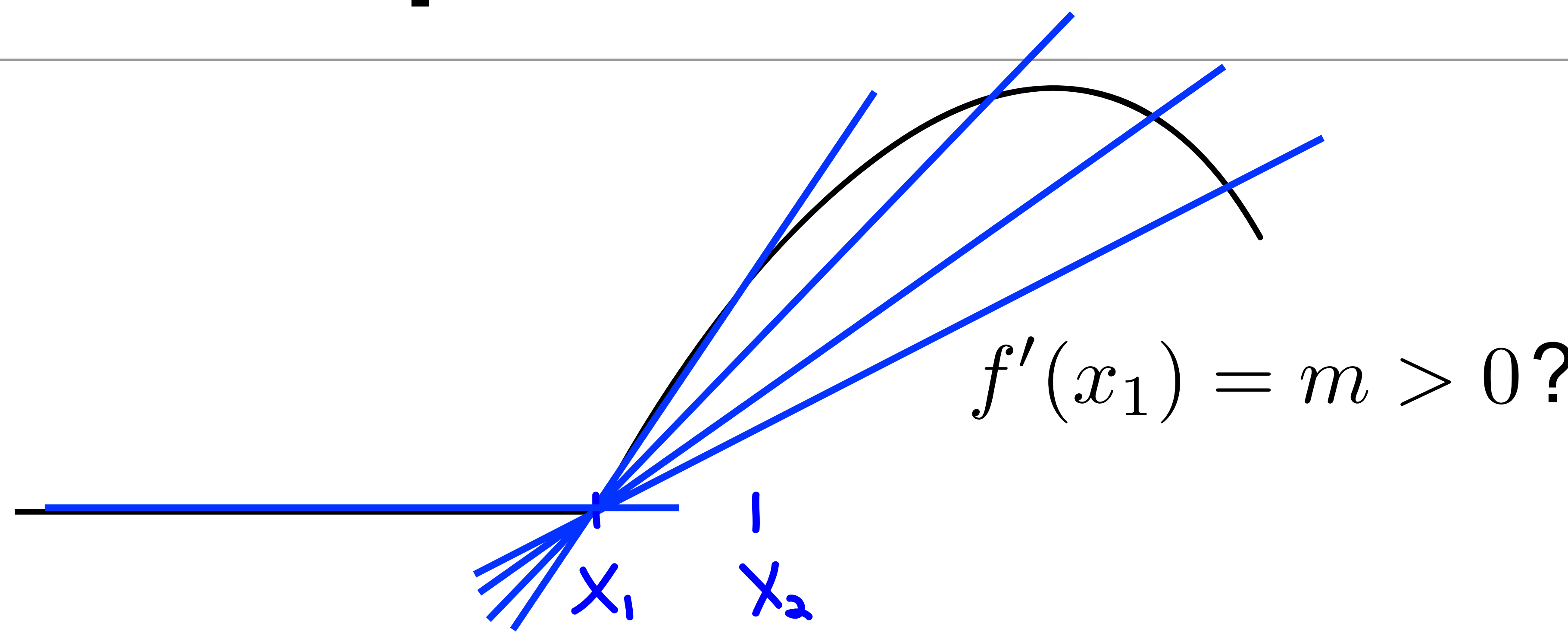

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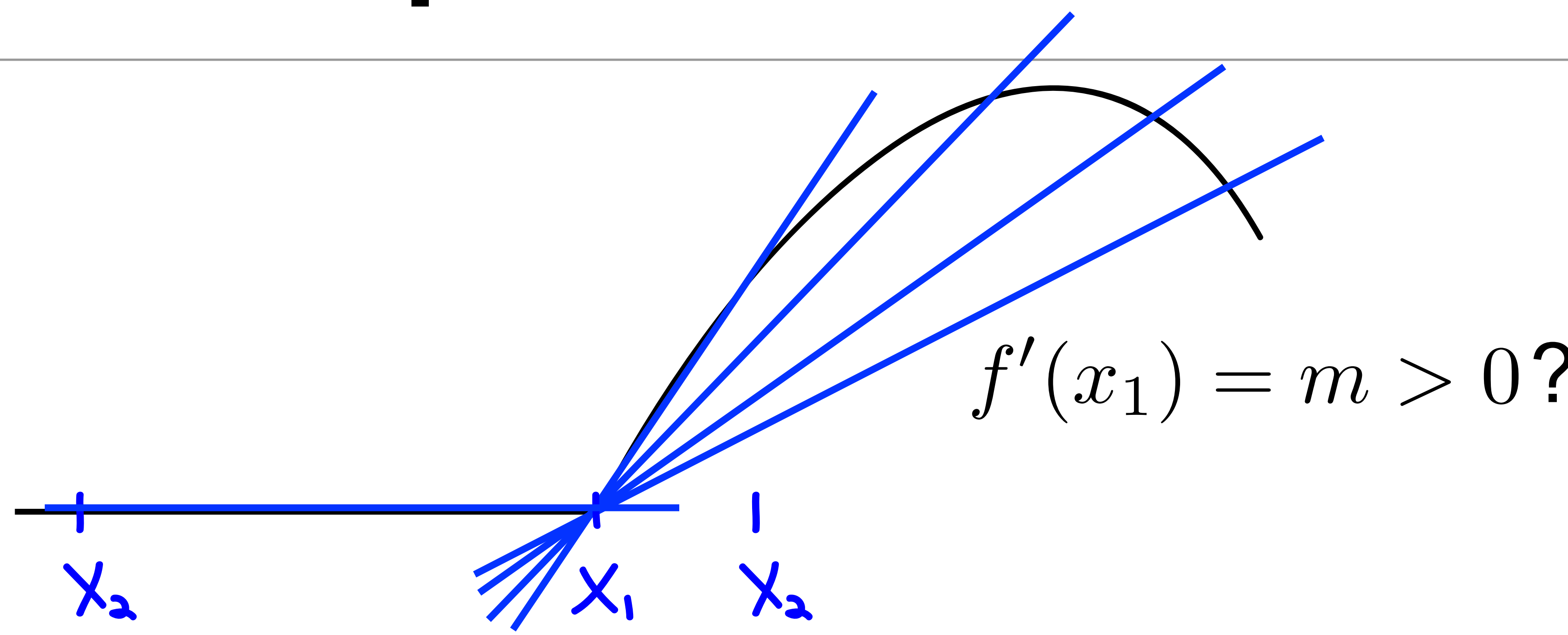

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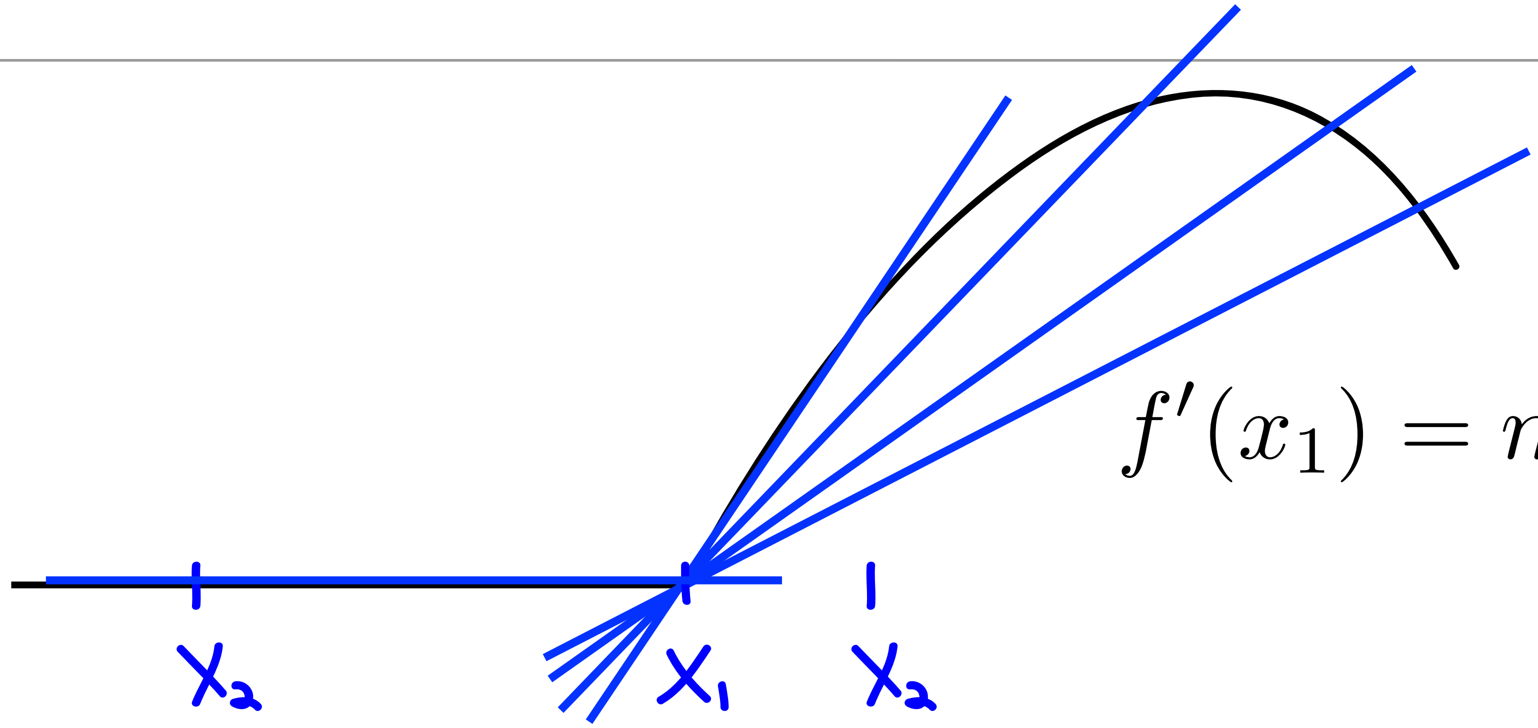


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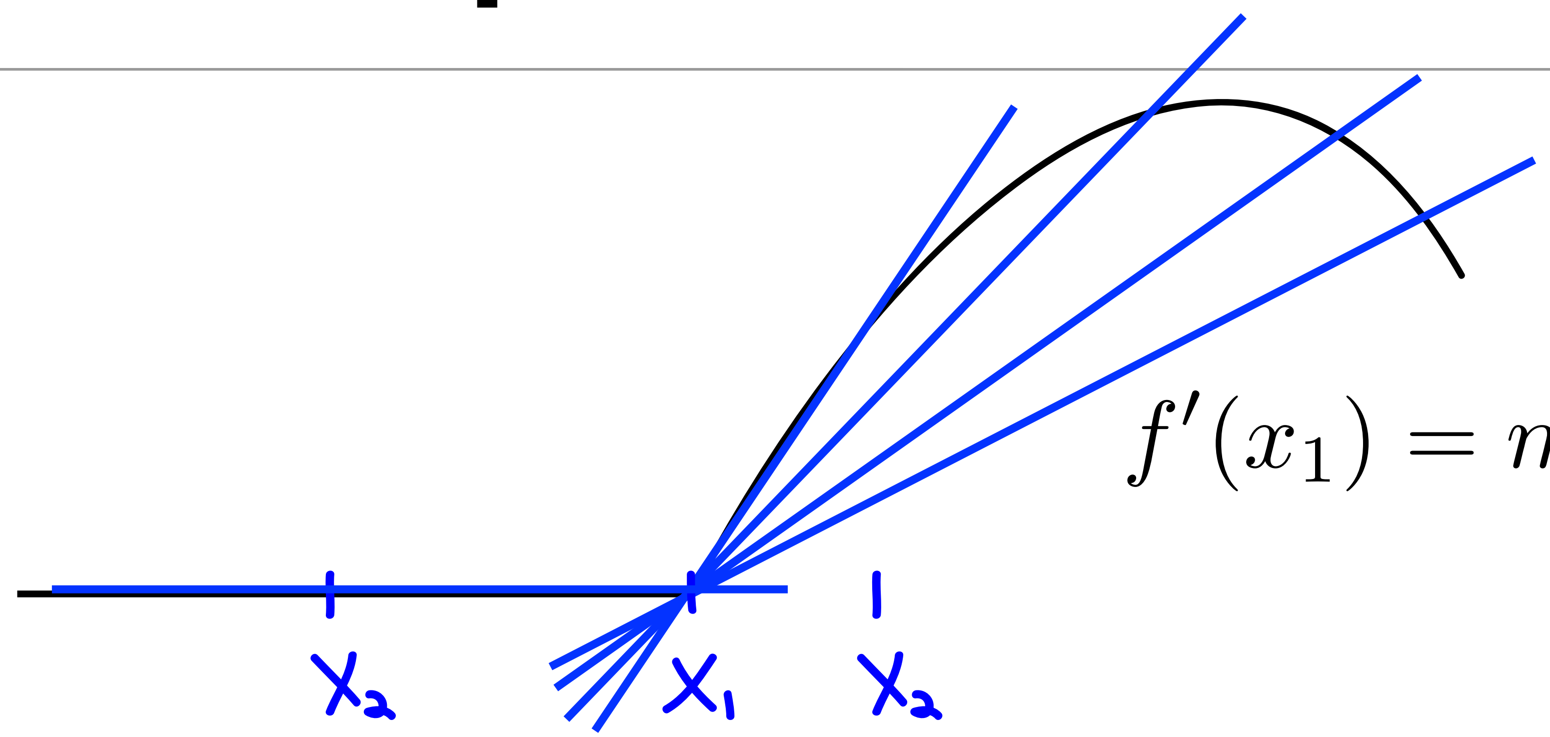


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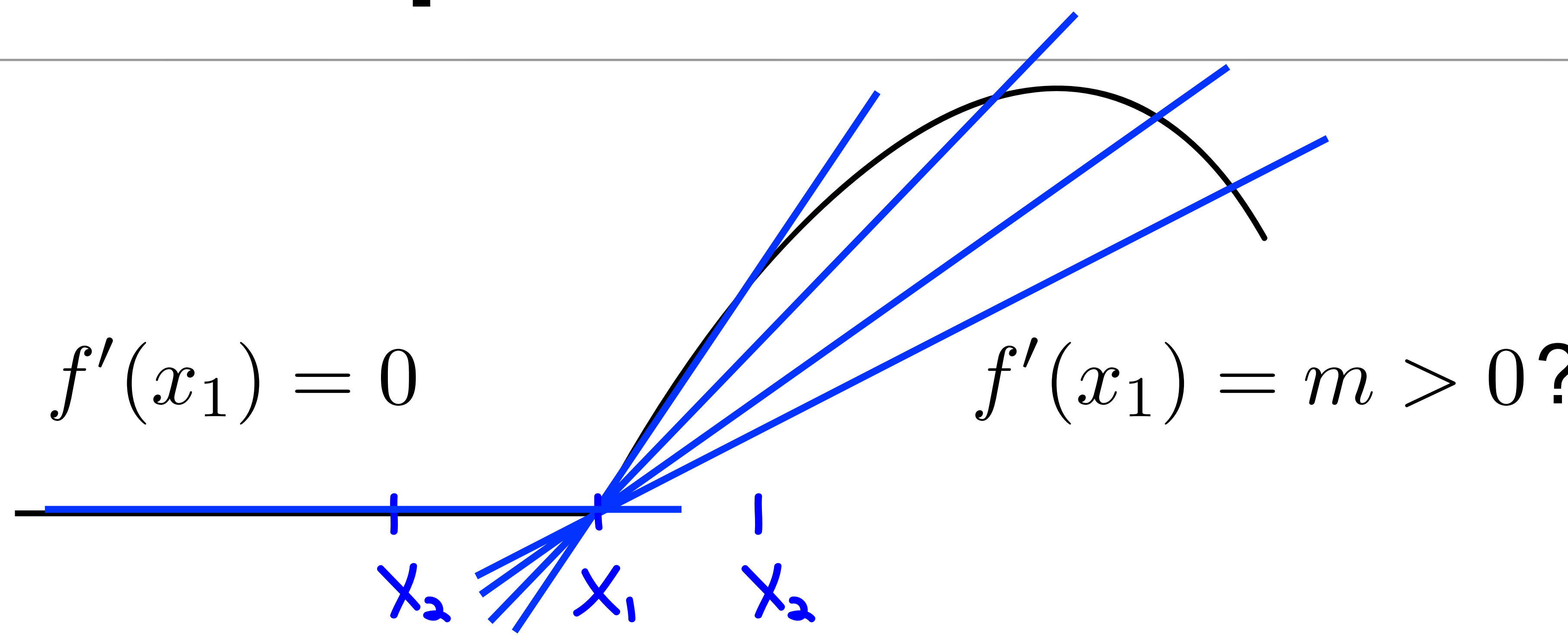

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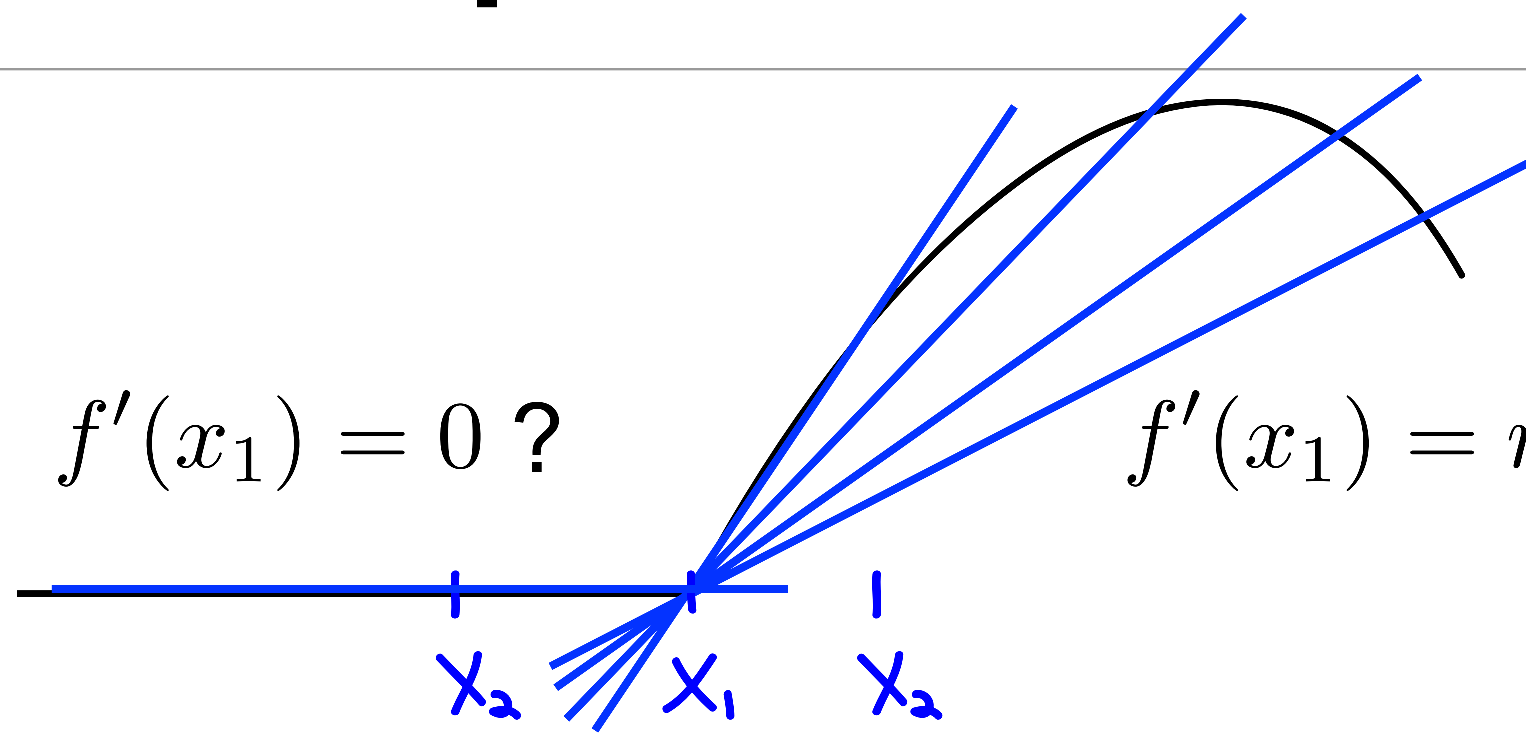

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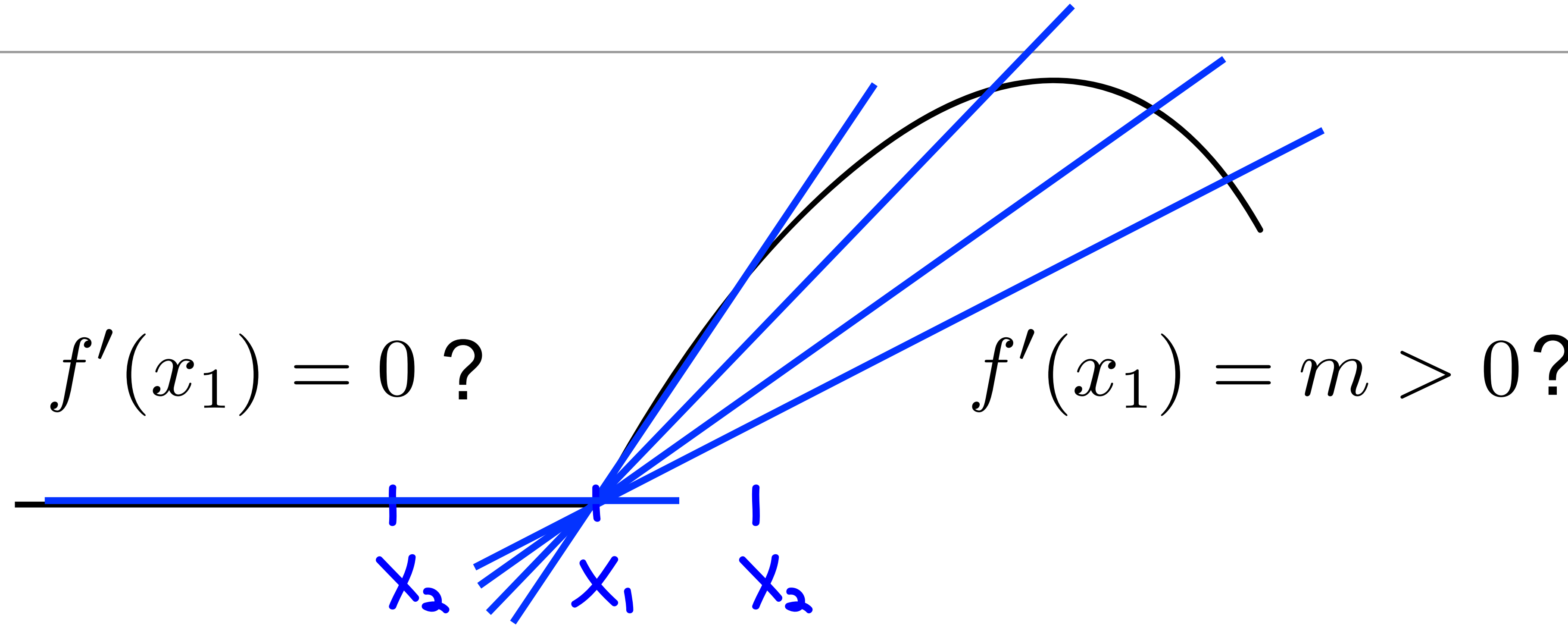


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(C) Both (A) and (B)

(D) The limit does not exist.

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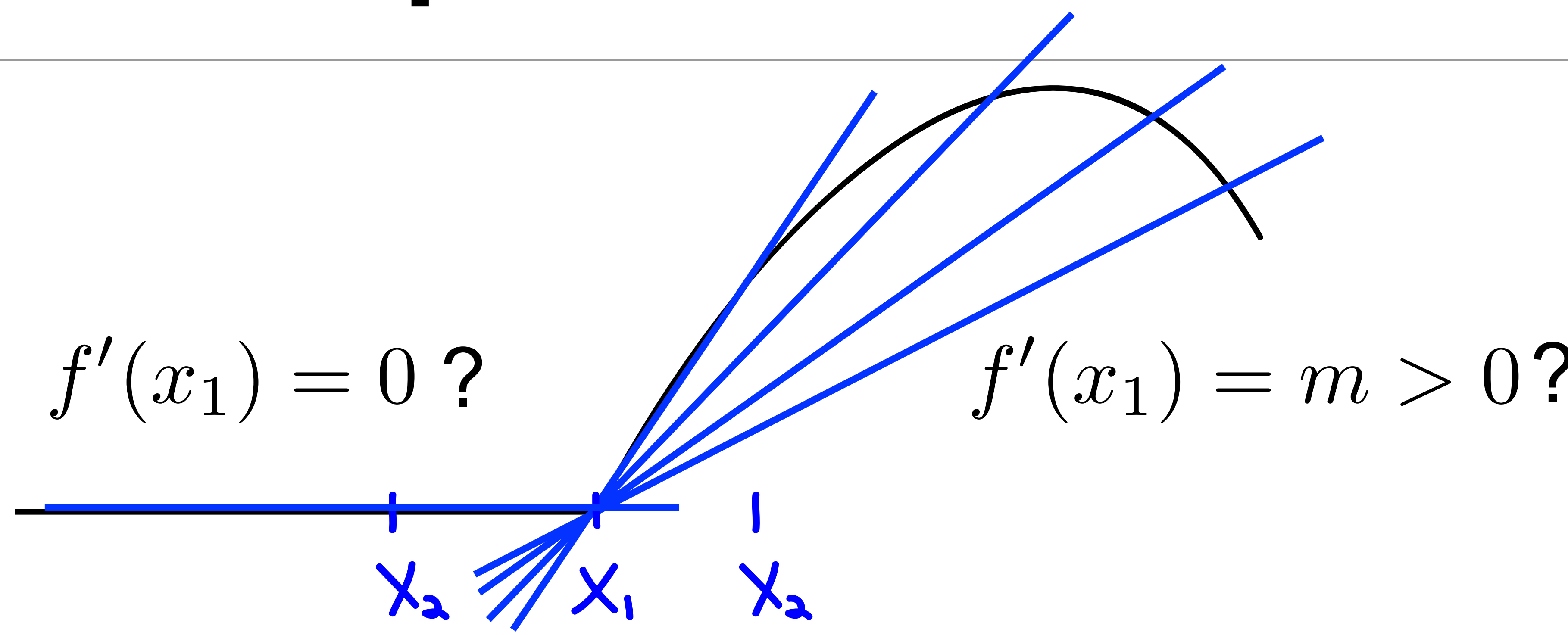


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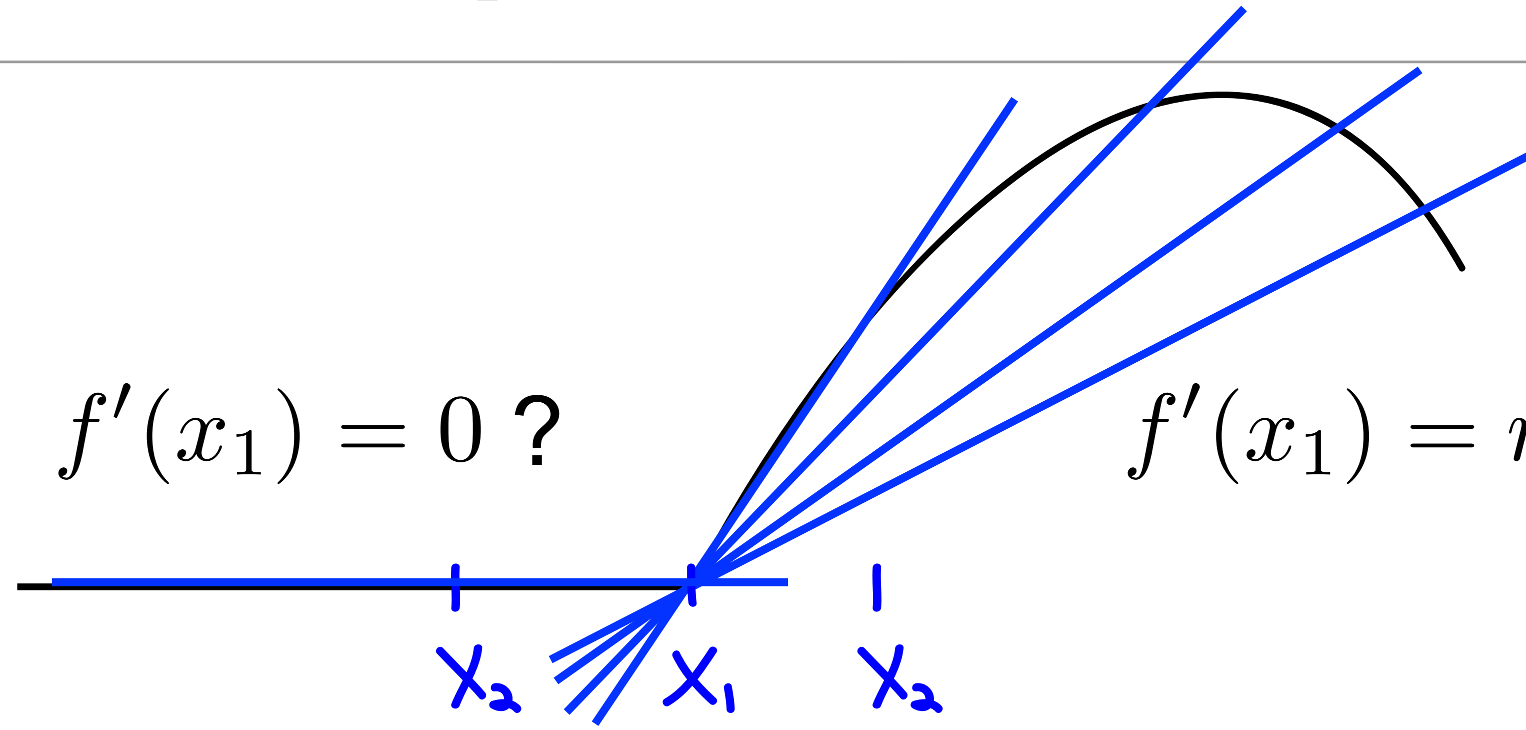


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Limits from left and right must agree for the limit to exist.

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# The derivative of f(x) at x=a...

(A) ...touches the function at x=**a** but does not cross it.

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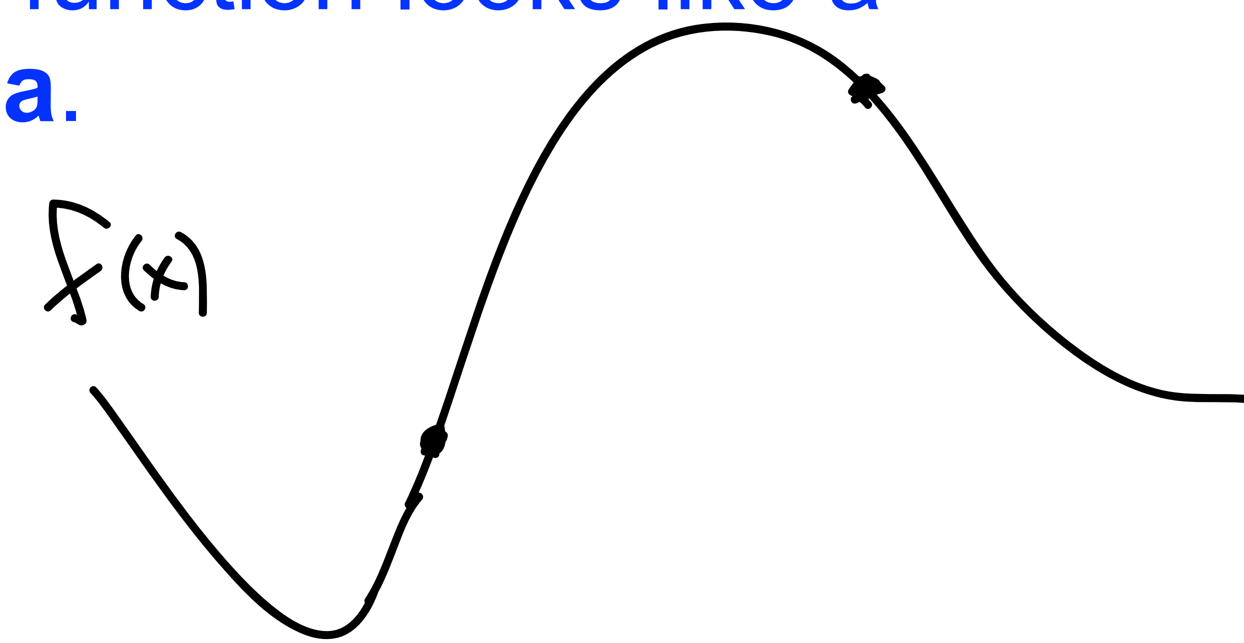

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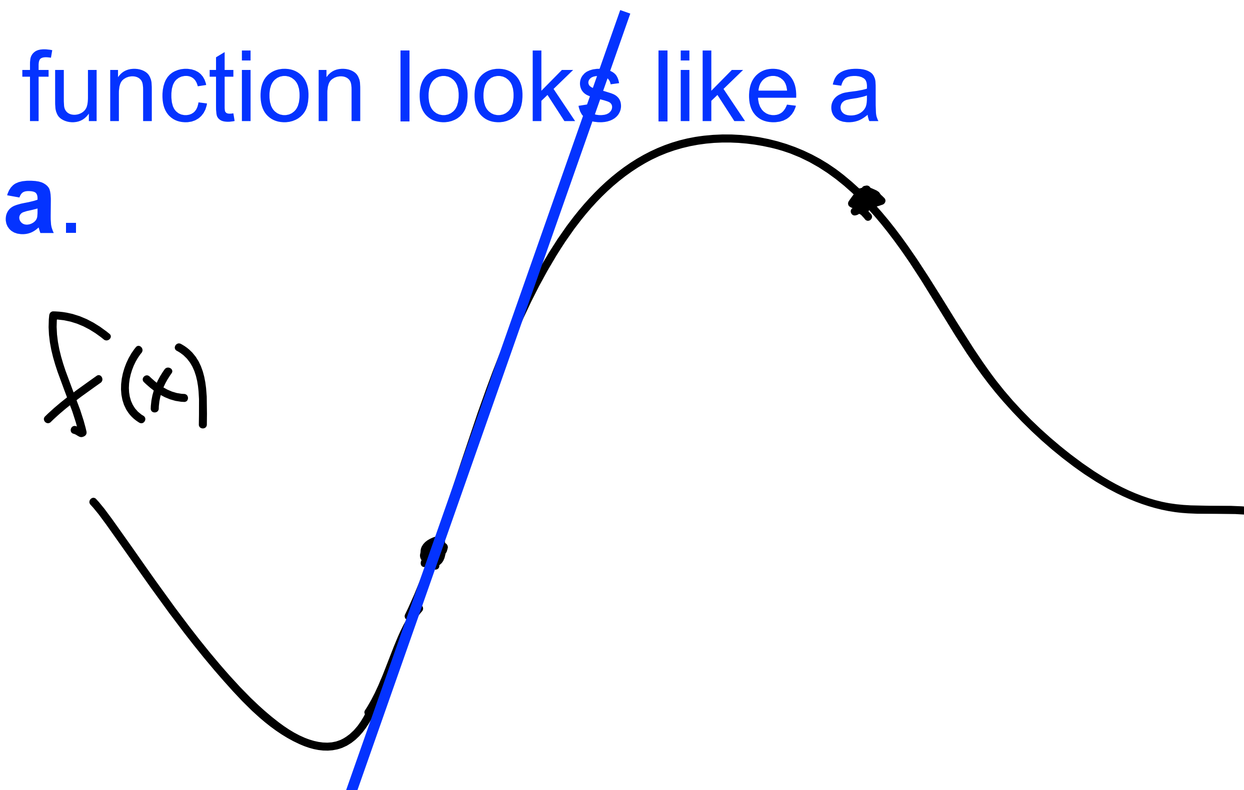

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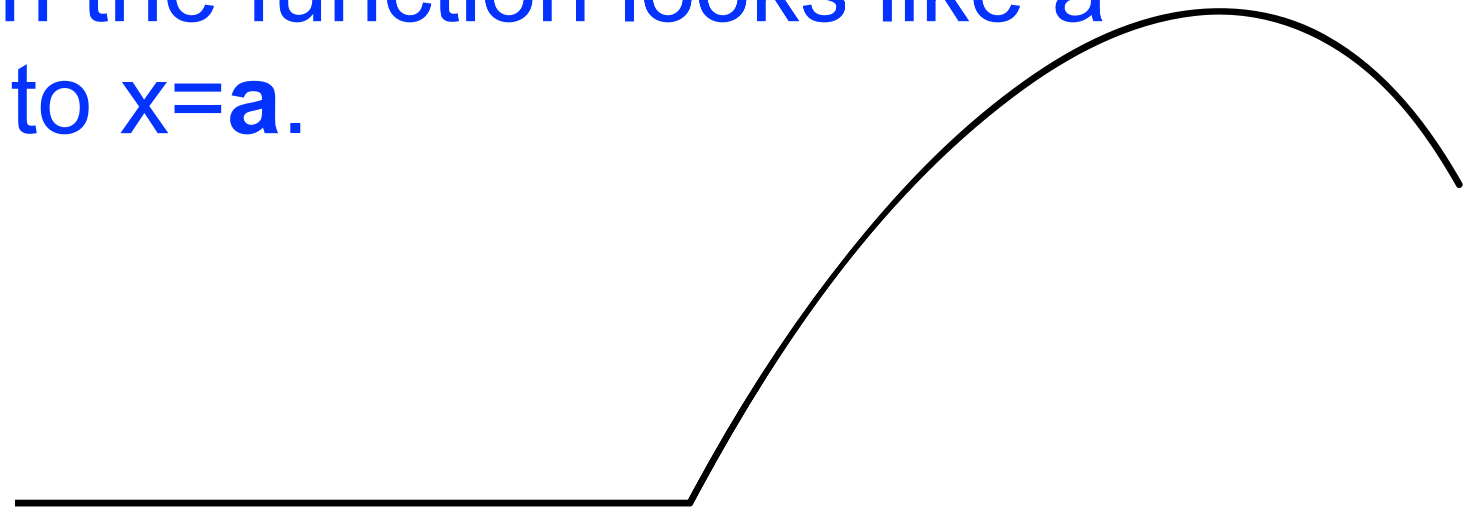

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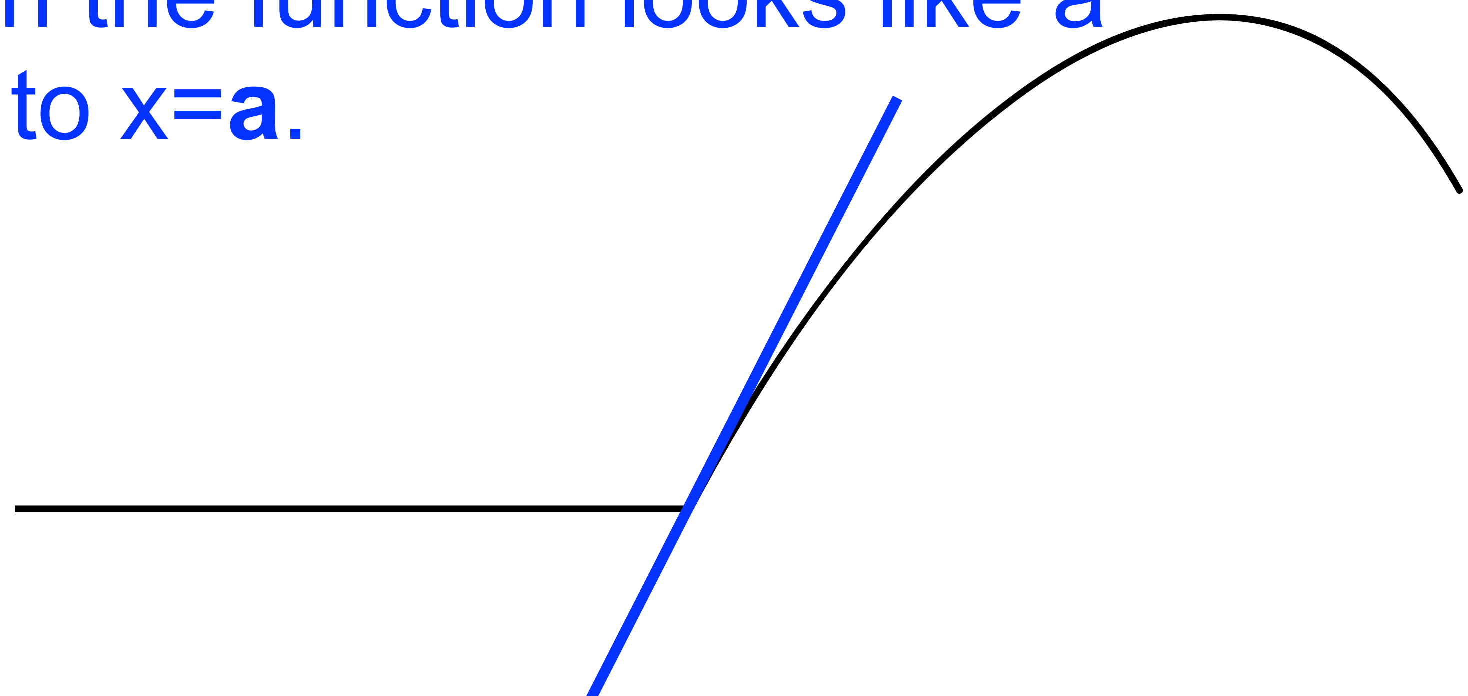

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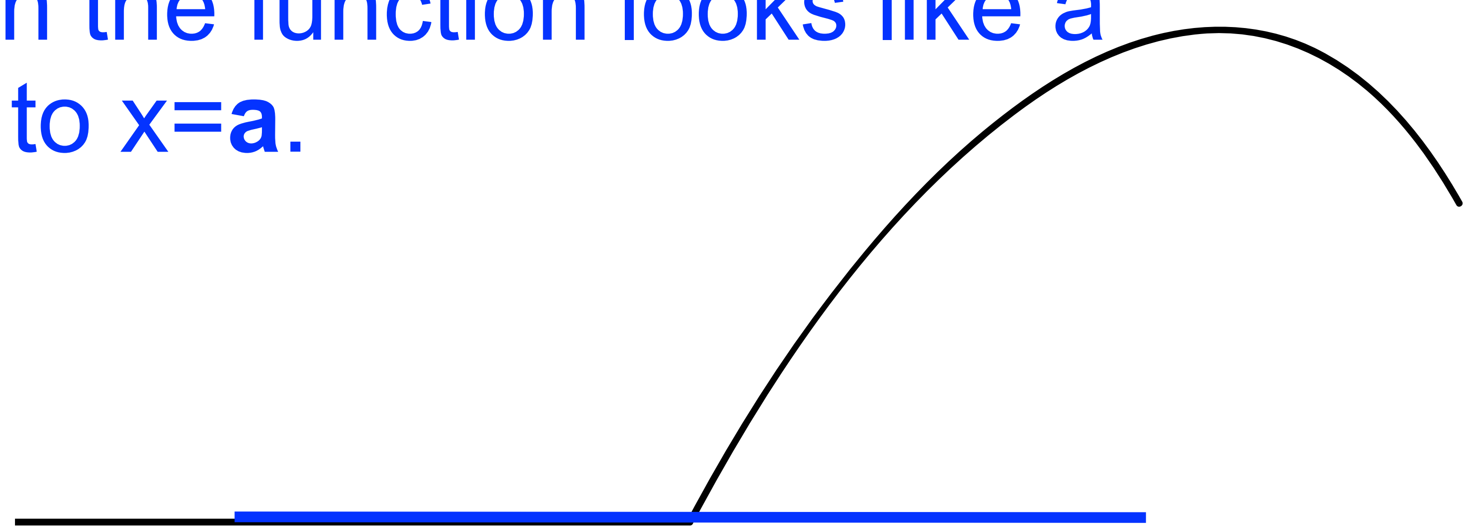

# To evaluate a limit

To evaluate $\lim_{x \to a} f(x)$, you plug in values closer and closer to ***a*** but you never get to ***a***. In fact, ***f(a)*** may not even be defined. If you always get the same number no matter how you approach **a**, then the limit exists.

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$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

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The limit involved in the derivative is only one special case. The limit $\lim_{x \longrightarrow a} f(x)$ is concerned with the **value** of the function near a.

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When a limit has the form

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

we're talking about the slope of f (in this case, at x=2).

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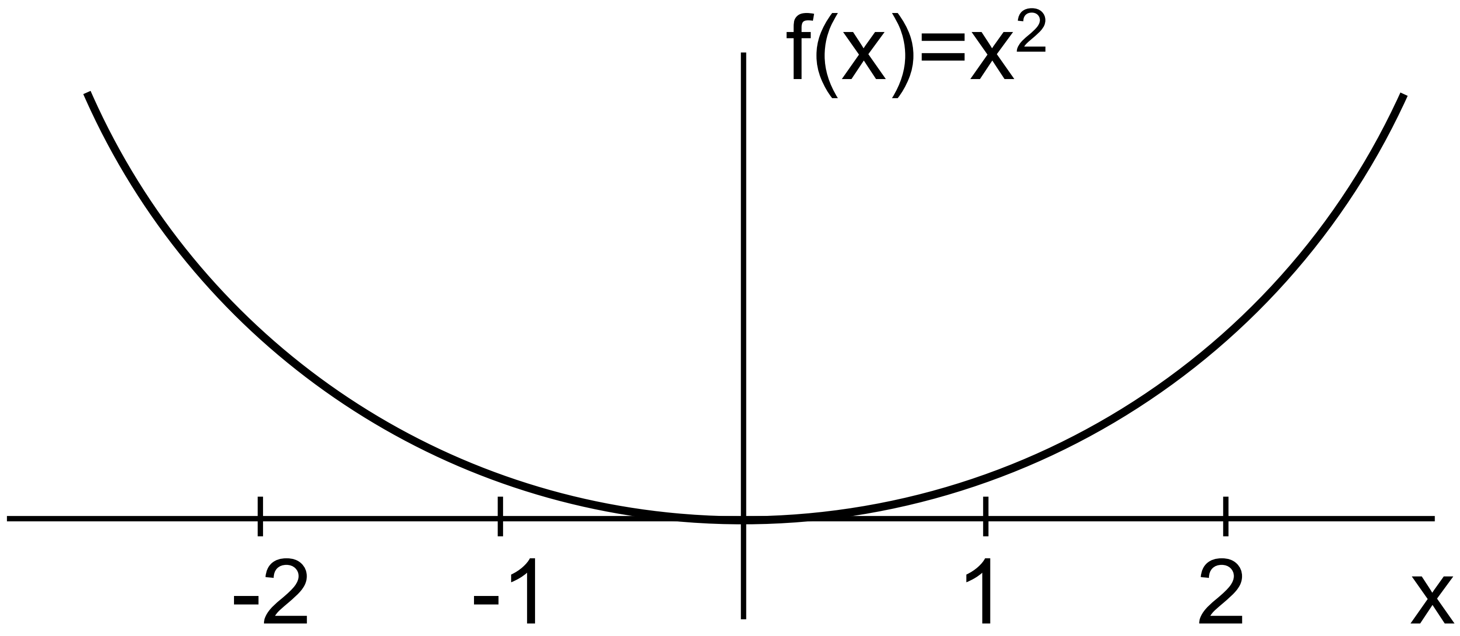

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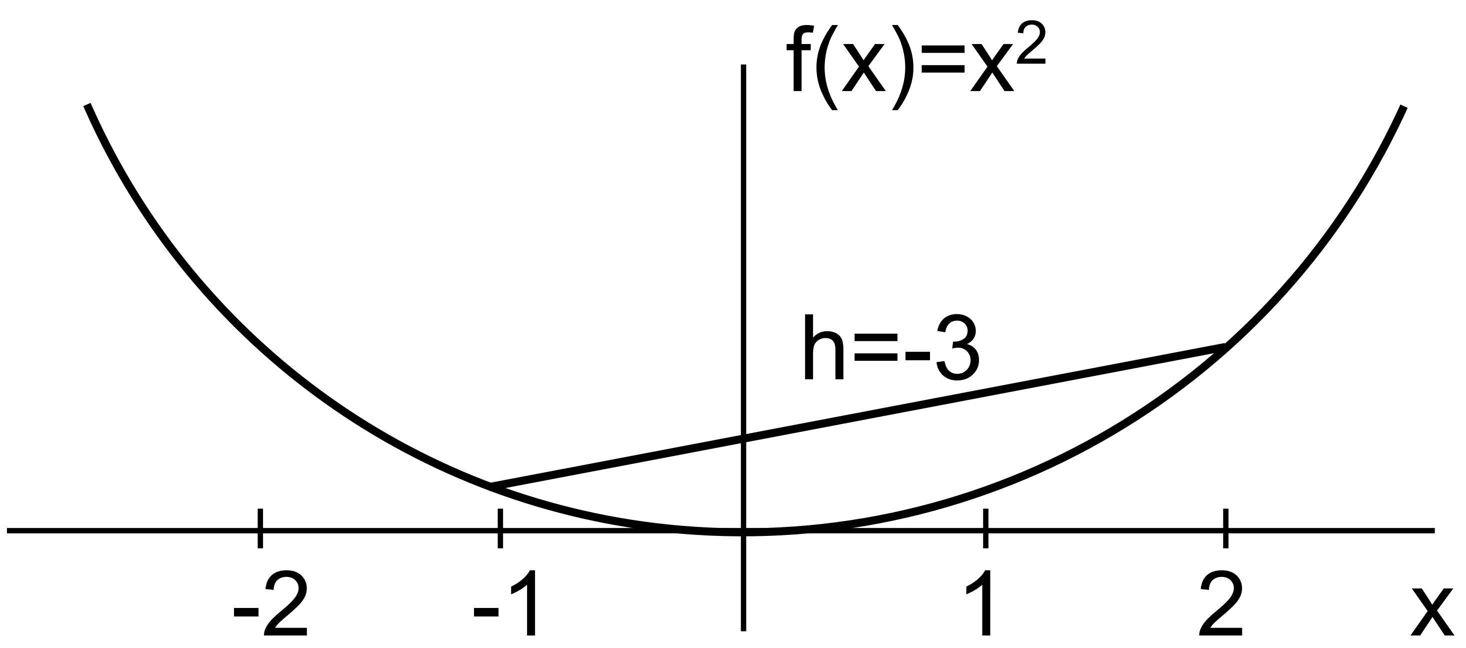


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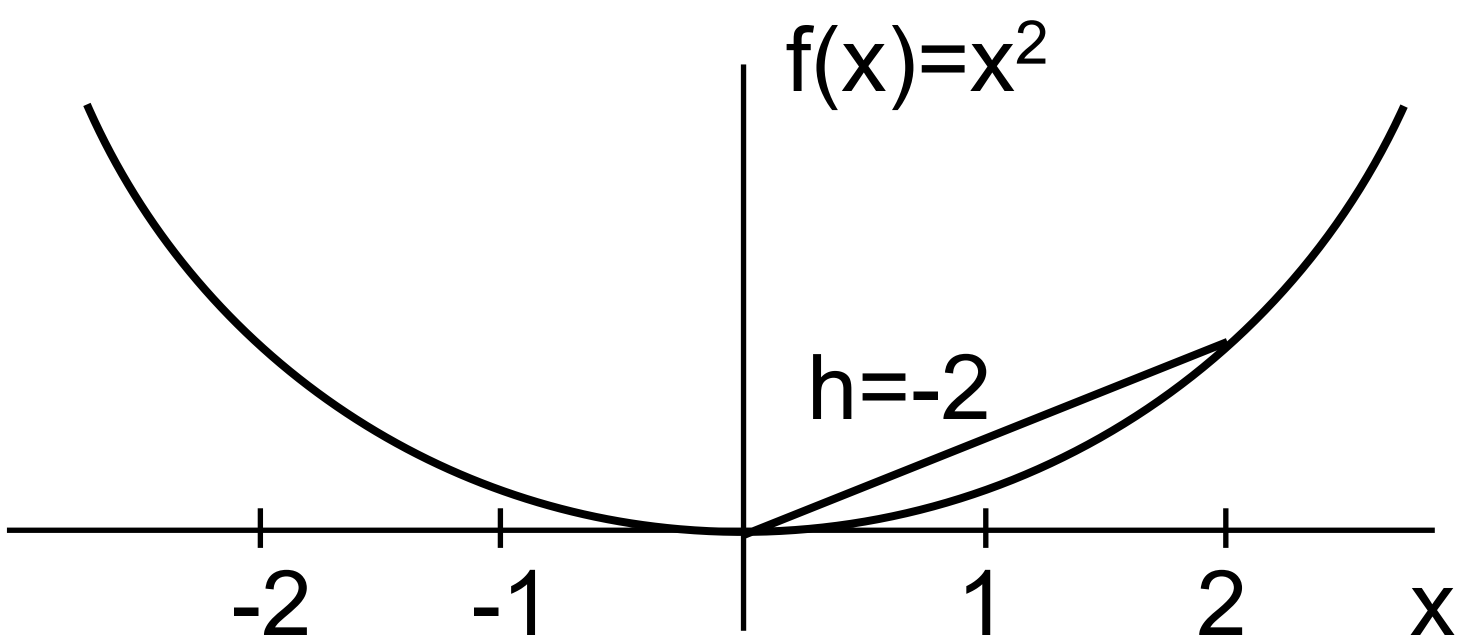


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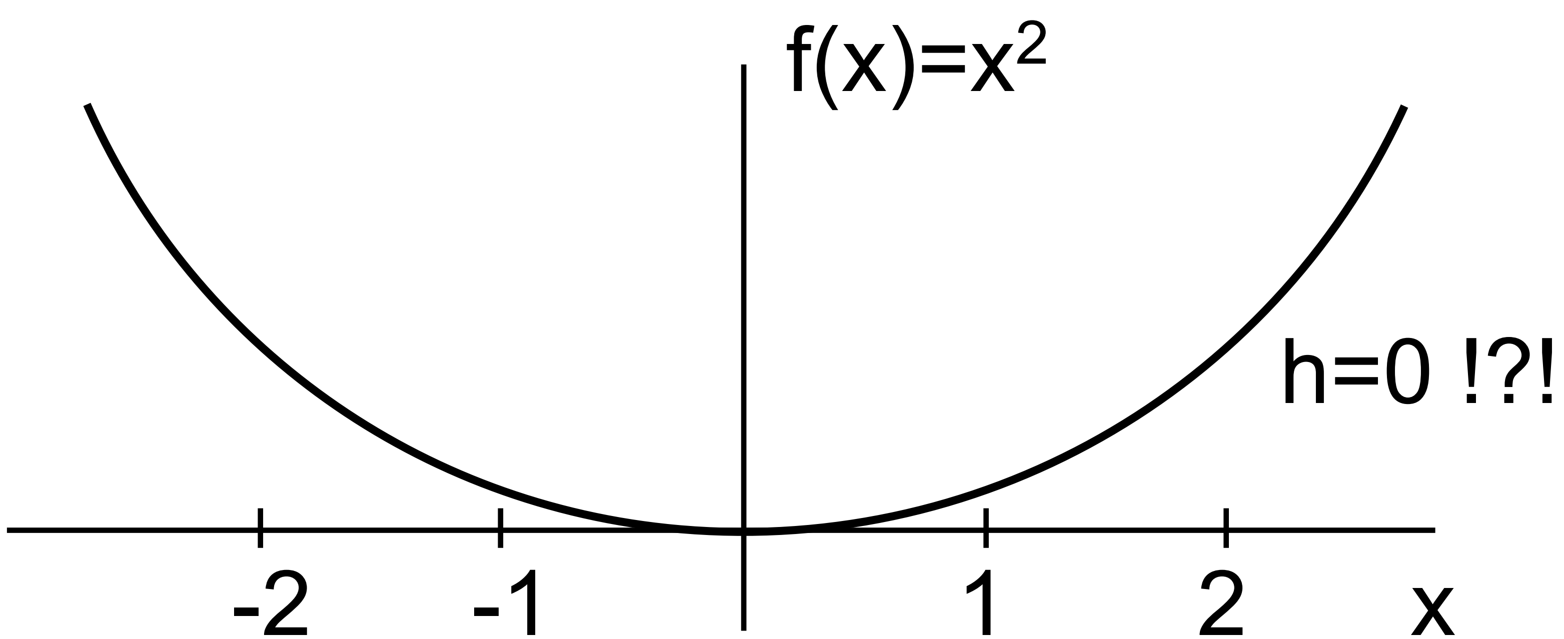


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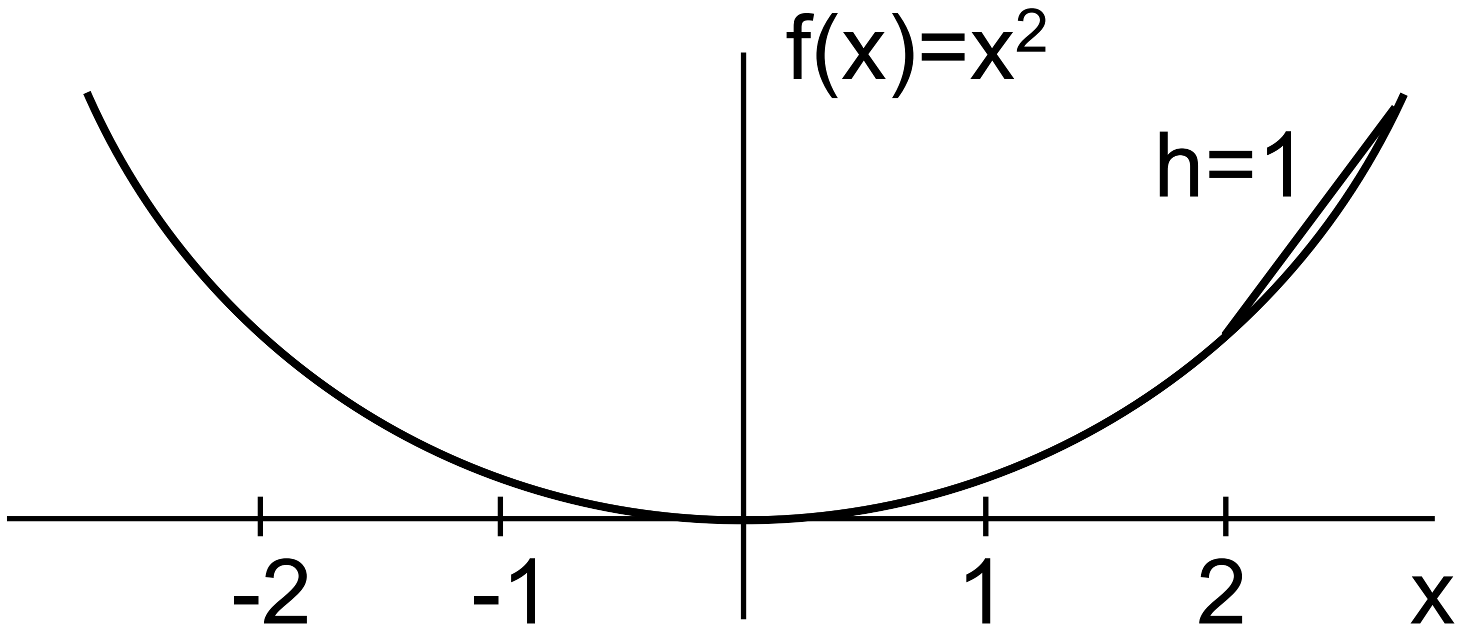


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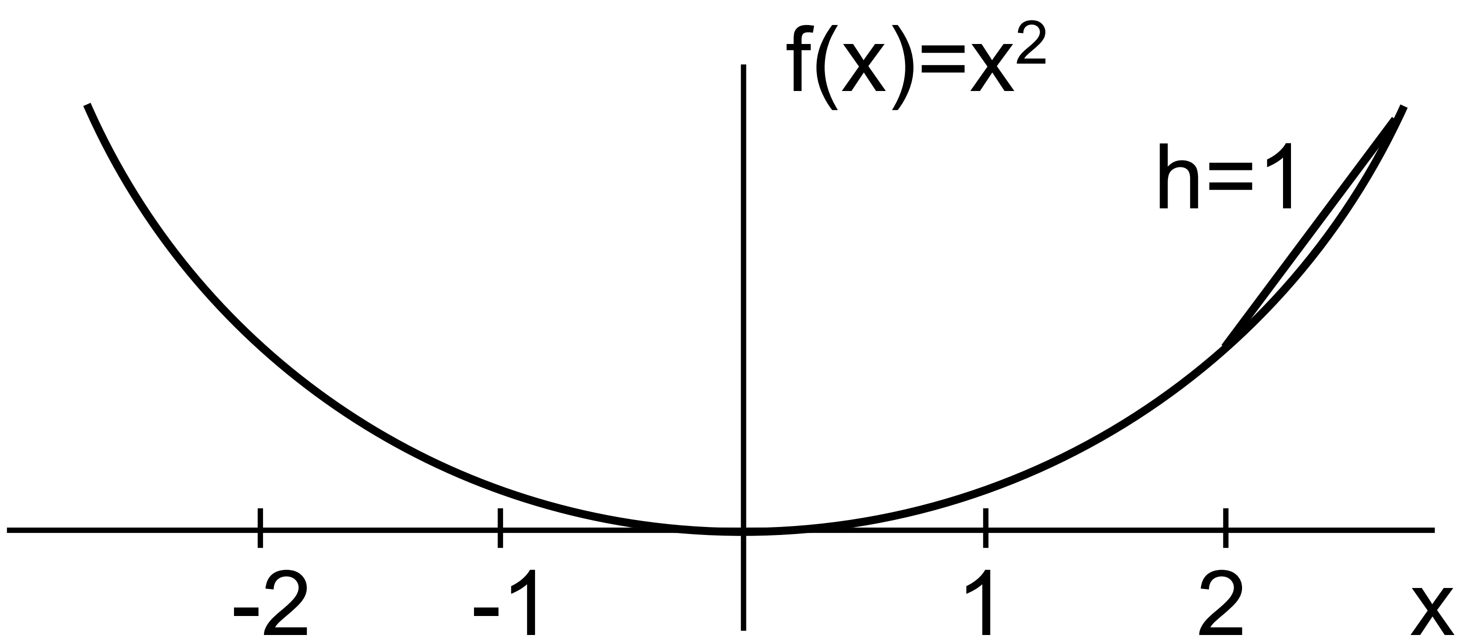


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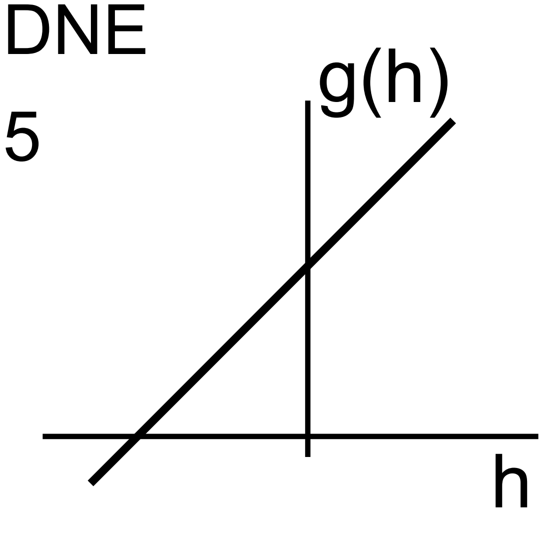

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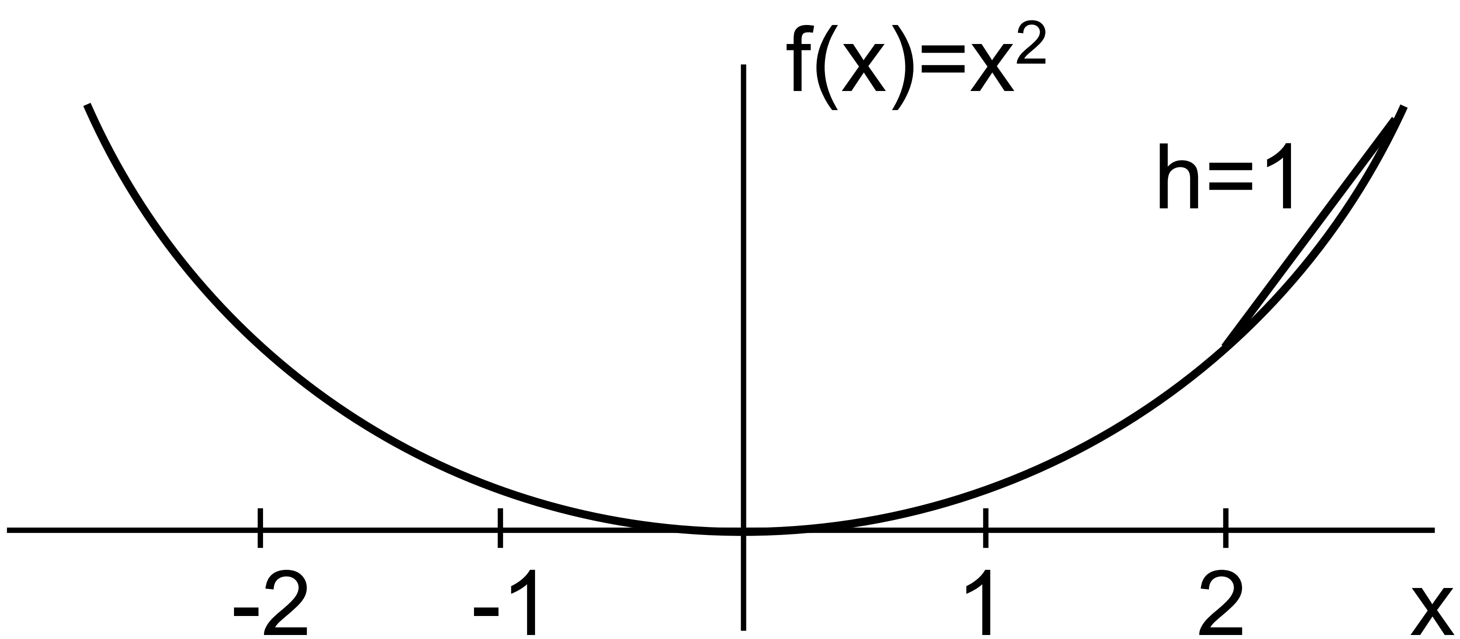


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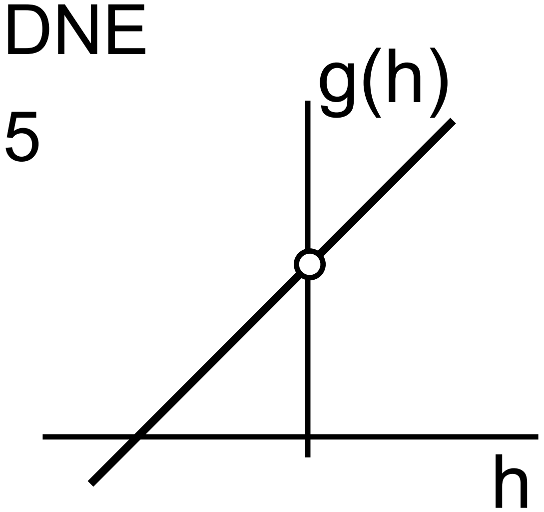

# A WeBWorK limit example

Guess the value of the limit (if it exists) by evaluating the function at values close to where the limit is to be done. If it does not exist, enter DNE below.

$$\lim_{h \to 0} \frac{\sin(\frac{\pi}{4} + h) - \sin(\frac{\pi}{4})}{h}$$

Limit:

Go to webwork Sandbox

# Calculate derivative from definition

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Calculate $f'(2)$ where $f(x) = 1/x$ on the board.

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Common notation mistake:

Do not drop the “lim” along the way!

First eliminate the 0/0 problem, evaluate, then drop “lim”.