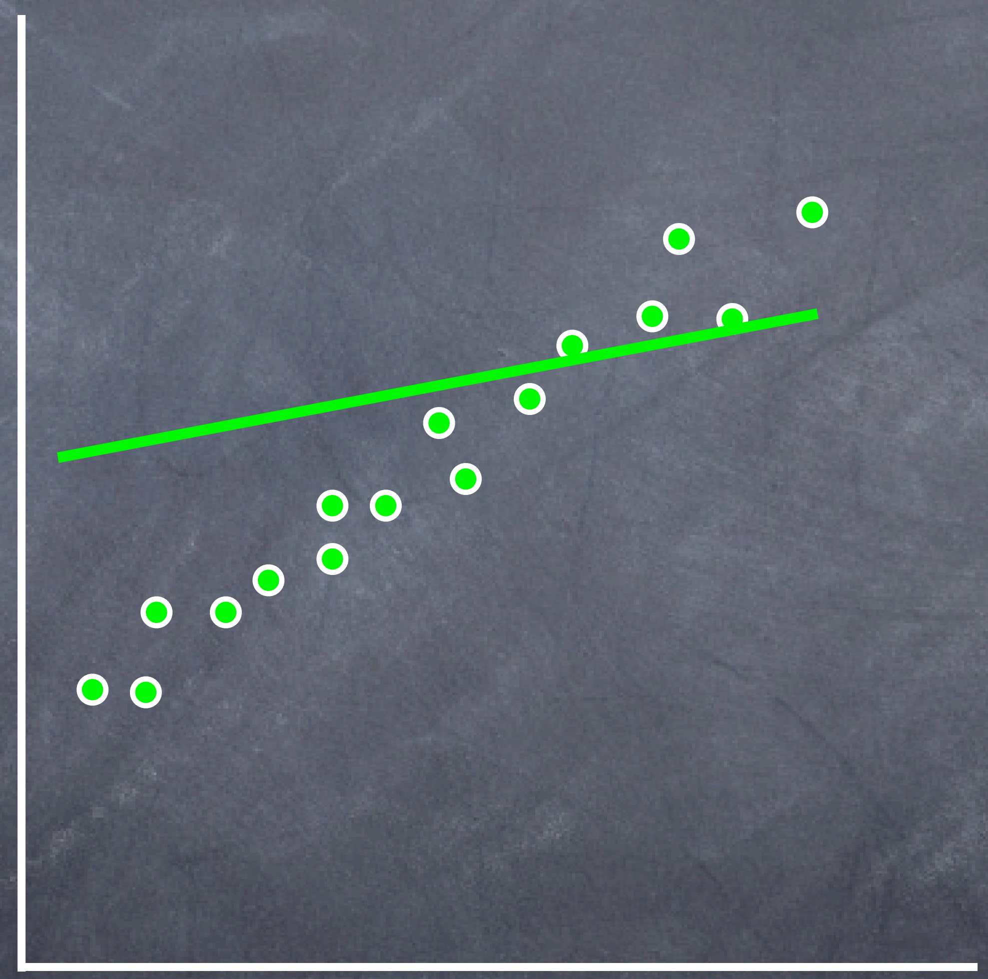
Today

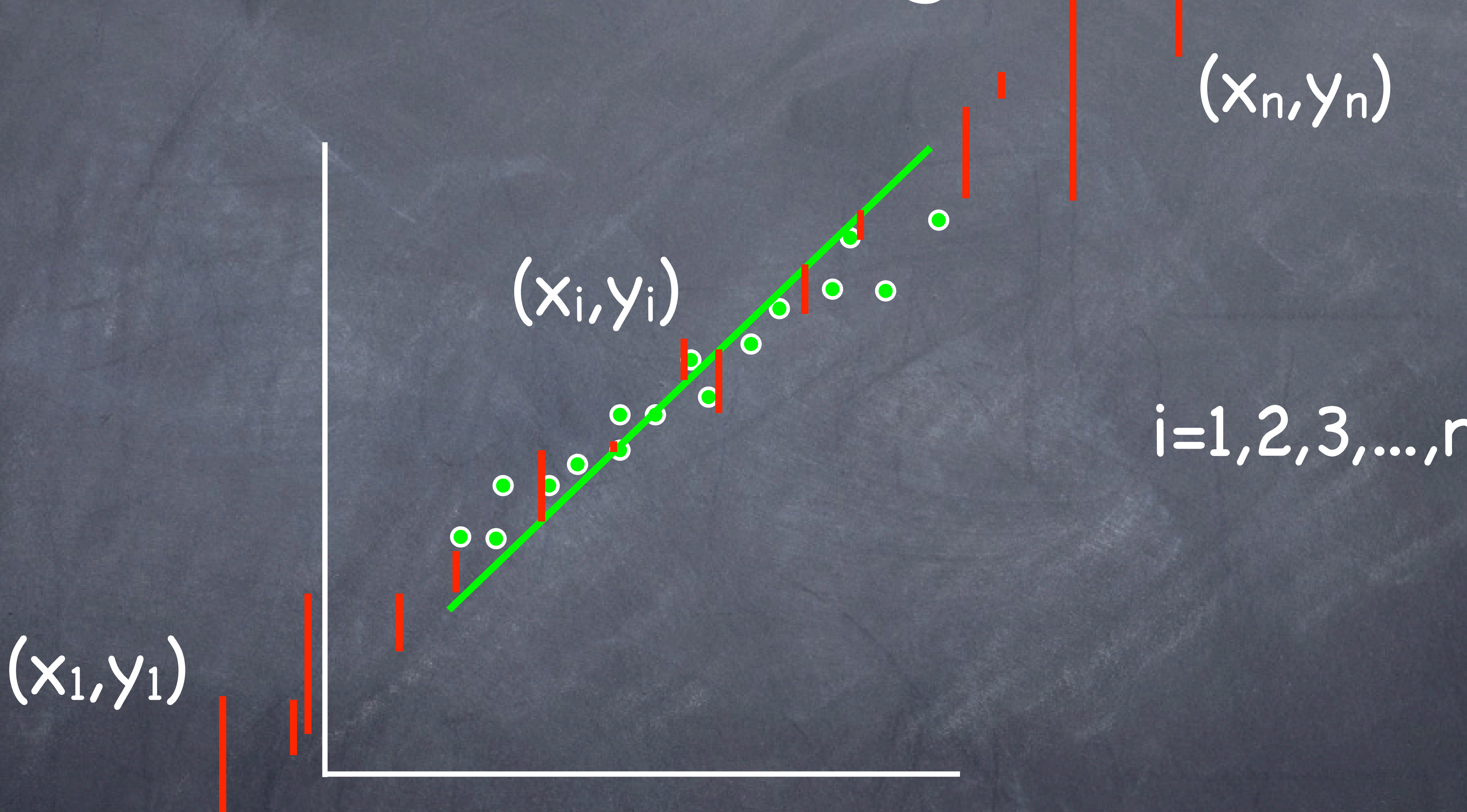
- Linear regression
  - aka linear least squares
  - aka fitting data with straight lines
- Another optimization example (if time allows)

Linear regression



How do we find the best line to fit through the data?

Minimizing



Each red bar is called a residual. We want the residuals to be as small as possible.

The residuals are...

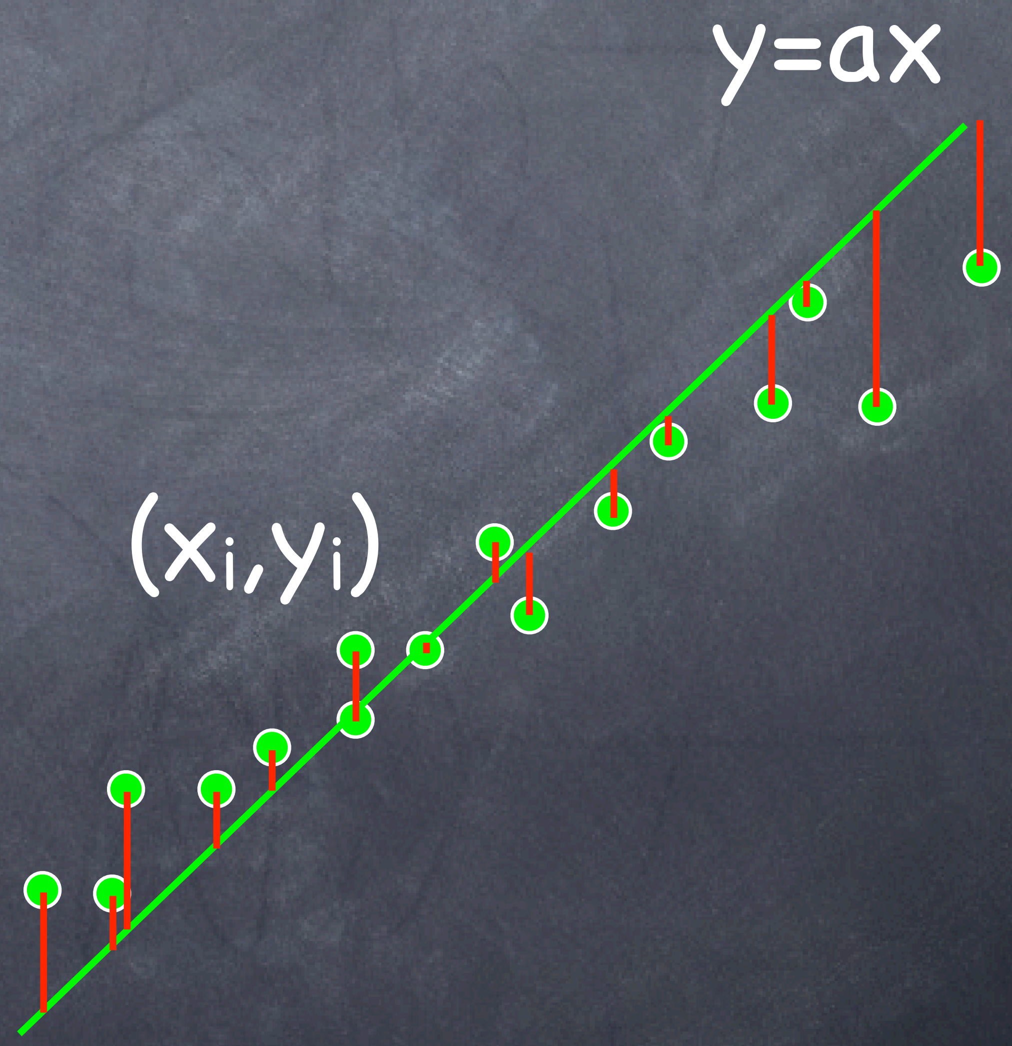
(A) $r_i = y_i^2 + x_i^2$

(B) $r_i = a^2(y_i^2 + x_i^2)$

(C) $r_i = y_i - a\ x_i$

(D) $r_i = y_i - x_i$

(E) $r_i = x_i - y_i$



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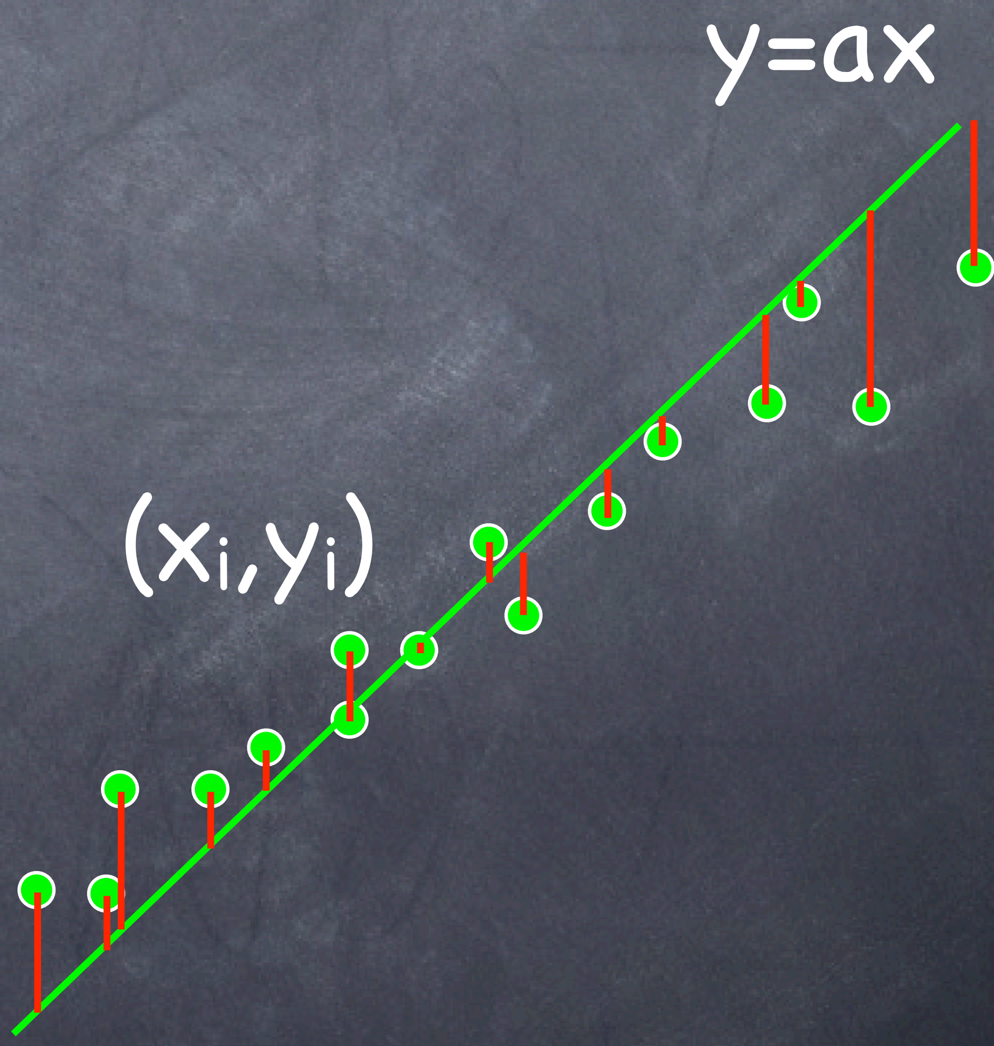

To minimize the residuals, we define the objective function…

(A) $f(a) = |y_1 - ax_1| + |y_2 - ax_2| + \ldots + |y_n - ax_n|$

(B) $f(a) = (y_1 - ax_1)^2 + (y_2 - ax_2)^2 + \ldots + (y_n - ax_n)^2$

(C) $f(a) = (y_1 - ax_1)(y_2 - ax_2)\ldots(y_n - ax_n)$

(D) $f(a) = \max((y_1 - ax_1), (y_2 - ax_2), \ldots, (y_n - ax_n))$

To minimize the residuals, we define the objective function...

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(B) $f(a) = (y_1 - ax_1)^2 + (y_2 - ax_2)^2 + ... + (y_n - ax_n)^2$

(C) $f(a) = (y_1 - ax_1)(y_2 - ax_2)...(y_n - ax_n)$

(D) $f(a) = \max((y_1 - ax_1),(y_2 - ax_2),...,(y_n - ax_n))$

(B) is called the "sum of squared residuals".
(A) and (D) are reasonable but not as good
(take a stats class to find out more).

Find a so that y=ax fits (4,5), (6,7) in the "least squares" sense.

Define f(a):

(A) $f(a) = |5-4a| + |7-6a|$

(B) $f(a) = (4-5a)^2 + (6-7a)^2$

(C) $f(a) = (5-4a)^2 + (7-6a)^2$

(D) $f(a) = (5-4-a)^2 + (7-6-a)^2$

Find a so that y=ax fits (4,5), (6,7) in the "least squares" sense.

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(D) $f(a) = (5-4-a)^2 + (7-6-a)^2$

Find a so that y=ax fits (4,5), (6,7) in the "least squares" sense.

Find the a that minimizes f(a):

(A) $a = 7/6$

(B) $a = 5/4$

(C) $a = (7/6 + 5/4) / 2$

(D) $a = 31/26$

Find a so that y=ax fits (4,5), (6,7) in the "least squares" sense.

Find the a that minimizes f(a):

(A) a = 7/6

(B) a = 5/4

(C) a = (7/6 + 5/4) / 2

(D) $a = 31/26 = (4 \cdot 5 + 6 \cdot 7) / (4^2 + 6^2)$

$= (x_1 \cdot y_1 + x_2 \cdot y_2) / (x_1^2 + x_2^2)$

Find a so that $y=ax$ fits (x_1,y_1), (x_2,y_2),…, (x_n,y_n) in the "least squares" sense.

Define $f(a)$:

(A) $f(a) = |y_1-ax_1| + |y_2-ax_2| + \ldots + |y_n-ax_n|$

(B) $f(a) = (y_1-ax_1)^2 + (y_2-ax_2)^2 + \ldots + (y_n-ax_n)^2$

(C) $f(a) = (ay_1-x_1)^2 + (ay_2-x_2)^2 + \ldots + (ay_n-x_n)^2$

(D) $f(a) = (y_1-a-x_1)^2 + (y_2-a-x_2)^2 + \ldots + (y_n-a-x_n)^2$

Find a so that y=ax fits (x_1,y_1), (x_2,y_2),…, (x_n,y_n) in the "least squares" sense.

Define f(a):

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(D) $f(a) = (y_1-a-x_1)^2 + (y_2-a-x_2)^2 + \ldots + (y_n-a-x_n)^2$

Notation

$$\sum_{i=1}^{n} q_i = q_1 + q_2 + \ldots + q_n$$

$$\sum_{i=1}^{n} (y_i - ax_i)^2 = (y_1 - ax_1)^2 + (y_2 - ax_2)^2 + \ldots + (y_n - ax_n)^2$$

Find a so that y=ax fits (x_1,y_1), (x_2,y_2),…, (x_n,y_n) in the "least squares" sense.

Find the a that minimizes f(a):

(A) $a = \sum_{i=1}^{n} y_i \Big/ \sum_{i=1}^{n} x_i$

(B) $a = \sum_{i=1}^{n} x_i \Big/ \sum_{i=1}^{n} y_i$

(C) $a = \sum_{i=1}^{n} x_i y_i \Big/ \sum_{i=1}^{n} x_i$

(D) $a = \sum_{i=1}^{n} x_i y_i \Big/ \sum_{i=1}^{n} x_i^2$

Find a so that y=ax fits (x_1,y_1), (x_2,y_2),..., (x_n,y_n) in the "least squares" sense.

Find the a that minimizes f(a):

(A) $a = \sum_{i=1}^{n} y_i \,/\, \sum_{i=1}^{n} x_i$

(C) $a = \sum_{i=1}^{n} x_i y_i \,/\, \sum_{i=1}^{n} x_i$

(B) $a = \sum_{i=1}^{n} x_i \,/\, \sum_{i=1}^{n} y_i$

(D) $a = \sum_{i=1}^{n} x_i y_i \,/\, \sum_{i=1}^{n} x_i^2$

For best fits using y=ax+b, see course notes supplement.

$$a = \frac{P_{avg} - \bar{x}\bar{y}}{X^2_{avg} - \bar{x}^2}$$

$$b = \bar{y} - a\bar{x}$$

$$P_{avg} = \frac{1}{n}\sum_{i=1}^{n} x_i y_i$$

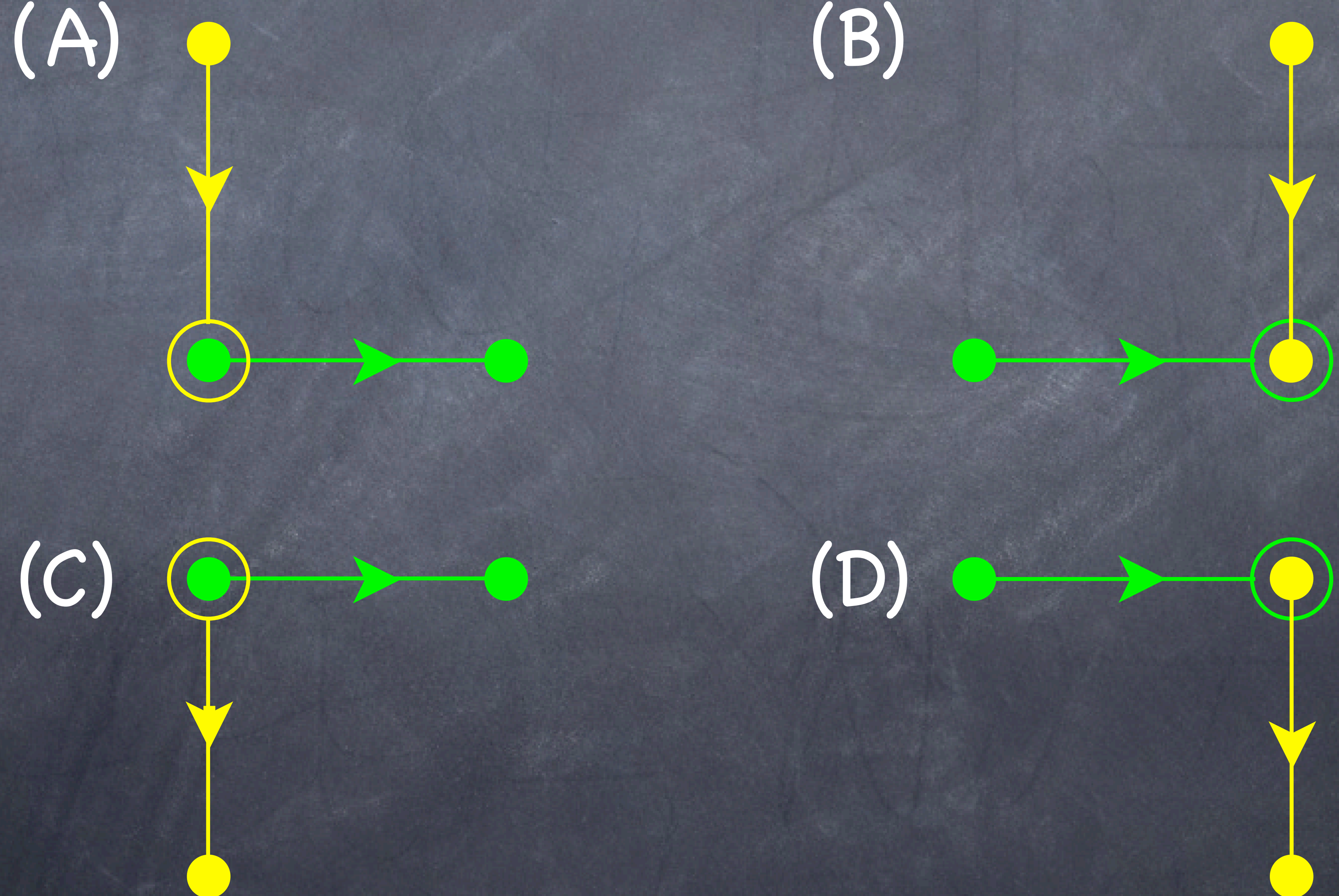
$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$X^2_{avg} = \frac{1}{n}\sum_{i=1}^{n} (x_i^2)$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

A boat leaves a dock at 2:00 P.M. and travels due south at a speed of 5 km/h. Another boat has been heading due east at 5 km/h and reaches the same dock at 3:00 P.M. How many minutes past 2:00 P.M. were the boats closest together?

Sketch:



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Two quantities relevant to solving this problem are:

(A) $x = 5/60\ t$, $y = 5/60\ (60-t)$.

(B) $x = 5(t-2)$, $y=5(3-t)$.

(C) $x = 5-2$, $y=5+3$.

(D) $x = 5t-2$, $y=5t-3$.

A boat leaves a dock at 2:00 P.M. and travels due south at a speed of 5 km/h. Another boat has been heading due east at 5 km/h and reaches the same dock at 3:00 P.M. How many minutes past 2:00 P.M. were the boats closest together?

Objective function to be minimized:

(A) $f(t) = 25|t| + 25|60-t|$

(B) $f(t) = 5/60 \text{ sqrt}(2t^2)$

(C) $f(t) = t^2 + (60-t)^2$

(D) $f(t) = \text{sqrt}(25(t-2)^2 + 25(3-t)^2)$

A boat leaves a dock at 2:00 P.M. and travels due south at a speed of 5 km/h. Another boat has been heading due east at 5 km/h and reaches the same dock at 3:00 P.M. How many minutes past 2:00 P.M. were the boats closest together?

Expectation: The boats will be closest together...

(A) at 2 pm.

(B) at 3 pm.

(C) sometime between 2 pm and 3 pm.

(D) before 2 pm.

(E) after 2 pm.

A boat leaves a dock at 2:00 P.M. and travels due south at a speed of 5 km/h. Another boat has been heading due east at 5 km/h and reaches the same dock at 3:00 P.M. How many minutes past 2:00 P.M. were the boats closest together?

Constraint:

(A) The minimum distance must occur between 2 pm and 3 pm.

(B) $x(t)^2+y(t)^2 = t^2/6$.

(C) $x(t) = 60-y(t)$.

(D) There isn't really a constraint for this problem.

A boat leaves a dock at 2:00 P.M. and travels due south at a speed of 5 km/h. Another boat has been heading due east at 5 km/h and reaches the same dock at 3:00 P.M. How many minutes past 2:00 P.M. were the boats closest together?

Answer (in minutes past 2 pm):

(A) $t = 0$.

(B) $t = 15$.

(C) $t = 30$.

(D) $t = 45$

(E) $t = 60$.