# Today

- Linear approximation
- Newton's method

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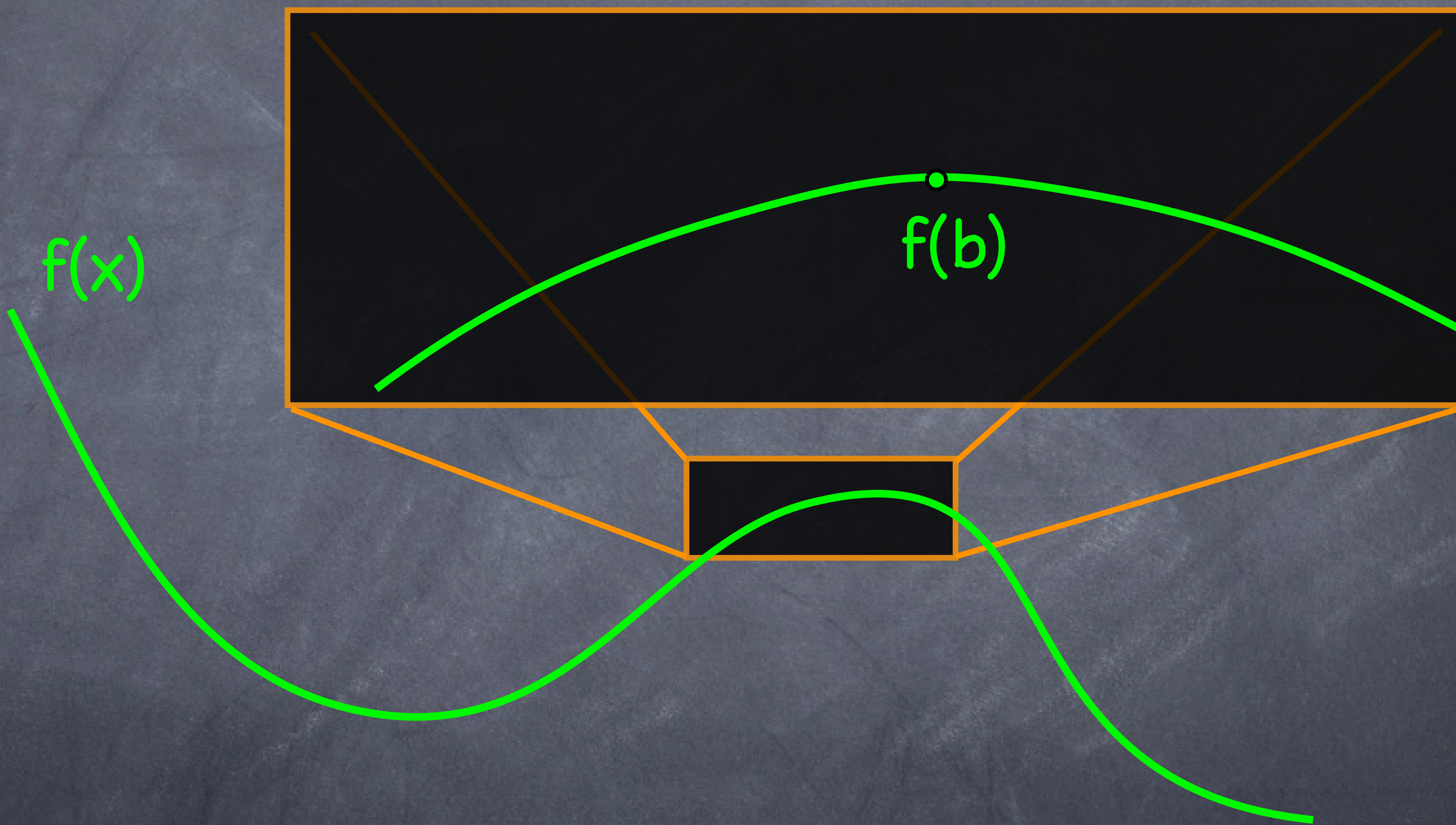
- Reminders:
  - OSH 3 due Monday (no questions on PL5.1)
  - Assignment 3 due tomorrow
  - Assignment 4a (midterm 1 content) Tues 7 am
  - Assignment 4b (not midterm 1 content) F 5pm
  - Office hours
    - Tu 10:30-11:30am
    - W 11:30-12:30, 2:30-3:30pm.

# Linear approximation




Suppose you want to know f(b) but it's hard to calculate.
If a is near b and both f(a) and f'(a) are easy to calculate, use tangent line to approximate f(b).

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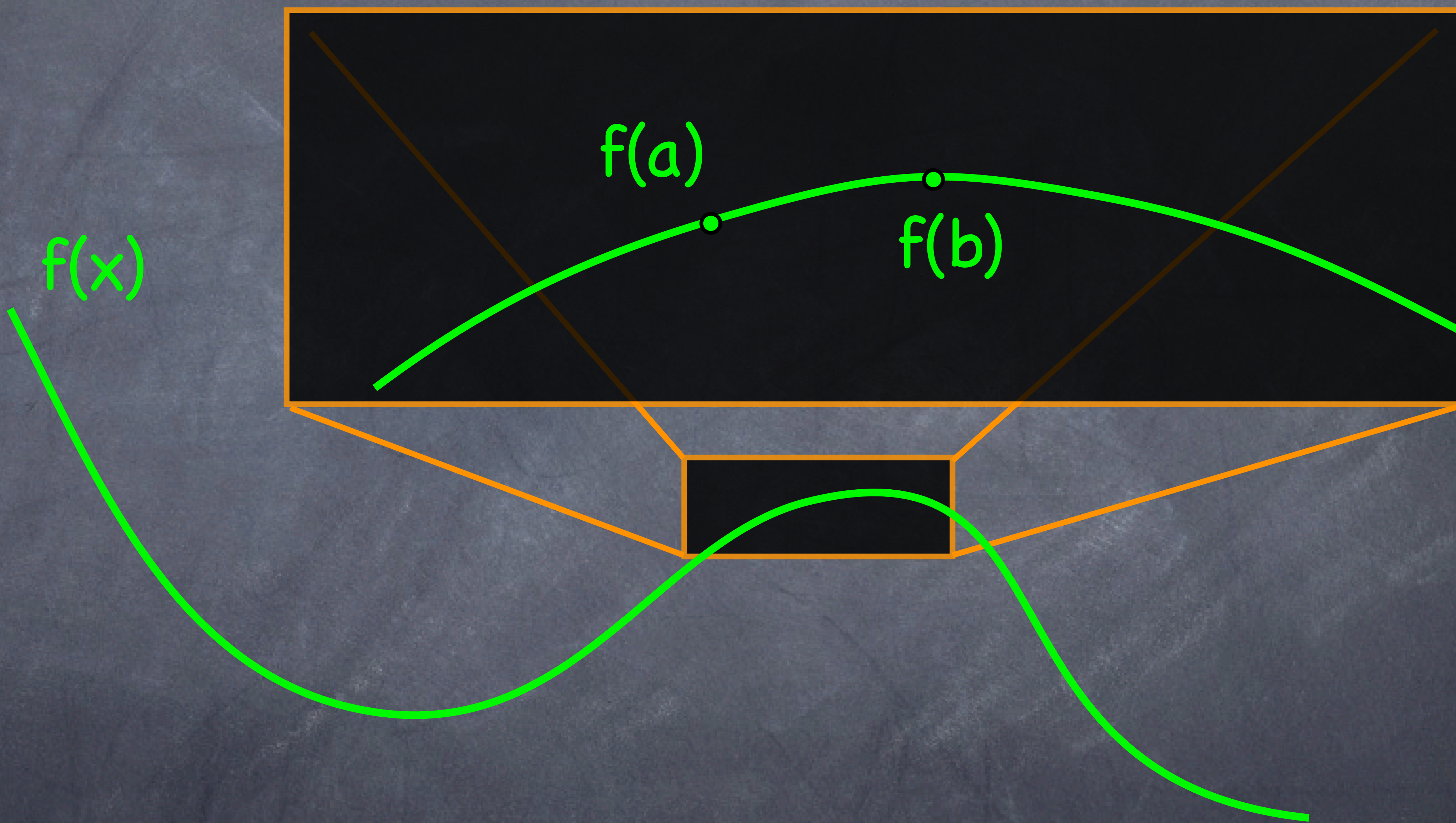


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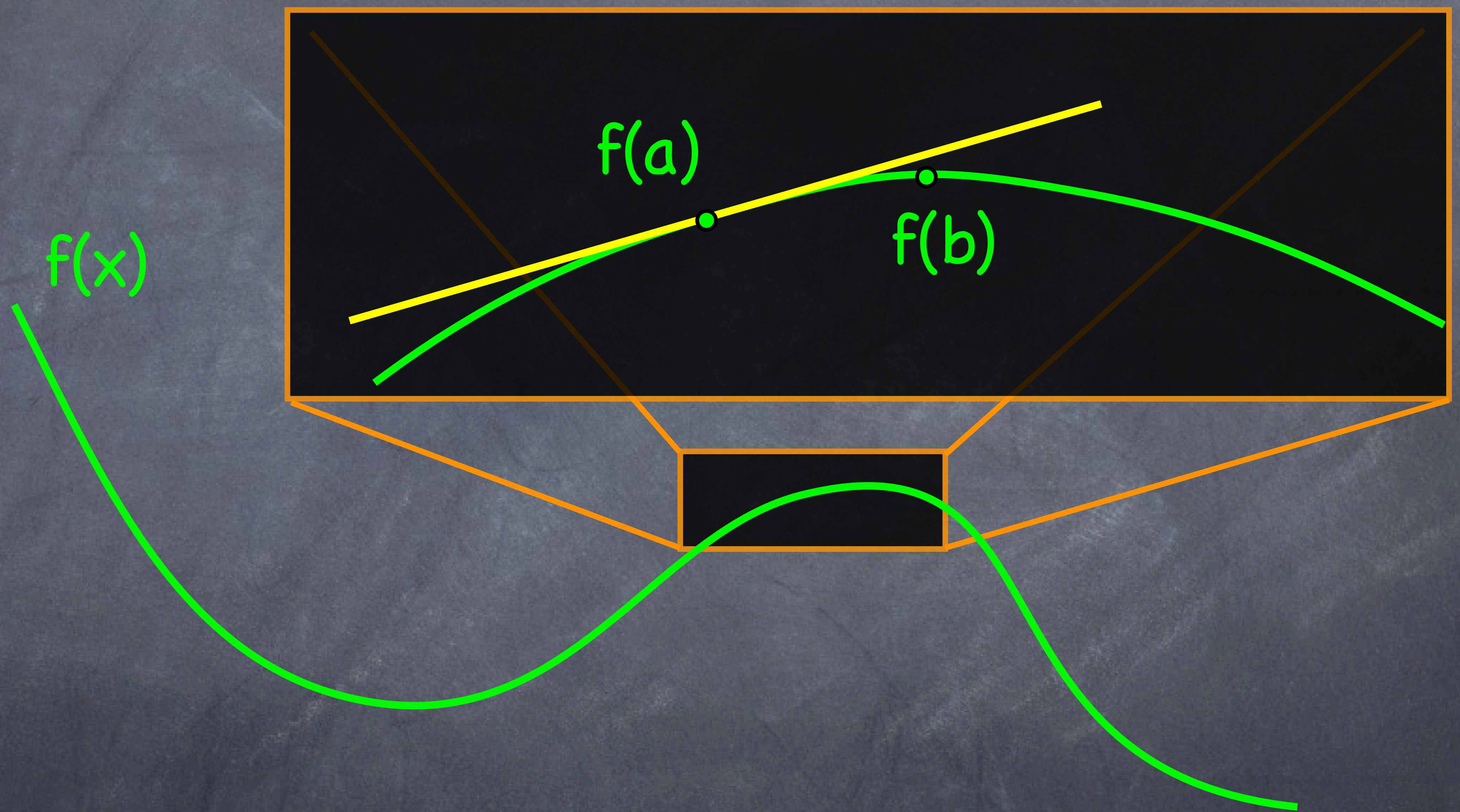


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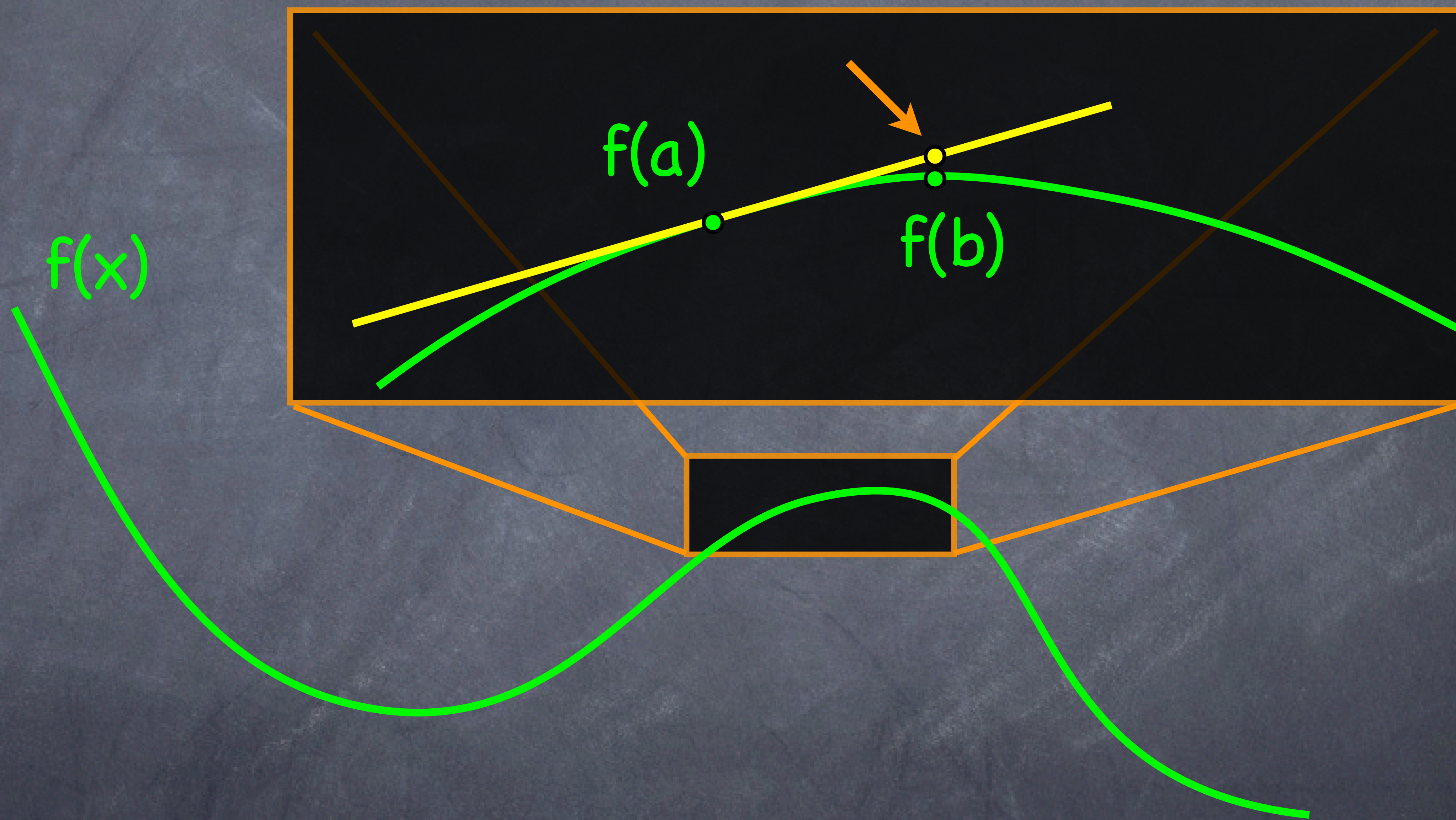


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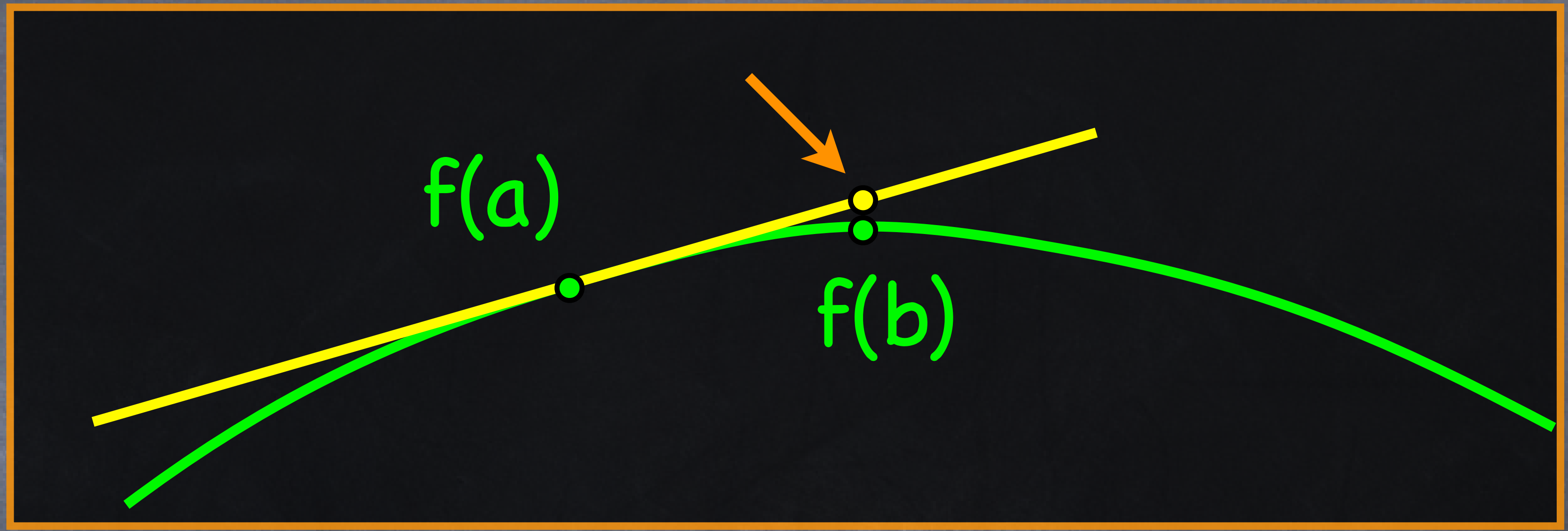


(A) $f(b) \approx f(b)+f'(b)(x-b)$

(B) $f(b) \approx f(a)+f'(a)(x-a)$

(C) $f(b) \approx f(a)+f'(a)(b-a)$

(D) $f(a) \approx f(b)+f'(b)(a-b)$

(E) Don't know.

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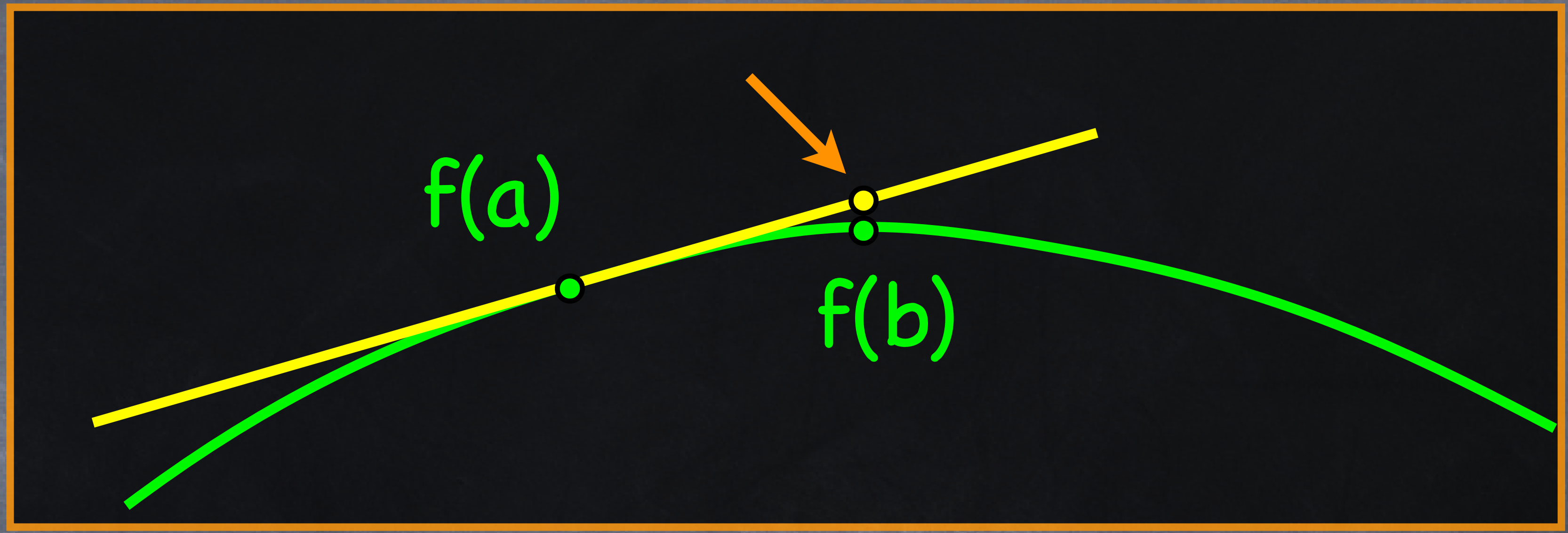


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$L(x) = f(a)+f'(a)(x-a)$

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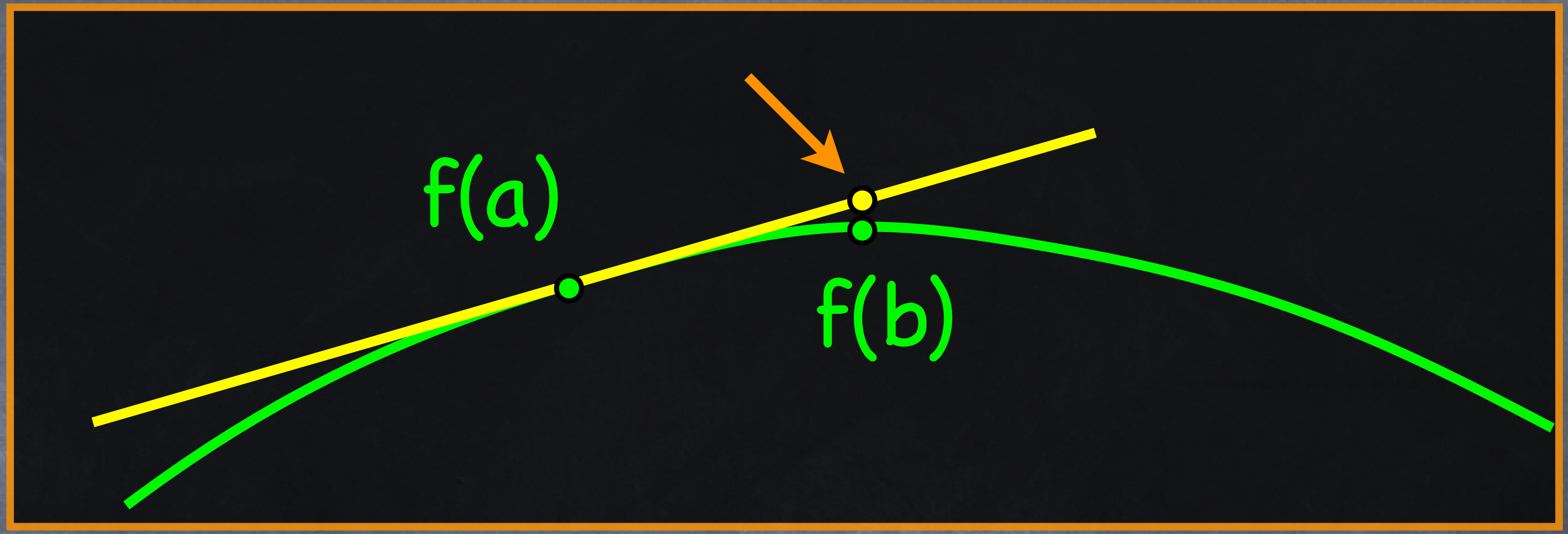


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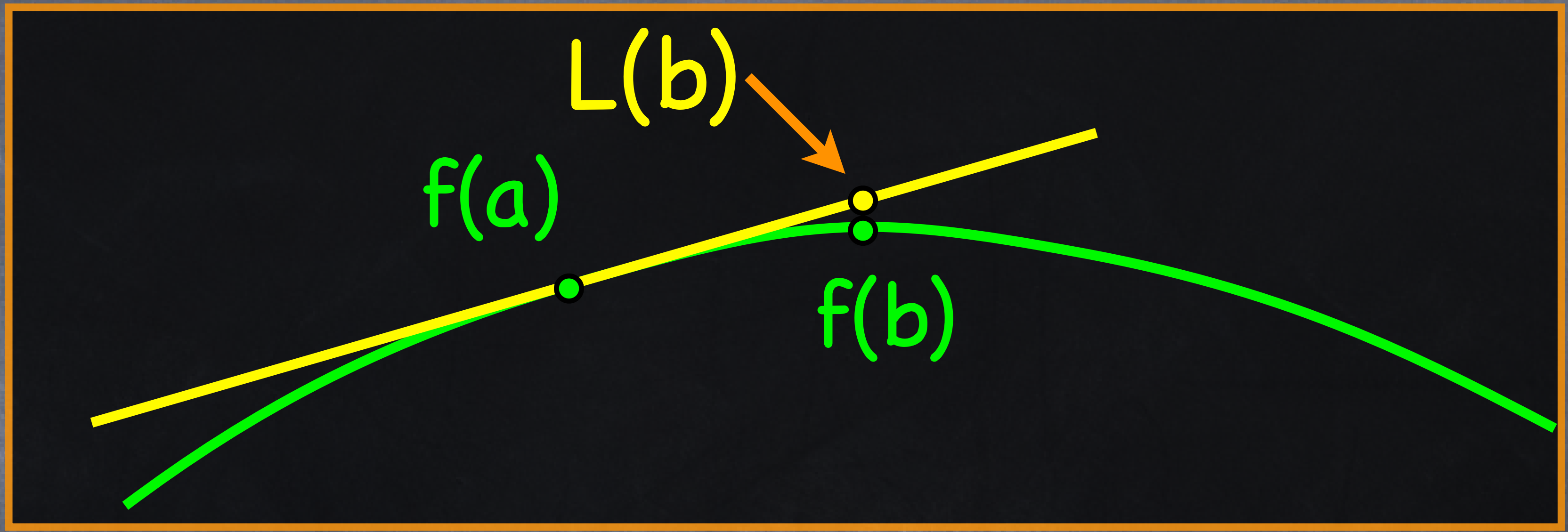


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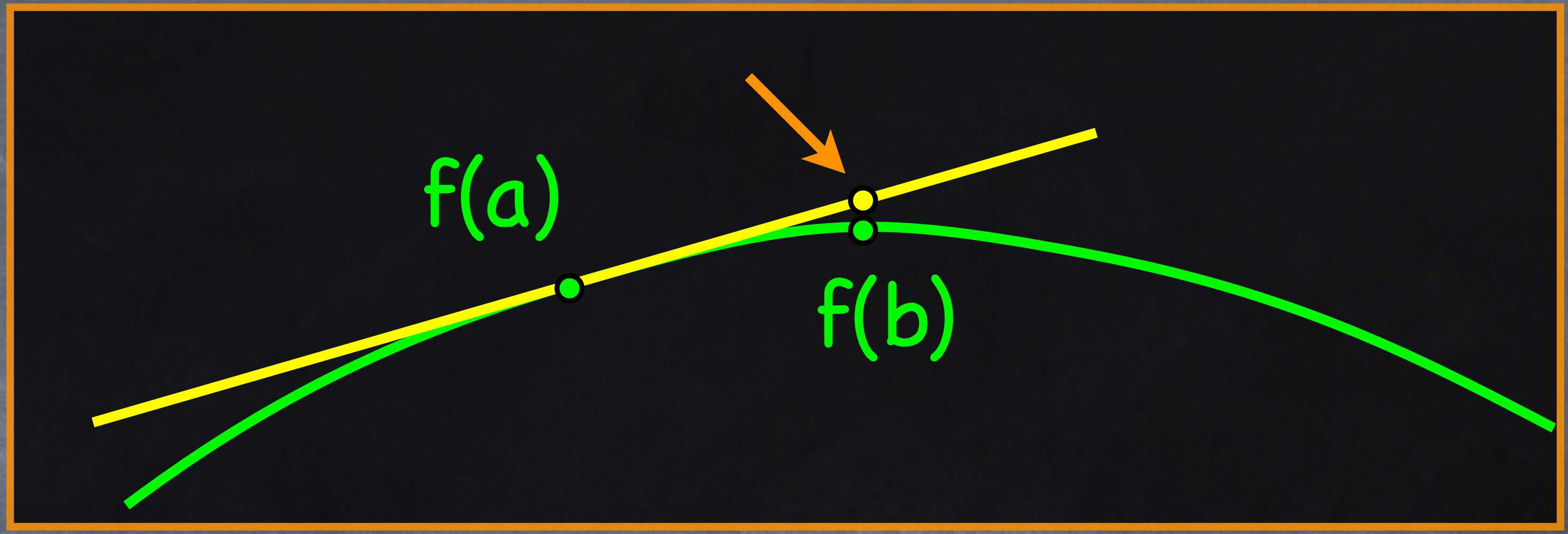


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$L(x) = f(a)+f'(a)(x-a)$

$L(b) = f(a)+f'(a)(b-a)$

# Use linear approximation to estimate $\sqrt{99}$

Step 1: Find the tangent line to f(x) = $\sqrt{x}$ at

(A) x = 1

(B) x = 10

(C) x = 99

(D) x = 100

(E) Don't know.

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Step 2: Plug ______ in to the tangent line equation

$L(x) = f'(a)(x-a) + f(a)$.

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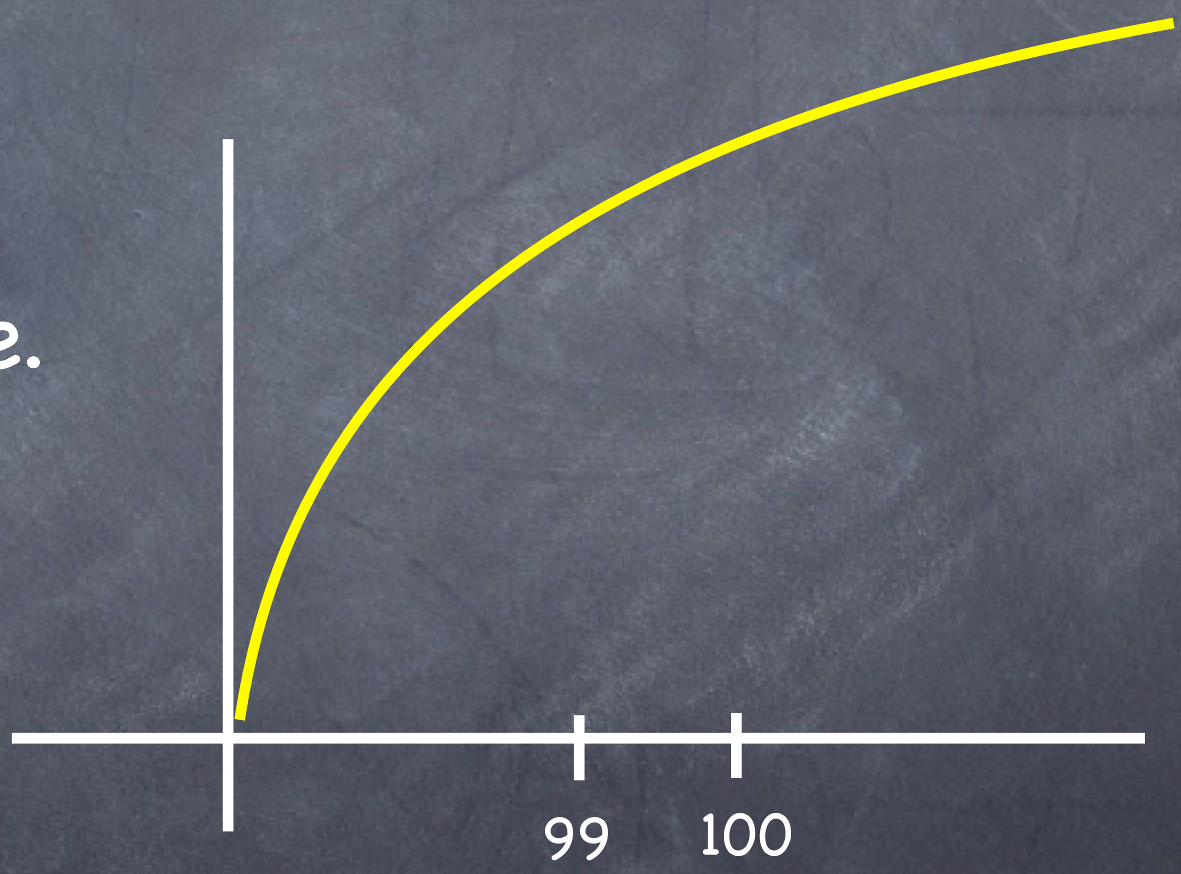

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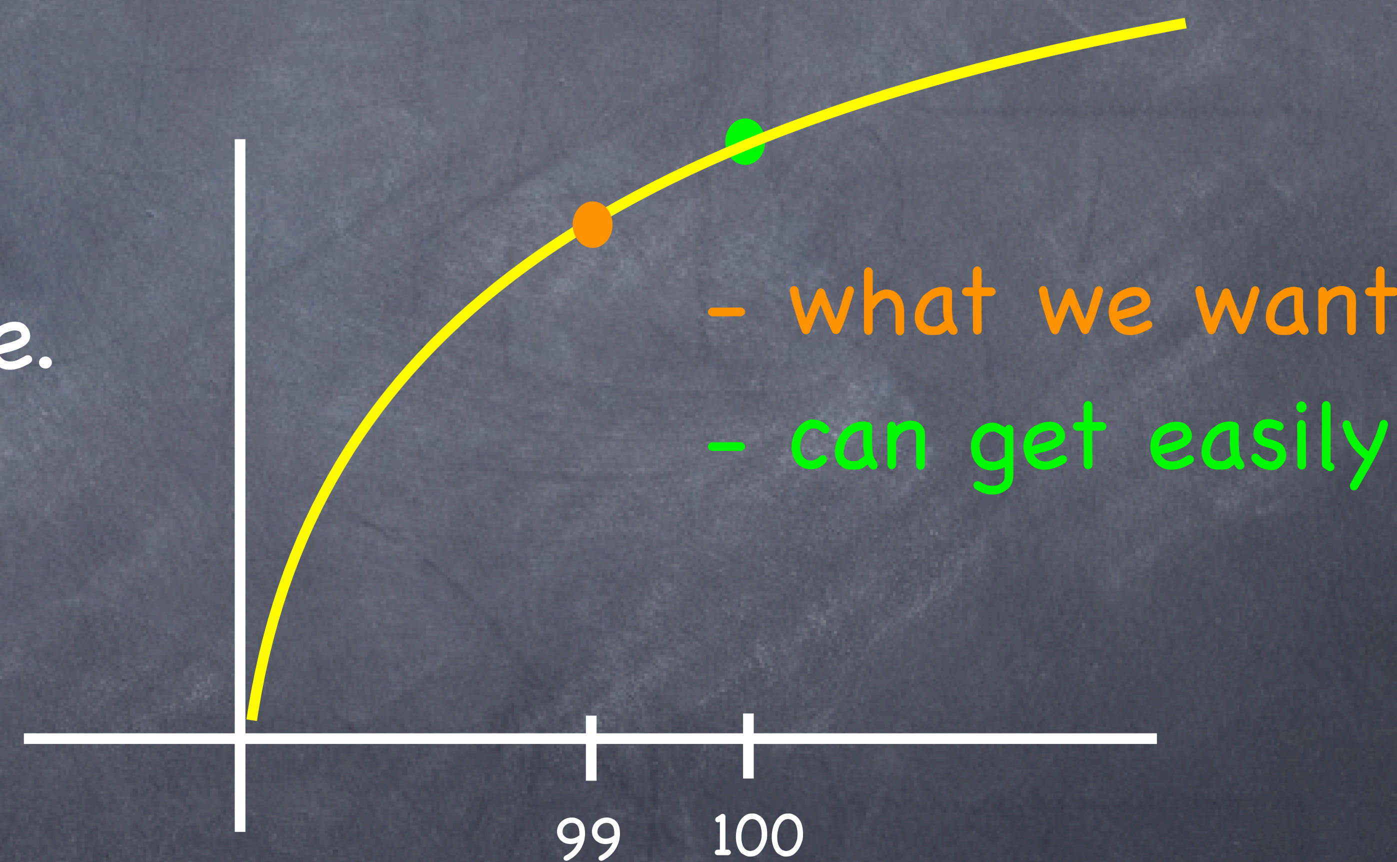

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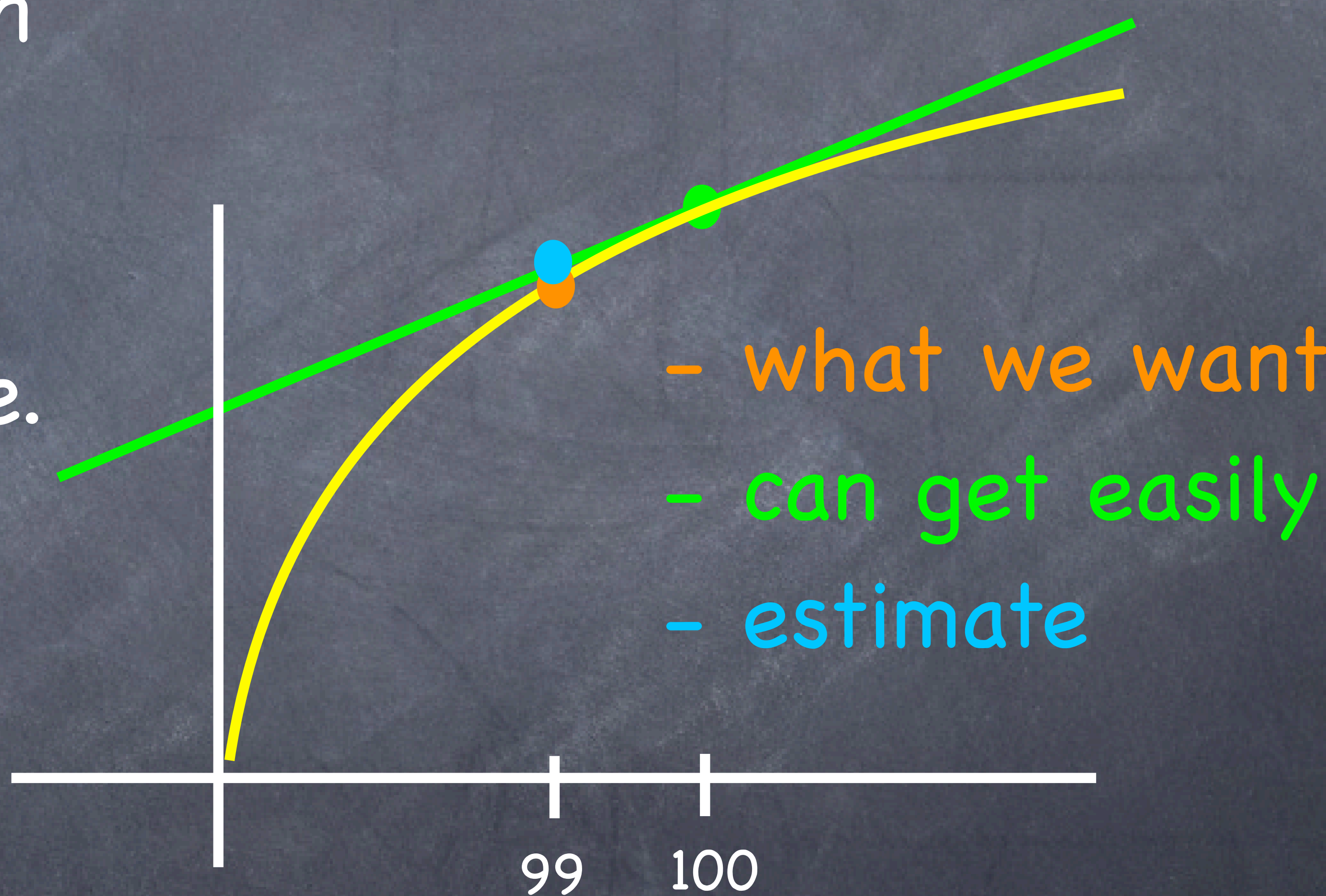

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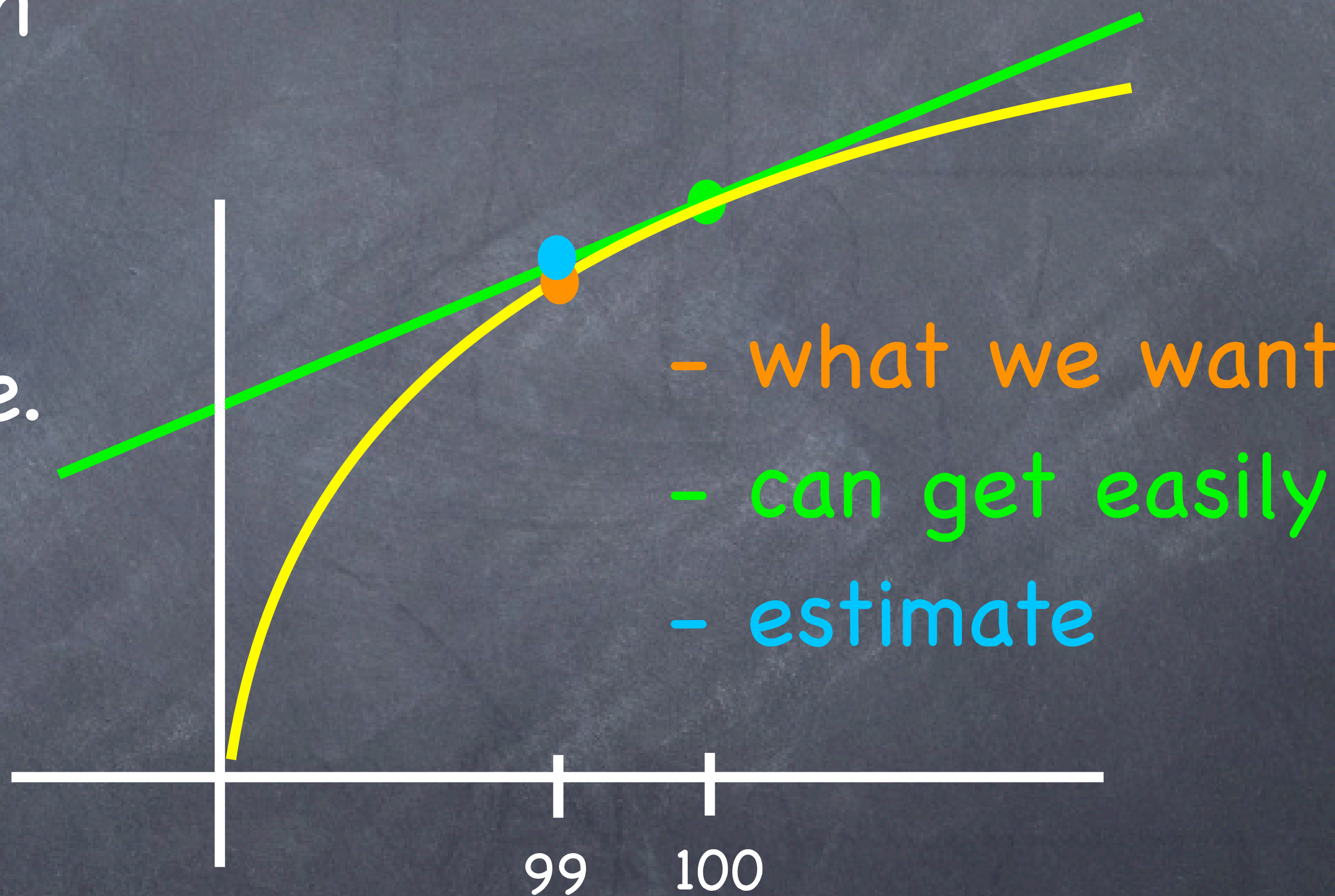


Blue is the estimate. B > O.

# Use linear approximation to estimate $\sqrt{99}$

(A) 9.94

(B) 9.95

(C) 9.96

(D) 9.97

(E) Don't know.

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(B) 9.95

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(D) 9.97

(E) Don't know.

- $f(x) = x^{1/2}$.
- b=99.
- a=100.
- $f(b) \approx f(a) + f'(a)(b-a)$

  $\approx 10 + 1/20\ (99-100)$

  $\approx 10 - 1/20 = 10 - 0.05 = 9.95.$

(You should be able to do this without a calculator on the midterm/exam!)

# Use linear approximation to estimate sin(3)

(A) 0

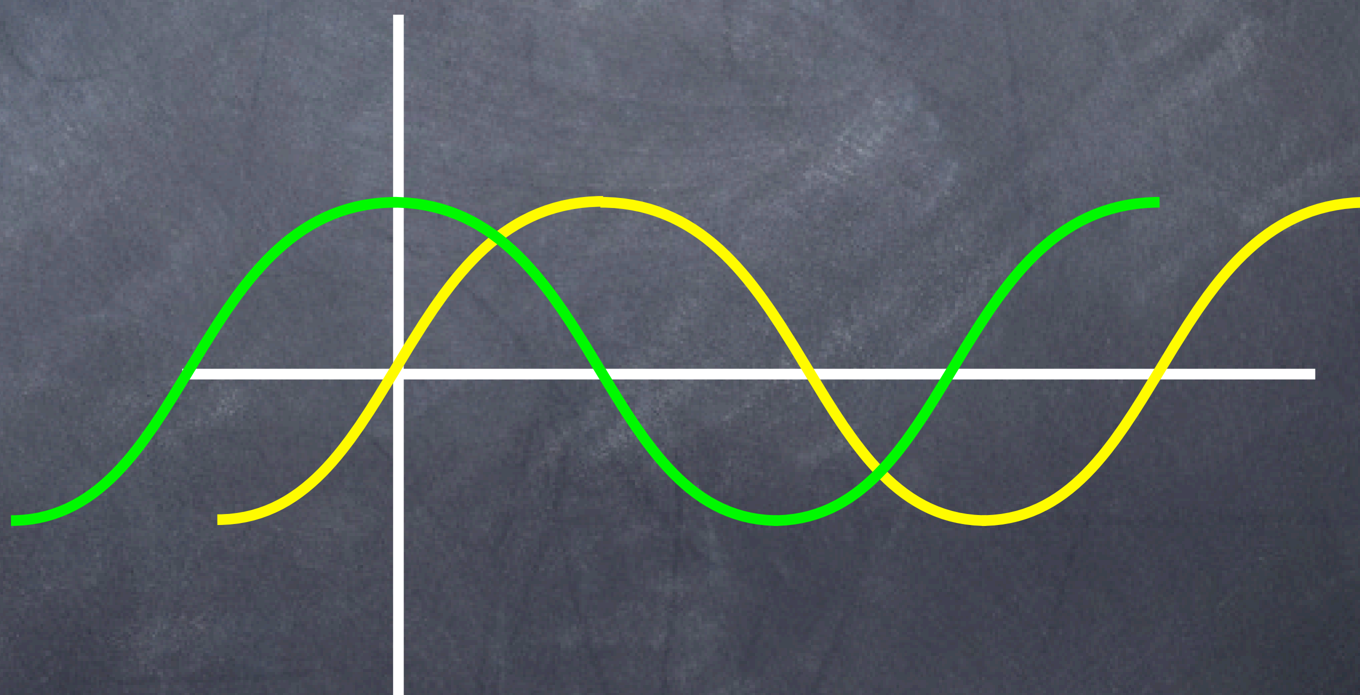
(B) $\pi$

(C) 0.141120...

(D) 0.14159...

(E) Don't know.

Recall: $(\sin(x))' = \cos(x)$



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- $f(x) = \sin(x)$.
- $b = 3$.
- $a = \pi$.
- $f(b) \approx f(a) + f'(a)(b-a)$

$\approx 0 + (-1)(3-\pi) = 0.14159...$

(You don't have to memorize $\pi$ for the midterm/exam.)

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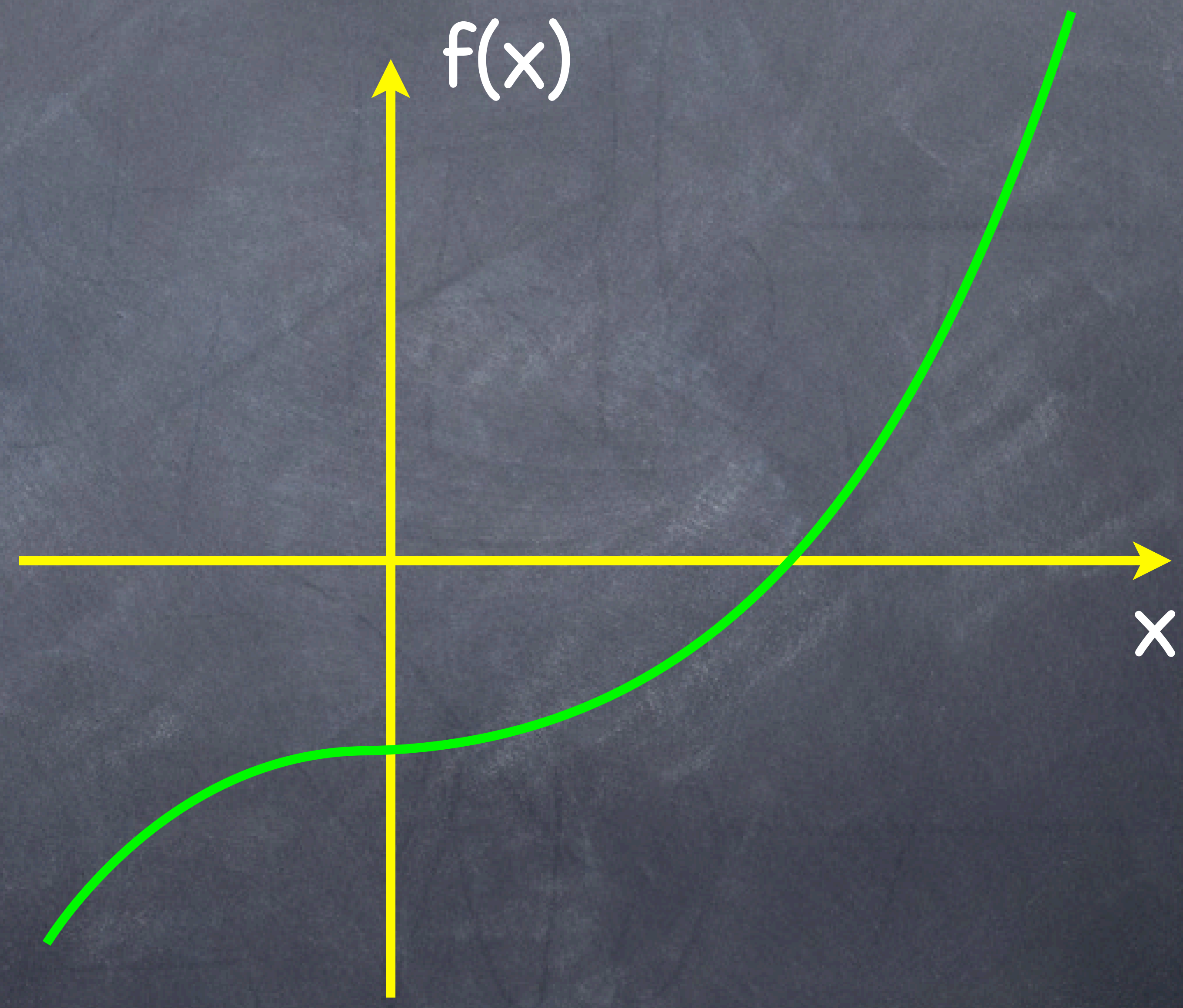
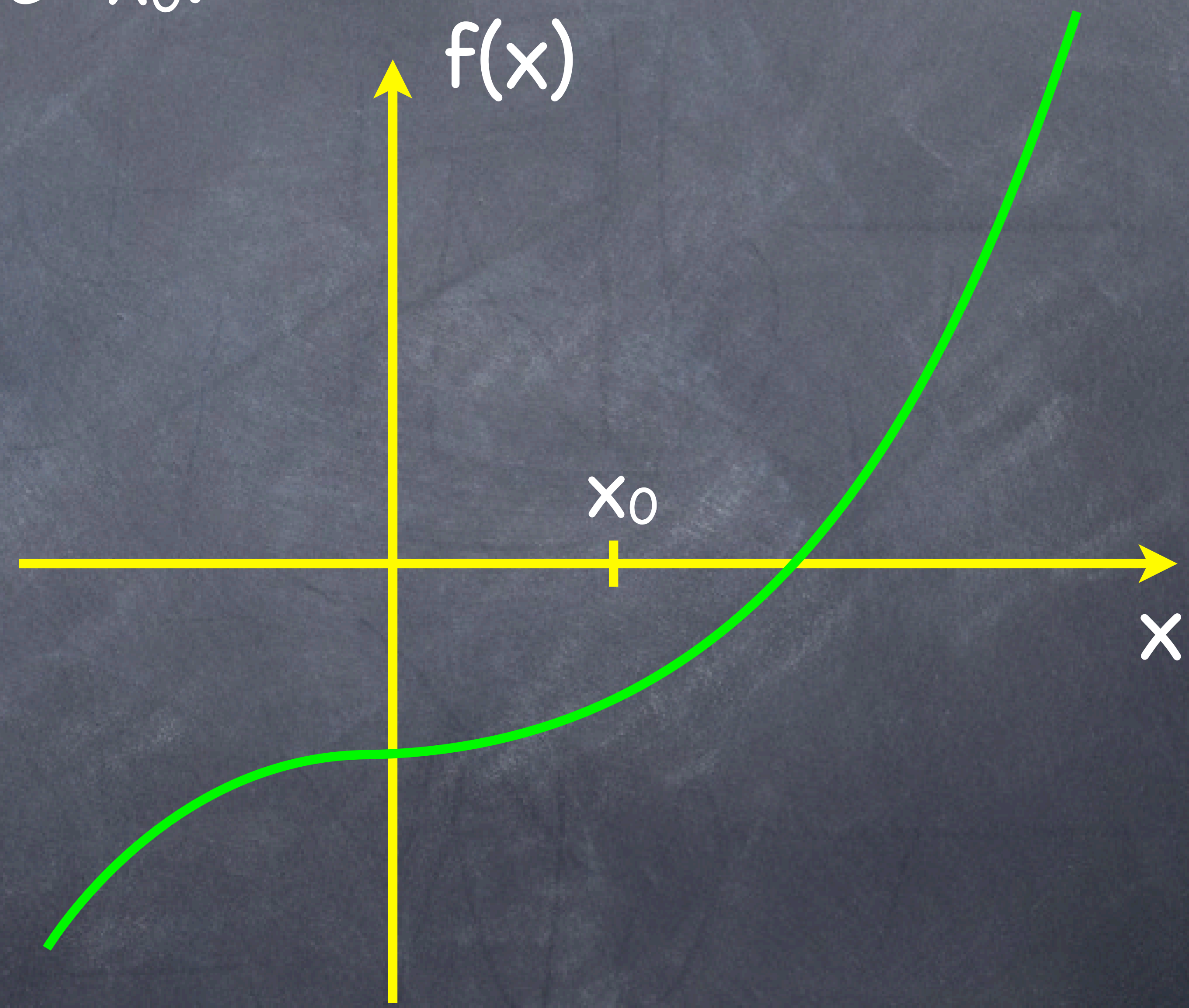
  - intersections of functions, $g(x)=h(x)$:
    - define $f(x) = g(x)-h(x)$,
  - irrational numbers: e.g. cuberoot(2):
    - define $f(x)=x^3-2$.

# Find the zero of $f(x)=x^3-2$.



# Find the zero of $f(x)=x^3-2$.

- Start with a "guesstimate" $x_0$.



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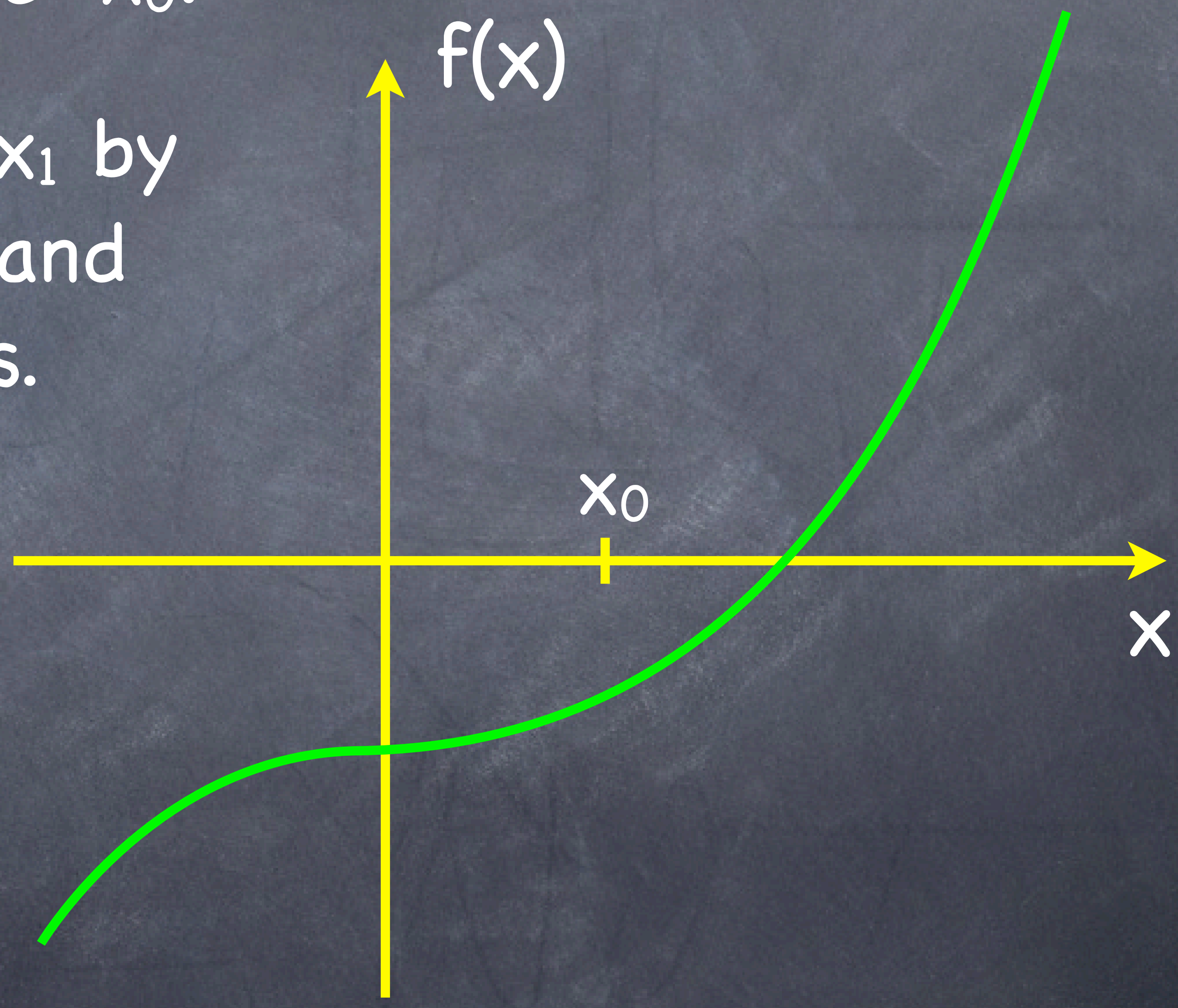

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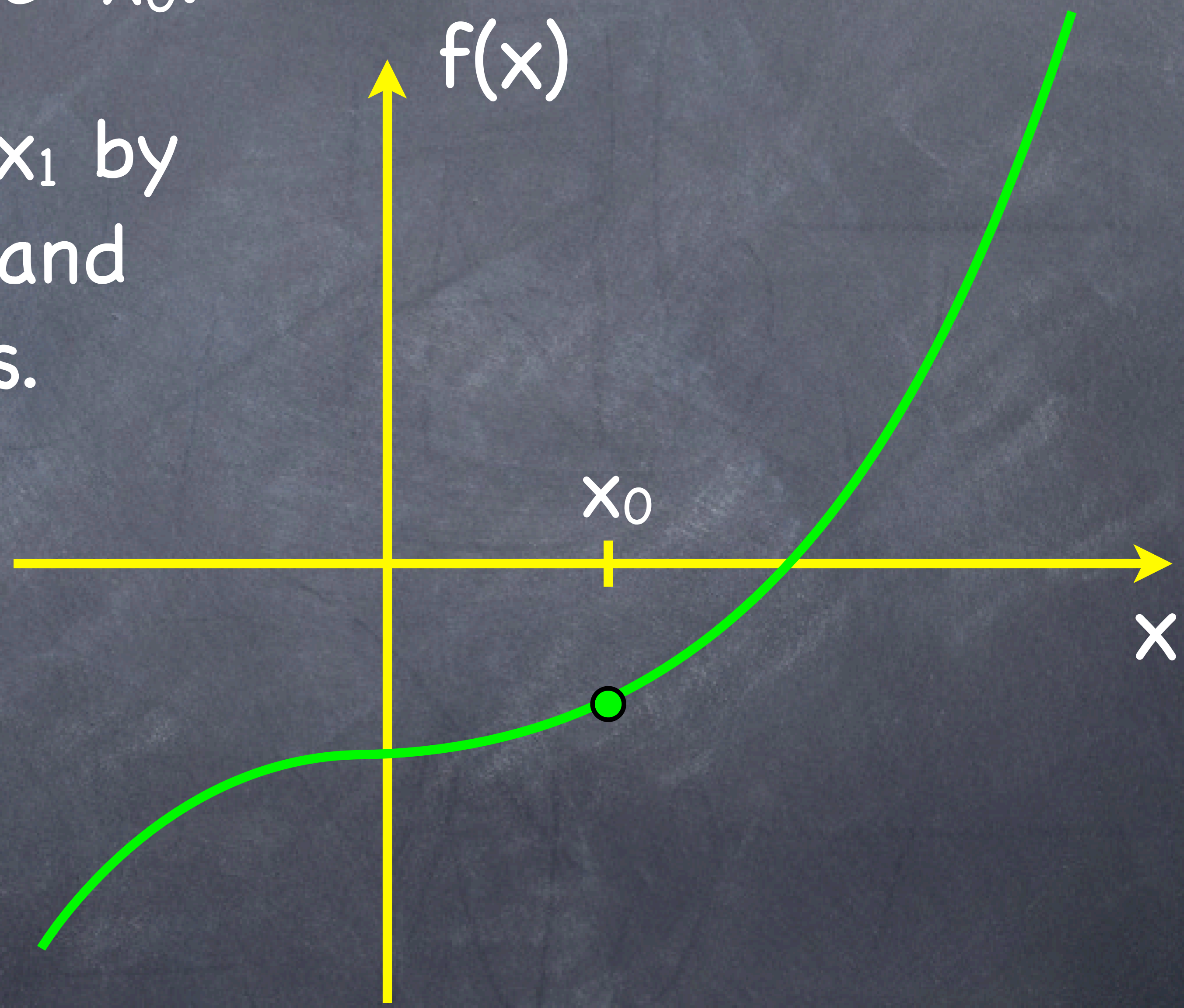

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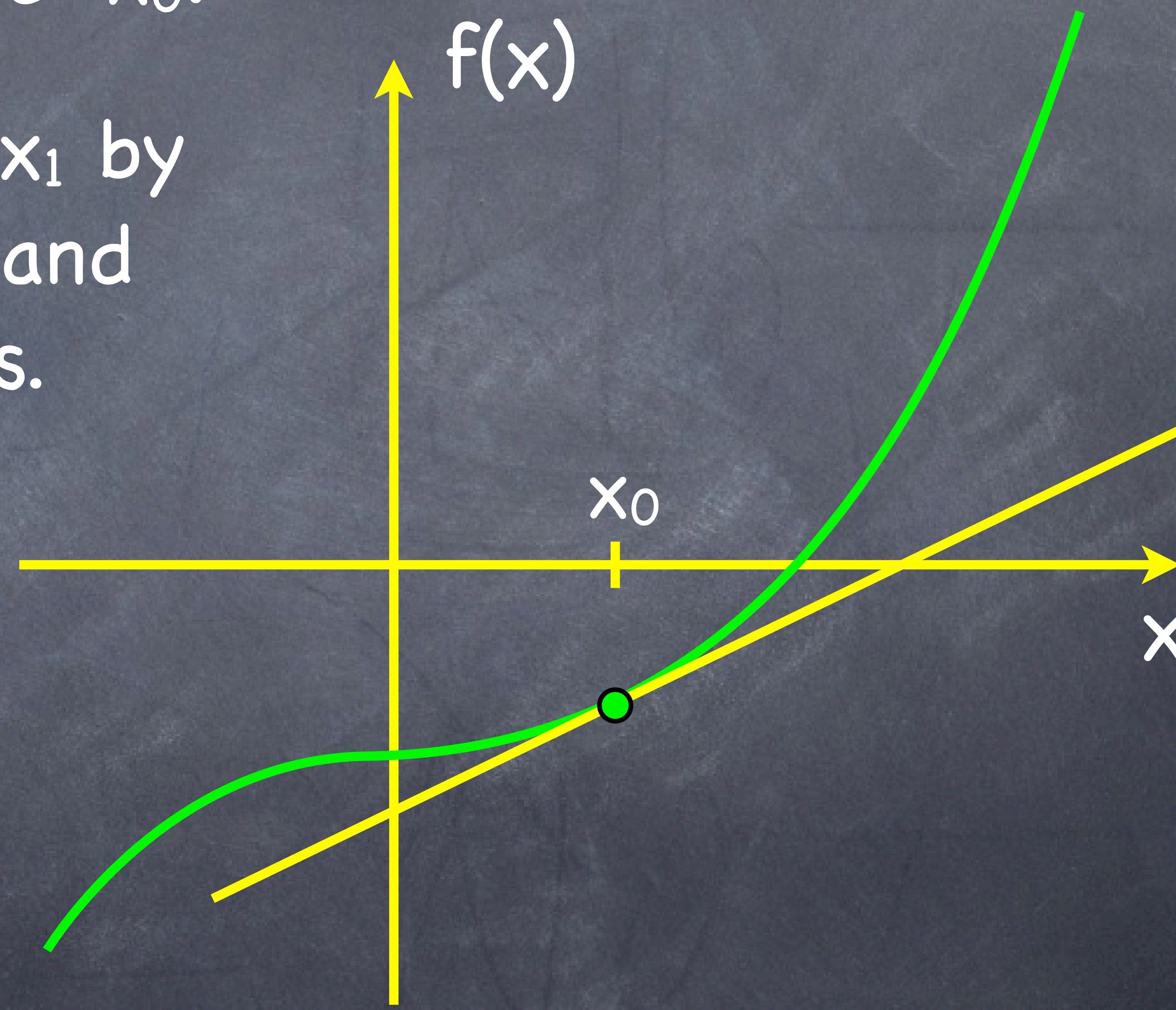

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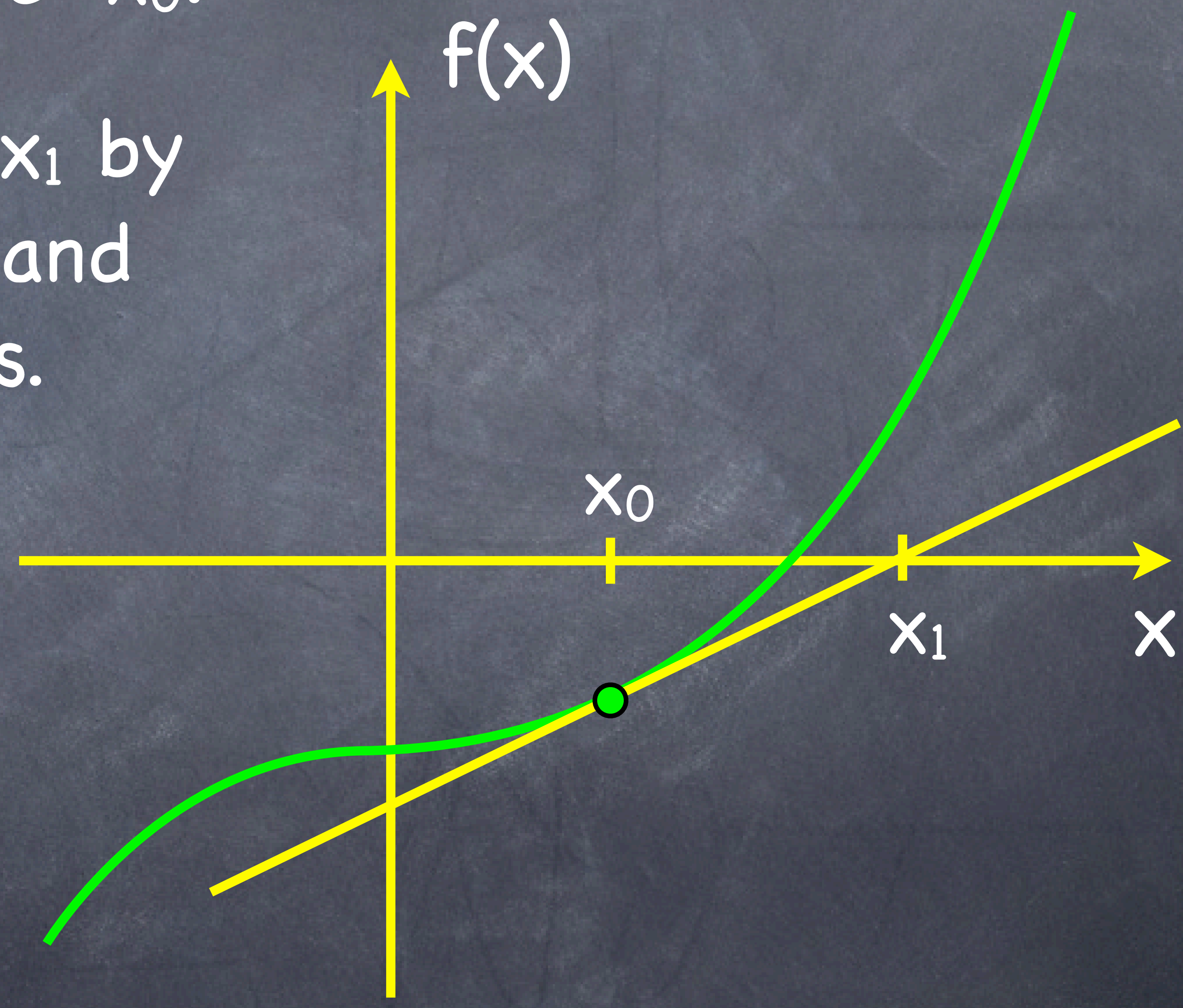

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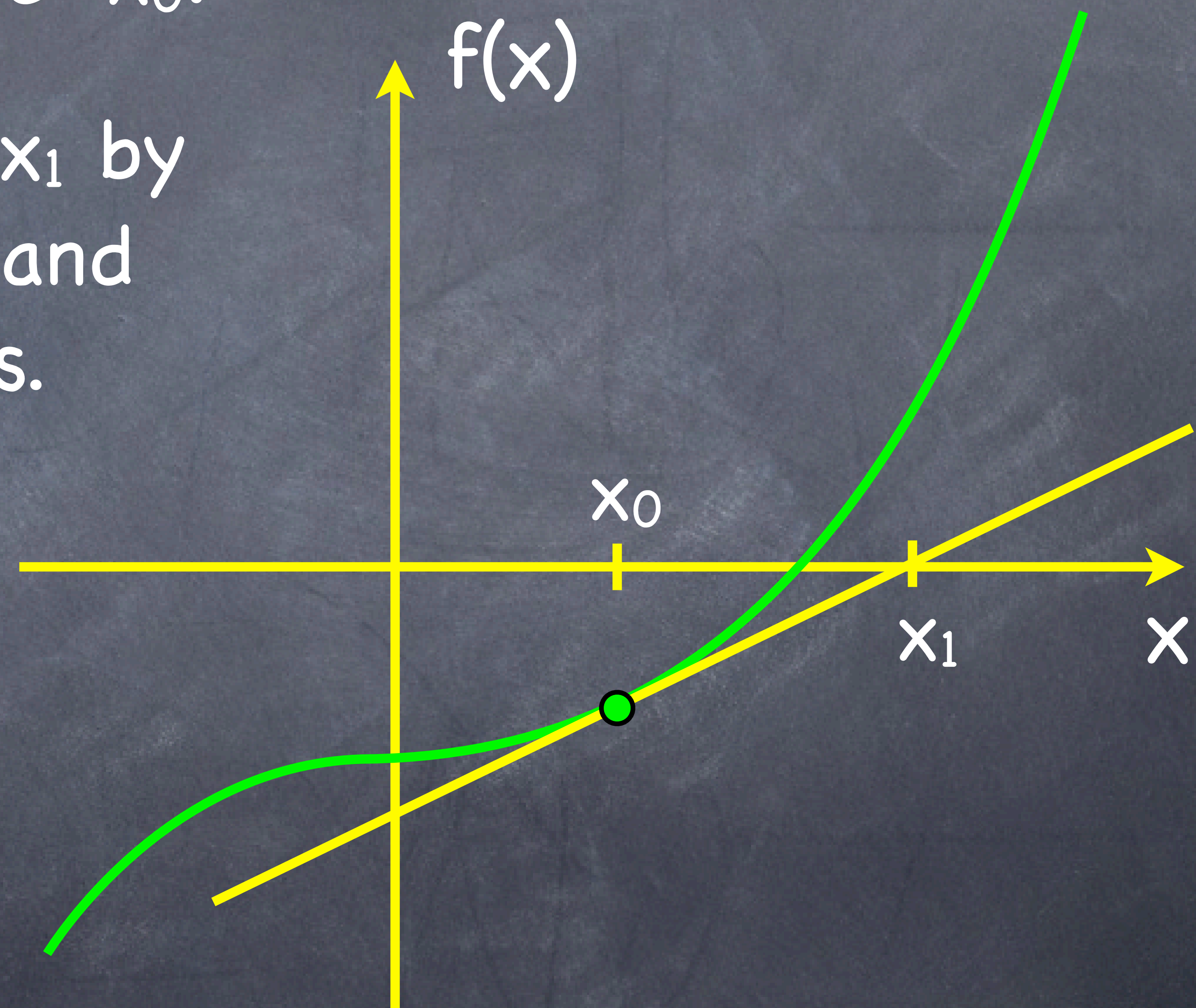
- Repeat to get $x_2$...



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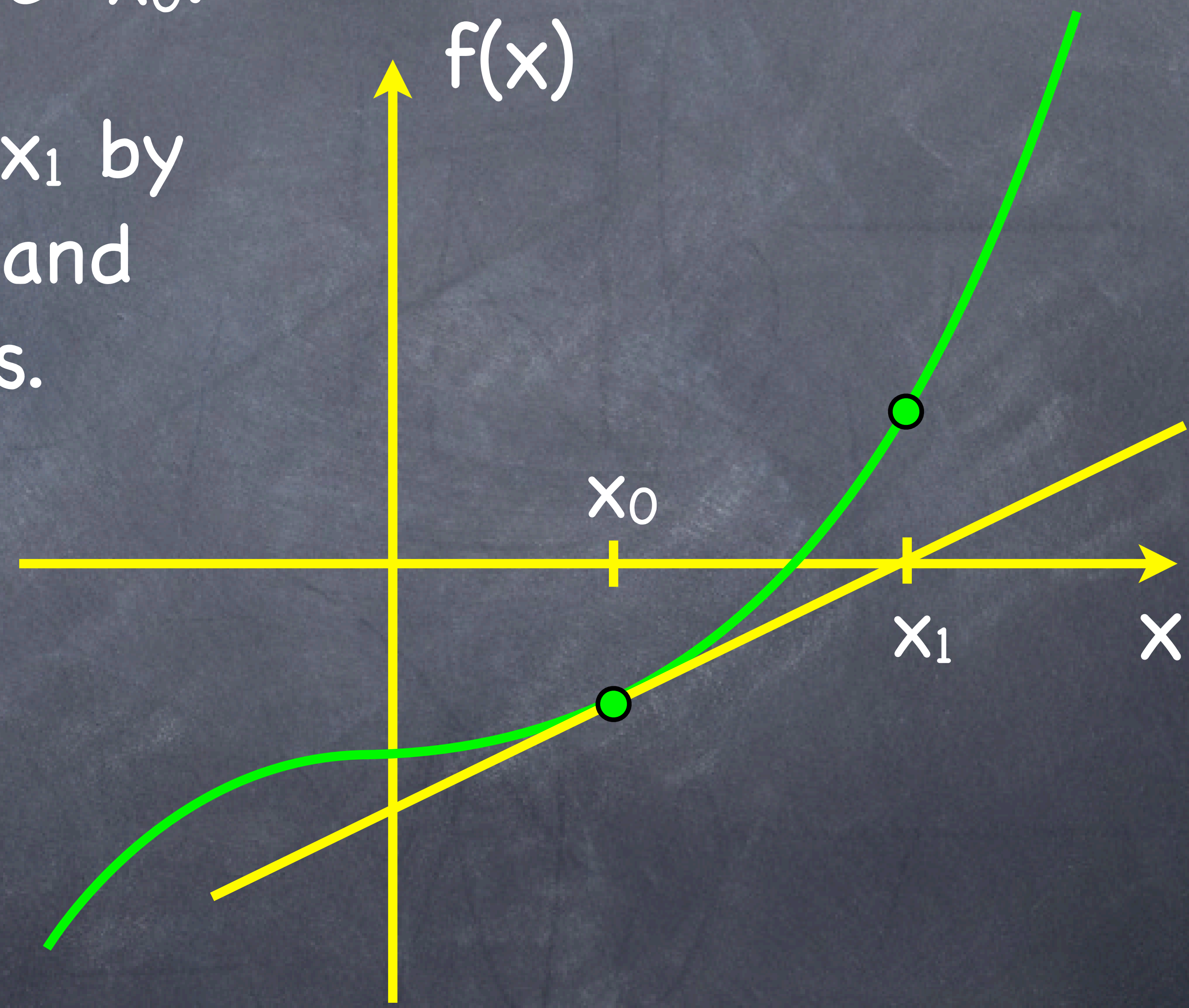

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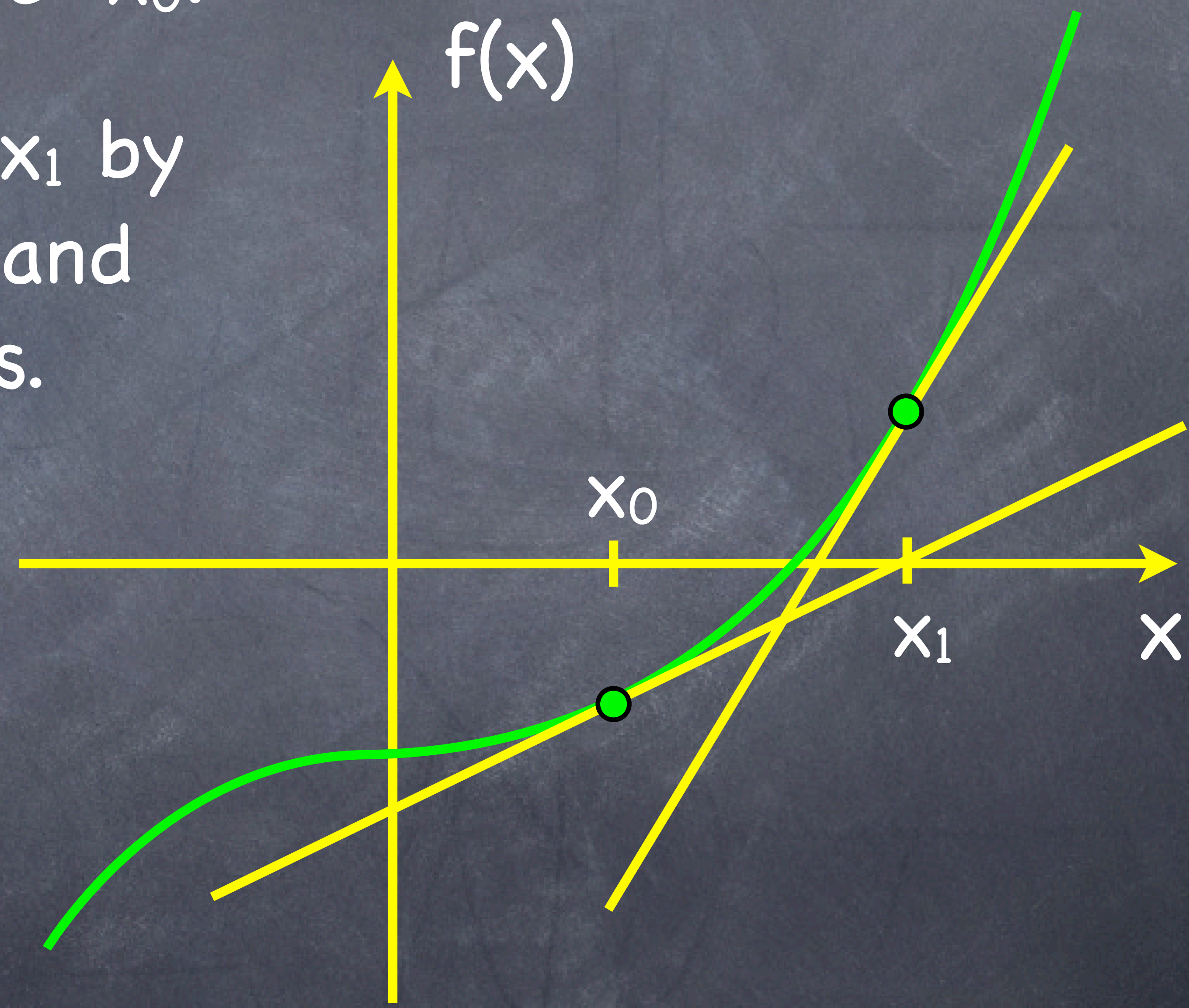

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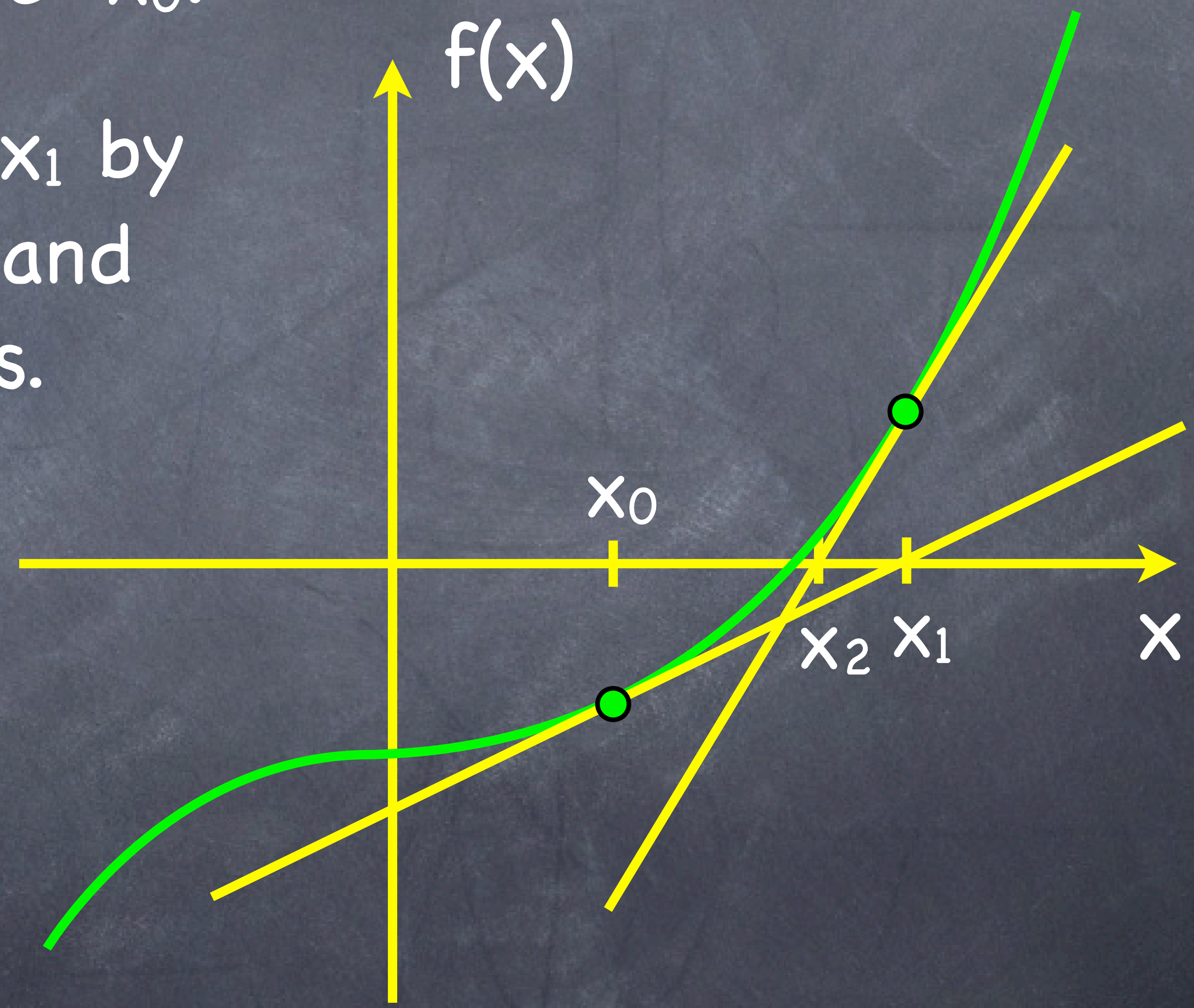
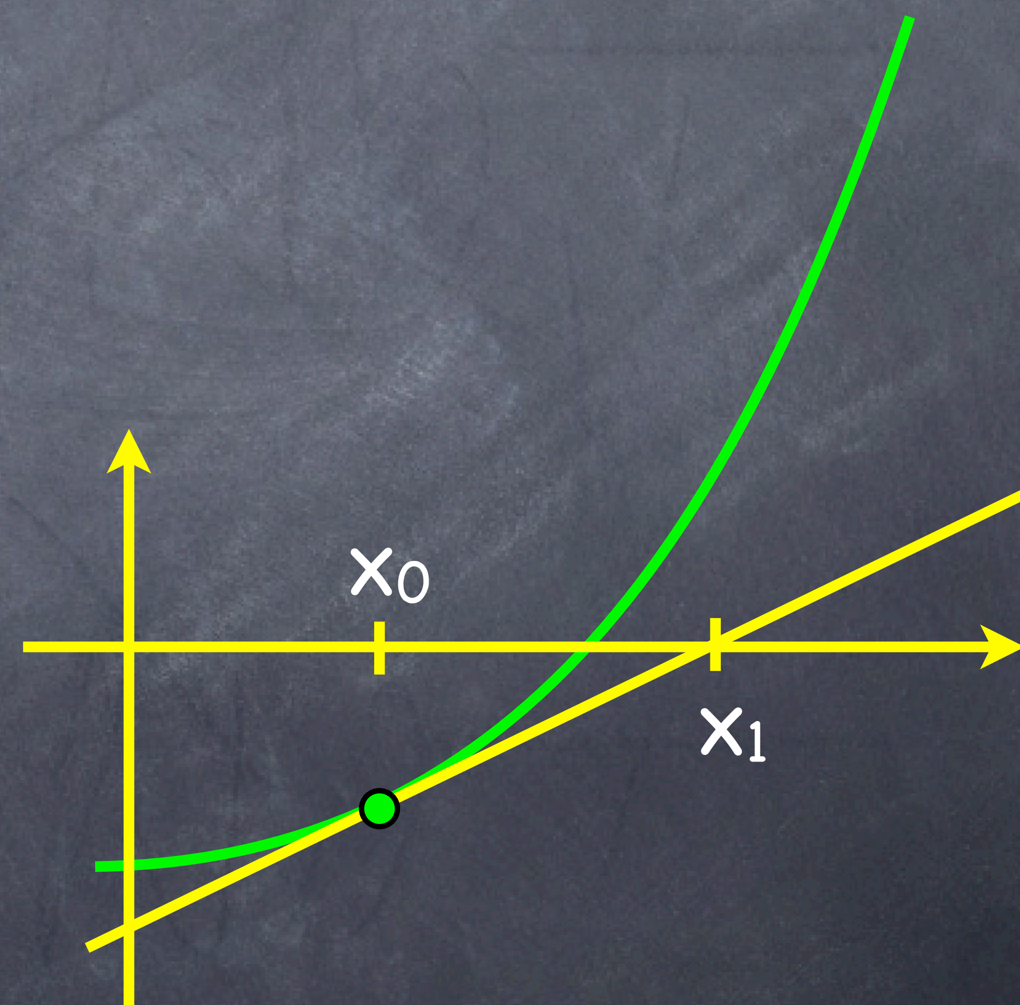


# Calculating successive estimates



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- First, find tangent line at $x_n$:
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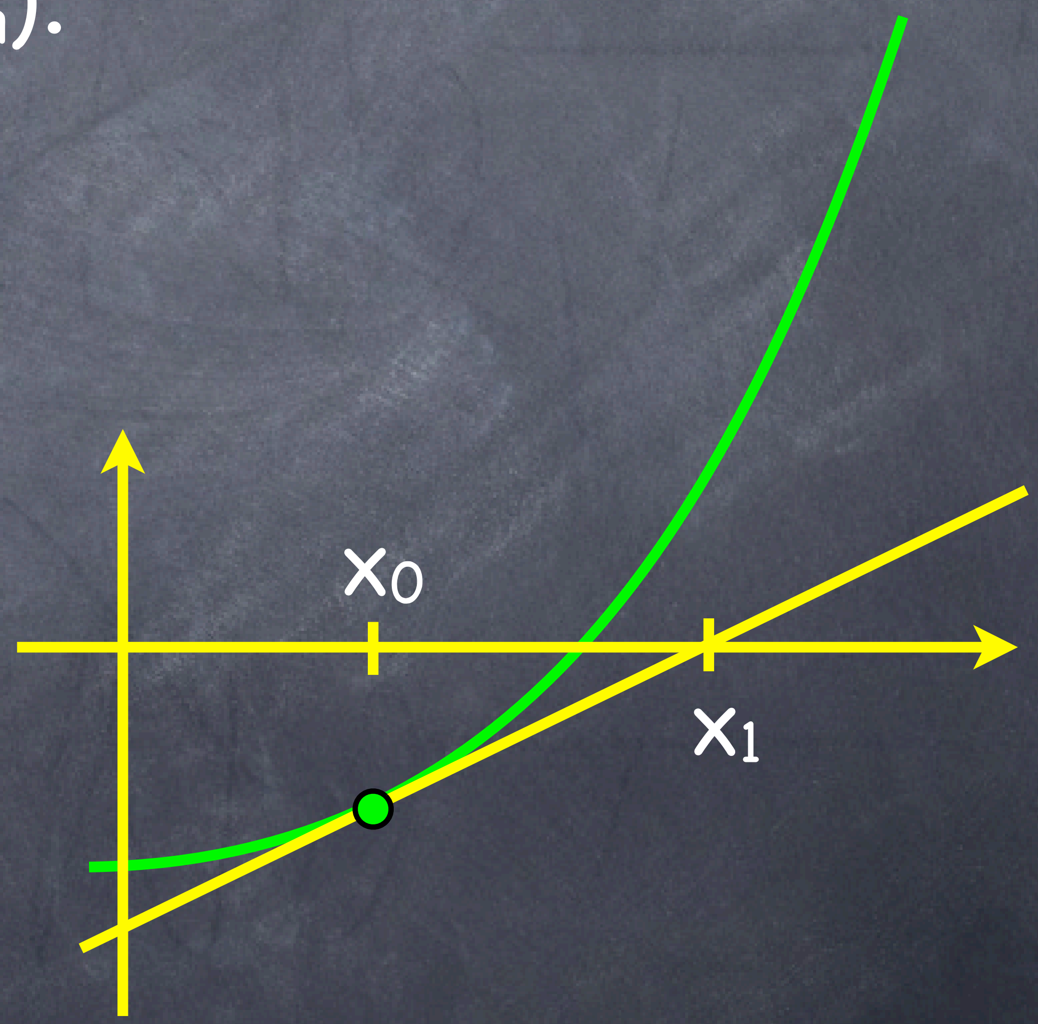

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- First, find tangent line at $x_n$:
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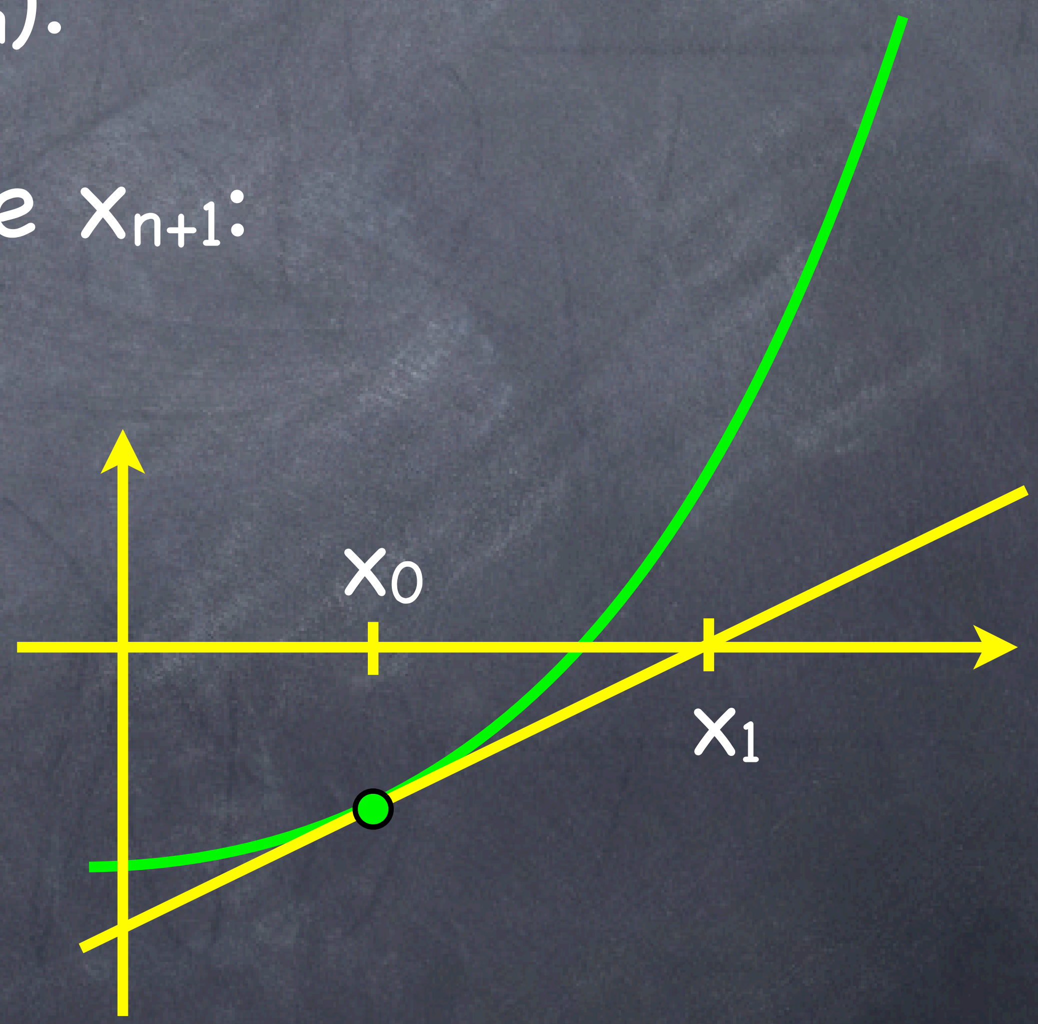
- Find x-intercept, that will be $x_{n+1}$:

(A) $x_{n+1} = x_n + f(x_n) / f'(x_n)$.

(B) $x_{n+1} = x_n - f(x_n) / f'(x_n)$.

(C) $x_{n+1} = x_n - f'(x_n) / f(x_n)$.

(D) $x_{n+1} = x_n + f'(x_n) / f(x_n)$.



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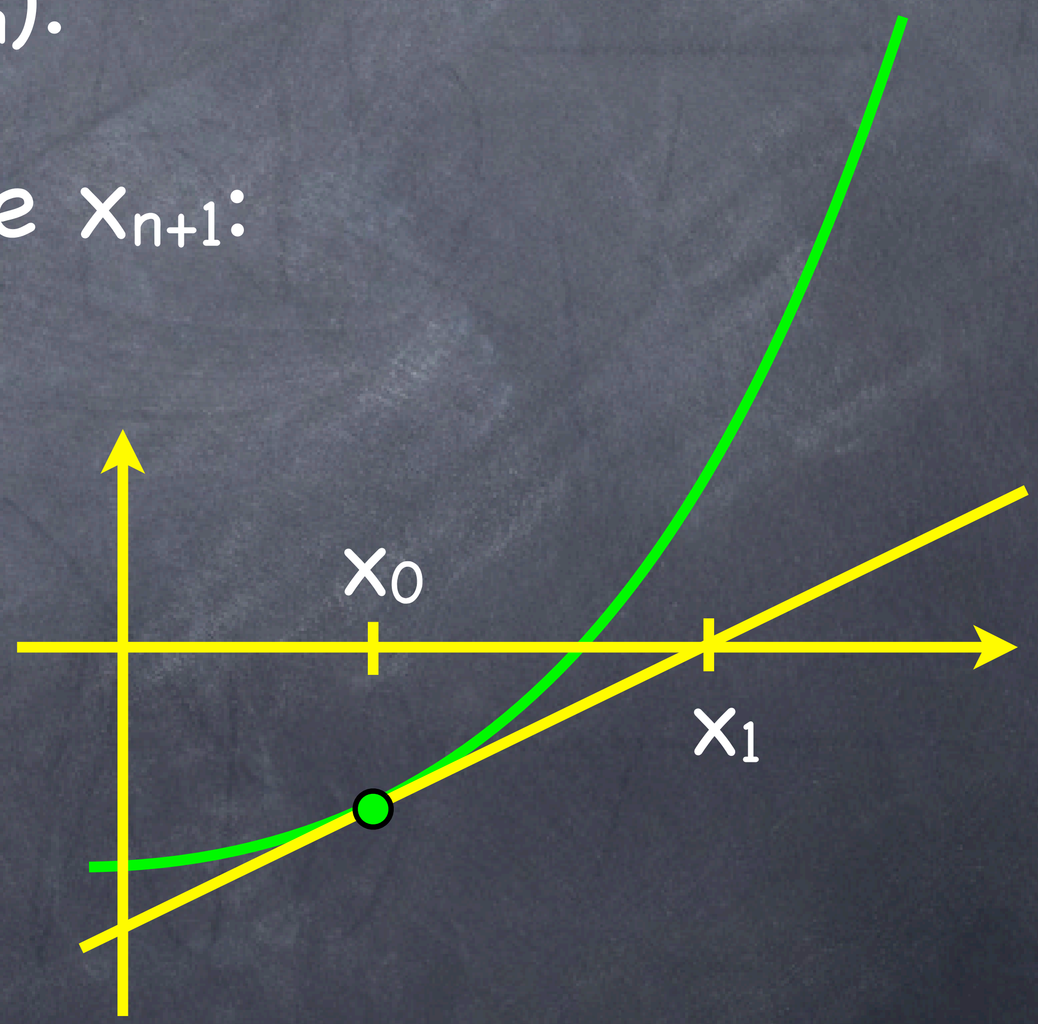

# To estimate $\sqrt{3}$, which function would you apply Newton's method to?

(A) $f(x) = x^{1/2}$

(B) $f(x) = x^{1/2} - 3$

(C) $f(x) = x^2$

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(E) $f(x) = (x-3)^{1/2}$

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(D) $f(x) = x^2 - 3$ <--- This one has a zero at $\sqrt{3}$.

(E) $f(x) = (x-3)^{1/2}$

# Estimate $\sqrt{3}$ using Newton's method with initial guess $x_0=2$.

(A) 7/4

(B) 97/56

(C) 1.7

(D) 1.73205080757

$$x_{n+1} = x_n - f(x_n) / f'(x_n).$$

Finished already? Now use linear approximation.
Which approach is better?

# Estimate $\sqrt{3}$ using Newton's method with initial guess $x_0=2$.

(A) 7/4 = 1.75 <--- $x_1$

(B) 97/56 = 1.73214 <--- $x_2$

(C) 1.7

(D) 1.73205080757 <--- first 11 digits of $\sqrt{3}$ .

# Desmos...

- How to choose $x_0$...

# From the 2011 final exam

8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of $2\text{m/s}^2$.

   i. (4 points) Write down a function $d(t)$ that is the distance from your car to the moose where $t = 0$ indicates the moment when you start backing away.

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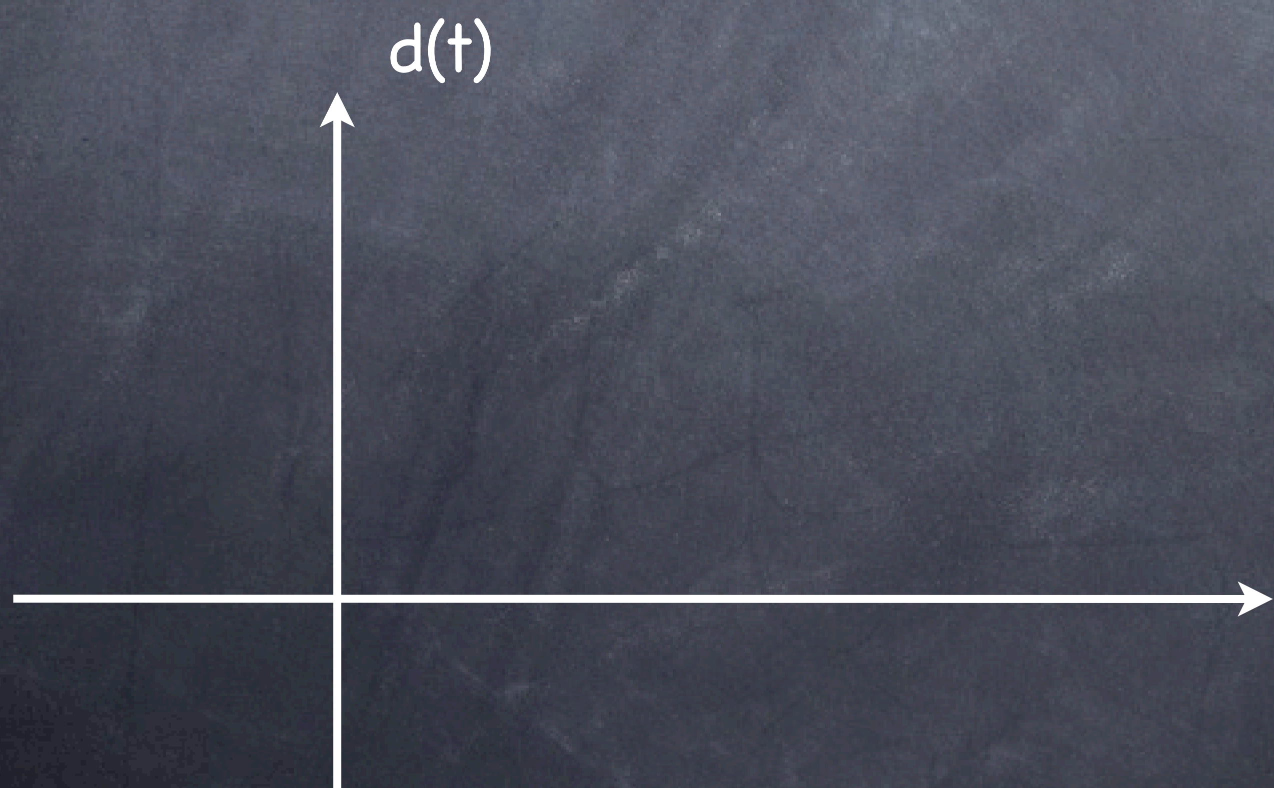

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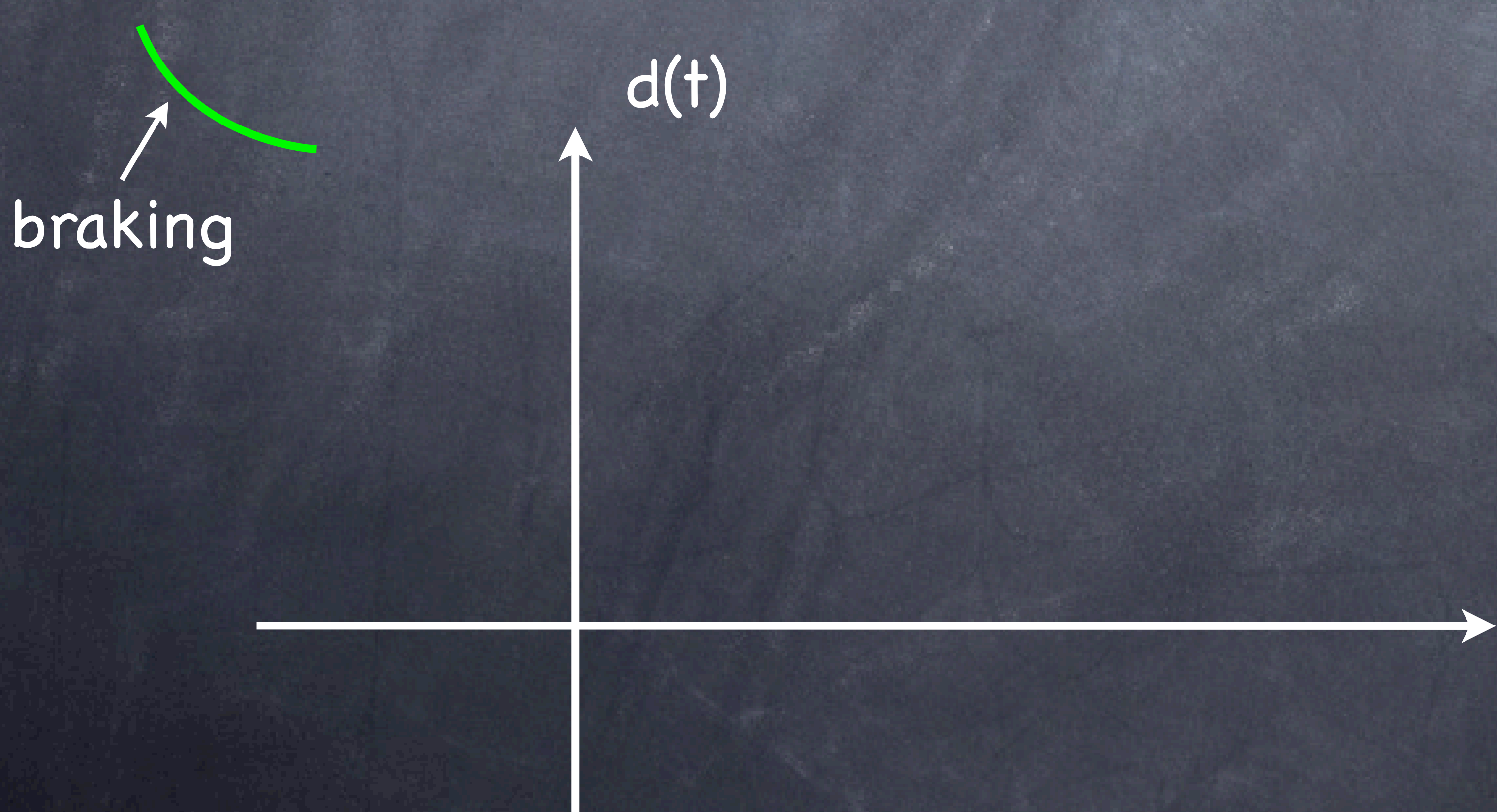

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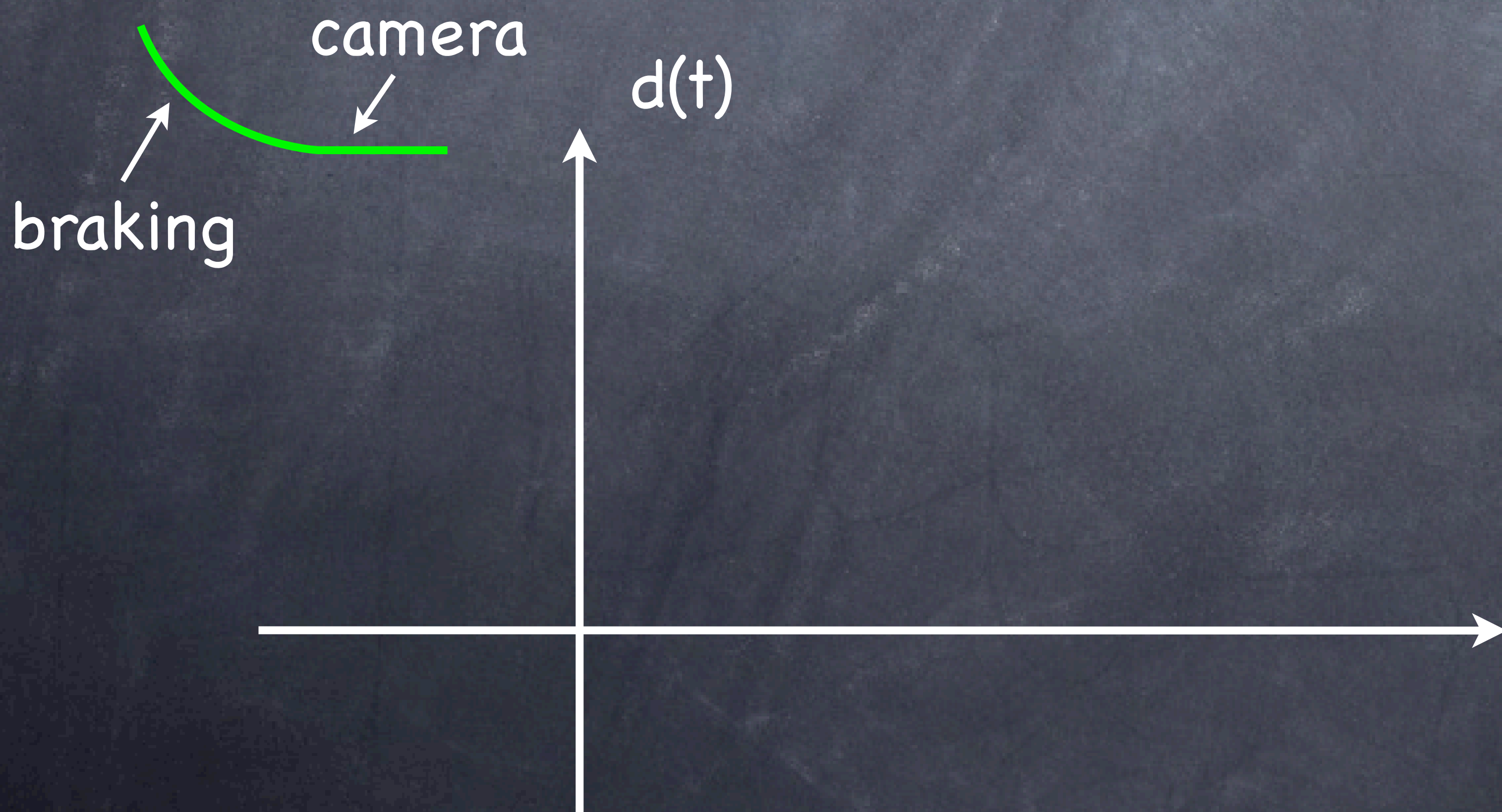

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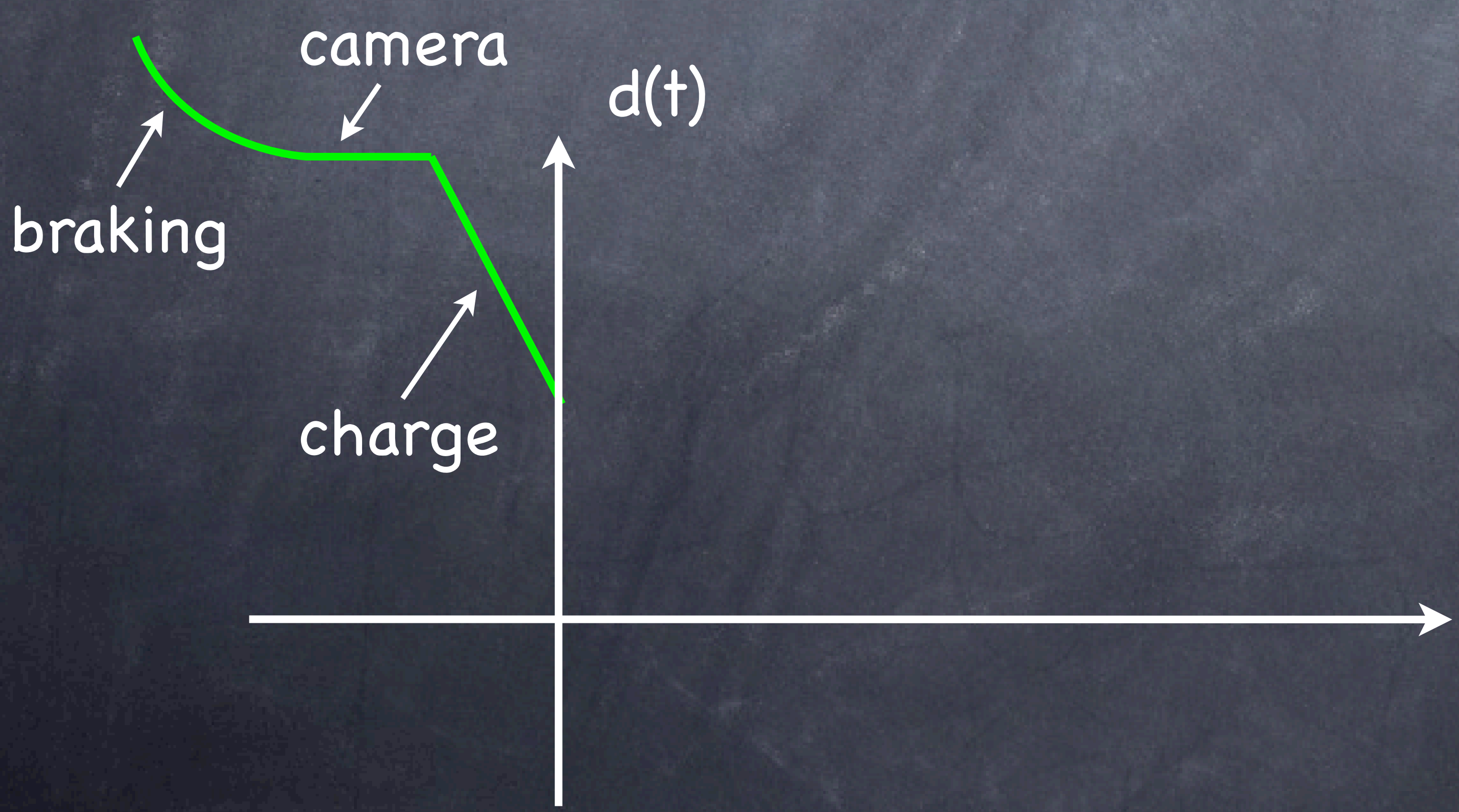

# From the 2011 final exam

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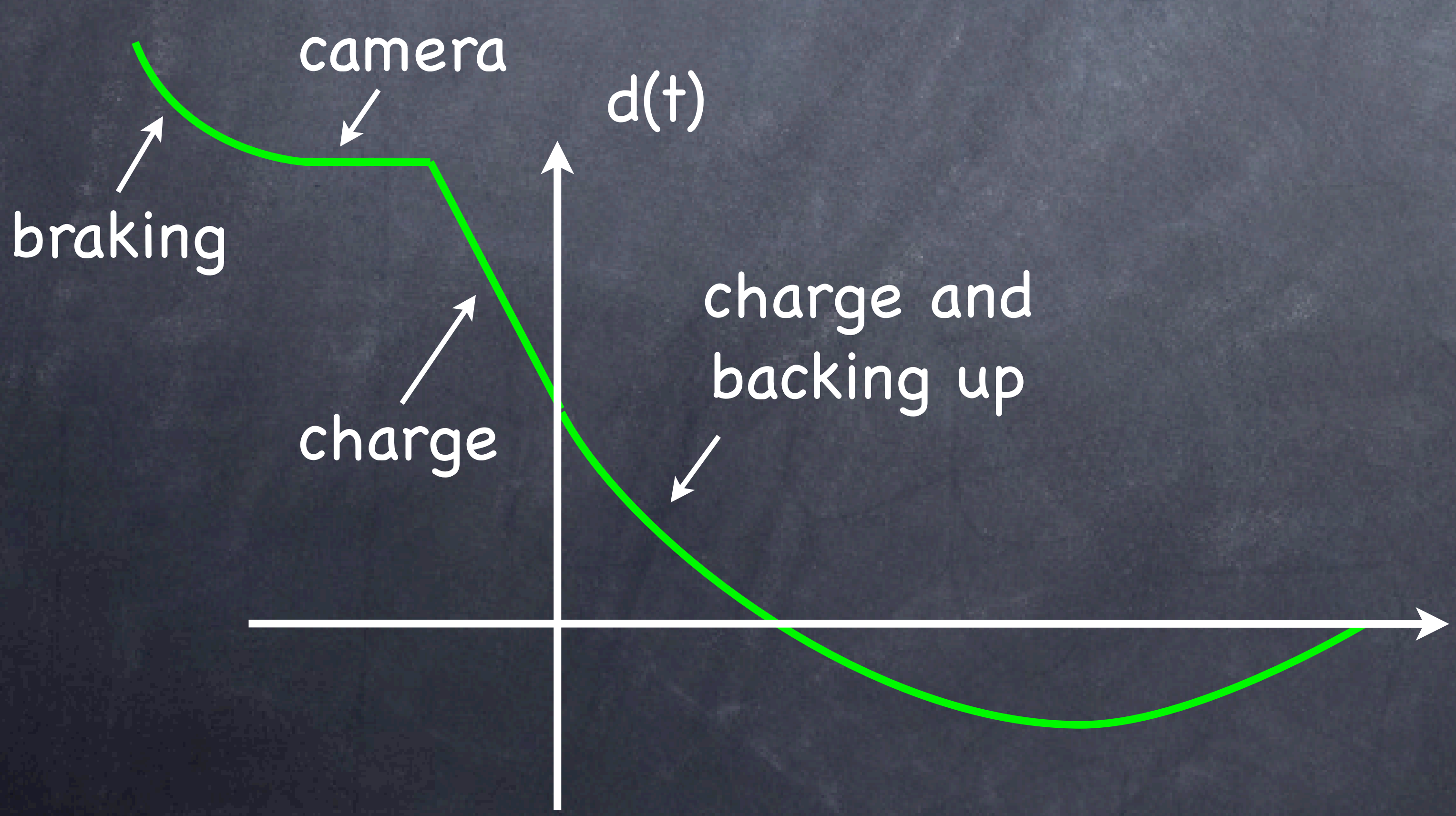

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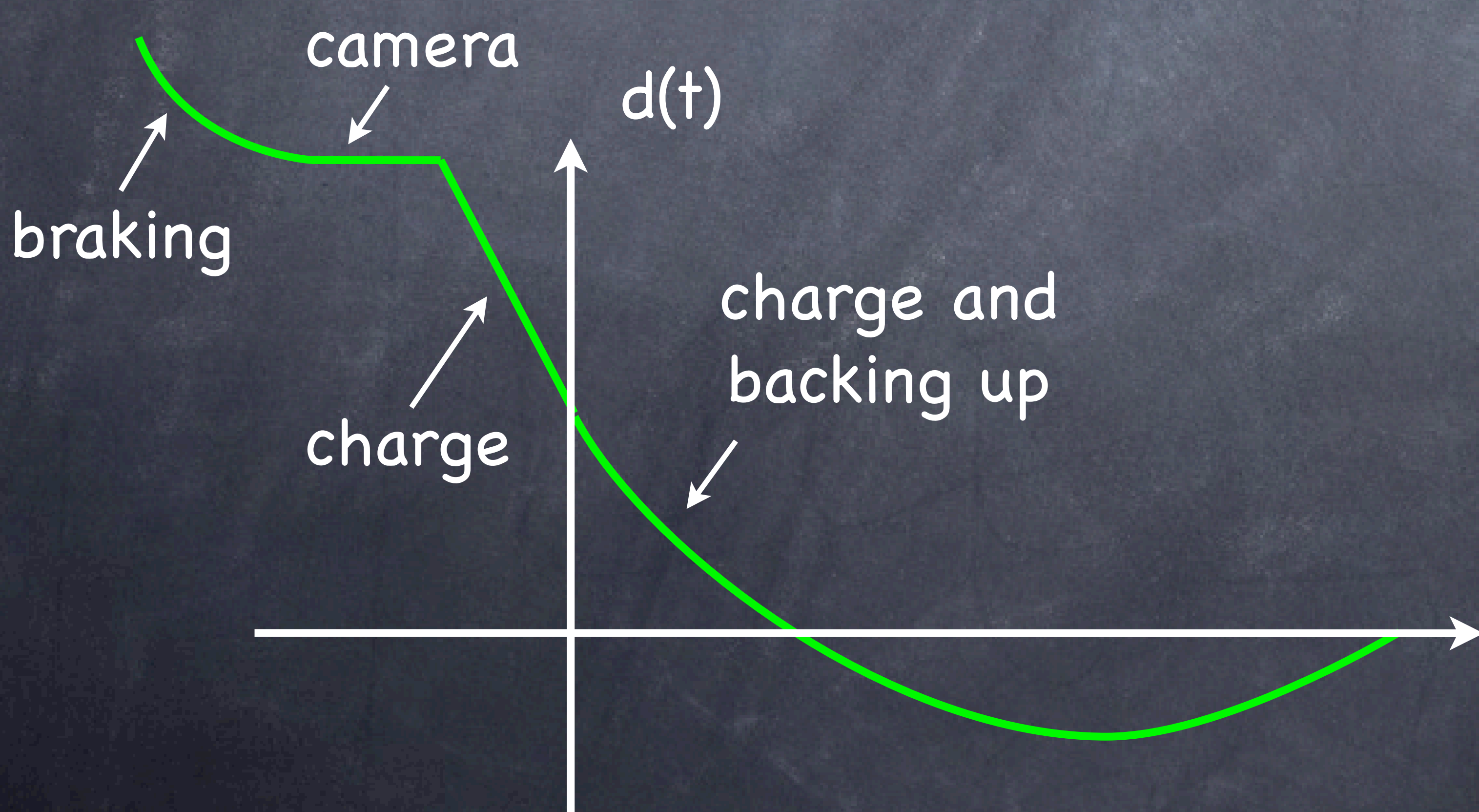


$v_{car}(t) = 2t$

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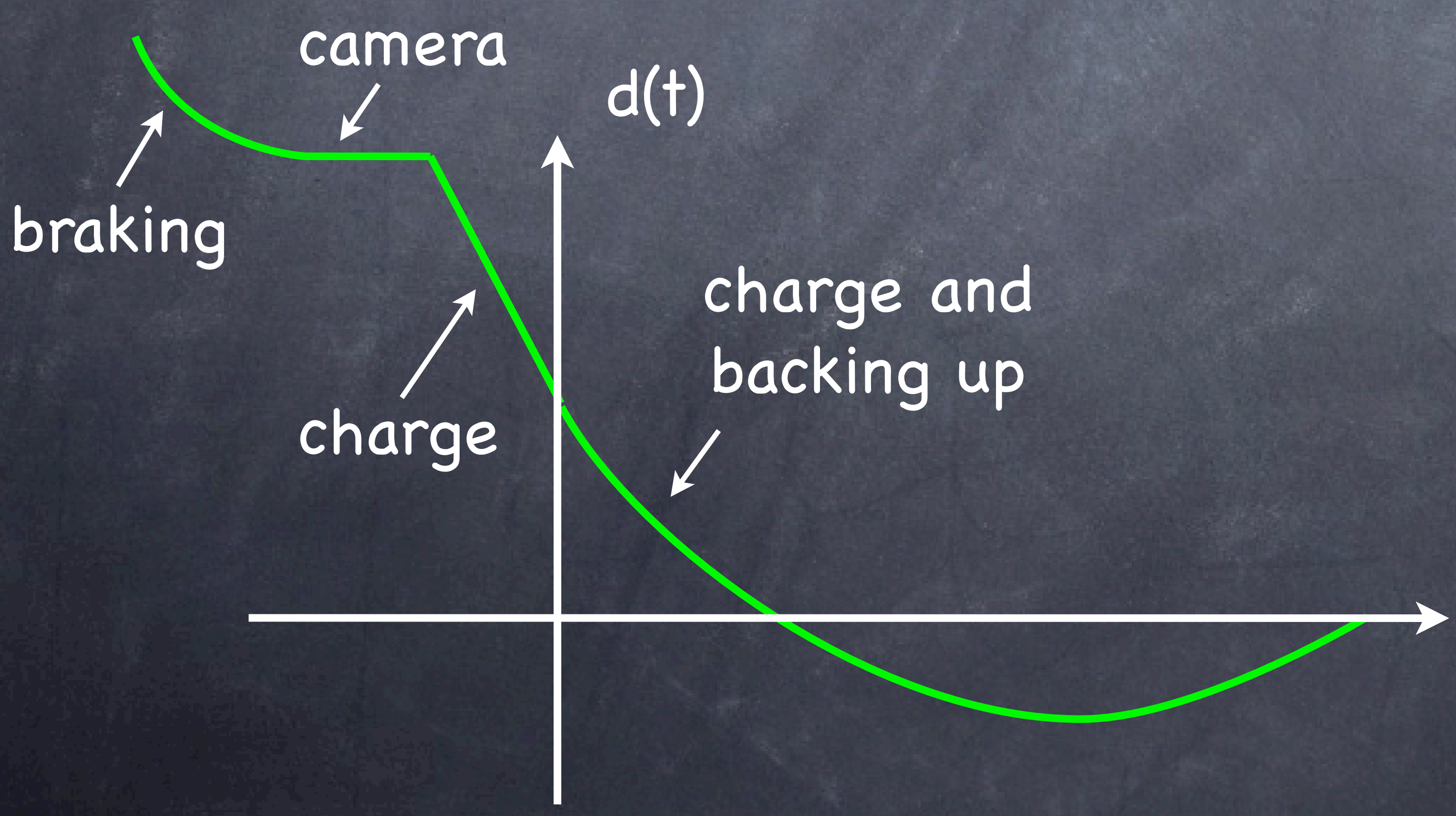


$v_{car}(t) = 2t$

$x_{car}(t) = t^2 + 12$

# From the 2011 final exam

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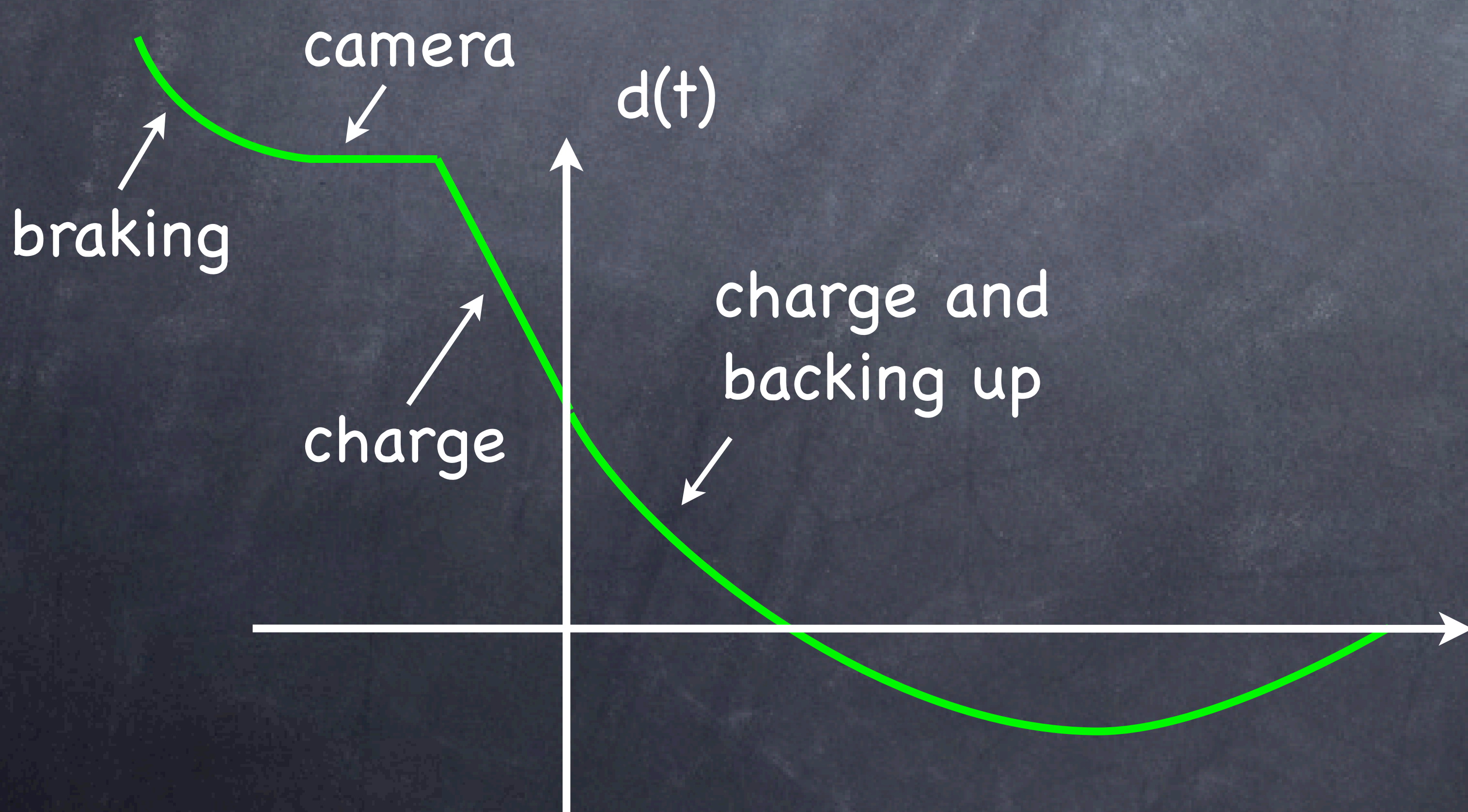


$v_{car}(t) = 2t$

$x_{car}(t) = t^2 + 12$

$v_{dist}(t) = 2t - 8$

$x_{dist}(t) = t^2 - 8t + 12$

# From the 2011 final exam

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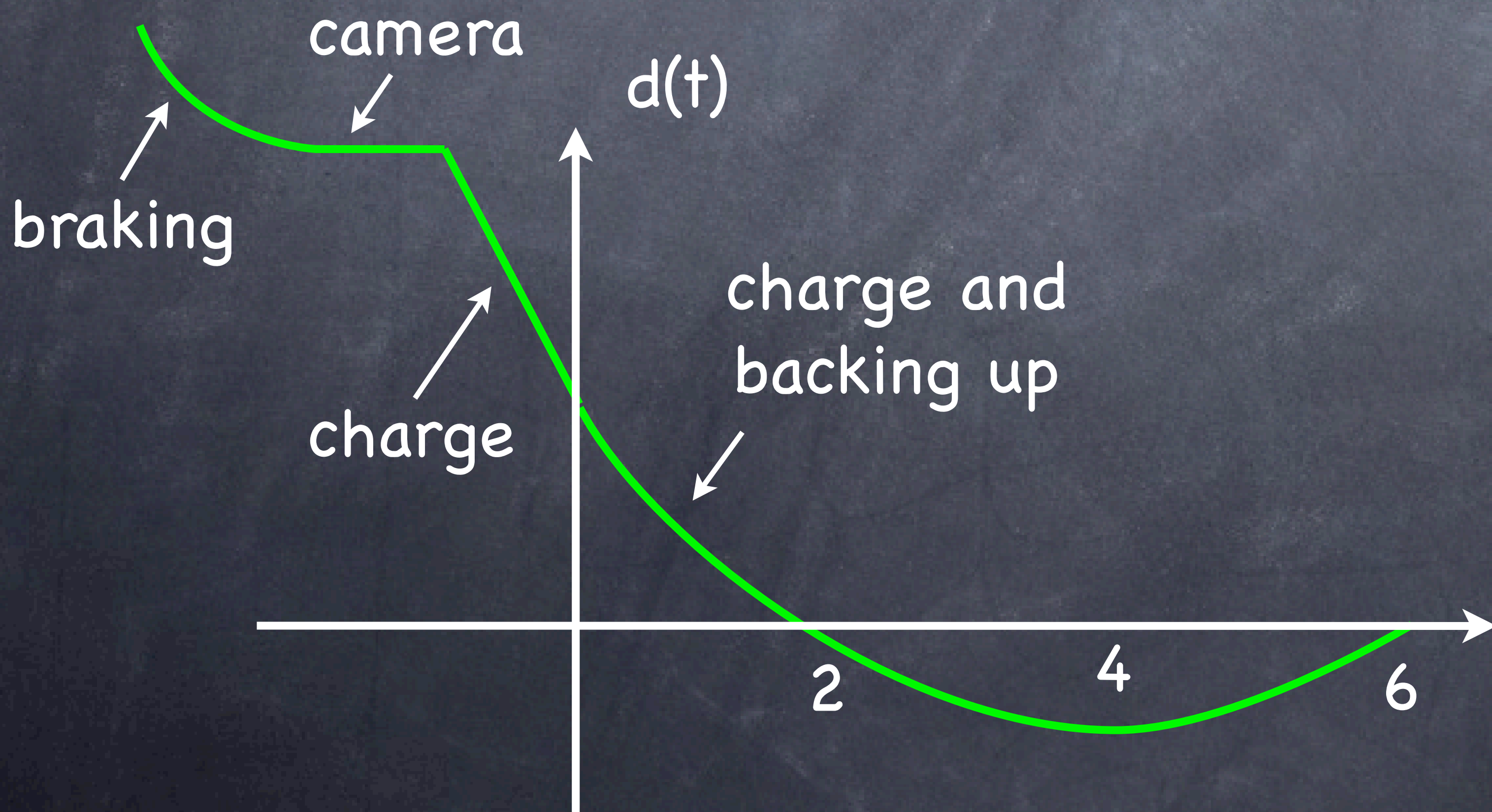


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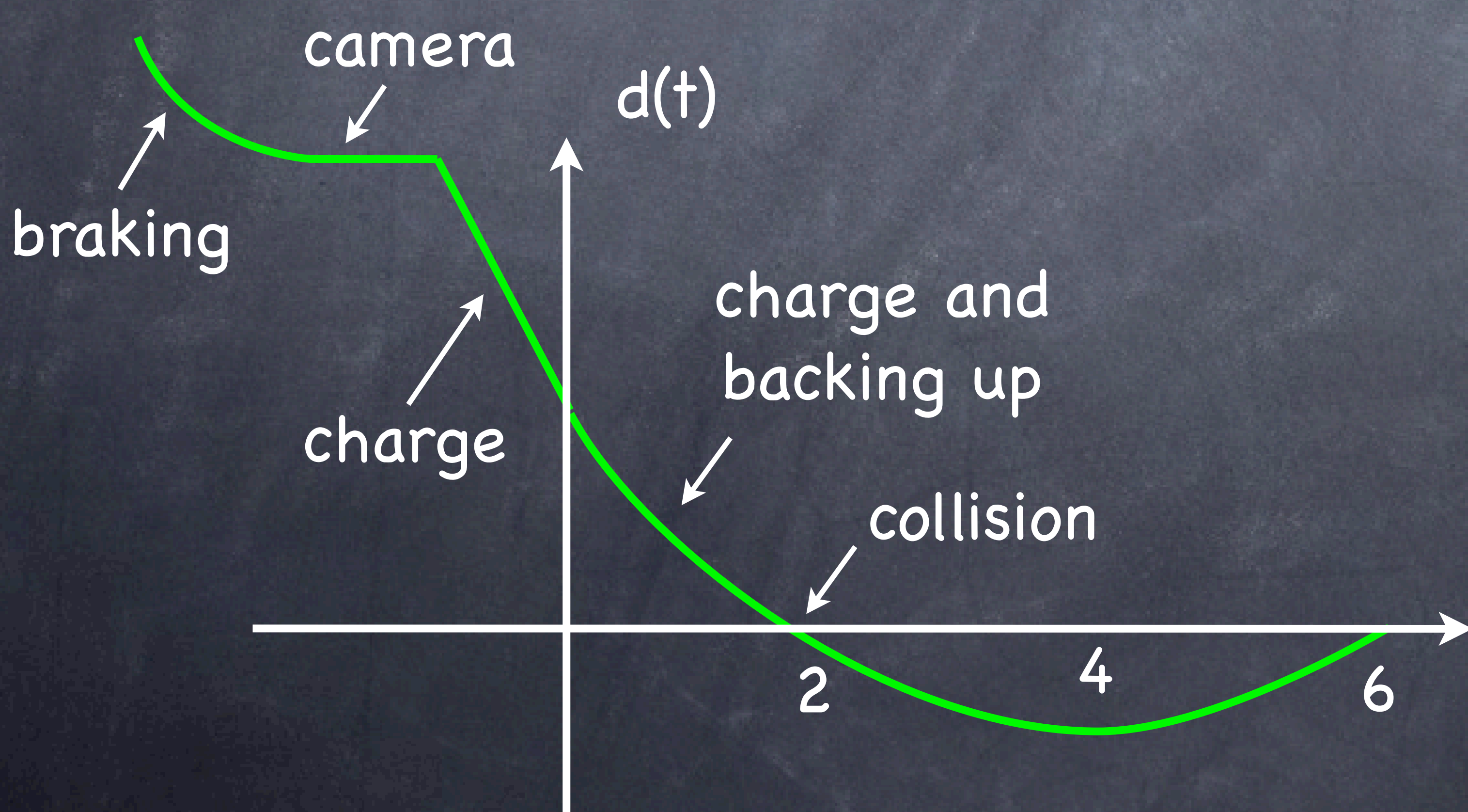


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# Use linear approximation to estimate $0.03^{1/3}$

(A) 0

(B) 28/90

(C) 79/240

(D) 0.310723

(E) infinity

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- $f(x) = x^{1/3}$.

# Use linear approximation to estimate $0.03^{1/3}$

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- $f(x) = x^{1/3}$.
- b=0.03

# Use linear approximation to estimate $0.03^{1/3}$

(A) 0

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- $f(x) = x^{1/3}$.
- b=0.03
- a = 0 <--- no good!

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- $f(x) = x^{1/3}$.
- b=0.03
- a = 0 <--- no good!
- Use $a=0.027=0.3^3$. ($0.4^3$ is ok too.)

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- b=0.03
- a = 0 <--- no good!
- Use a=0.027=$0.3^3$. ($0.4^3$ is ok too.)
- f(b) ≈ f(a) + f'(a)(b-a)

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- b=0.03
- a = 0 <--- no good!
- Use $a=0.027=0.3^3$. ($0.4^3$ is ok too.)
- $f(b) \approx f(a) + f'(a)(b-a)$

  $\approx 3/10 + 100/27\ (3/100-27/1000)$

# Use linear approximation to estimate $0.03^{1/3}$

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  $\approx 28/90$

(You should be able to do this without a calculator but we probably wouldn't ask you to on the midterm/exam!)