Exercises for Math 102

University of British Columbia
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2. Review of Simple Functions

Exercise 1. A kilogram weighs about 2.2 pounds.

(a) Write a formula for the function, f, which give's an object's mass in kilograms, k, as a function of its weight in pounds, p.

(b) Find a formula for the inverse function of f. What does this inverse function tell you, in practical terms?

Exercise 2. Find the inverse function of $f(x) = \frac{x+1}{2x+1}$

Exercise 3. A right circular cone with height h and radius r has volume $V = \frac{1}{3}\pi r^2 h$. A right circular cone with height $h = 50cm$ is to be constructed. The volume of this cone is to be 600 cm^3. Use an inverse function to find the radius of the base of the cone .

Exercise 4. Consider the function with domain $[-1, \infty)$ and range $(-\infty, \infty)$ defined piecewise by

$$y = f(x) = \begin{cases} (x-2)^2, & x \geq 1, \\ x, & -1 \leq x \leq 1. \end{cases}$$

(a) Draw the graph of this function. (*Hint: To draw the graph over $x \geq 1$, apply the appropriate geometric transformations to the function $y = f(x) = x^2$.*)

(b) Find all zeroes of this function.

Exercise 5. Find all points of intersection of the two polynomials $y = p_1(x) = x^{100} + x + 2$ and $y = p_2(x) = x^{100} + x^2 + 1$.

Exercise 6. Give an accurate sketch plot of each function and its inverse function, drawn on the same axes:

(a) $f(x) = -(x/2) + 2$ on $-2 \leq x \leq 2$.

(b) $g(x) = x^2 + 1$ on $0 \leq x \leq 2$. [Hint: what is the domain of the inverse function?]

(c) $h(x) = x^2 + 2x$ on $-1 \leq x \leq 1$.

Exercise 7. Find all (real) zeros of the polynomials. a is a constant number.

(a) $f(x) = x^2 - a^2$

(b) $g(x) = x^3 - a^3$

(c) $h(x) = x^4 - a^4$

(d) $j(x) = x^n - a^n$ for n a positive integer.

Exercise 8. Sketch the graph of the function $y = ax^2 - x^5$ for constant values of the constant a, $a \neq 0$. Comment on the behaviour close to zero ($x = 0$) and far away from zero.

Exercise 9. Find the inverse function $f^{-1}(x)$ for the function

$$f(x) = (x-2)^{\frac{1}{3}} + 1,$$

where $f(x)$ is *defined on the set of nonnegative real numbers* $\{x|\ x \geq 0\}$, i.e. for the definition of f we take only nonnegative x. Determine the explicit expression for f^{-1}, and its domain and range. Sketch the graph of both f and f^{-1}.

Optional Exercise 10. In general, a function is neither odd nor even but every function $f(x)$ can be written as the sum of an odd and an even function, $f(x) = f_{\text{even}}(x) + f_{\text{odd}}(x)$. Find $f_{\text{even}}(x)$, $f_{\text{odd}}(x)$ for the following functions:

(a) $f(x) = 3x^3 - x^2 + 4x - 1$. Any conclusions about the sum of even or odd functions?

(b) $f(x) = \exp(x)$.

The hint in section 2.5.1 of the lecture notes might be useful to get you started.