Today

- A comment on derivative notation.
- Power rule.
- Rules for differentiating sum, products and quotients of functions.
- Antiderivatives of power functions

A comment on derivative notation

$$y = f(x)$$

Leibniz → $$\frac{dy}{dx} = f'(x)$$ ← Newton

$$\left.\frac{dy}{dx}\right|_{x=2} = f'(2)$$

Power rule

$$f(x) = x^2$$

Find f' at x=2 (using the definition of the derivative).

$$f'(2) = \lim_{h\to 0} \frac{(2+h)^2 - 2^2}{h}$$

$$= \lim_{h\to 0} \frac{4 + 4h + h^2 - 4}{h}$$

$$= \lim_{h\to 0} \frac{4h + h^2}{h} = 4$$

Power rule

$$f(x) = x^2$$

Find f′ at all points x at the same time

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2}{h} = 2x$$

Power rule

$$f(x) = x^3$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3\cancel{h}x^2 + 3h^{\cancel{2}}x + h^{\cancel{3}2} - \cancel{x^3}}{\cancel{h}}$$

$$= 3x^2$$

Power rule

$$f(x) = x^n$$

$$f'(x) = nx^{n-1}$$

Suppose $f(x) = g(x) + k(x)$ and that

$$g(2) = 3,\ k(2) = 1,\ g'(2) = 2,\ k'(2) = 5.$$

- What is f'(2)?

(A) 4

(B) 7

(C) 10

(D) 11

Suppose $f(x) = g(x) + k(x)$ and that $g(2) = 3,\ k(2) = 1,\ g'(2) = 2,\ k'(2) = 5.$

- What is f'(2)?

(A) 4

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(C) 10

(D) 11

Suppose $f(x) = g(x)k(x)$ and that

$$g(2) = 3,\ k(2) = 1,\ g'(2) = 2,\ k'(2) = 5.$$

- What is f'(2)?

(A) 3

(B) 10

(C) 11

(D) 17

Suppose $f(x) = g(x)k(x)$ and that $g(2) = 3,\ k(2) = 1,\ g'(2) = 2,\ k'(2) = 5.$

- What is f'(2)?

(A) 3

(B) 10

(C) 11

(D) 17

Try g(x)=x and k(x)=x^2.

If f'(x)=g'(x)k'(x) then

f(x) = (x) (x^2) so f'(x)= (1) (2x)=2x.

But f(x)=x^3 and power rule says

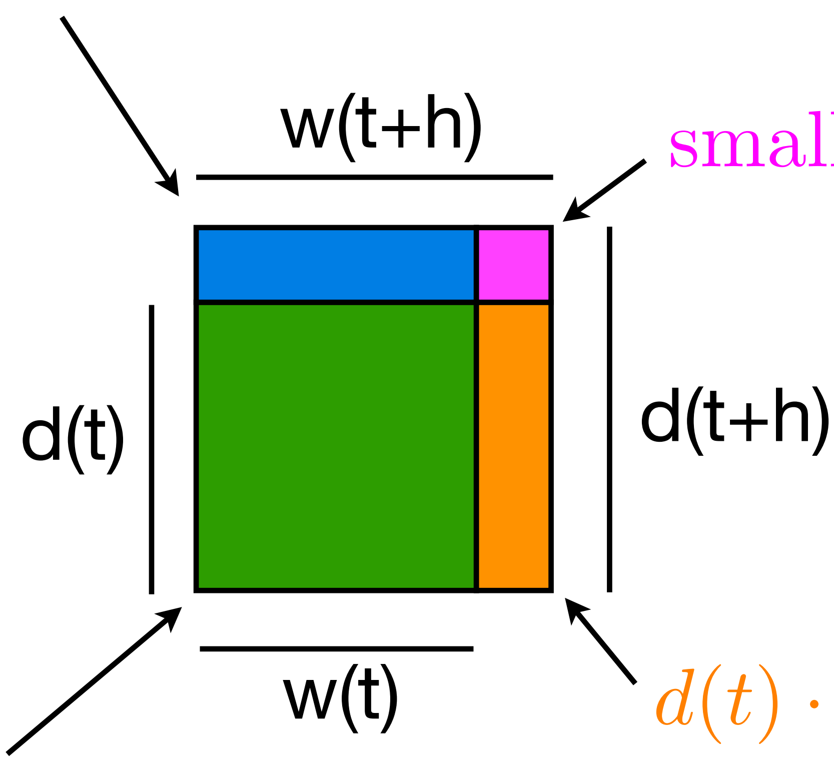
f'(x) = $3x^2$.

So g'(x)k'(x) can't be right.

What is the correct derivative for f(x)=g(x)k(x)?

$$A(t) = d(t)w(t)$$

$$A(t+h) = A(t) + (d(t+h) - d(t)) \cdot w(t)$$
$$+ d(t) \cdot (w(t+h) - w(t)) + \text{small corner}$$



$$\frac{A(t+h) - A(t)}{h} \approx \frac{(d(t+h) - d(t)) \cdot w(t)}{h} + \frac{d(t) \cdot (w(t+h) - w(t))}{h}$$

$$A'(t) = d'(t)w(t) + d(t)w'(t)$$

Rules for differentiation - summary

- Addition rule:
  - $f(x) = g(x) + h(x)$
  - $f'(x) = g'(x) + h'(x)$
- Product rule:
  - $f(x) = g(x)h(x)$
  - $f'(x) = g'(x)h(x) + g(x)h'(x)$
- Quotient rule (can be justified once we cover chain rule):
  - $f(x) = g(x) / h(x) = g(x) (h(x))^{-1}$ <---- apply product and chain rules or
  - $f'(x) = [g'(x)h(x) - g(x) h'(x)] / g(x)^2$

Suppose $f(x) = g(x)/k(x)$ and that $g(2) = 3,\ k(2) = 1,\ g'(2) = 2,\ k'(2) = 5.$

- What is f'(2)?

  (A) -13

  (B) -13/25

  (C) 17

  (D) 17/25

Suppose $f(x) = g(x)/k(x)$ and that $g(2) = 3,\ k(2) = 1,\ g'(2) = 2,\ k'(2) = 5.$

- What is f'(2)?

(A) -13

(B) -13/25

(C) 17

(D) 17/25

Antiderivatives - going backward

If $f'(x) = 6x^2 + 4x - 1$, then

(A) $f(x) = 12x + 4$

(B) $f(x) = 2x^3 + 2x^2 - x$

(C) $f(x) = 2x^3 + 2x^2 - x + 2$

(D) $f(x) = 2x^3 + 2x^2 - x + C$

Antiderivatives - going backward

If $f'(x) = 6x^2 + 4x - 1$, then

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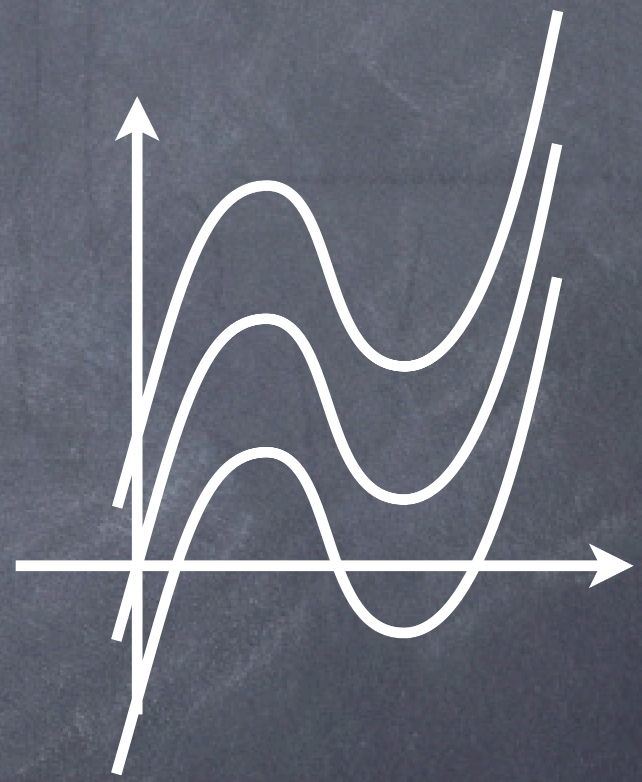

Slopes at each x don't change with vertical shift.

If $f'(x) = x^n$, which of the following could be f(x)?

(A) $f(x) = \frac{1}{n+1}x^{n+1}$

(B) $f(x) = \frac{1}{n+1}x^{n+1} + C$

(C) $f(x) = nx^{n-1}$

(D) $f(x) = nx^{n-1} + C$

(E) $f(x) = x^n + C$

If $f'(x) = x^n$, which of the following could be f(x)?

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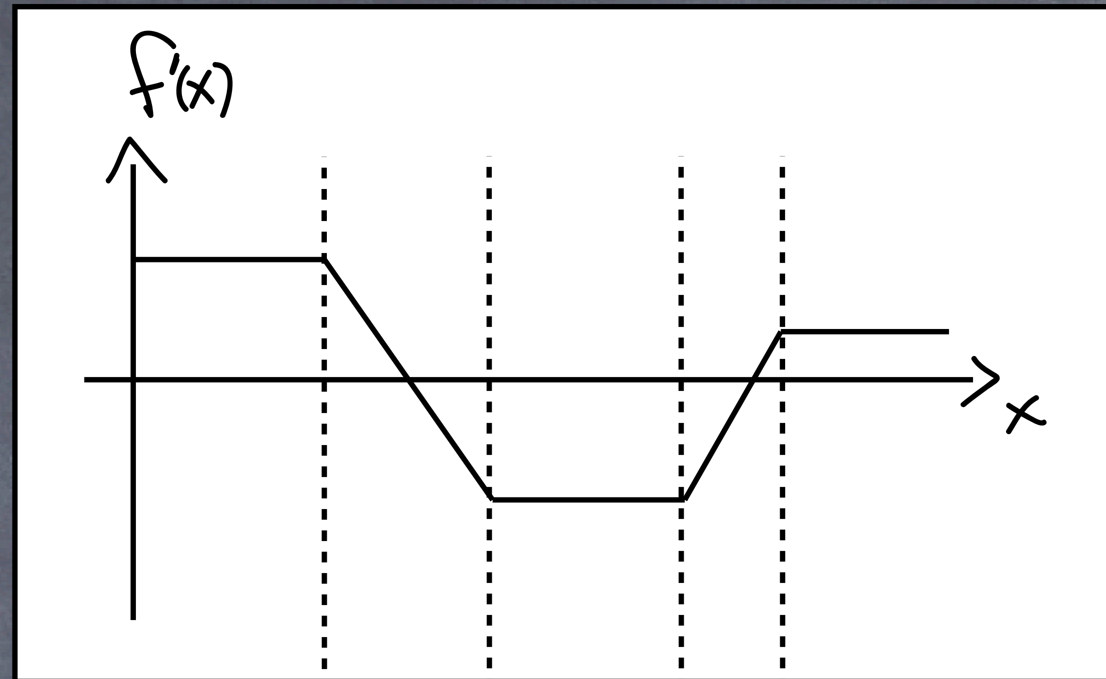
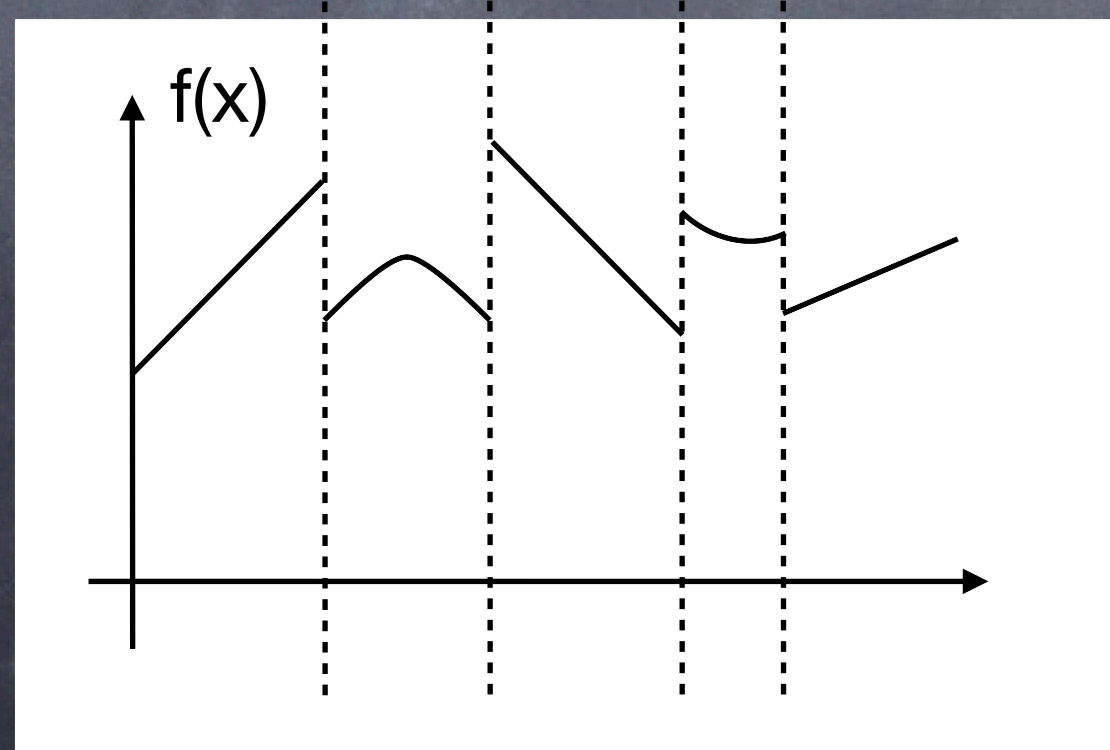


This is f′(x). Draw f(x).



Only determined up to a vertical shift.

Position-Velocity-Acceleration

- If $x(t)$ is position as a function of time,
  - velocity $v(t) = x'(t)$,
  - acceleration $a(t) = v'(t) = x''(t)$.
- Constant acceleration a:
  - $v(t) = at + C = at + v_0$
  - $x(t) = a/2\ t^2 + v_0 t + D = a/2\ t^2 + v_0 t + x_0$

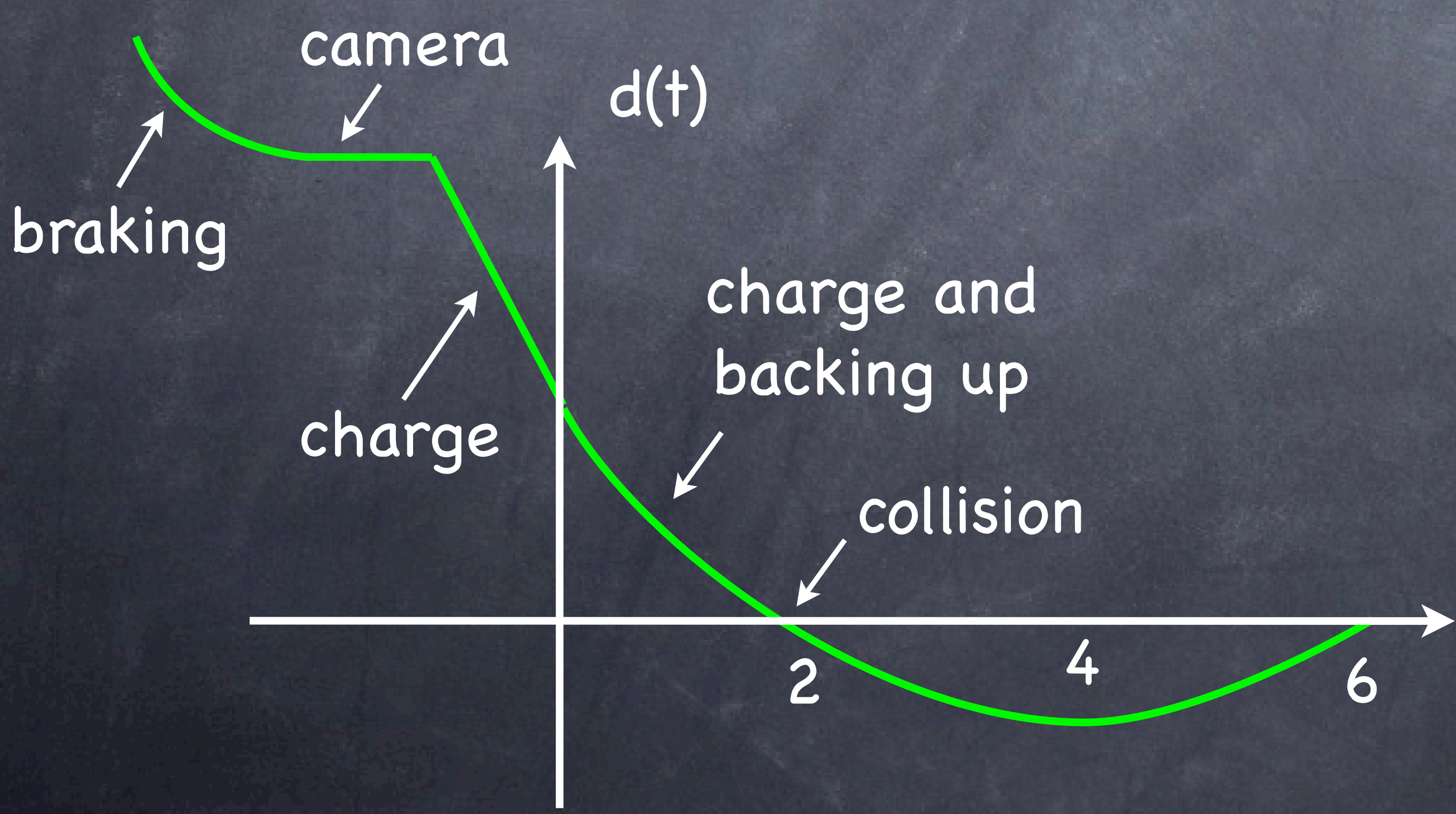
Examples of approximately constant acceleration

- Ball dropping near surface of planet
- Fireworks

From the 2011 final exam

8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s^2.

   i. (4 points) Write down a function $d(t)$ that is the distance from your car to the moose where $t = 0$ indicates the moment when you start backing away.



$v_{car}(t) = 2t$

$x_{car}(t) = t^2 + 12$

$v_{dist}(t) = 2t - 8$

$x_{dist}(t) = t^2 - 8t + 12$

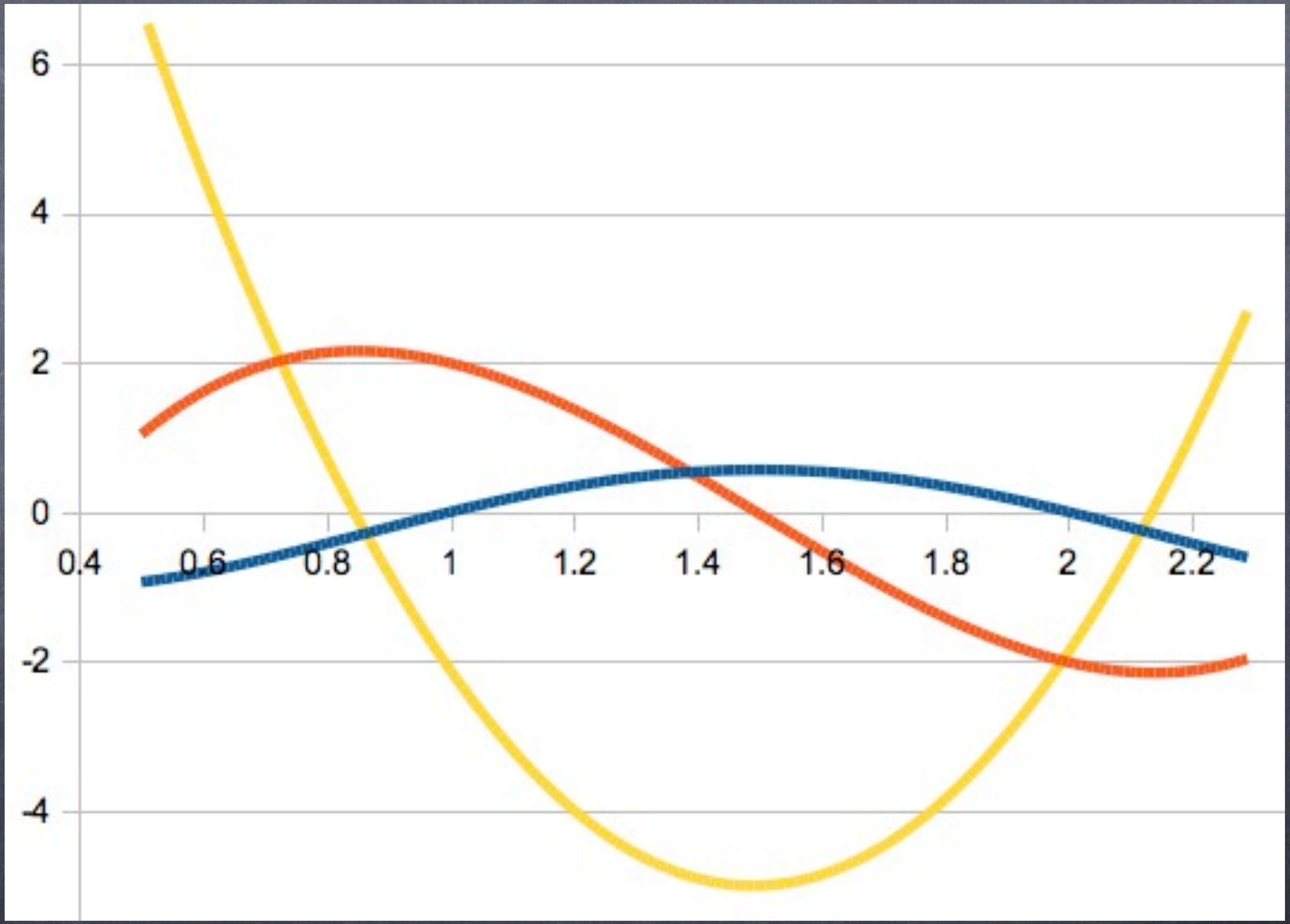
Which is x, v, a?

(A) x, v, a

(B) x, v, a

(C) x, v, a

(D) x, v, a



Check max/mins --> zeros, check inc/dec --> +/-.

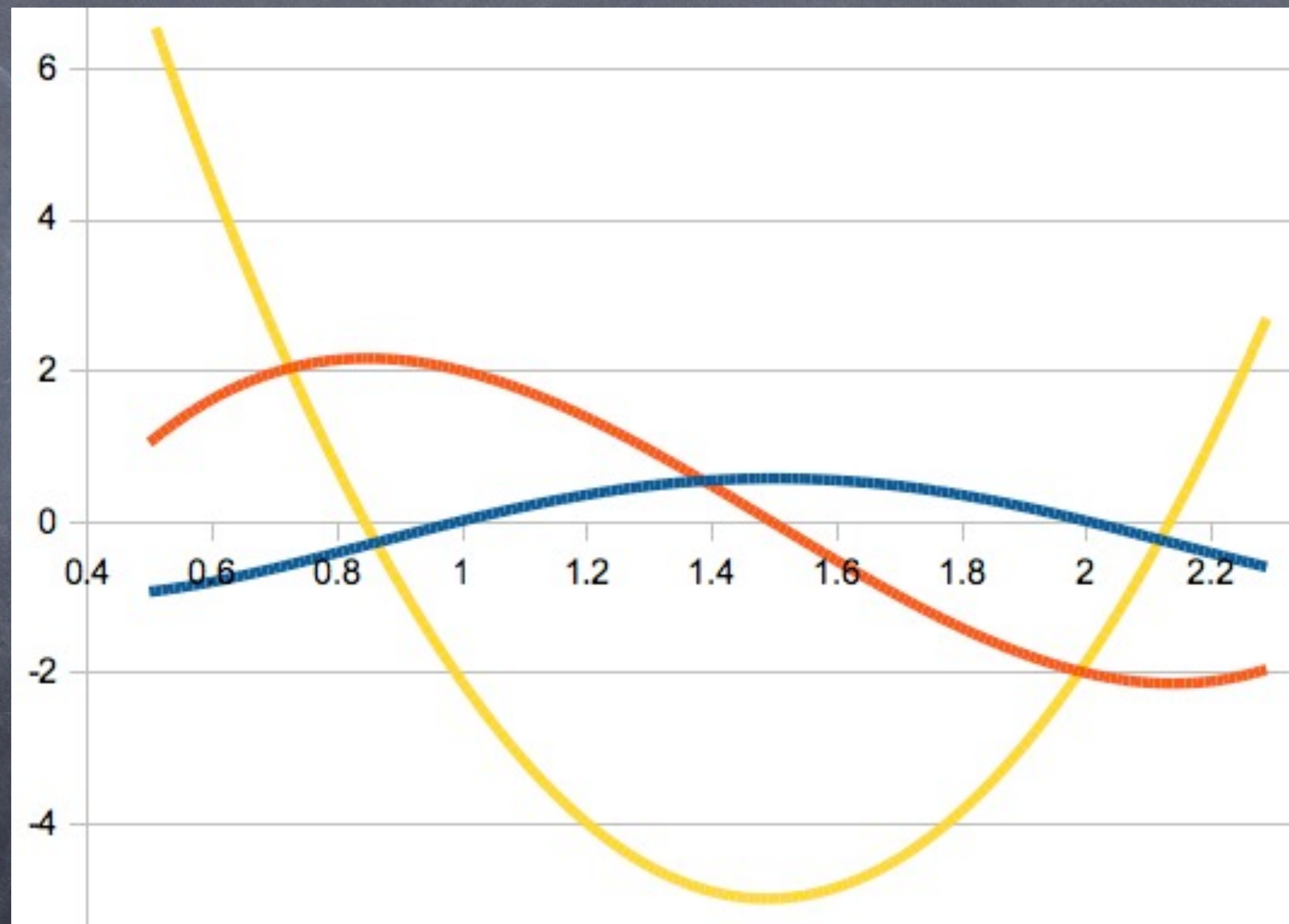
Which is x, v, a?

(A) x, v, a

(B) x, v, a

(C) x, v, a

(D) x, v, a



Check max/mins --> zeros, check inc/dec --> +/-.

Product rule: If $k(x)=f(x)g(x)$ then $k'(x) = ?$

- (A) $f'(x)g(x)$
- (B) $f(x)g'(x)$
- (C) $f'(x)g(x) + f(x)g'(x)$
- (D) $f'(x)g'(x)$

Example: k(x)=(x^5-2x^3+x^2+3)(3x^3-x^2+1)

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Example: $k(x)=(x^5-2x^3+x^2+3)(3x^3-x^2+1)$

Quotient rule: If $k(x)=f(x)/g(x)$ then $k'(x) = ?$

- (A) $f'(x)/g'(x)$
- (B) $[\ f'(x)g(x) - f(x)g'(x)\ ] / g(x)^2$
- (C) $f'(x)g(x) + f(x)g'(x)$
- (D) $f'(x)/g(x)$

Example: k(x)=2x^2/(3x+1)

Quotient rule: If $k(x)=f(x)/g(x)$ then $k'(x) = ?$

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- (B) $[\ f'(x)g(x) - f(x)g'(x)\ ]\ /\ g(x)^2$
- (C) $f'(x)g(x) + f(x)g'(x)$
- (D) $f'(x)/g(x)$

Example: k(x)=2x^2/(3x+1)

What is k′(x) if $k(x) = \dfrac{2x^2}{3x+1}$?

(A) $k'(x) = \dfrac{4x}{3}$

(B) $k'(x) = \dfrac{4x}{3x+1} - \dfrac{2x^2}{3}$

(C) $k'(x) = \dfrac{6x^2+4x}{(3x+1)^2}$

(D) $k'(x) = \dfrac{4x}{3x+1} - \dfrac{2x^2}{(3x+1)^2}$

What is k'(x) if $k(x) = \dfrac{2x^2}{3x+1}$?

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