Today...

- Approximations and the shapes of graphs.
- Hill functions.
- Motivating limits: secant lines, tangent lines.

Which of the following is a safe approximation to make?

(A) If a is small then we can say $a \approx 0$.

(B) If a is small then we can say $ab \approx 0$.

(C) If a is small then we can say $a+b \approx b$.

(D) If a is small compared to b then we can say $a+b \approx b$.

Explain notation $a<<b$.

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Explain notation $a \ll b$.

Comparisons and approximation must be based on relative sizes!

For each of the following, (A) True, (B) False . . . You line up some bricks to make a wall one brick high.

- One brick is a small number of bricks (i.e. “a is small”).
- The wall is small ($a \approx 0$).
- If you make the wall 20 times as high, it is still small ($ab \approx 0$).
- Add one row of bricks to a wall 20 bricks high. The new wall about the same size as the old wall. ($a+b \approx b$)

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For each of the following, (A) True, (B) False . . . You line up some bricks to make a wall one brick high.

- One brick is a small number of bricks (i.e. “a is small”). True.
- The wall is small ($a \approx 0$). Depends.
- If you make the wall 20 times as high, it is still small ($ab \approx 0$). Depends.
- Add one row of bricks to a wall 20 bricks high. The new wall about the same size as the old wall. ($a+b \approx b$) True.

When $0 < x \ll b$, then $x + b$ can be approximated by...

(A) b

(B) x

(C) infinity

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When 0 < x << b, then $f(x) = \dfrac{ax^n}{b^n + x^n}$

can be approximated by...

(A) a

(B) $\dfrac{a}{b^n}$

(C) $a\left(\dfrac{x}{b}\right)^n$

(D) 0

(E) 1

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When x >> b, then x + b can be approximated by...

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(B) x

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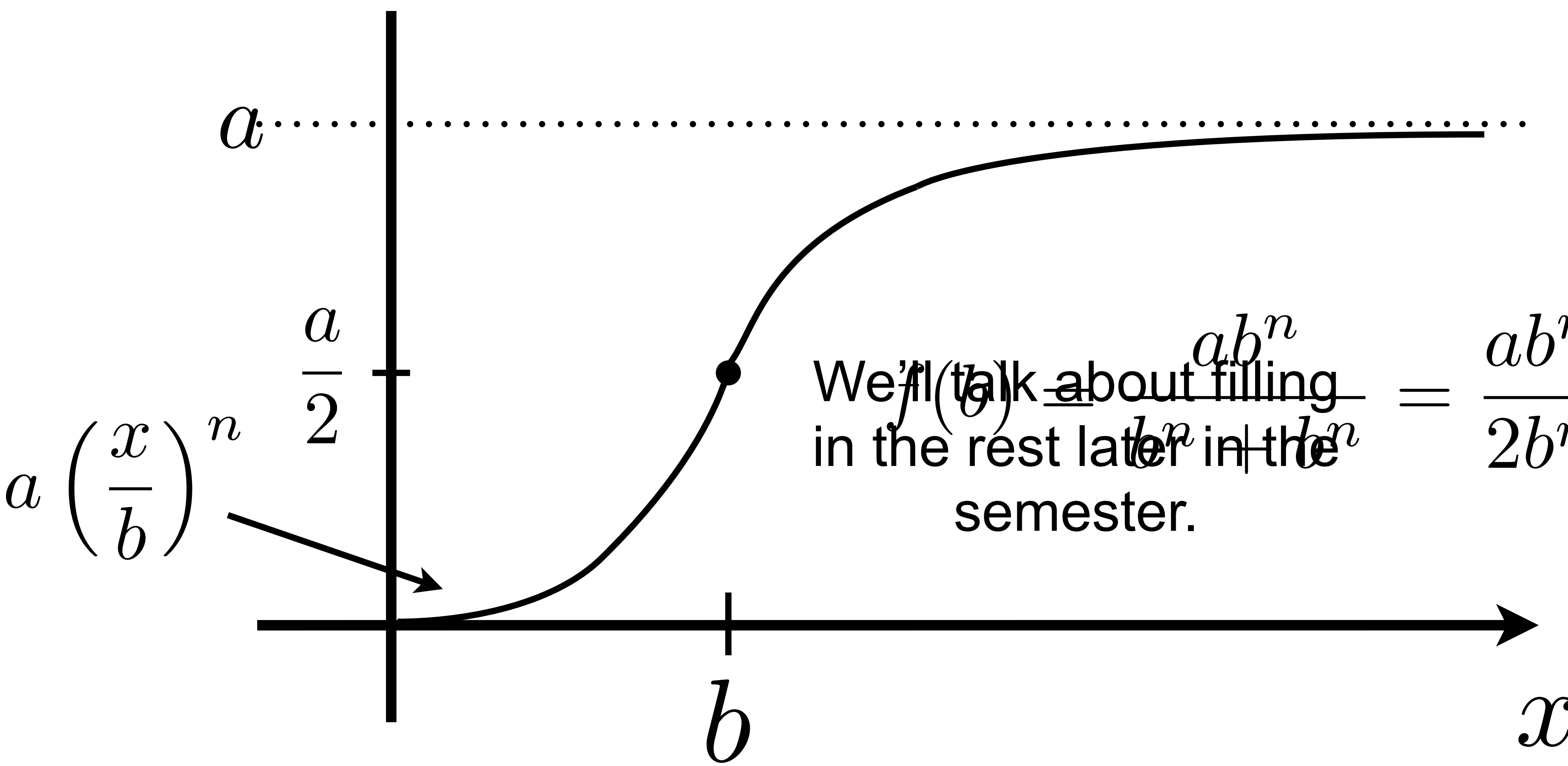
Implications for graphing

$$f(x) = \frac{ax^n}{b^n + x^n}$$



$$f(b) = \frac{ab^n}{b^n + b^n} = \frac{ab^n}{2b^n} = \frac{a}{2}$$

We'll talk about filling in the rest later in the semester.

Comparing Hill functions with different n values

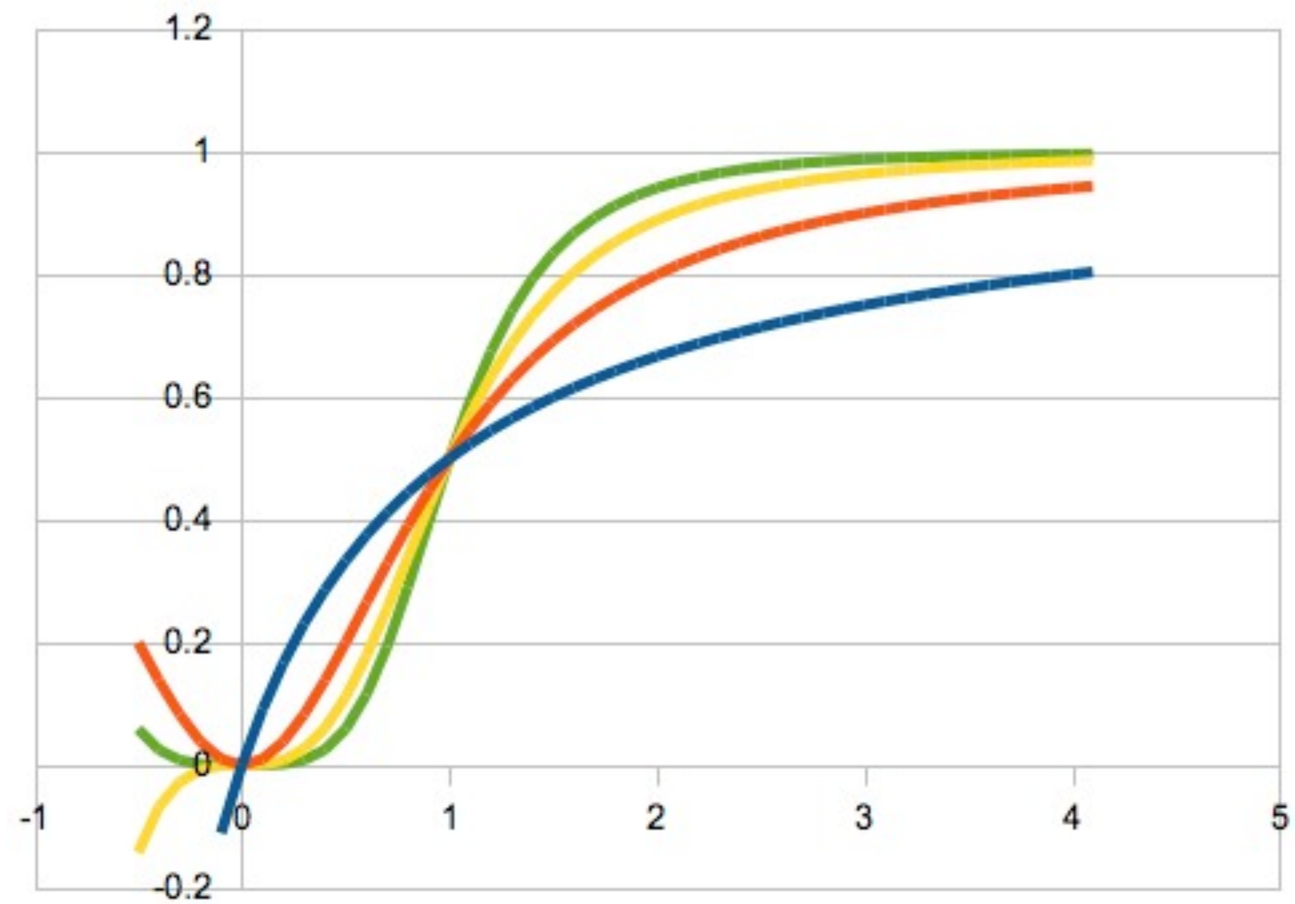
$$f(x) = \frac{ax^n}{b^n + x^n}$$

(A) Green: n=2, yellow: n=3, red: n=4, blue: n=5.

(B) Green: n=4, yellow: n=3, red: n=2, blue: n=1.

(C) Green: n=5, yellow: n=4, red: n=3, blue: n=2.

(D) Either (B) or (C) (not enough info).



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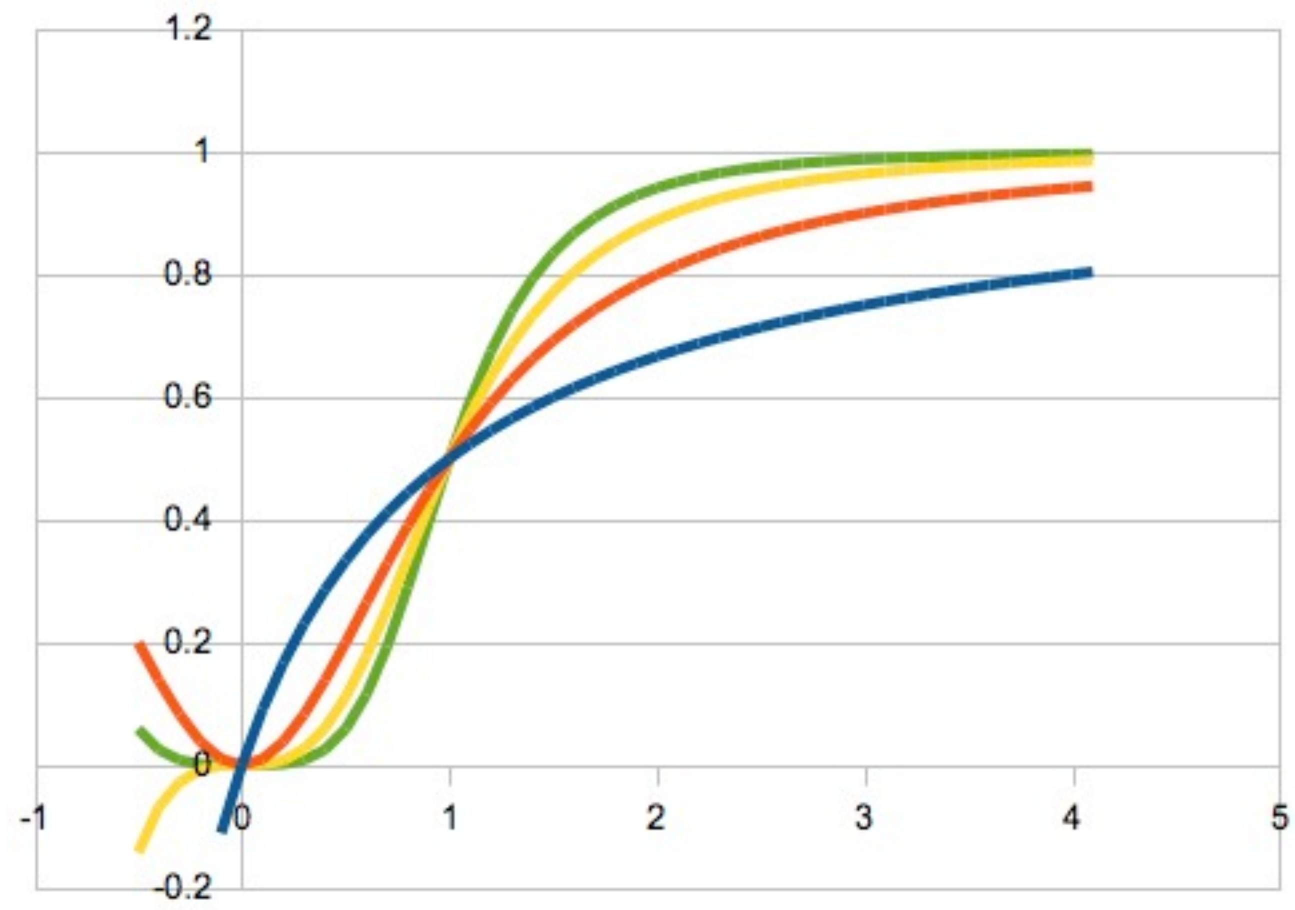


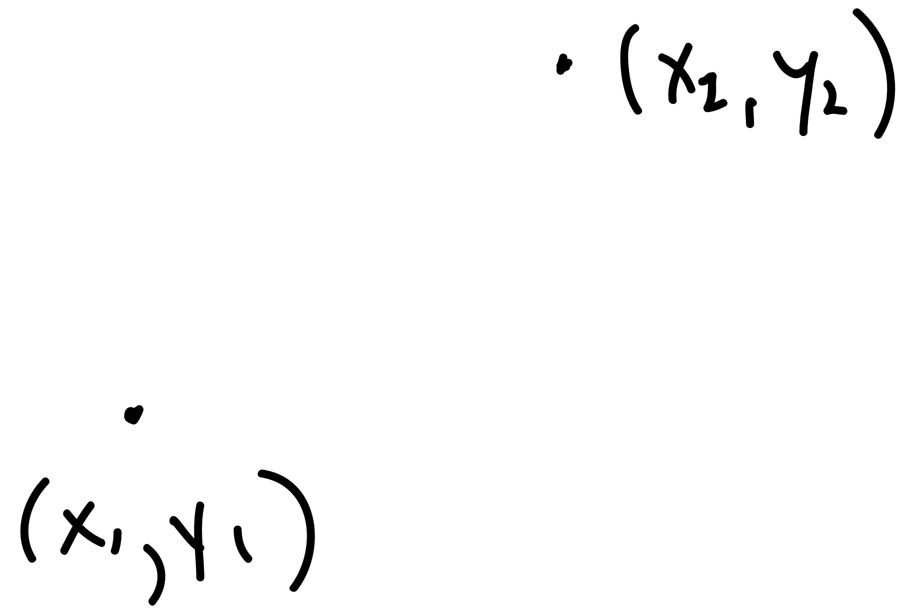
What is the slope of the line connecting the points?

(A) $m=(x_1-x_2)/(y_1-y_2)$

(B) $m=(x_2-x_1)/(y_1-y_2)$

(C) $m=(y_1-y_2)/(x_1-x_2)$

(D) $m=(y_2-y_1)/(x_2-x_1)$



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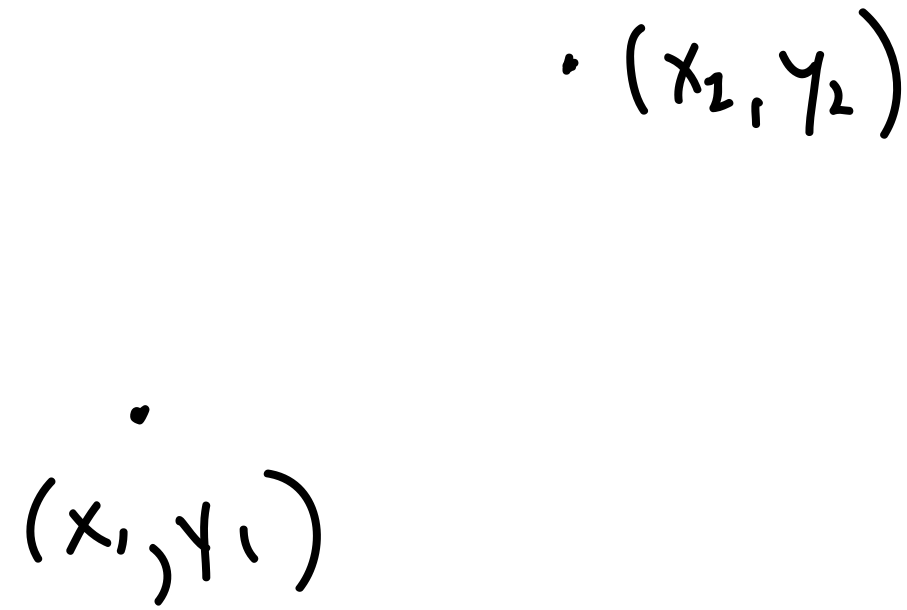


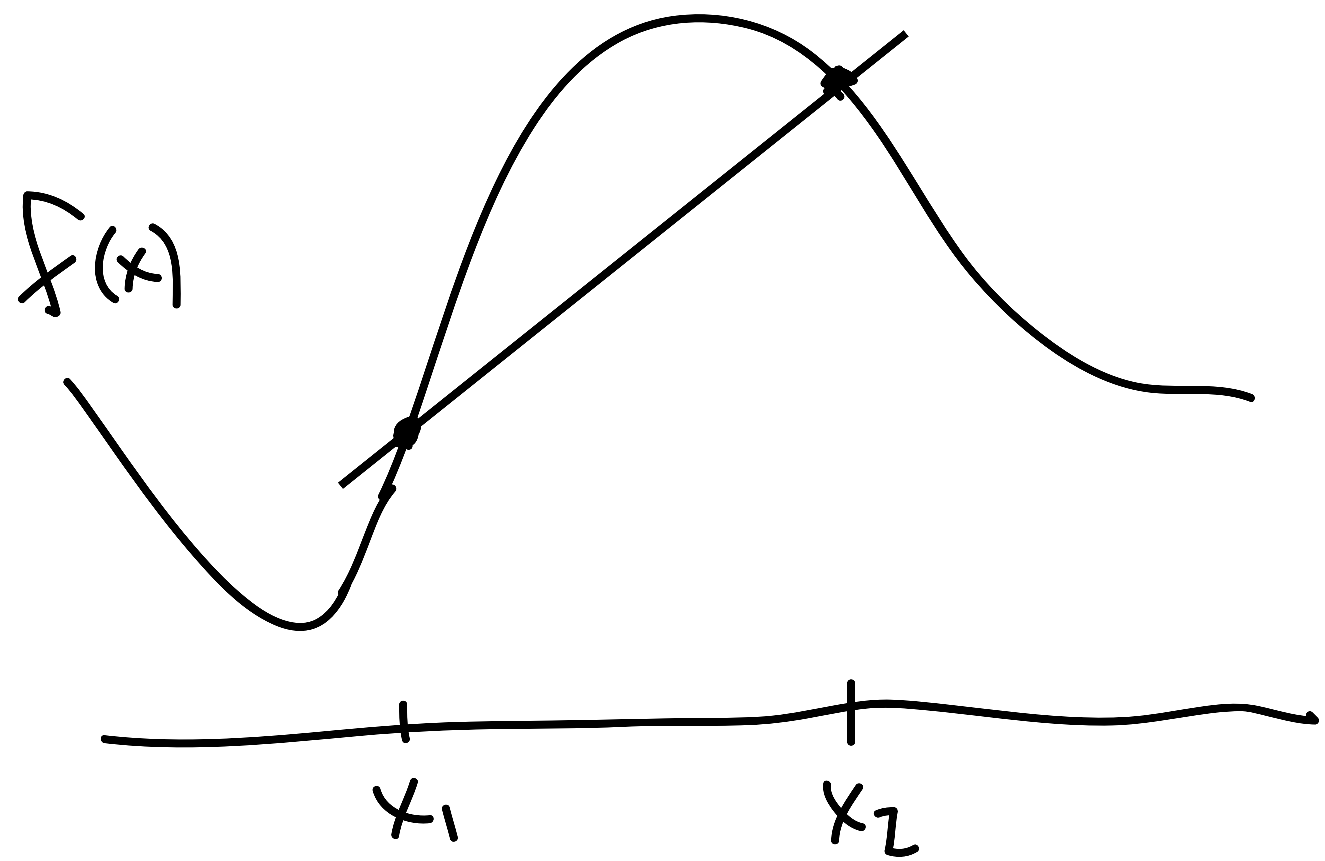
What is the slope of the secant line to the graph of f(x)?

(A) $m=(f(x_1)-f(x_2))/(x_2-x_1)$

(B) $m=(f(x_2)-f(x_1))/(x_2-x_1)$

(C) $m=(x_1-x_2)/(f(x_1)-f(x_2))$

(D) $m=(x_2-x_1)/(f(x_1)-f(x_2))$



Slope of secant line = average rate of change from x_1 to x_2.

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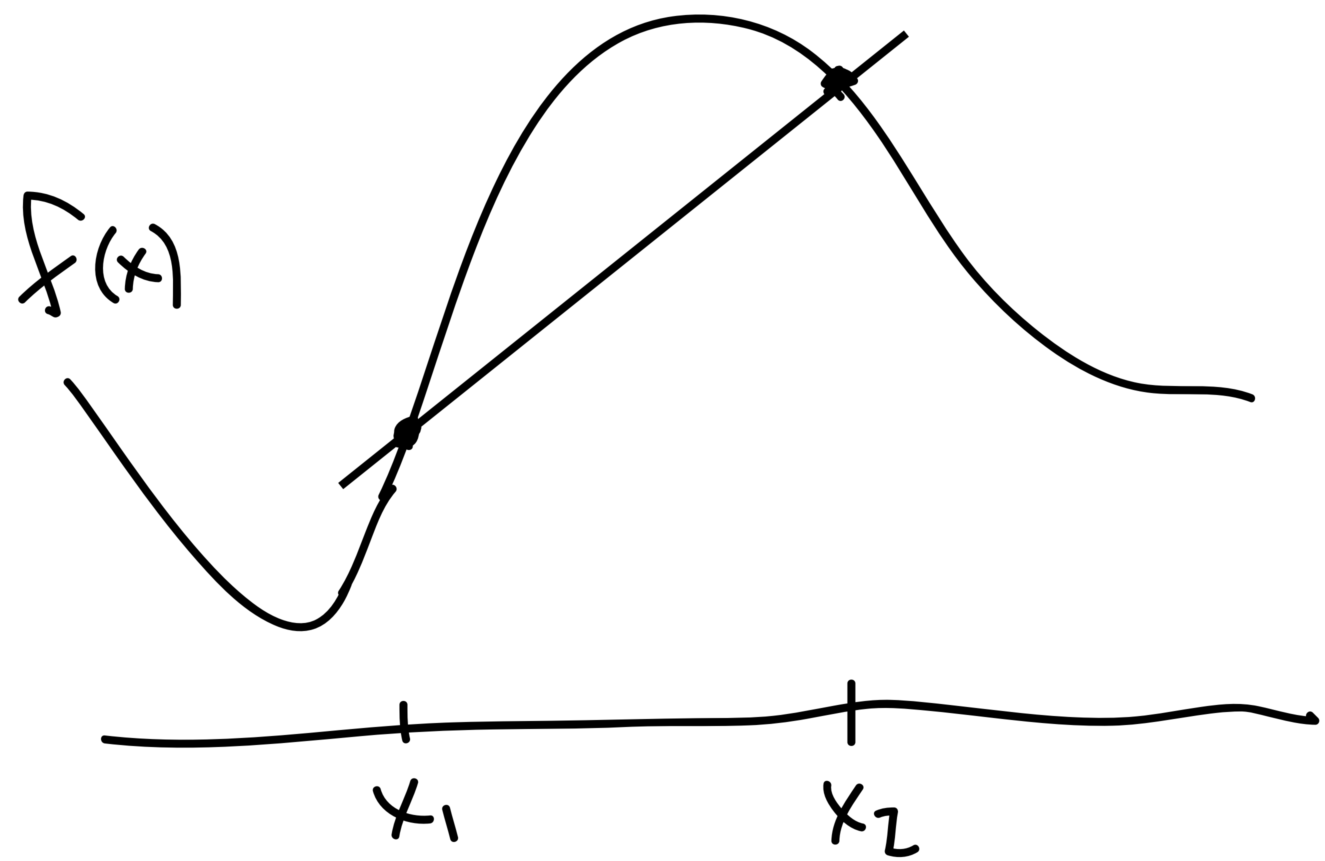
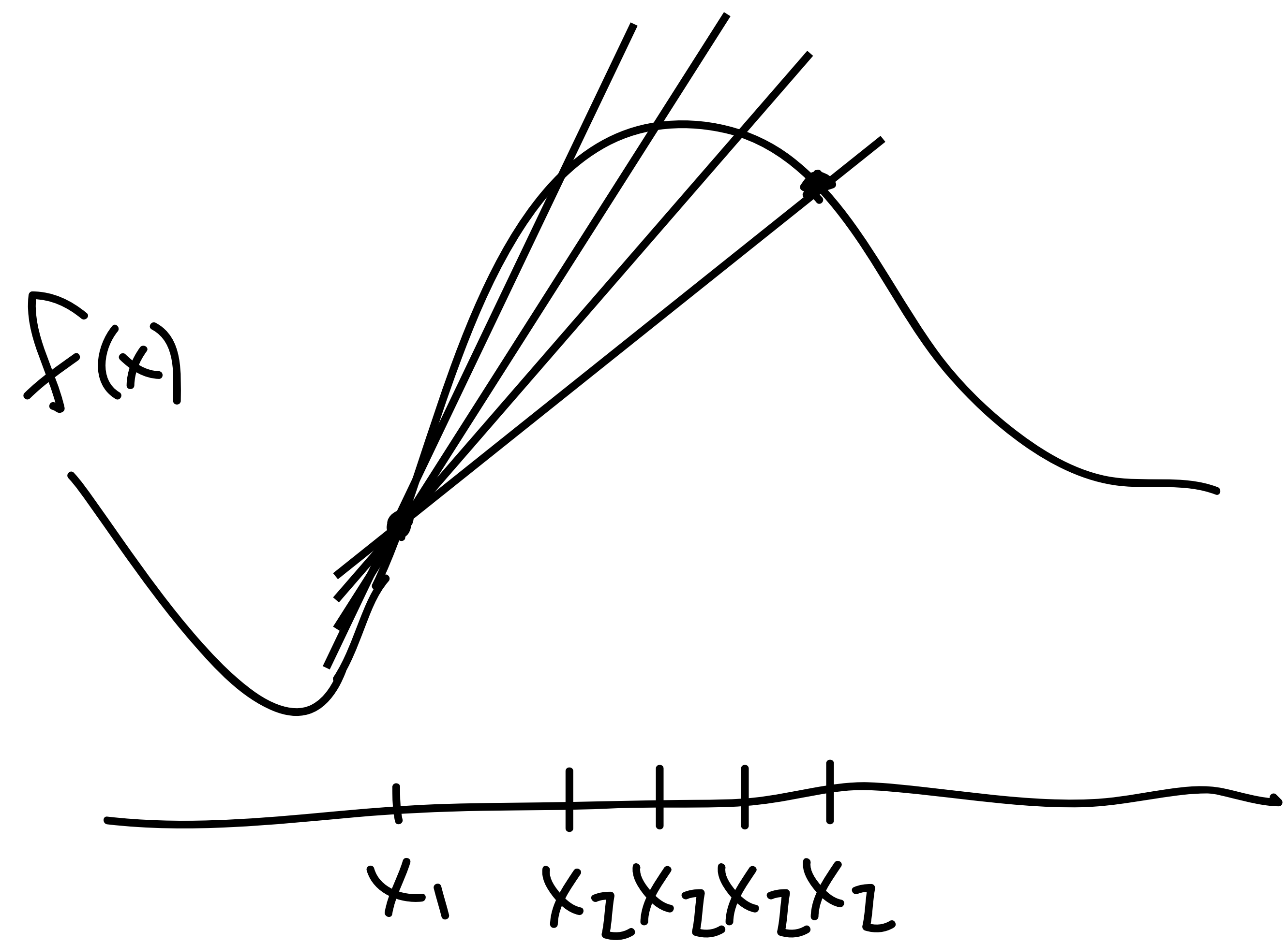


Slope of secant line = average rate of change from x_1 to x_2.

What if you want the rate of change AT x_1?

Take a point x_2 so that the secant line is closer to the "secant line" AT x_1.



Alternate notation: let $x_2=x_1+h$ so that

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1+h) - f(x_1)}{x_1 + h - x_1}$$

If we take h values closer and closer to 0...

- The secant line approaches the tangent line.
- The slope of the secant line approaches the slope of the tangent line.
- We call the resulting slope the derivative at x_1.
- We now have to learn how to take limits!

$$\text{slope at } x_1 = f'(x_1) = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$