Name:______________________________ Quiz Score:____/20

Student Number:______________________________

Answer questions in the space provided. Show your work.

1.

$$f(x) = \frac{-x^2+3x}{2x^3+x}$$

(a) (2 points) For $|x| << 1$, $f(x) \approx cx^n$ with constant c and integer n. What are c and n?

$-x^2+3x \approx 3x$, $2x^3+x \approx x$

$\Rightarrow f(x) \approx \frac{3x}{x} = 3$ (for $x \neq 0$)

$\Rightarrow \boxed{c=3, \quad n=0}$

(b) (2 points) For $|x| >> 1$, $f(x) \approx cx^n$ with constant c and integer n. What are c and n?

$-x^2+3x \approx -x^2$, $2x^3+x \approx 2x^3$

$\Rightarrow f(x) \approx \frac{-x^2}{2x^3} = -\frac{1}{2x} = -\frac{1}{2}x^{-1}$

$\Rightarrow \boxed{c=-1/2 \quad n=-1}$

(c) (2 points) Determine $\lim_{x\to 0} f(x)$.

$$\lim_{x\to 0}\left(\frac{-x^2+3x}{2x^3+x}\right) = \lim_{x\to 0}\left(\frac{3x}{x}\right) = \lim_{x\to 0}(3) = \boxed{3}$$

(d) (2 points) Determine $\lim_{x\to 1} f(x)$.

$f(x)$ is continuous at $x=1$ $\Rightarrow \lim_{x\to 1} f(x) = f(1)$

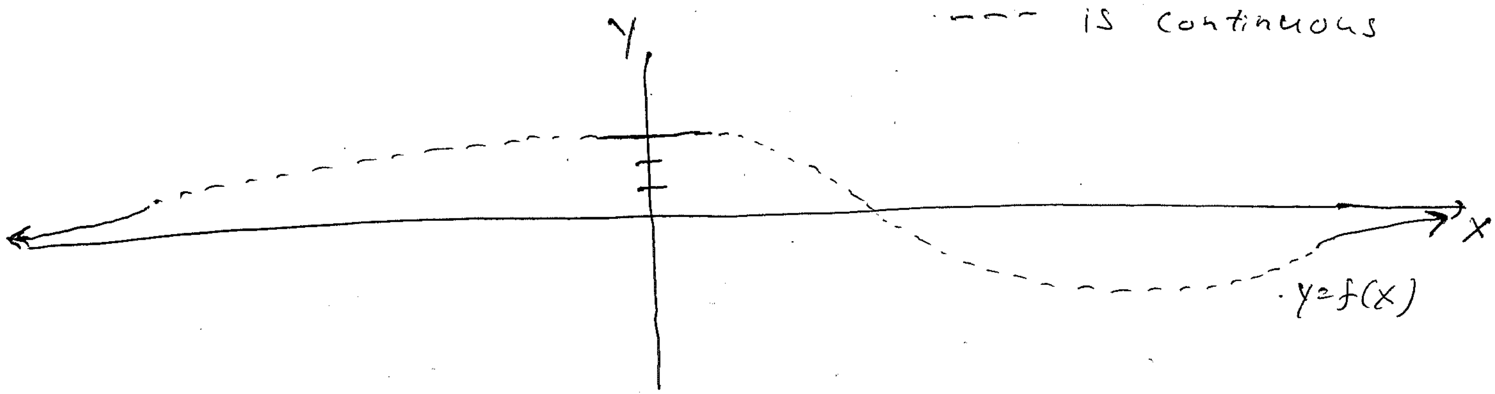
$$\lim_{x\to 1}\left(\frac{-x^2+3x}{2x^3+x}\right) = \frac{-(1)^2+3(1)}{2(1)^3+(1)} = \frac{-1+3}{2+1} = \boxed{2/3}$$

(e) (2 points) Determine $\lim_{x\to\infty} f(x)$.

$$\lim_{x\to\infty}\left(\frac{-x^2+3x}{2x^3+x}\right) = \lim_{x\to\infty}\left(\frac{-x^2}{2x^3}\right) = \lim_{x\to\infty}\left(\frac{-1}{2x}\right) = \boxed{0}$$

(f) (2 points) In a solid line, sketch the graph of $f(x)$ for small x ($|x| << 1$) and for large x ($|x| >> 1$). Based solely on the continuity of $f(x)$, fill in the remainder of your sketch with a dashed line. [Do not determine precise behaviour of f(x): zeros, minimums, maximums, or inflection points]

- - - is continuous



$\underbrace{\quad}$ $f(x) \approx -\frac{1}{2x}$ $x << -1$

$\underbrace{\quad}$ $f(x) \approx 3$ $|x| << 1$

$\underbrace{\quad}$ $f(x) \approx -\frac{1}{2x}$ $x >> 1$

2. (a) (4 points) For a differentiable function $f(x)$, what is the definition of $f'(x)$ in the form of a limit?

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

(b) (4 points) For $f(x) = x^2 + x + 1$, determine $f'(x)$ from the limit definition of the derivative.

$$f'(x) = \lim_{h \to 0} \left(\frac{(x+h)^2 + (x+h) + 1 - (x^2 + x + 1)}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{x^2 + 2xh + h^2 + x + h + 1 - x^2 - x - 1}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{2xh + h^2 + h}{h} \right)$$

$$= \lim_{h \to 0} \left(2x + h + 1 \right)$$

$$= 2x + 1$$