MATH 102 - FINAL EXAM

University of British Columbia

December 18, 2015.

Last name: _____ First name: _____

Student number:

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Section number:

- ⊖ 101 - MWF 10-11 am - Eric Cytrynbaum
- ⊖ 102 - MWF 8-9 am - Weiwei Ao
- ⊖ 103 - MWF 1-2 pm - Yue-Xian Li
- ⊖ 104 - MWF 1-2 pm - Hexi Ye
- ⊖ 105 - TTh 9:30-11 am - Omar Antolin-Camarena
- ⊖ 106 - MWF 9-10 am - Leah Keshet
- ⊖ 107 - MWF 8-9 am - Will Carlquist
- ⊖ 110 - TTh 2-3:30 pm - Kseniya Garaschuk

↑ **Fill in bubble corresponding to your section.**

← **Fill in bubble-number below each number in your student number.**

This page will be removed for marking and data entry so you must put your personal information on the next page as well.
DO NOT REMOVE THIS PAGE YOURSELF.

Multiple choice answers:

Question:	Answer:	Question:	Answer:
1	A B C D E	5	A B C D E
2	A B C D E	6	A B C D E
3	A B C D E	7	A B C D E
4	A B C D E	8	A B C D E

For marking purposes only - DO NOT WRITE IN THIS BOX:

9	0 1 2 3 4 5 6 7 8 9	14	0 1 2 3 4 5 6 7 8 9
10	0 1 2 3 4 5 6 7 8 9	15	0 1 2 3 4 5 6 7 8 9
11	0 1 2 3 4 5 6 7 8 9	16	0 1 2 3 4 5 6 7 8 9
12	0 1 2 3 4 5 6 7 8 9	17	0 1 2 3 4 5 6 7 8 9
13	0 1 2 3 4 5 6 7 8 9	18	0 1 2 3 4 5 6 7 8 9

19 0 1 2 3 4 5 6 7 8 9 10 11 12 13

MATH 102 - FINAL EXAM

University of British Columbia

December 18, 2015.

Last name: First name:

Student number:

Section number:

Number of pages: 11 (including both cover pages and blank page)
Exam type: *Closed book*
Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Multiple choice

Each multiple choice question is worth **2 pts**. No partial points will be given for work shown.
Enter your answers using the bubbles on the front page.

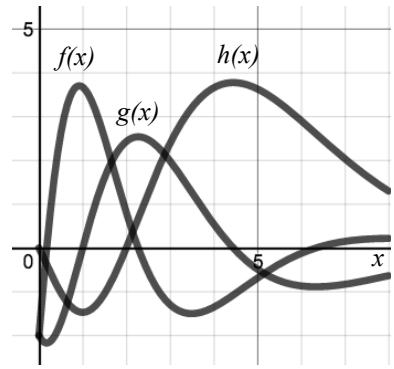
1. According to the graphs in the figure below, which of the following is true?

   (a) $f(x) = g'(x) = h''(x)$

   (b) $f'(x) = g''(x) = h(x)$

   (c) $f''(x) = g(x) = h'(x)$

   (d) $f''(x) = g'(x) = h(x)$



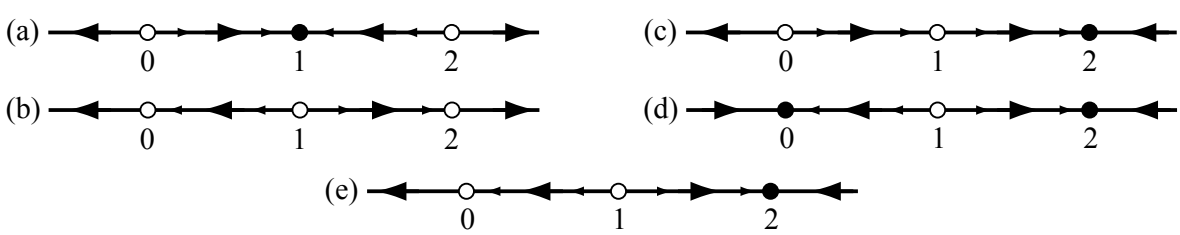
2. The line tangent to the graph of $f(x) = \cos(x)$ at $x = 0$

   (a) is $x = 0$.
   (b) is $y = 1$.
   (c) is $y = x$.
   (d) does not exist.
   (e) is not unique. There are infinitely many tangent lines.

3. Let f^{-1} be the inverse function of $f(x)$. Assume that $f(0) = 1$ and $f'(0) = 2$. Find the tangent line $y = mx + b$ to $f^{-1}(x)$ at $x = 1$.

   (a) $y = x + 2$ (b) $y = 2x + 1$ (c) $y = \frac{1}{2}(x - 1)$ (d) $y = \frac{1}{2}(1 - x)$ (e) $y = 2(x - 1)$

4. Which of the following phase lines represents the differential equation $y' = -y(y-1)(y-2)$

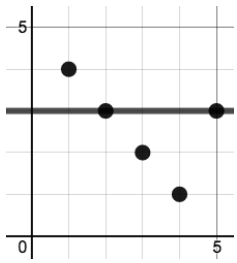


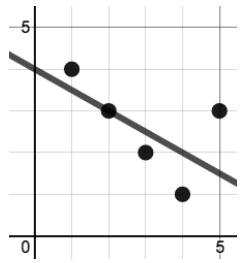
Enter your answers using the bubbles on the front page.

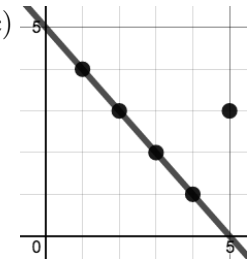
Multiple choice (continued)

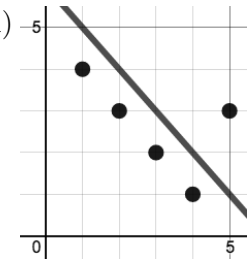
Enter your answers using the bubbles on the front page.

5. Which of the following lines $y = ax + b$ provides the best fit to the data in the least squares sense?

(a) 

(b) 

(c) 

(d) 

6. If we assume that the amount of food gained in a food patch during time t is

$$f(t) = F_{max}\frac{t^2}{k^2 + t^2},$$

then

(a) The total amount of food in the patch is F_{max}/k^2.
(b) Getting food from the patch is initially a slow process but then it speeds up for a while.
(c) It takes a time $t = k$ to get all the energy out of the patch.
(d) It takes a time $t = k/2$ to get all the energy out of the patch.
(e) There is unlimited energy in the patch.

7. Rainbow Trout in Deer Lake can no longer reproduce due to habitat destruction. On November 1st when the population is estimated at 800 fish, locals start stocking the lake with fish at a constant rate of 100 fish per day and fish them out at a rate proportional to their population size. In the options below, $r > 0$. Which of the following equations describes the size of the fish population?

(a) $F(t) = 100t - 800e^{-rt}$
(b) $\dfrac{dF}{dt} = (100 - r)F$
(c) $F(t) = 100/r + (800 - 100/r)e^{-rt}$
(d) $\dfrac{dF}{dt} = r(100 - F)$
(e) $\dfrac{dF}{dt} = 100t - rF$

8. Ecologists observe that the product of a species' population density, D, and its average metabolic rate, M, is a constant across species. Thus, $DM = C$. If a species evolves over time with its population density increasing at a constant rate, what do you expect to be happening to the species' metabolic rate?

(a) Metabolic rate increases at a constant rate.
(b) Metabolic rate increases at a non-constant rate.
(c) Metabolic rate decreases at a constant rate.
(d) Metabolic rate decreases at a non-constant rate.

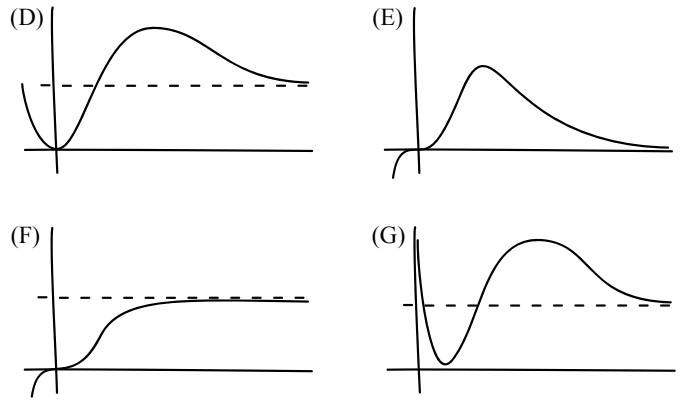
Enter your answers using the bubbles on the front page.

Written-answer problems

Show your work. Enter your answer in the box provided.

9. **[3 pts]** Match the function ((a), (b), (c)) to its sketch (one of (D), (E), (F), (G)).

(a) $\dfrac{3x^2+x^3}{3+2x^3}$ (b) $\dfrac{x^3}{3+2x^3}$ (c) $\dfrac{x^3+x^4}{1+2x^5}$

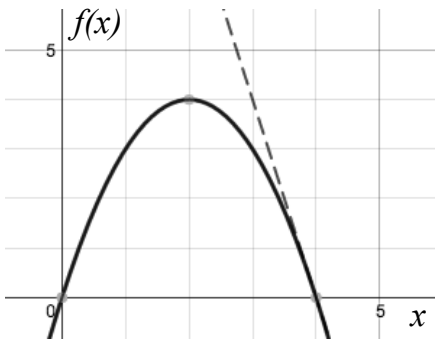


(a) → ☐

(b) → ☐

(c) → ☐

10. **[3 pts]** Consider the function $f(x)$ (solid curve) and its tangent line at $x = 4$ (dashed). The graph of $f(x)$ is symmetric about $x = 2$. Calculate the quantities below.

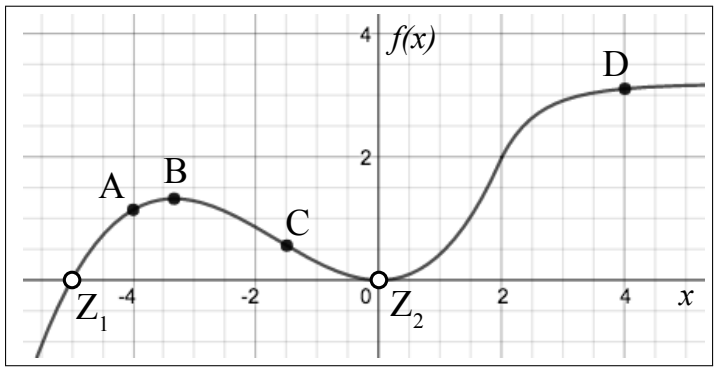


$\lim_{h\to 0} \dfrac{f(4+h)-f(4)}{h} =$ ☐ .

If $g(x) = f(f(x))$, then $g'(4) =$ ☐ .

11. **[4 pts]** Consider each of the labeled points (solid dot) on the graph of $f(x)$ as a starting point for Newton's method. To which zero of the function $f(x)$ (empty dots Z_1, Z_2, or neither) will Newton's method converge for each one? You may assume that the graph of the function continues off the edges of the graph with no significant change in direction.

Initial point	zero (Z_1, Z_2, or neither)
A	
B	
C	
D	



Do not write in these boxes - for marking purposes only. 9: ☐ 10: ☐ 11: ☐

12. **[3 pts]** The function $f(x) = 3\sqrt{x} + 20$ models the median height (in inches) of boys who are x months of age. Suppose Lucas' growth follows this pattern. Use linear approximation to predict Lucas' height on his 2nd birthday (i.e. age 24 months).

Lucas' approximate height at age 2: $\boxed{}$.

13. **[4 pts]** What is the absolute minimum of the function $f(x) = -x^3 + 3x + 1$ on the interval $[0, 3]$?

$x_{min} = \boxed{}$

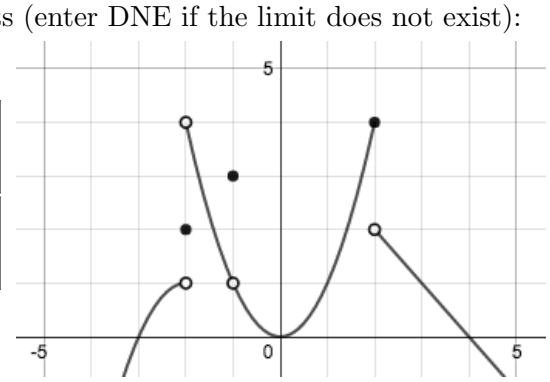
$f(x_{min}) = \boxed{}$

14. **[6 pts]**

(a) Use the graph below to determine the following limits (enter DNE if the limit does not exist):

$\lim_{x\to -2^-} f(x) = \boxed{}$ $\lim_{x\to -2} f(x) = \boxed{}$

$\lim_{x\to -1} f(x) = \boxed{}$ $\lim_{x\to 2} f(x) = \boxed{}$



(b) At which values of x is the function discontinuous?

$x = \boxed{}$

Do not write in these boxes - for marking purposes only. 12: $\boxed{}$ 13: $\boxed{}$ 14: $\boxed{}$

15. **[4 pts]** Someone released a group of 10 rabbits on campus and they started reproducing at a rate proportional to the population size with constant of proportionality of 3 per year. After 5 years, the rabbits continued to reproduce in the same manner but a coyote moved in and began eating rabbits at a rate proportional to the population size with a constant of proportionality of 9 per year. The population of rabbits is given by the function

$$P(t) = \begin{cases} C_0 e^{at} & 0 \leq t < 5, \\ C_1 e^{bt} & t \geq 5. \end{cases}$$

Give values for a, C_0, b, and C_1. Your answers can be in terms of exponentials and/or logarithms.

$a =$ $\boxed{}$ $C_0 =$ $\boxed{}$ $b =$ $\boxed{}$ $C_1 =$ $\boxed{}$

16. **[4 pts]** A squirrel sitting 6 m up in a tree is watching a coyote walk past the tree. The squirrel measures the angle formed between a vertical line directly below her and the line connecting her and the coyote and finds that it is changing at a rate of 1/12 radians per second when the coyote is 8 m away from the base of the tree. How fast is the coyote walking?

Coyote's speed = $\boxed{}$

Do not write in these boxes - for marking purposes only. 15: $\boxed{}$ 16: $\boxed{}$

17. **[4 pts]** Use implicit differentiation to calculate the derivative of $y = \arctan(x)$. No points will be given for simply stating the answer. Your answer should be in terms of x and should not include any trig functions.

$y' =$ $\boxed{}$

18. **[6 pts]** A dangerous infectious disease spreads through Vancouver as described by the differential equation

$$I' = \beta I(N - I) - \mu I = \beta I\left(N - \frac{\mu}{\beta} - I\right)$$

where $\beta > 0$ is the transmission rate constant, N is the total population size (constant), $\mu > 0$ is the recovery rate and $I(t)$ is the number of infected individuals. Assume that $N > \mu/\beta$.

(a) Find the steady state(s) in terms of the parameters β, N, and μ.

Steady state(s): $\boxed{}$

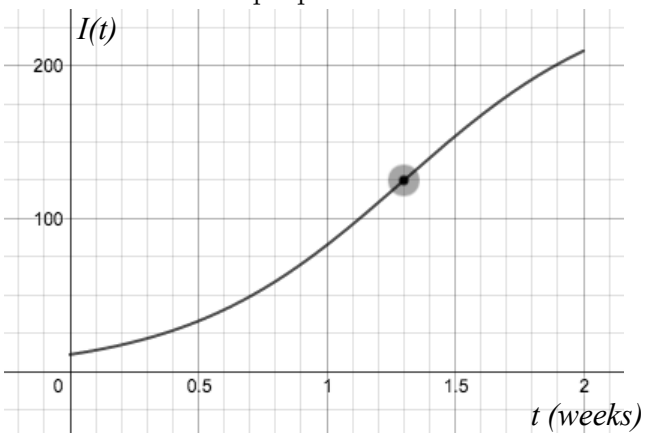
(b) At what value of I is the infection rate (dI/dt) largest?

$I =$ $\boxed{}$

Do not write in this box - for marking purposes only. 17: $\boxed{}$

(c) Sketch the phase line for the differential equation.

(d) By collecting data from local hospitals, you manage to plot the graph of number of infected individuals as a function of time from the onset of the epidemic until two weeks later. You notice an inflection point in the data and mark it with a black dot. What is the steady state number of infected people? Your answer should be a number.



Steady state number of infected people:

19. **[13 pts]** In deciding how long a resident's shift in the emergency room should be, the Chief of Staff at Vancouver General Hospital would like to minimize the average rate at which errors are made. Let $E(t)$ be the number of errors made by a resident from the start of a shift until t hours into the shift. The instantaneous rate of change of errors made is $E'(t) = 4 - t + \frac{1}{16}t^2$.

(a) During what interval of time is $E'(t)$ increasing? decreasing?

Decreasing:

Increasing:

(b) What is the total number of errors, $E(t)$, made t hours into a shift?

$E(t) =$

Do not write in this box - for marking purposes only. 18:

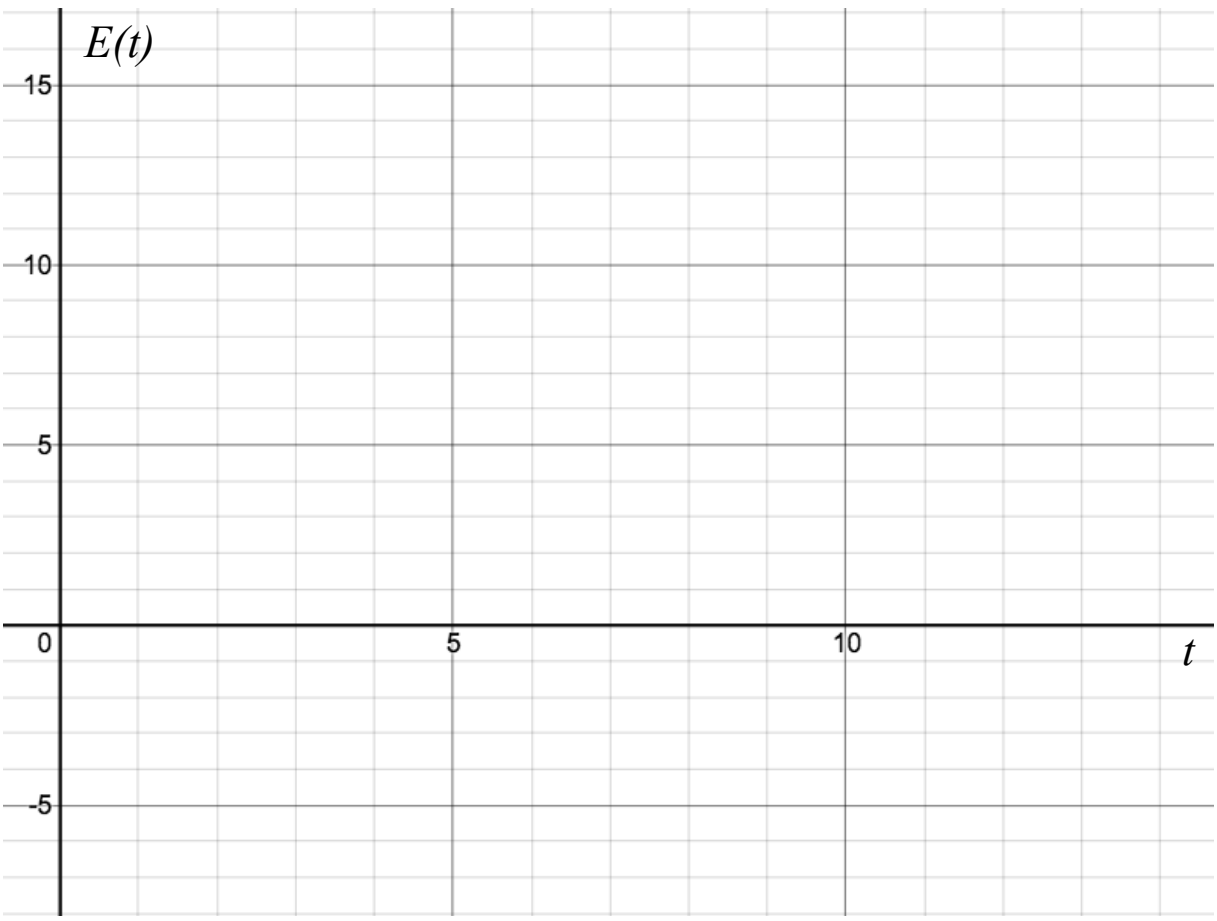
(c) What is the average rate of change of $E(t)$ from the start of a shift ($t = 0$) up until time t?

$A(t) =$ ☐

(d) How long should a resident's shift be in order to minimize the average rate of change of errors made (i.e. minimize $A(t)$)?

$t_{min} =$ ☐

(e) Sketch $E(t)$. Label any minima, maxima and/or inflection points. On the same axes, draw a line that shows when the average error rate is minimized.



Note: there is a set of axes on the last page that you can use to draw a practice sketch.

Do not write in this box - for marking purposes only. 19: ☐

This page may be used for rough work. It will not be marked.