# Today

- Trig derivatives
- Related rates with trig
  - Zebra Danio
  - Hands on a clock
- Reminders:
  - Friday is the last day of classes :o
  - Exam: Dec 6 @ 3:30 pm - SRC ABC

Before taking Math 102, I was aware that mathematics can be applied to problems in the Life Sciences.

(A) Strongly agree.

(B) Agree.

(C) Neutral

(D) Disagree

(E) Strongly disagree

After taking Math 102, my awareness that mathematics can be applied to problems in the Life Sciences has increased.

(A) Strongly agree.

(B) Agree.

(C) Neutral

(D) Disagree

(E) Strongly disagree

# Derivative of $f(x)=\sin(x)$

# Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} ( f(x+h) - f(x) )/ h$

# Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h \to 0} ( f(x+h) - f(x) ) / h$

  $= \lim_{h \to 0} ( \sin(x+h) - \sin(x) ) / h$

# Derivative of $f(x)=\sin(x)$

- $f'(x) = \lim_{h\to 0} ( f(x+h) - f(x) )/ h$

  $= \lim_{h\to 0} ( \sin(x+h) - \sin(x) ) /h$

  $= \lim_{h\to 0} ( \sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x) ) /h$

# Derivative of $f(x)=\sin(x)$

- $f'(x) = \lim_{h\to 0} ( f(x+h) - f(x) )/ h$

$= \lim_{h\to 0} ( \sin(x+h) - \sin(x) ) /h$

$= \lim_{h\to 0} ( \sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x) ) /h$

$= \lim_{h\to 0} ( \sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

# Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} ( f(x+h) - f(x) )/ h$

  $= \lim_{h\to 0} ( \sin(x+h) - \sin(x) ) /h$

  $= \lim_{h\to 0} ( \sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x) ) /h$

  $= \lim_{h\to 0} ( \sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

  $= \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

  $+ \cos(x) \lim_{h\to 0} \sin(h) /h$

# Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} ( f(x+h) - f(x) )/ h$

$= \lim_{h\to 0} ( \sin(x+h) - \sin(x) ) /h$

$= \lim_{h\to 0} ( \sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x) ) /h$

$= \lim_{h\to 0} ( \sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

$= \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

$+ \cos(x) \lim_{h\to 0} \sin(h) /h$

See what h=0.0001 gives...

# Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} ( f(x+h) - f(x) )/ h$

$= \lim_{h\to 0} ( \sin(x+h) - \sin(x) ) /h$

$= \lim_{h\to 0} ( \sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x) ) /h$

$= \lim_{h\to 0} ( \sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

$= \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

$+ \cos(x) \lim_{h\to 0} \sin(h) /h$

$= \sin(x) \times 0 + \cos(x) \times 1 = \cos(x).$

# Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} ( f(x+h) - f(x) )/ h$

$= \lim_{h\to 0} ( \sin(x+h) - \sin(x) ) /h$

$= \lim_{h\to 0} ( \sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x) ) /h$

$= \lim_{h\to 0} ( \sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

$= \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

$+ \cos(x) \lim_{h\to 0} \sin(h) /h$

$= \sin(x) \times 0 + \cos(x) \times 1 = \cos(x).$

Note: this last step requires a bunch of work to show.

# Derivative of $g(x)=\cos(x)$

- $g'(x) = -\sin(x)$
- See last lecture's posted slides.

# Other trig functions

The derivative of cot(x) is

(A) csc(x)cot(x)

(B) -csc(x)cot(x)

(C) $csc^2(x)$

(D) $-csc^2(x)$

(E) $sec^2(x)$

# Other trig functions

The derivative of cot(x) is

(A) csc(x)cot(x)

(B) -csc(x)cot(x)

(C) $\csc^2(x)$

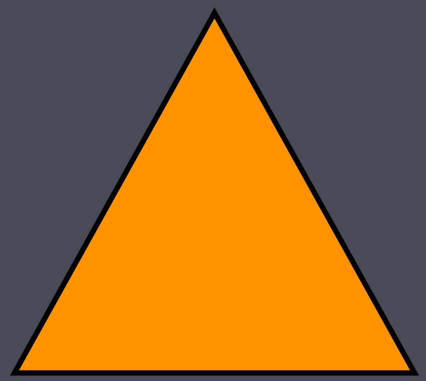
(D) $-\csc^2(x)$

(E) $\sec^2(x)$

Rewrite
cot(x) = cos(x)/sin(x)
and use quotient rule.

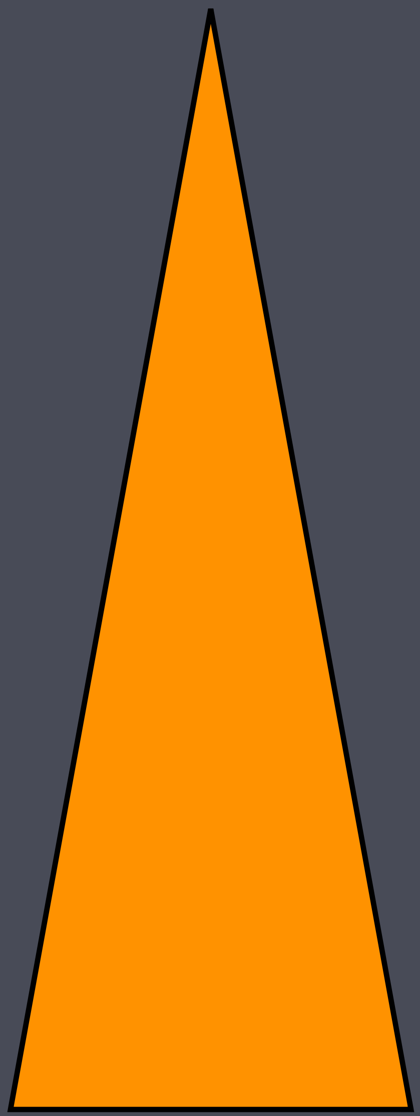
# Trig-related rates

- These usually come down to a triangle that changes in time. For example...



# Trig-related rates

- These usually come down to a triangle that changes in time. For example...



If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

Relate the two changing quantities (h and θ):

(A) $\sin(\theta) = 2/h$

(B) $\sin(\theta/2) = 1/h$

(C) $\sin(\theta/2) = 1/\mathrm{sqrt}(1+h^2)$

(D) $\tan(\theta) = 2/h$

(E) $\tan(\theta/2) = 1/h$

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

Relate the two changing quantities (h and θ):

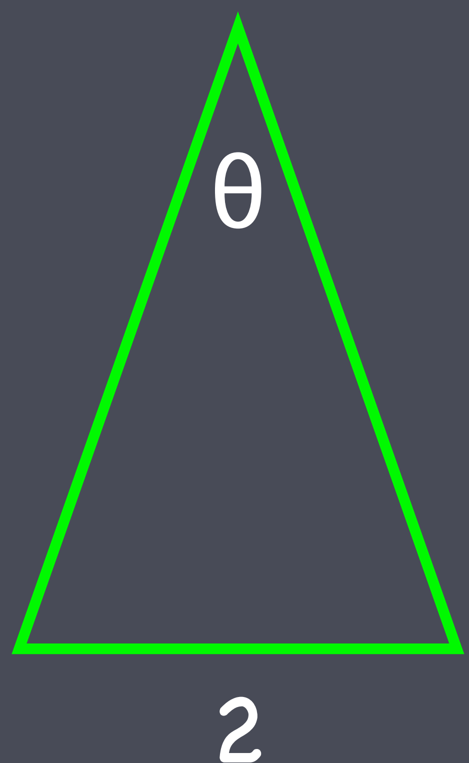
(A) $\sin(\theta) = 2/h$

(B) $\sin(\theta/2) = 1/h$

(C) $\sin(\theta/2) = 1/\mathrm{sqrt}(1+h^2)$

(D) $\tan(\theta) = 2/h$

(E) $\tan(\theta/2) = 1/h$



If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

Relate the two changing quantities (h and θ):

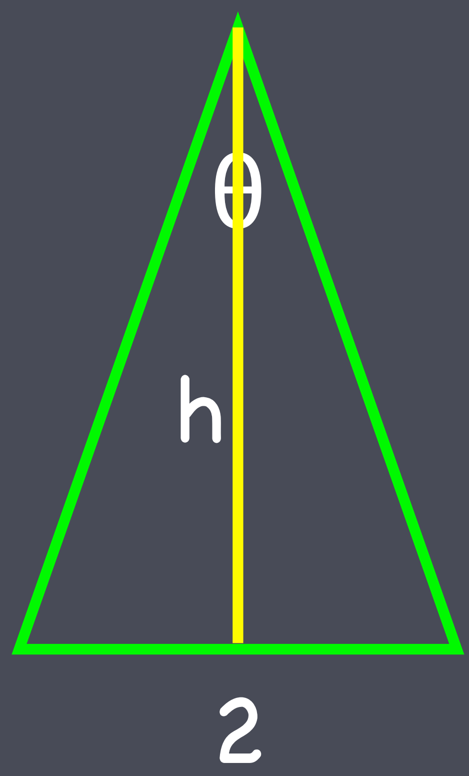
(A) $\sin(\theta) = 2/h$

(B) $\sin(\theta/2) = 1/h$

(C) $\sin(\theta/2) = 1/\mathrm{sqrt}(1+h^2)$

(D) $\tan(\theta) = 2/h$

(E) $\tan(\theta/2) = 1/h$



If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

Relate the two changing quantities (h and θ):

(A) $\sin(\theta) = 2/h$

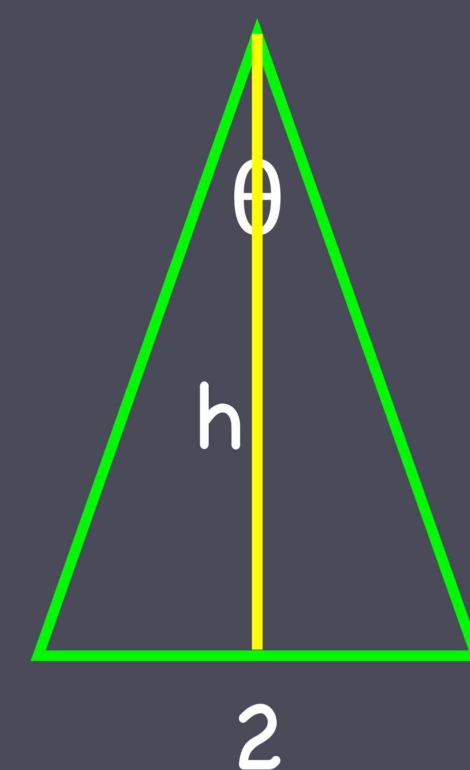
(B) $\sin(\theta/2) = 1/h$

(C) $\sin(\theta/2) = 1/\text{sqrt}(1+h^2)$

This will get messy.

(D) $\tan(\theta) = 2/h$

(E) $\tan(\theta/2) = 1/h$



If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

- Take derivatives to relate their rates of change ($h'$ and $\theta'$):
    - $\tan(\theta/2) = 1/h$

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

- Take derivatives to relate their rates of change (h' and θ'):
  - $\tan(\theta/2) = 1/h$
  - $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

- Take derivatives to relate their rates of change (h' and θ'):
  - $\tan(\theta/2) = 1/h$
  - $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$
  - $\theta' = -2\ h'/(h^2 \sec^2(\theta/2)) = -2\ h' \cos^2(\theta/2)\ /h^2$

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

- Take derivatives to relate their rates of change (h' and θ'):
  - $\tan(\theta/2) = 1/h$
  - $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$
  - $\theta' = -2\ h'/(h^2 \sec^2(\theta/2)) = -2\ h' \cos^2(\theta/2)\ /h^2$

    $= -2 \cos^2(\theta/2)$

If the height of an isosceles triangle with base 2m changes at a rate $h'=3$ m/s, how quickly is the angle opposite the base changing when $h=\sqrt{3}$ m?

- Take derivatives to relate their rates of change ($h'$ and $\theta'$):
  - $\tan(\theta/2) = 1/h$
  - $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$
  - $\theta' = -2\ h'/(h^2 \sec^2(\theta/2)) = -2\ h' \cos^2(\theta/2)\ /h^2$

    $= -2 \cos^2(\theta/2)$

$\theta = \ldots$ (A) $\pi/6$ (B) $\pi/4$ (C) $\pi/3$ (D) $2\pi/3$ (E) $\pi$.

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

- Take derivatives to relate their rates of change (h' and θ'):
  - $\tan(\theta/2) = 1/h$
  - $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$
  - $\theta' = -2\ h'/(h^2 \sec^2(\theta/2)) = -2\ h' \cos^2(\theta/2)\ /h^2$

    $= -2 \cos^2(\theta/2)$

θ = ... (A) π/6 (B) π/4 (C) π/3 (D) 2π/3 (E) π.

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

- Take derivatives to relate their rates of change (h' and θ'):
  - $\tan(\theta/2) = 1/h$
  - $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$
  - $\theta' = -2\ h'/(h^2 \sec^2(\theta/2)) = -2\ h' \cos^2(\theta/2)\ /h^2$

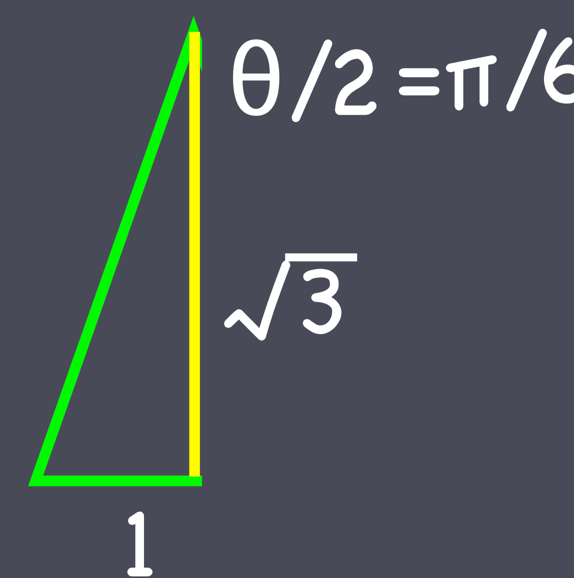
    $= -2 \cos^2(\theta/2)$


θ = … (A) π/6 (B) π/4 (C) π/3 (D) 2π/3 (E) π.

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

- Take derivatives to relate their rates of change (h' and θ'):
  - $\tan(\theta/2) = 1/h$
  - $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$
  - $\theta' = -2\ h'/(h^2 \sec^2(\theta/2)) = -2\ h' \cos^2(\theta/2)\ /h^2$

    $= -2 \cos^2(\theta/2)$

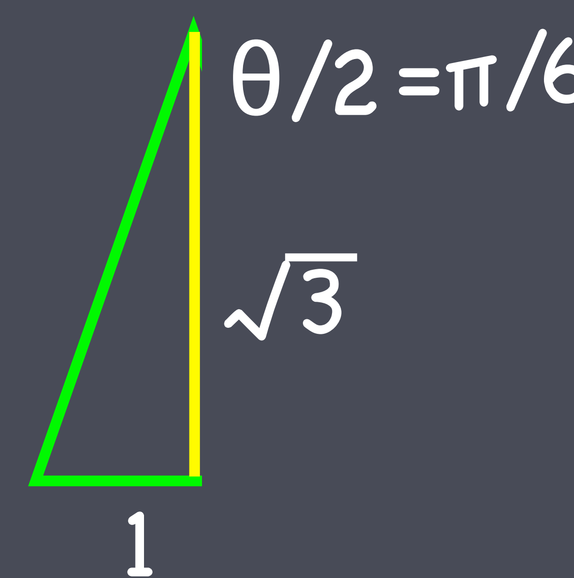



θ = ... (A) π/6 (B) π/4 (C) π/3 (D) 2π/3 (E) π.

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

- Take derivatives to relate their rates of change (h' and θ'):
  - $\tan(\theta/2) = 1/h$
  - $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$
  - $\theta' = -2\ h'/(h^2 \sec^2(\theta/2)) = -2\ h' \cos^2(\theta/2)\ /h^2$

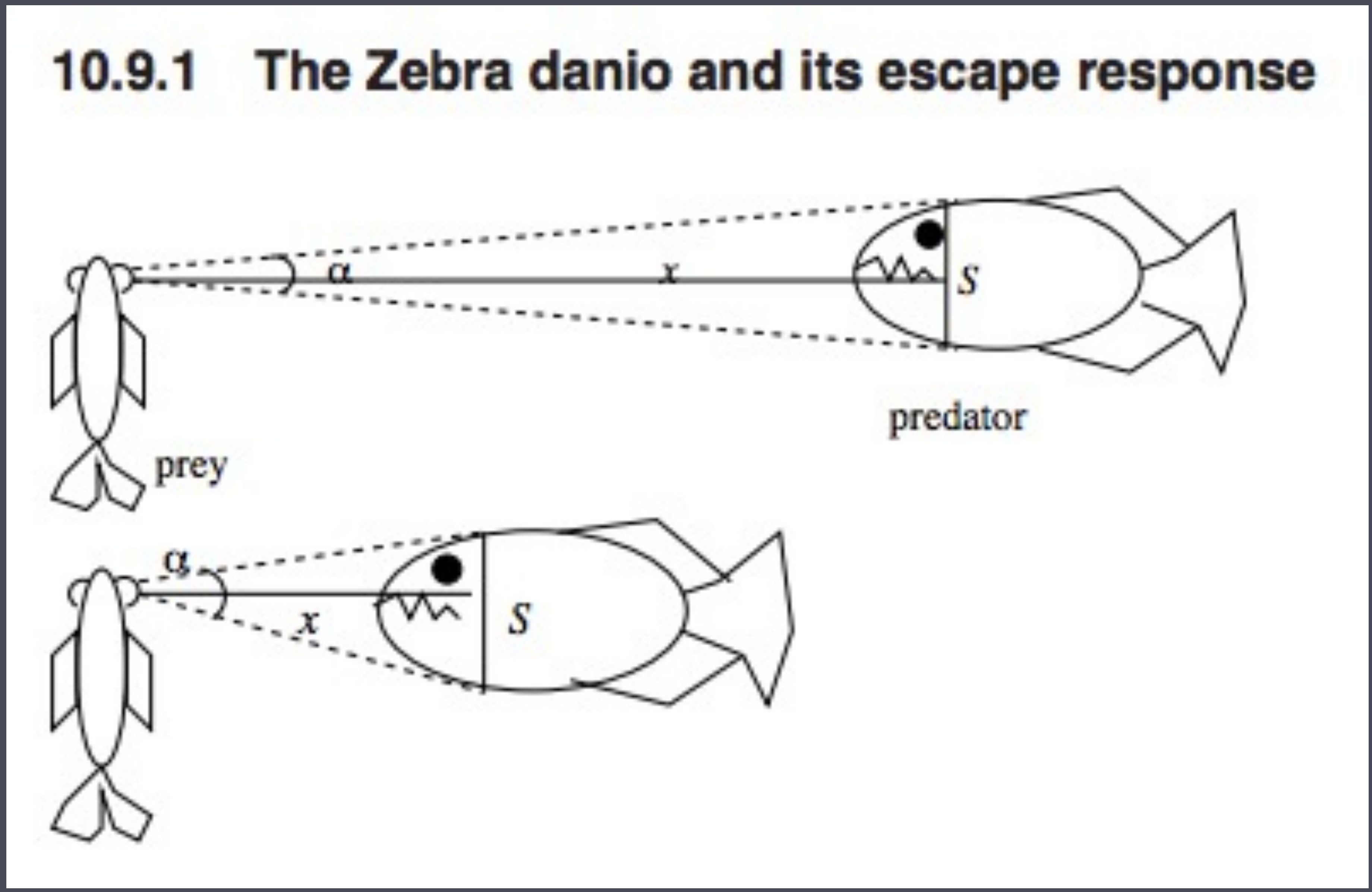
    $= -2 \cos^2(\theta/2) \quad = -3/2$ radians/s




θ = ... (A) π/6 (B) π/4 (C) π/3 (D) 2π/3 (E) π.

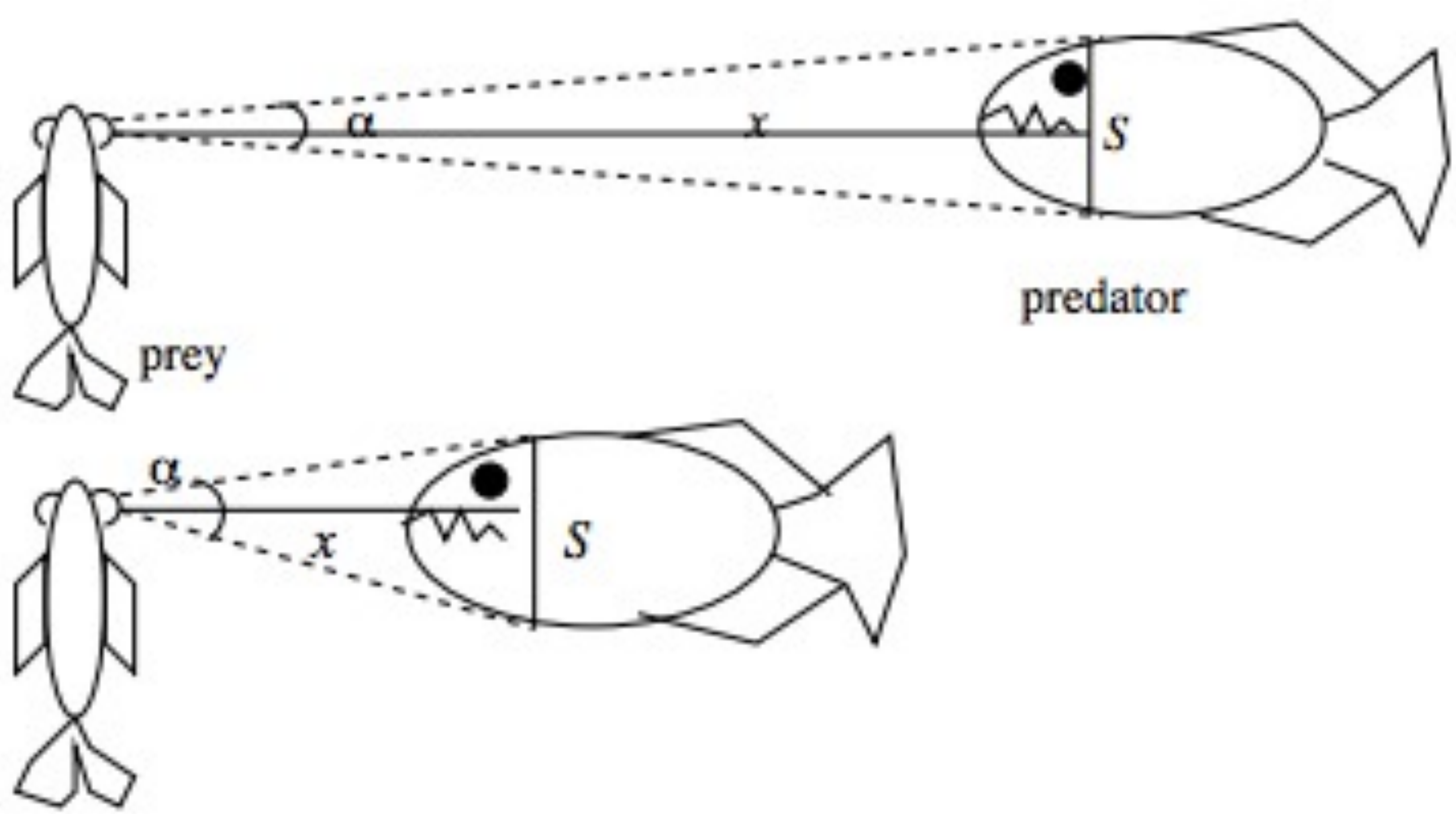
# Zebra Danio escape response

# Zebra Danio escape response



# Zebra Danio escape response

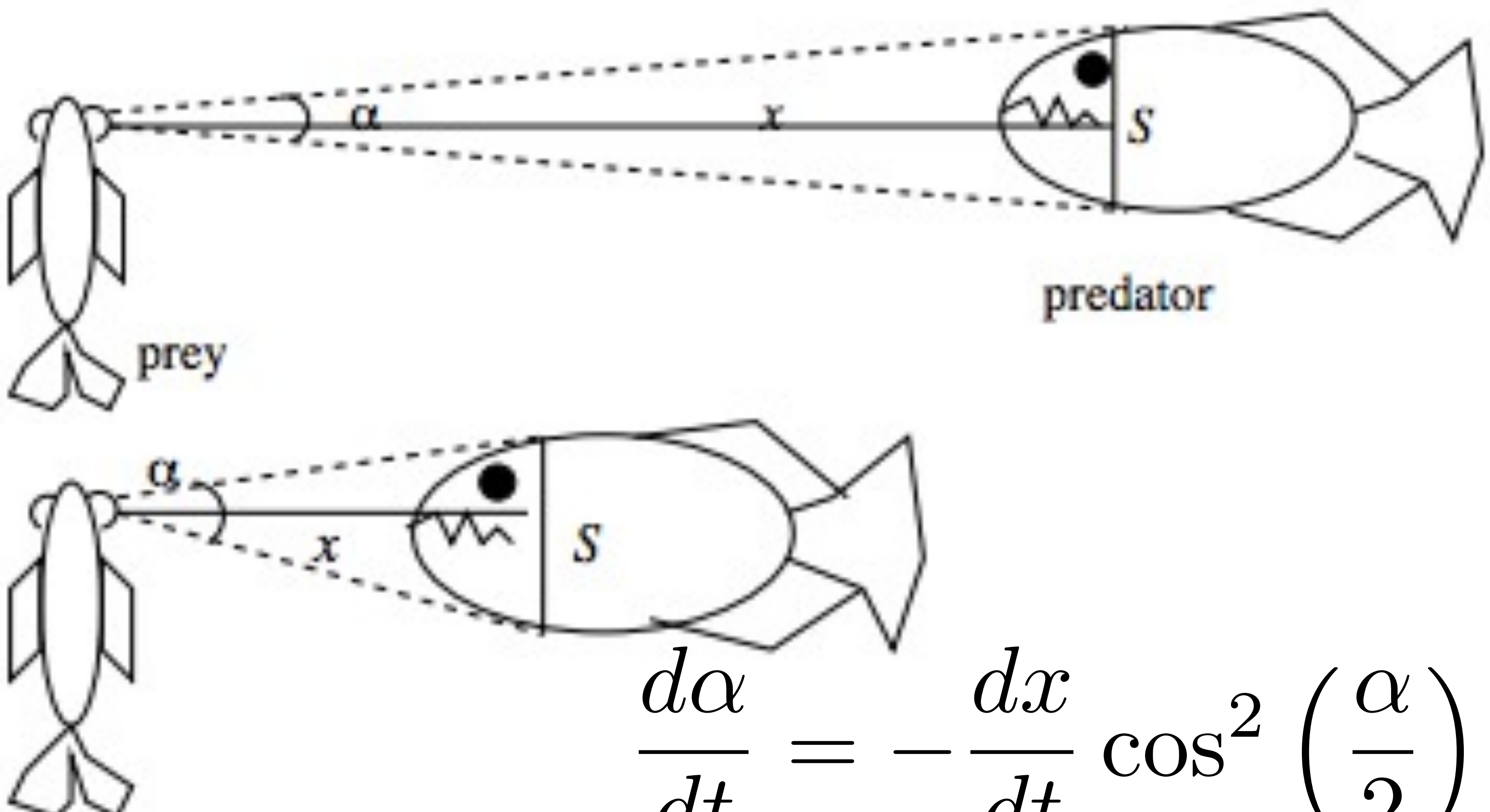
### 10.9.1 The Zebra danio and its escape response




ZD tries to escape when α′ is above a threshold value.

# Zebra Danio escape response

**10.9.1 The Zebra danio and its escape response**




$$\frac{d\alpha}{dt} = -\frac{dx}{dt}\cos^2\left(\frac{\alpha}{2}\right)\frac{S}{x^2}$$

ZD tries to escape when $\alpha'$ is above a threshold value.

# What is $\cos^2(a)$ when $\tan(a)=p/q$?

(A) $(p^2+q^2) / q^2$

(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) $p^2/q^2$

# What is $\cos^2(a)$ when $\tan(a)=p/q$?

(A) $(p^2+q^2) / q^2$

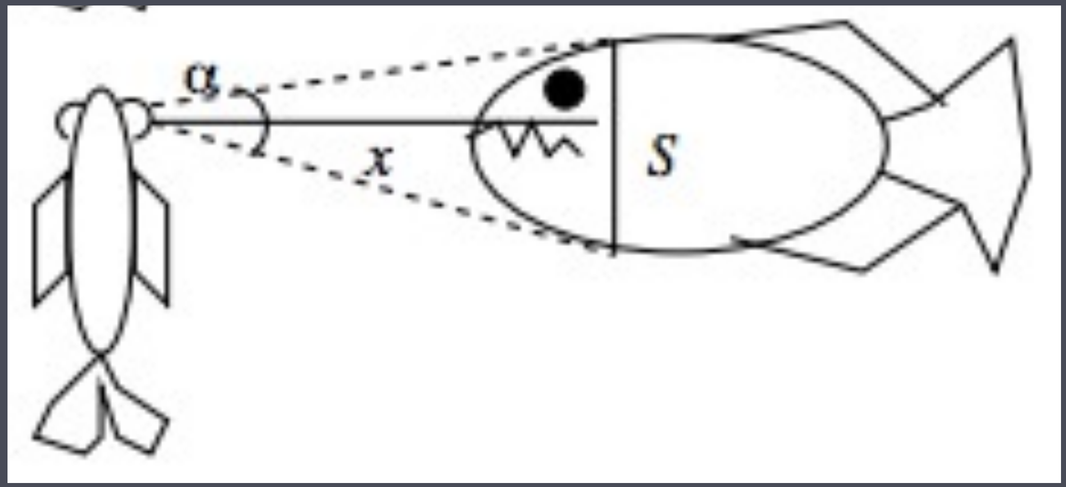
(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) $p^2/q^2$

# What is $\cos^2(a)$ when $\tan(a)=p/q$?




(A) $(p^2+q^2) / q^2$

(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) $p^2/q^2$

# What is $\cos^2(a)$ when $\tan(a)=p/q$?

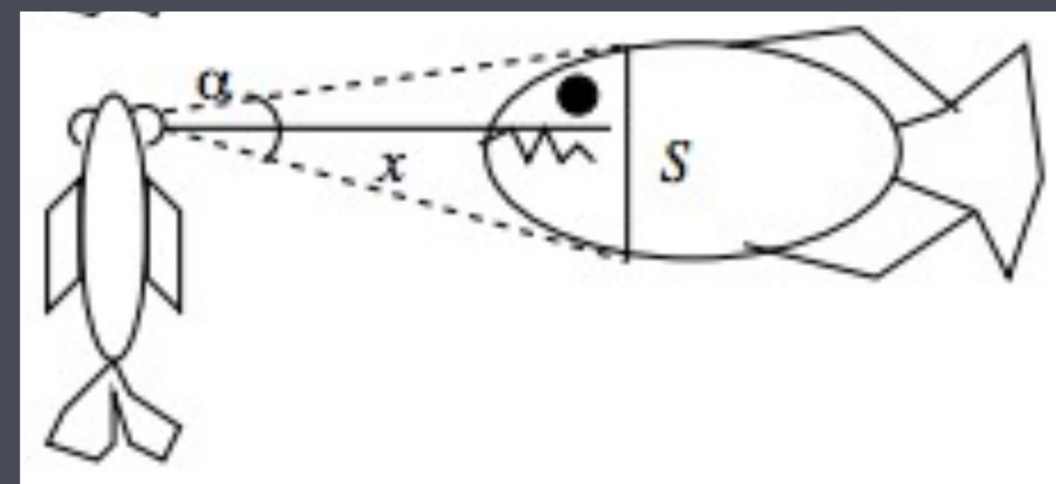
(A) $(p^2+q^2) / q^2$

(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) $p^2/q^2$




$$\frac{d\alpha}{dt} = -\frac{dx}{dt}\cos^2\left(\frac{\alpha}{2}\right)\frac{S}{x^2}$$

# What is $\cos^2(a)$ when $\tan(a)=p/q$?

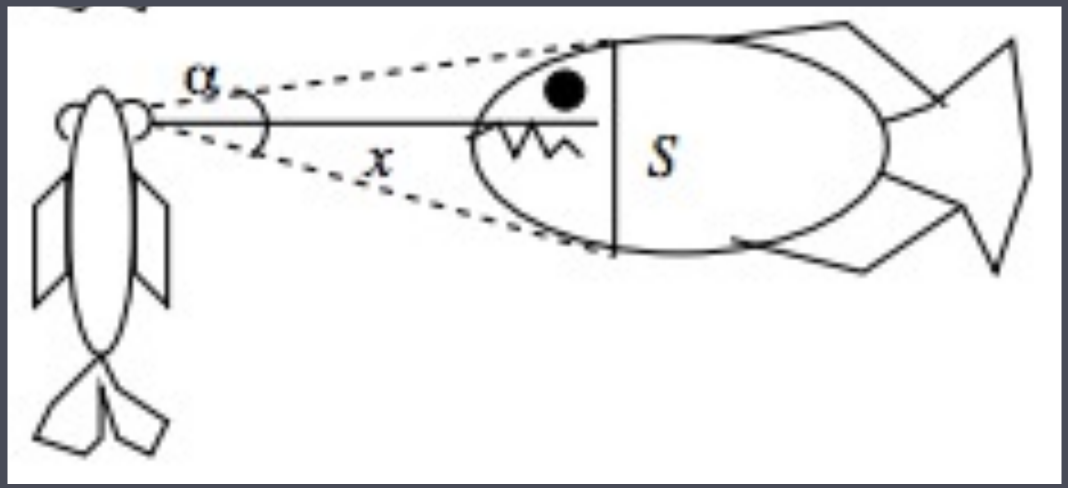
(A) $(p^2+q^2) / q^2$

(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) $p^2/q^2$




$$\frac{d\alpha}{dt} = -\frac{dx}{dt}\cos^2\left(\frac{\alpha}{2}\right)\frac{S}{x^2}$$

$$= -\frac{dx}{dt}\frac{x^2}{x^2+\frac{S^2}{4}}\frac{S}{x^2}$$

# What is $\cos^2(a)$ when $\tan(a)=p/q$?

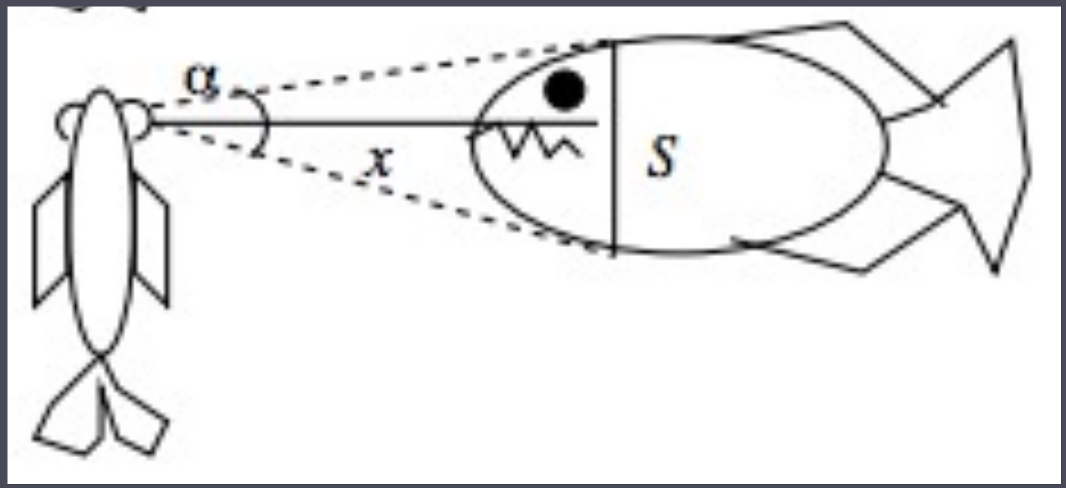
(A) $(p^2+q^2) / q^2$

(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) $p^2/q^2$




$$\frac{d\alpha}{dt} = -\frac{dx}{dt}\cos^2\left(\frac{\alpha}{2}\right)\frac{S}{x^2}$$

$$= -\frac{dx}{dt}\frac{x^2}{x^2+\frac{S^2}{4}}\frac{S}{x^2}$$

$$= -\frac{dx}{dt}\frac{S}{x^2+\frac{S^2}{4}}$$

# What is $\cos^2(a)$ when $\tan(a)=p/q$?

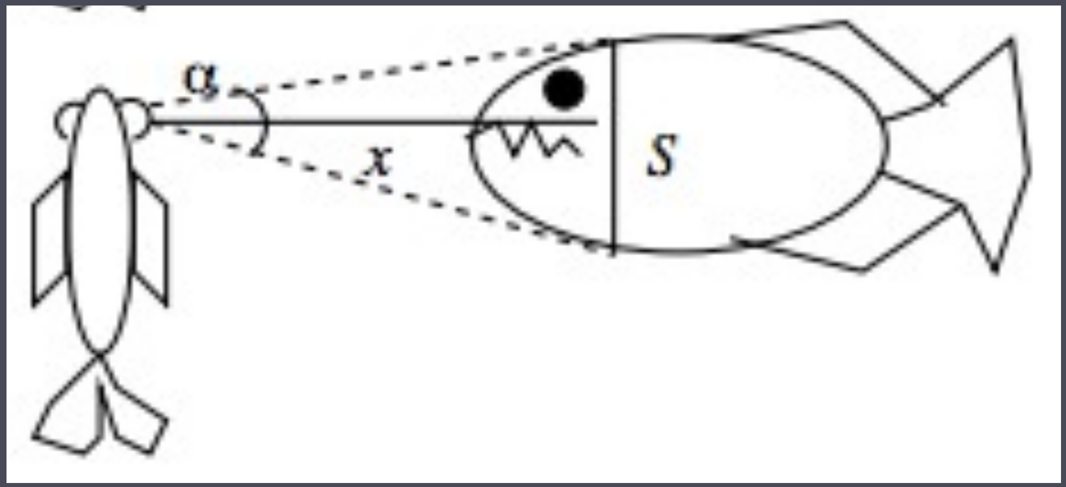
(A) $(p^2+q^2) / q^2$

(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) $p^2/q^2$




$$\frac{d\alpha}{dt} = -\frac{dx}{dt}\cos^2\left(\frac{\alpha}{2}\right)\frac{S}{x^2}$$

$$= -\frac{dx}{dt}\frac{x^2}{x^2+\frac{S^2}{4}}\frac{S}{x^2}$$

$$= -\frac{dx}{dt}\frac{S}{x^2+\frac{S^2}{4}} = v\frac{S}{x^2+\frac{S^2}{4}}$$

Assuming the Zebra Danio reacts to a rapidly changing optical angle α, it will try to escape from...

(A) ...a very large predator (large S).

(B) ...a very small predator (small S).

(C) ...a predator that is far away (large x).

(D) ...a slow-moving predator (small v).

(E) ...a fast-moving predator (large v).

$$\frac{d\alpha}{dt} = v\frac{S}{x^2 + \frac{S^2}{4}}$$

Assuming the Zebra Danio reacts to a rapidly changing optical angle α, it will try to escape from...

(A) ...a very large predator (large S).

(B) ...a very small predator (small S).

(C) ...a predator that is far away (large x).

(D) ...a slow-moving predator (small v).

(E) ...a fast-moving predator (large v).

$$\frac{d\alpha}{dt} = v\frac{S}{x^2 + \frac{S^2}{4}}$$

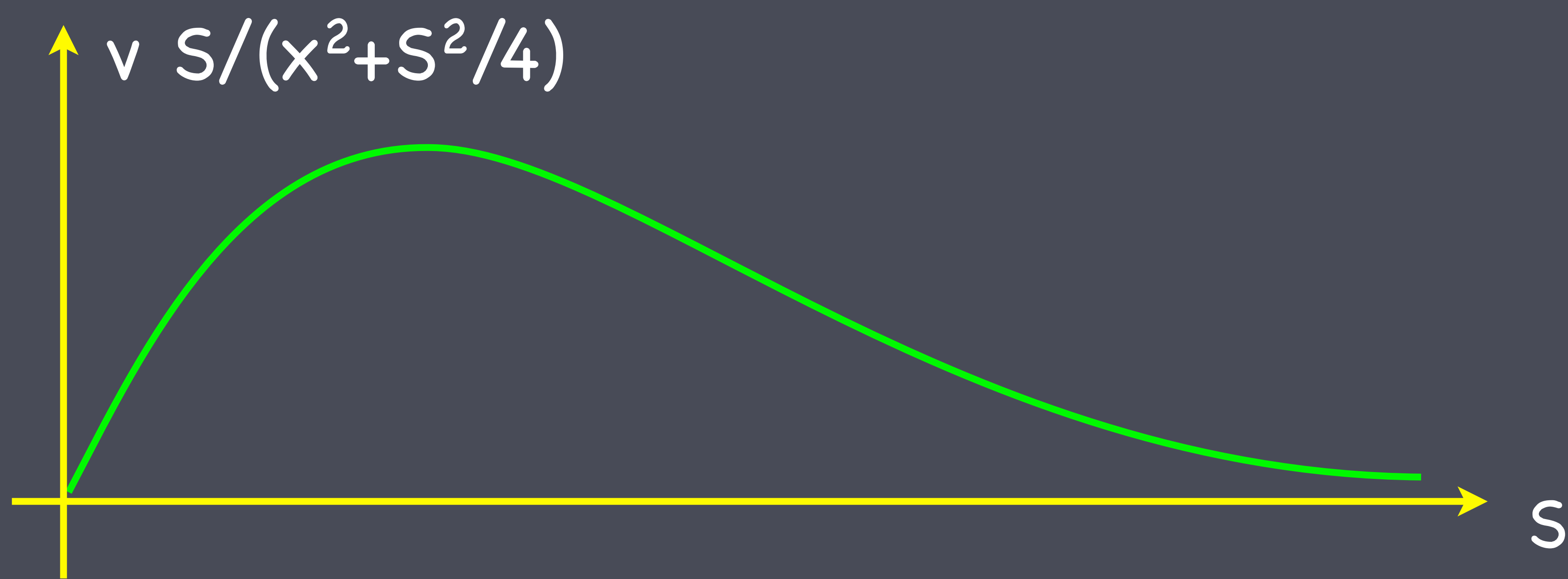
# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator distance x constant, plot $\alpha' = v\, S/(x^2+S^2/4)$ as function of S.

(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then…

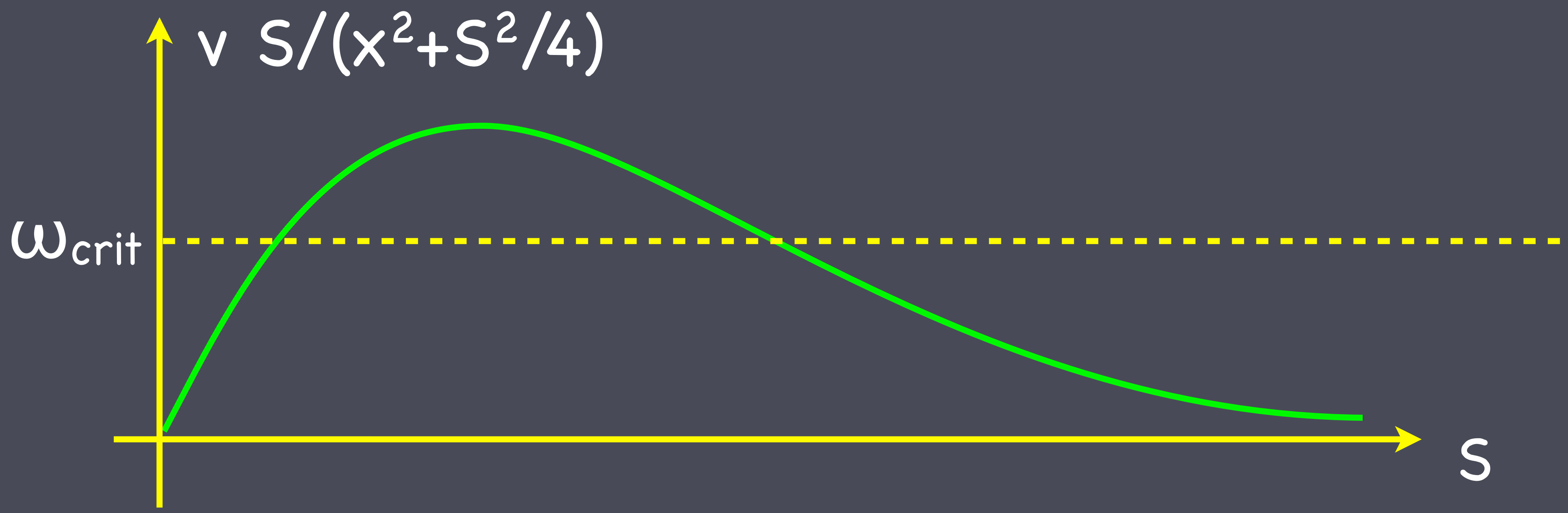
Hold predator distance x constant, plot $\alpha' = v\, S/(x^2+S^2/4)$ as function of S.




(A) …a very large predator.
(B) …a very small predator.
(C) …a predator that is far away.
(D) …a slow-moving predator.
(E) …a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

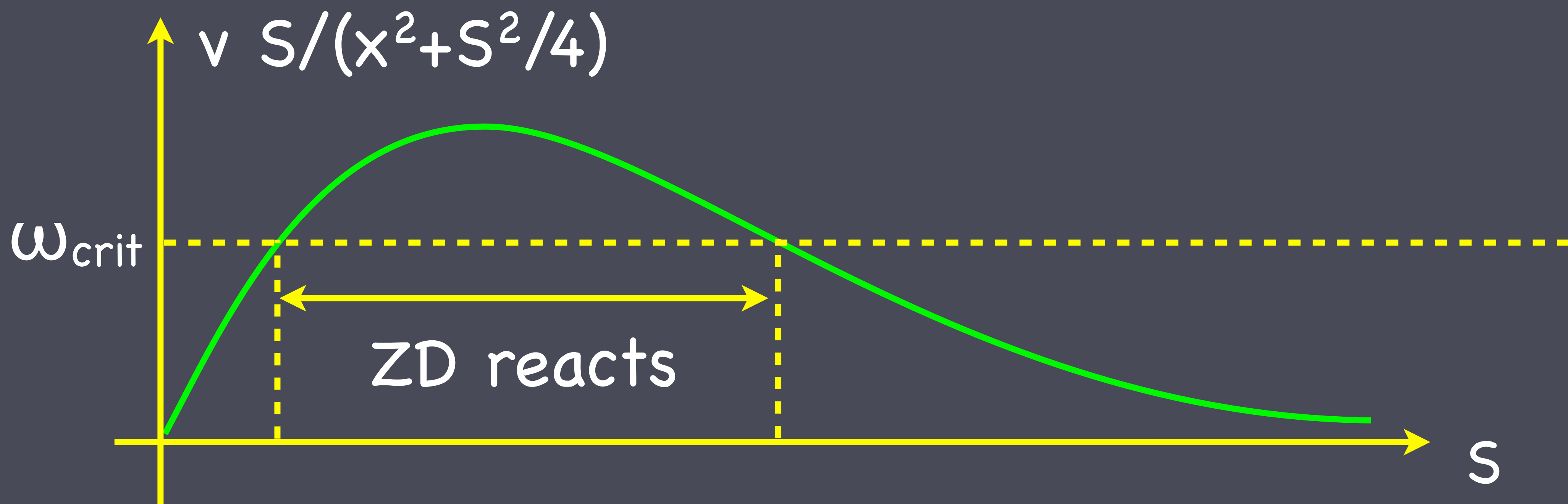
Hold predator distance x constant, plot $\alpha' = v\, S/(x^2+S^2/4)$ as function of S.




(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

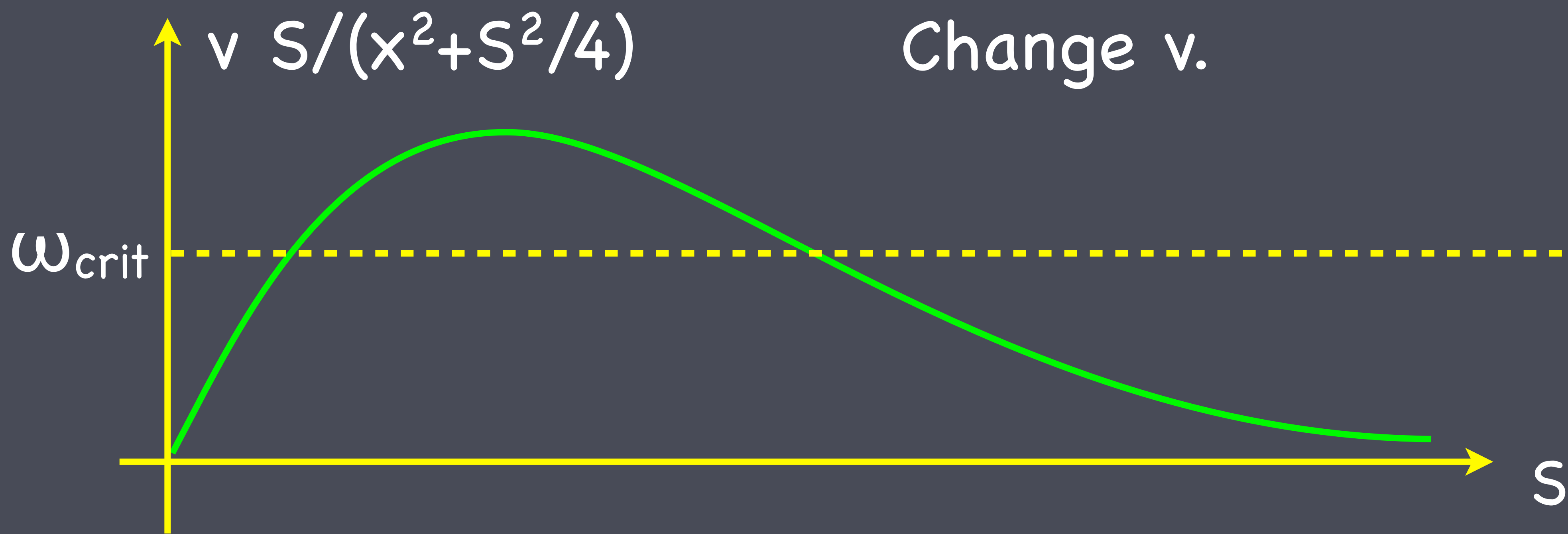
Hold predator distance x constant, plot
$\alpha' = v\, S/(x^2+S^2/4)$ as function of S.




(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator distance x constant, plot $\alpha' = v\, S/(x^2+S^2/4)$ as function of S.



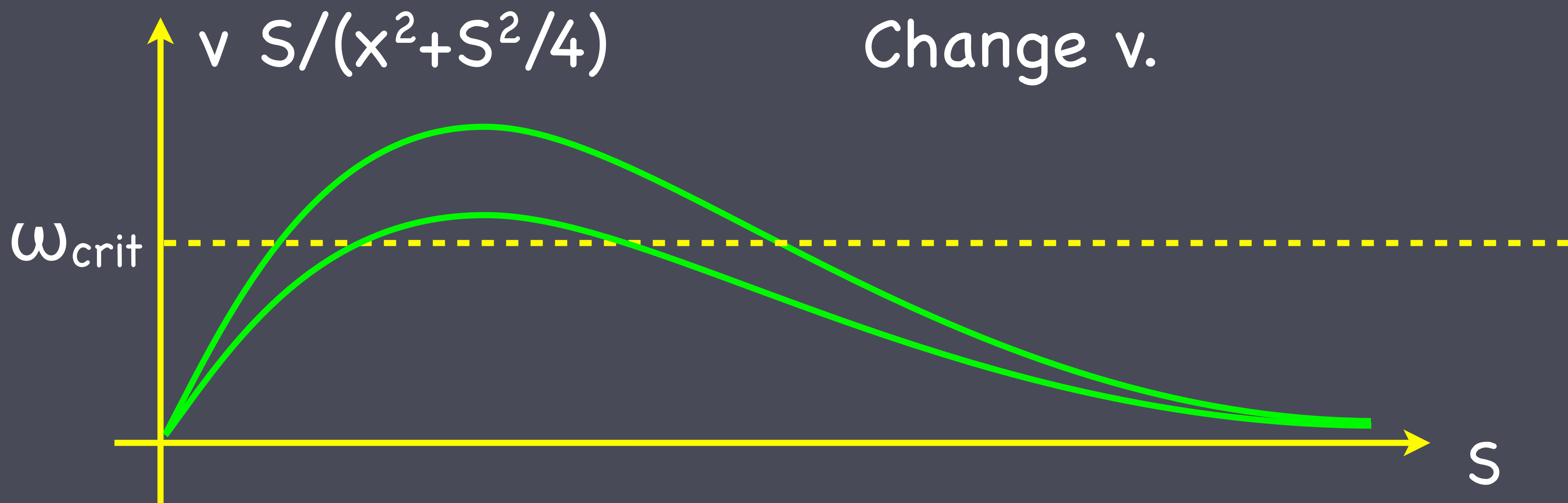

Change v.

(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator distance x constant, plot $\alpha' = v\, S/(x^2+S^2/4)$ as function of S.

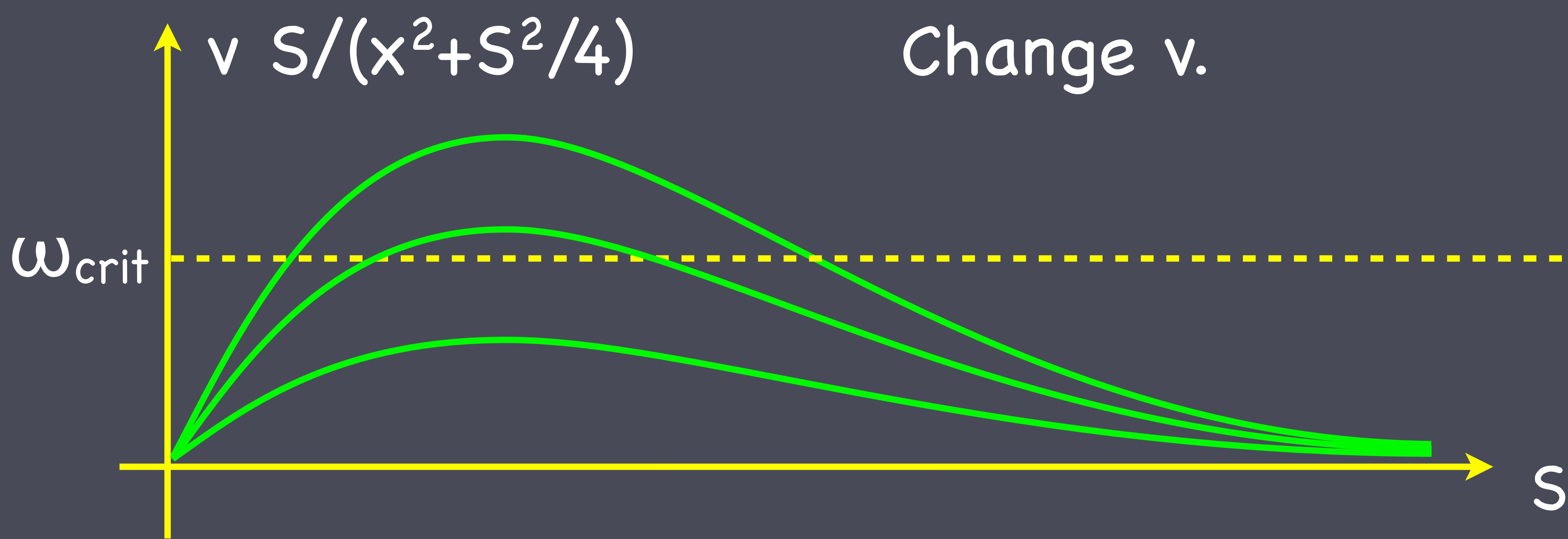
Change v.




(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

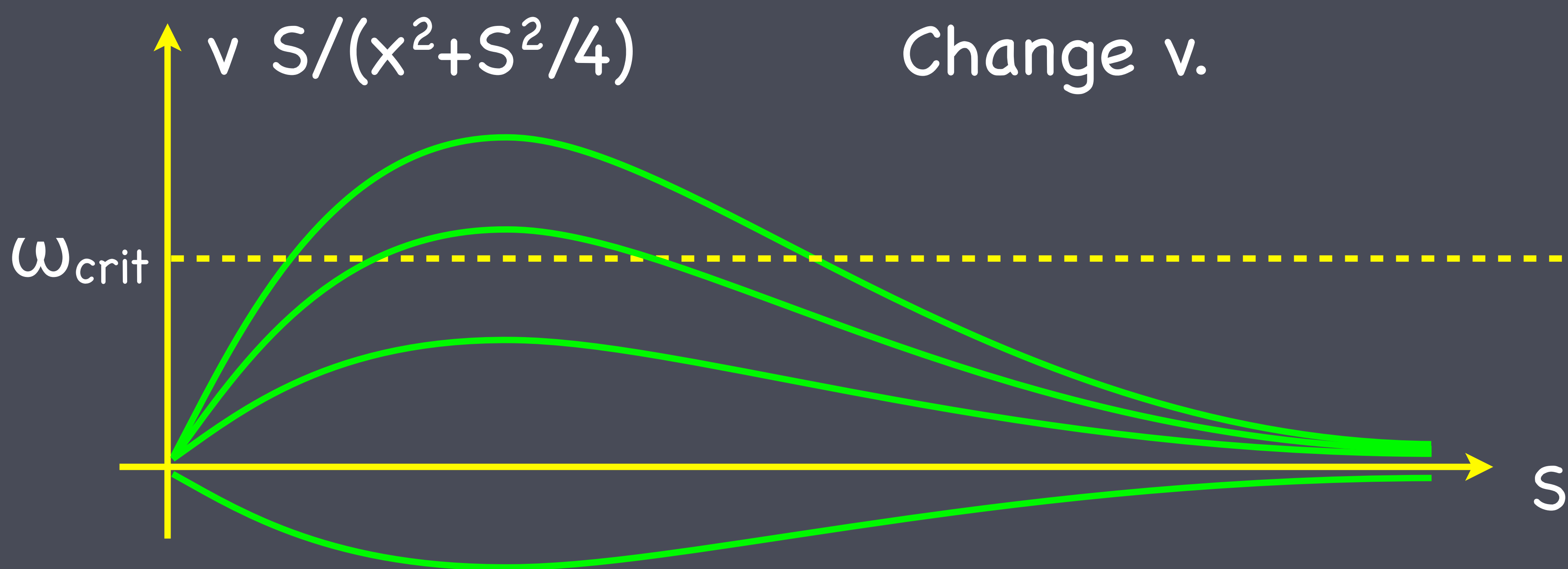
Hold predator distance x constant, plot
$\alpha' = v\, S/(x^2+S^2/4)$ as function of S.




(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

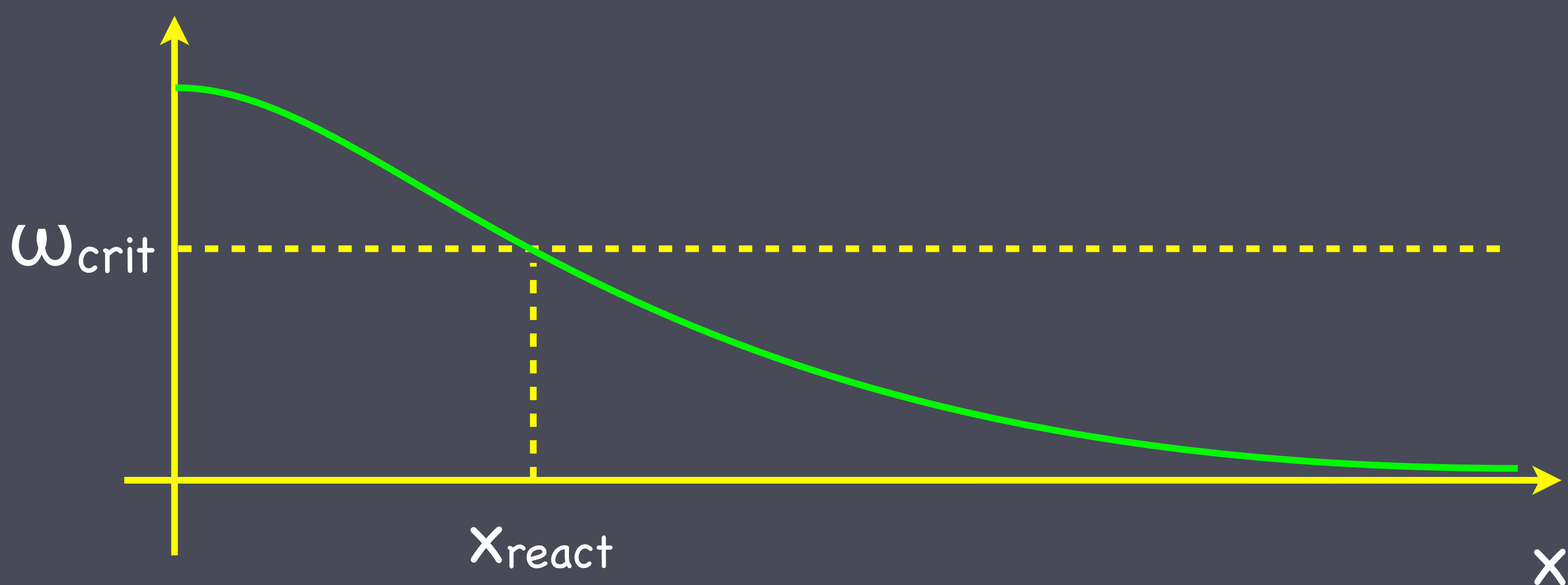
Hold predator distance x constant, plot $\alpha' = v\, S/(x^2+S^2/4)$ as function of S.




(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

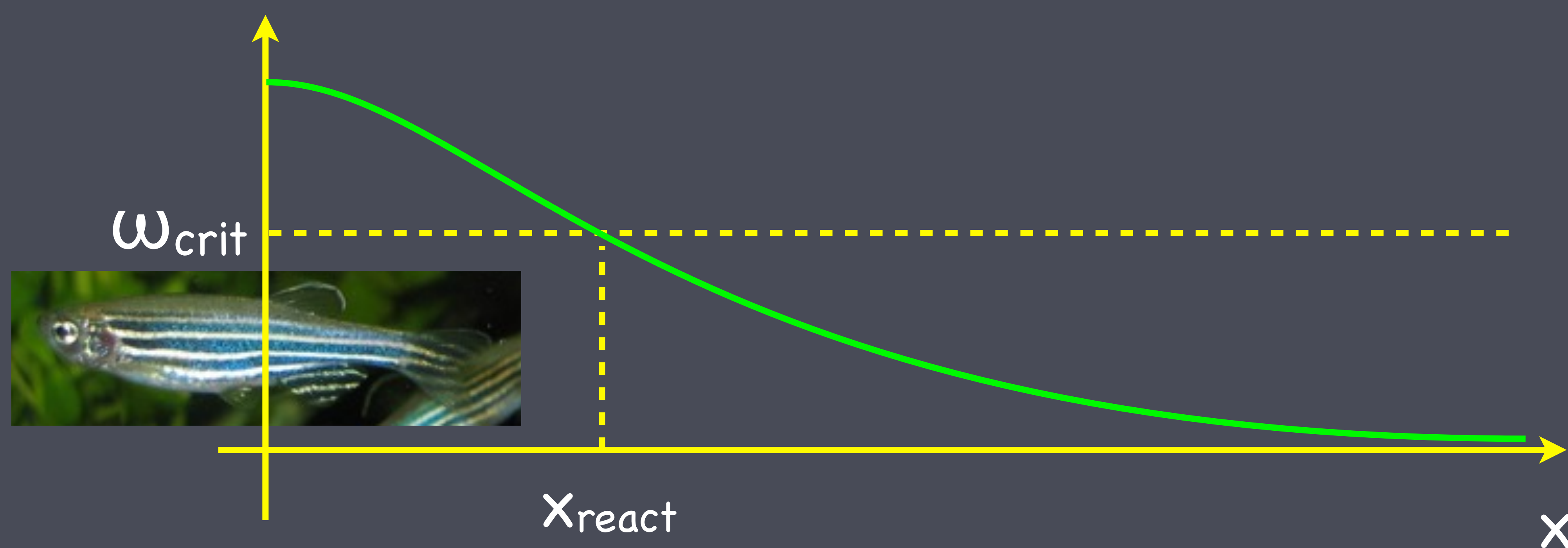
Hold predator size S constant, plot
$\alpha' = v\, S/(x^2+S^2/4)$ as function of x.




(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

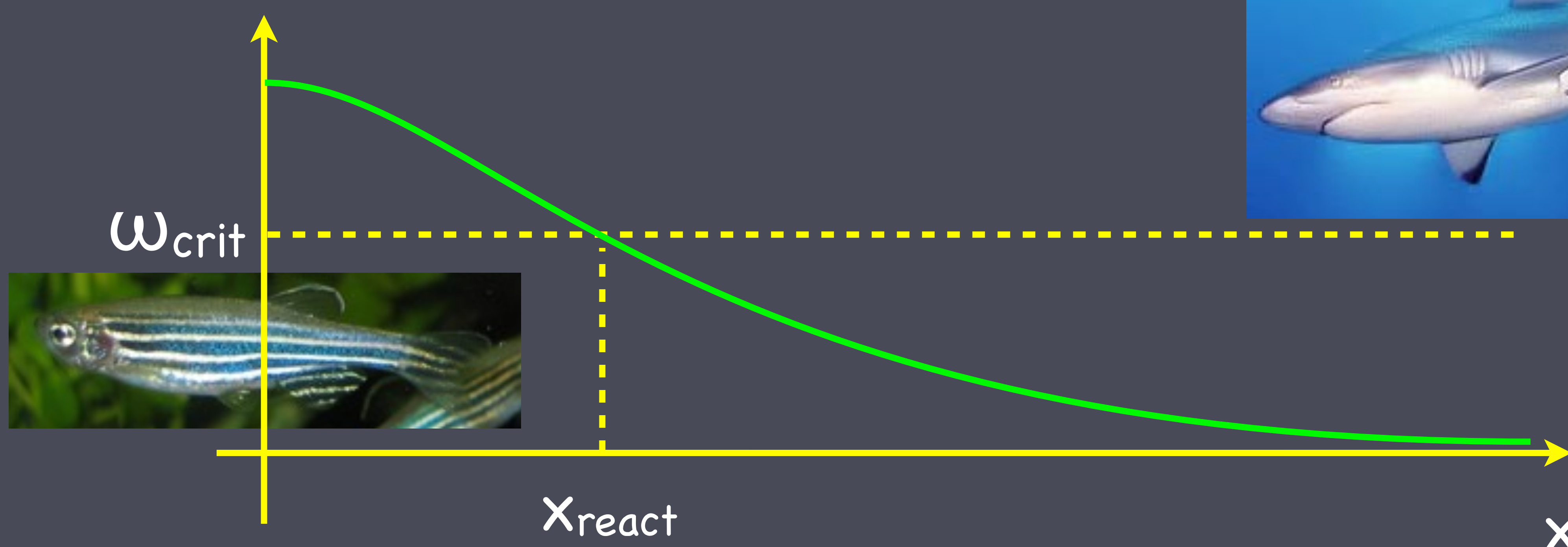
Hold predator size S constant, plot
$\alpha' = v\, S/(x^2 + S^2/4)$ as function of x.




(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator size S constant, plot
$\alpha' = v\, S/(x^2+S^2/4)$ as function of x.

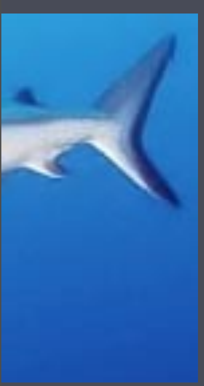


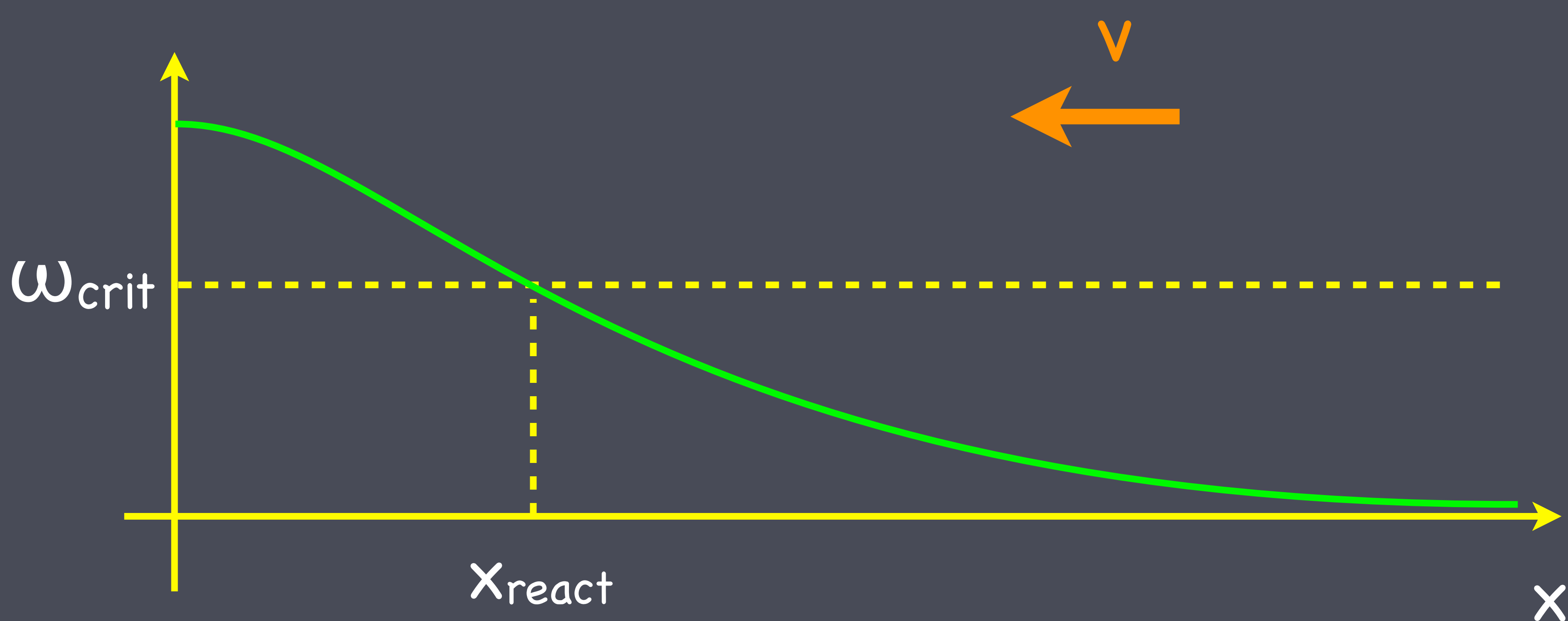

(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

Shark image - http://en.wikipedia.org/wiki/File:Tibur%C3%B3n.jpg

# If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator size S constant, plot
$\alpha' = v\, S/(x^2+S^2/4)$ as function of x.



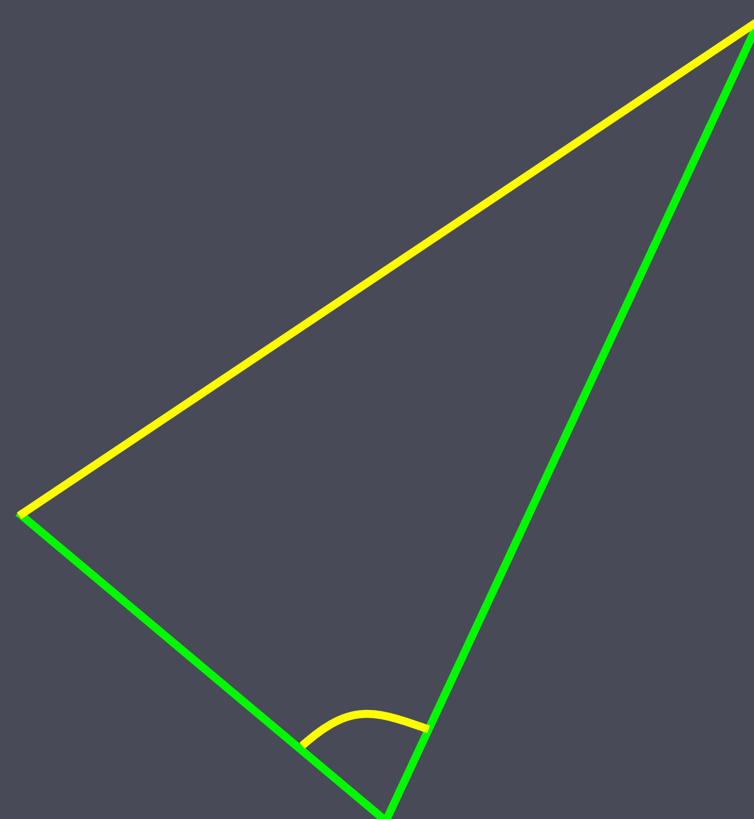



(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

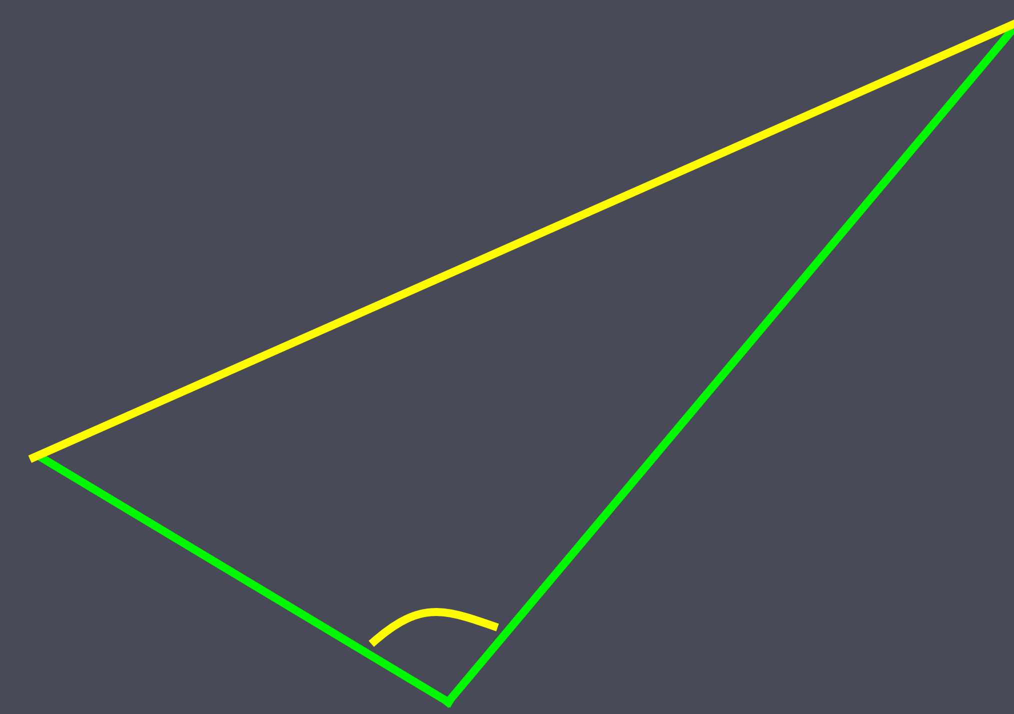
Shark image - http://en.wikipedia.org/wiki/File:Tibur%C3%B3n.jpg

# Triangle with two sides of fixed length, angle between them changes.

# Triangle with two sides of fixed length, angle between them changes.



# Triangle with two sides of fixed length, angle between them changes.



# Triangle with two sides of fixed length, angle between them changes.

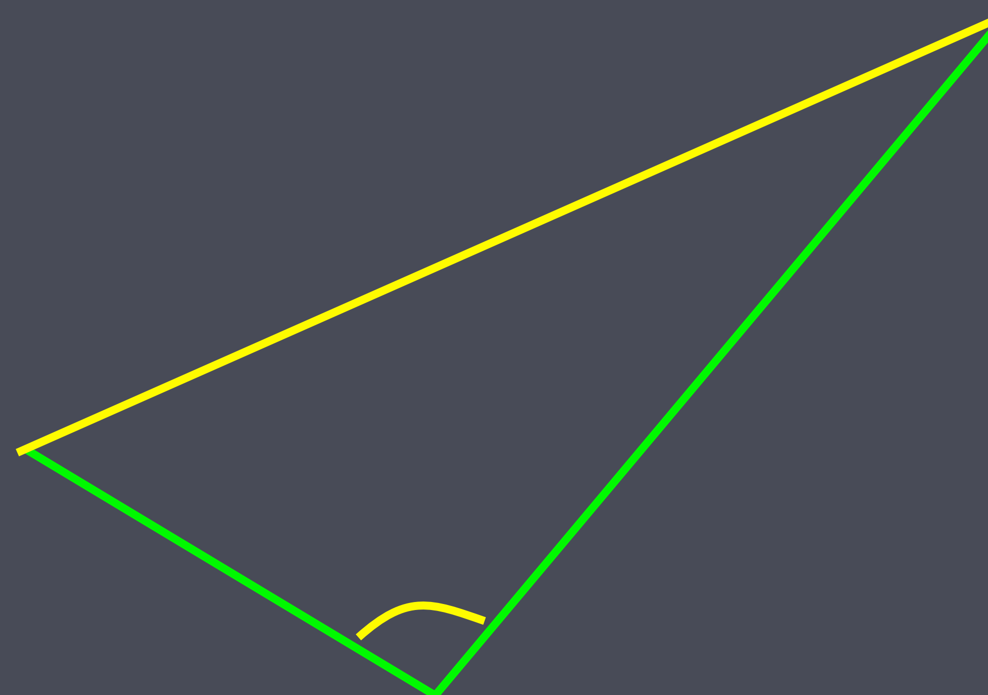
Relate the two changing quantities:

(A) $a^2 = b^2+c^2$

(B) $a^2 = b^2+c^2 - 2bc\cos(\theta)$

(C) $a/\sin(A) = b/\sin(B)$

(D) $\sin(\theta) = a/b$



# Triangle with two sides of fixed length, angle between them changes.

Relate the two changing quantities:

(A) $a^2 = b^2+c^2$

(B) $a^2 = b^2+c^2 - 2bc\cos(\theta)$

(C) $a/\sin(A) = b/\sin(B)$

(D) $\sin(\theta) = a/b$