What do derivatives tell us about functions?

Math 102 Section 106
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Announcements

- New & Improved Anonymous Feedback Form: https://goo.gl/forms/JJ3XWYcafxgfZerR2 (Link on Section 106 Wiki Page)
- Office Hours:
  - M 4-5 in Math Annex 1118
  - W 3-4 in Math Annex 1118
  - Thursday: 3-4 in LSK 300B

Today:
- Newton’s Method recap
- Sketching with calculus
Announcements

▶ Quiz 2 on Friday
▶ You will need to practice Newton’s Method & Linear Approximation
▶ Check out Extra Practice on WW
▶ Quiz Groups: computer/tablet for spreadsheet or internet access
▶ Midterm Signup Form (link under Wiki: Midterm Information)
▶ Old practice midterms (also on Wiki: Midterm Information)
Newton’s Method

Q1. We wish to find a good approximation to $7^{2/3}$ using Newton’s Method. What function $f(x)$ could we use to help us compute this value?

A. $f(x) = x^{2/3} - 7$
B. $f(x) = x^2 - 343$
C. $f(x) = x^{3/2} - 7$
D. $f(x) = x^3 - 49$

$x = 7^{2/3} \iff x^3 - 49 = 0$
Newton’s Method

Q2. We wish to find a good approximation to $7^{\frac{2}{3}}$ using Newton’s Method and $f(x) = x^3 - 49$. What value of $x_0$ would you chose as your starting estimate if you were planning to do the calculations by hand?

A. $x_0 = 4$
B. $x_0 = 7$
C. $x_0 = 8$
D. $x_0 = \sqrt{512}$

$x_0 = 8^{\frac{2}{3}} = 4$
Newton’s Method

Spreadsheet demo with

\[ f(x) = x^3 - 49, \quad x_0 = 4. \]
Newton’s Method

The 10 best ways to *not* pick $x_0$:

www.buzzfeed.com/math102/newton.html

Kidding...

https://www.desmos.com/calculator/0ukfgz1f4y
What do derivatives tell us about a function?

- We will cover a lot of definitions today...
- ... practice and applications we reserve for Wednesday.
Increasing vs. decreasing

- A function $f(x)$ is increasing on some interval if for any points $a$ and $b$ in the interval with $a < b$, we have $f(a) < f(b)$.

- A function $f(x)$ is decreasing on some interval if for any points $a$ and $b$ in the interval with $a < b$, we have $f(a) > f(b)$.

- Note: no reference to $f'(x)$!
What about $f'(x)$?

- If $f'(x) > 0$ on some interval, then $f(x)$ is increasing on that interval.
- If $f'(x) < 0$ on some interval, then $f(x)$ is decreasing on that interval.
- If $f'(x) = 0$ on some interval, then $f(x)$ is constant on that interval.
- Want justification for these facts? Look up the Mean Value Theorem.
Local extrema (minima or maxima)

- A point \( a \) is a local minimum of a function \( f(x) \) provided that \( f(x) > f(a) \) for all \( x \neq a \) on an interval around \( a \).

Q3.

Which of the following is a local minimum?

- (A)
- (B)
- (C)

- If the function is differentiable at the minimum, then it must look like (A).
Critical Points

- A critical point (CP) of \( f(x) \) is a point \( a \) at which \( f'(a) = 0 \) or \( f'(a) \) is not defined even though \( f(a) \) is defined.

Q4. If \( f'(x) \) changes sign at a CP, then the CP is an extremum (min/max) of \( f(x) \).

A. True
B. False
(Q4) This is called the first derivative test.
Q5. If \( f'(x) \) goes from \(-\) to \(0\) to \(+\), then \( x = a \) is a maximum of \( f(x) \).

A. True
B. False
Q6. If $f'(x)$ is differentiable at $x = a$, then $x = a$ is an extremum when $f''(a) \neq 0$.

A. True

B. False

(Q6) This is called the second derivative test.
Q7. If \( f''(x) > 0 \), then \( f'(x) \) goes from \(-\) to 0 to \(+\), and \( x = a \) is a minima of \( f(x) \).

A. True
B. False
A critical point is not an extremum when...

- The first derivative test fails:
  - $f(x) = x^3 \Rightarrow f'(x) = 3x^2$.
  - $f'(0) = 0$, but $f'(x) > 0$ for all $x$!

- The second derivative test fails:
  - If $f(x) = x^4$, then $f''(0) = 0$ but $x = 0$ is a minimum.
  - If $g(x) = x^5$, then $g''(0) = 0$, but $x = 0$ is neither a maximum nor a minimum.
What about $f''(x)$?

- We say a function is concave up on some interval if $f'(x)$ is increasing on that interval.
  - When $f''(x)$ exists, this is the same as $f''(x) > 0$.
- We say a function is concave down on some interval if $f'(x)$ is decreasing on that interval.
  - When $f''(x)$ exists, this is the same as $f''(x) < 0$. 
Inflection Points

- An **inflection point** of \( f(x) \) is a point at which the concavity changes from up to down or from down to up.

Q8. If \( f''(a) = 0 \), then \( x = a \) is an inflection point of \( f(x) \).
   A. True
   B. False

- If \( f(x) = 2x \), then \( f''(x) = 0 \) for all \( x \) but \( f \) doesn’t even have any local maxima or minima!
Example

Find the zeros, critical points, and inflection points of the function \( g(x) = x^5 - x^3 \).

- \( g'(x) = 5x^4 - 3x^2 = x^2(5x^2 - 3) = 0 \) when \( x = 0, \pm \sqrt{3}/5 \)
- FDT: \( g'(x) \approx -3x^2 \) for \( x \ll 1 \), so \( g'(x) \) doesn't change sign!
- Plugging in values (\( g'(-1) \) risks going too far from \( x = 0 \))
- \( \sqrt{3}/5 \) is a maximum, and \( -\sqrt{3}/5 \) is a minimum.
- SDT: \( g''(x) = 20x^3 - 6x = 2x(10x^2 - 3) \), so \( g''(0) = 0 \)
- \( x = 0 \) is a mystery point!
Summary

- Definitions today...
- ...examples and practice on Wednesday.
Answers

1. C or D
2. A
3. A B and C
4. A
5. B
6. A
7. A
8. B
Related exam problems

1. Find the zeros, critical points, and inflection points of the function

\[ f(x) = -x^4 - 2x^3. \]

2. Find all minima, maxima, and infection points of \( f(x) = x^4 - x^2 \). Sketch the graph of \( f(x) \).

3. Consider the function \( f(x) = x^4 + ax^3 - x^2 \). Find all values of \( a \) for which \( f \) has no inflection points, or show that no such values exist.