Today

- Chain rule
- Related rates examples

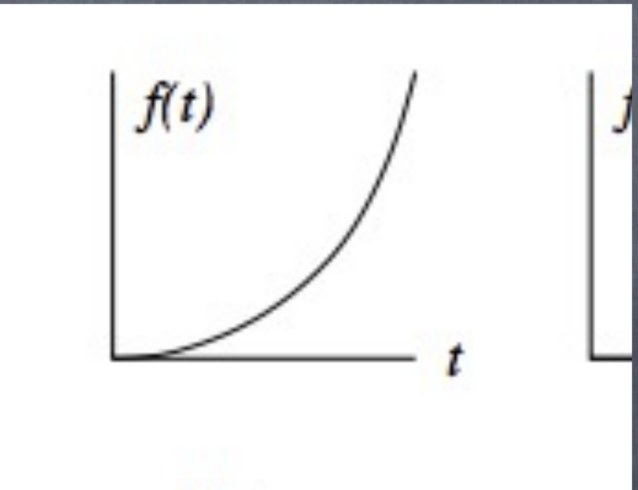
Choose $f(t_p) = t_p^2$

Find t_p that maximizes $R_{avg} = nt_p^2 / (nt_p + t_0)$

(A) $t_p = -2nt_0$

(B) $t_p = 0$

(C) Never leave.



(4)

Think and/or sketch before you calculate.

Choose $f(t_p) = t_p^2$

Find t_p that maximizes $R_{avg} = n t_p^2 / (n t_p + t_0)$

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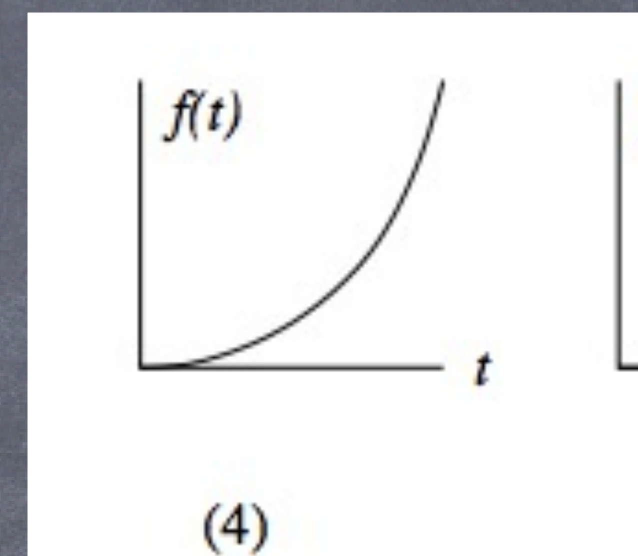

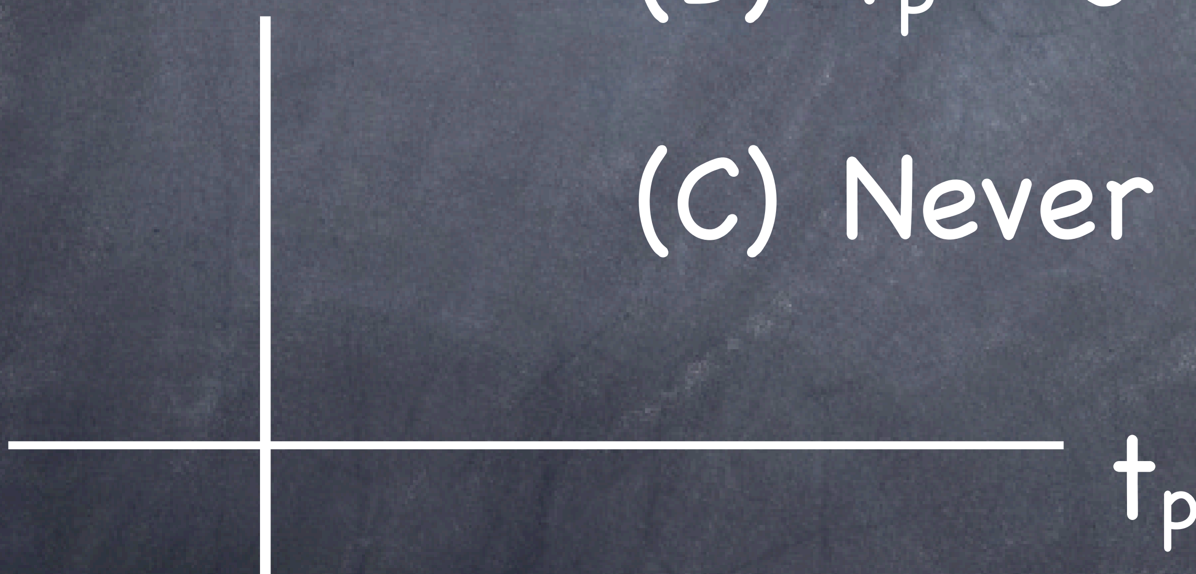

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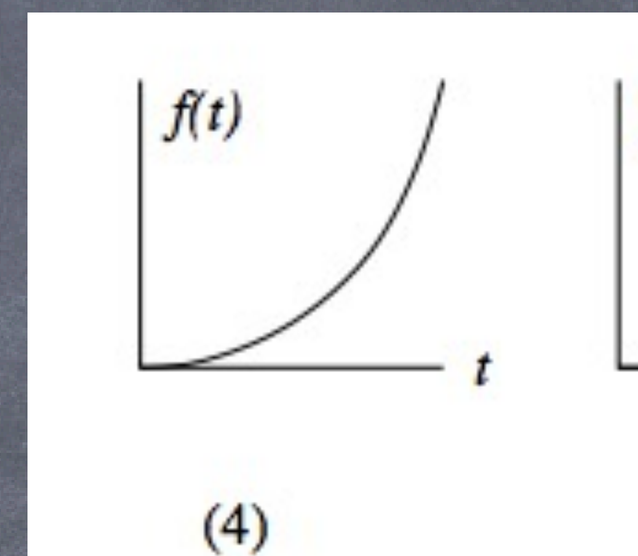

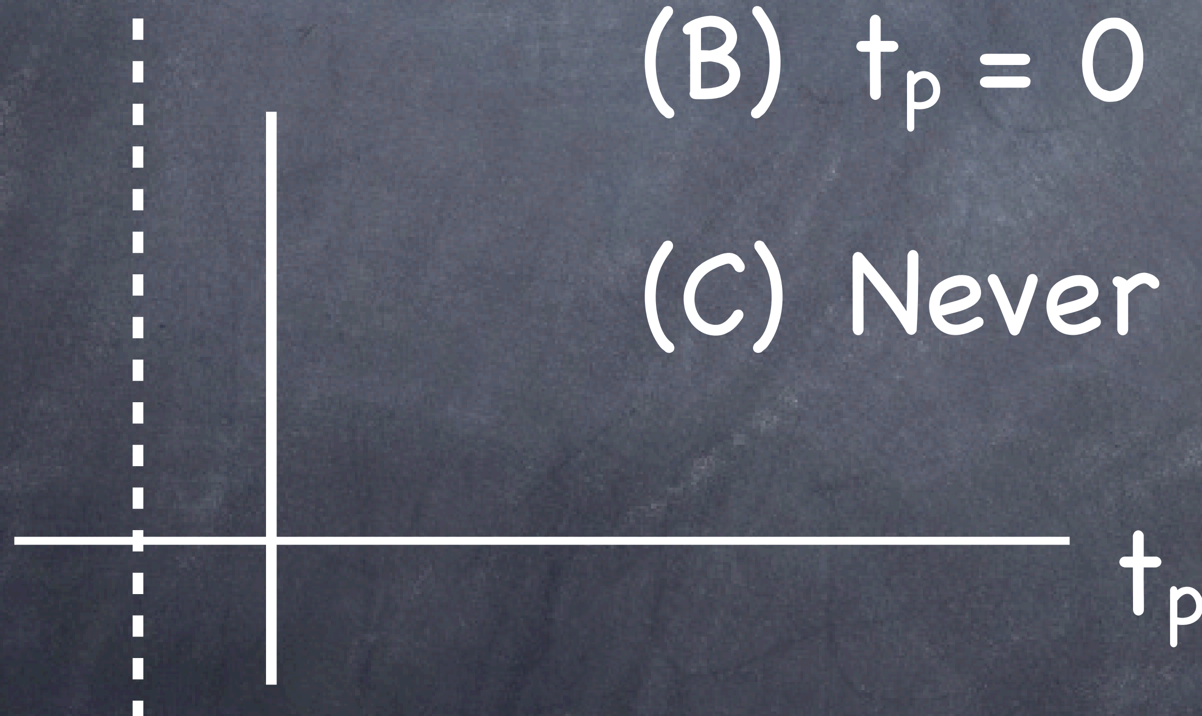

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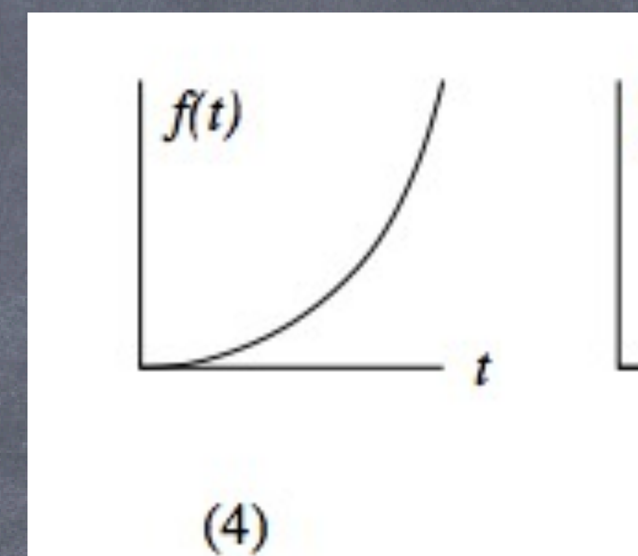

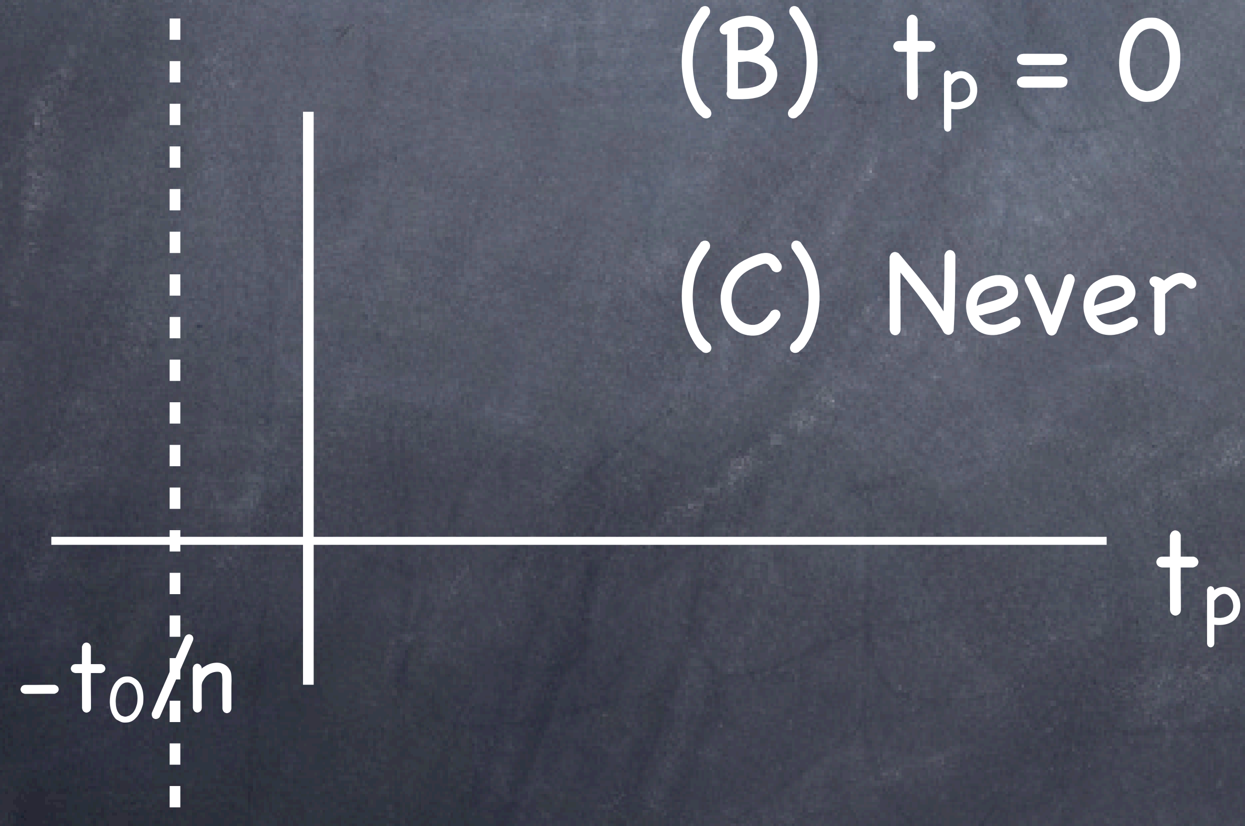

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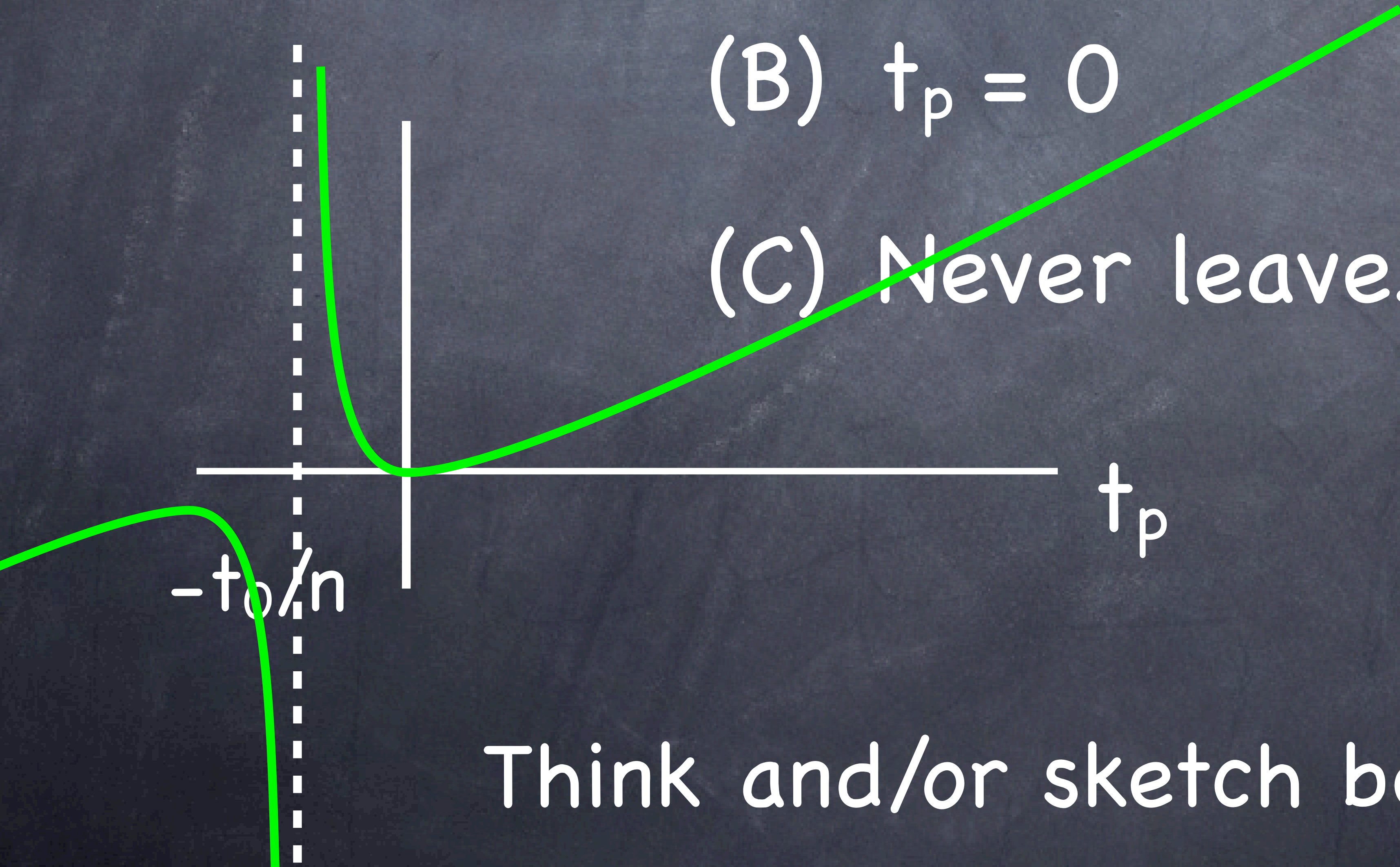

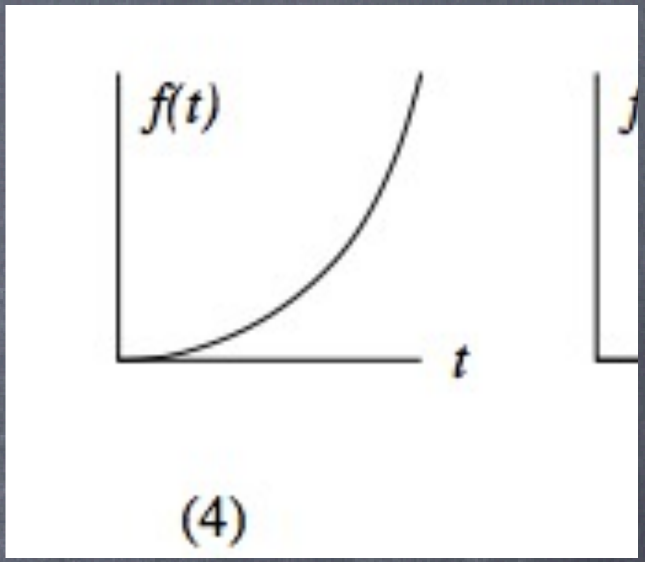

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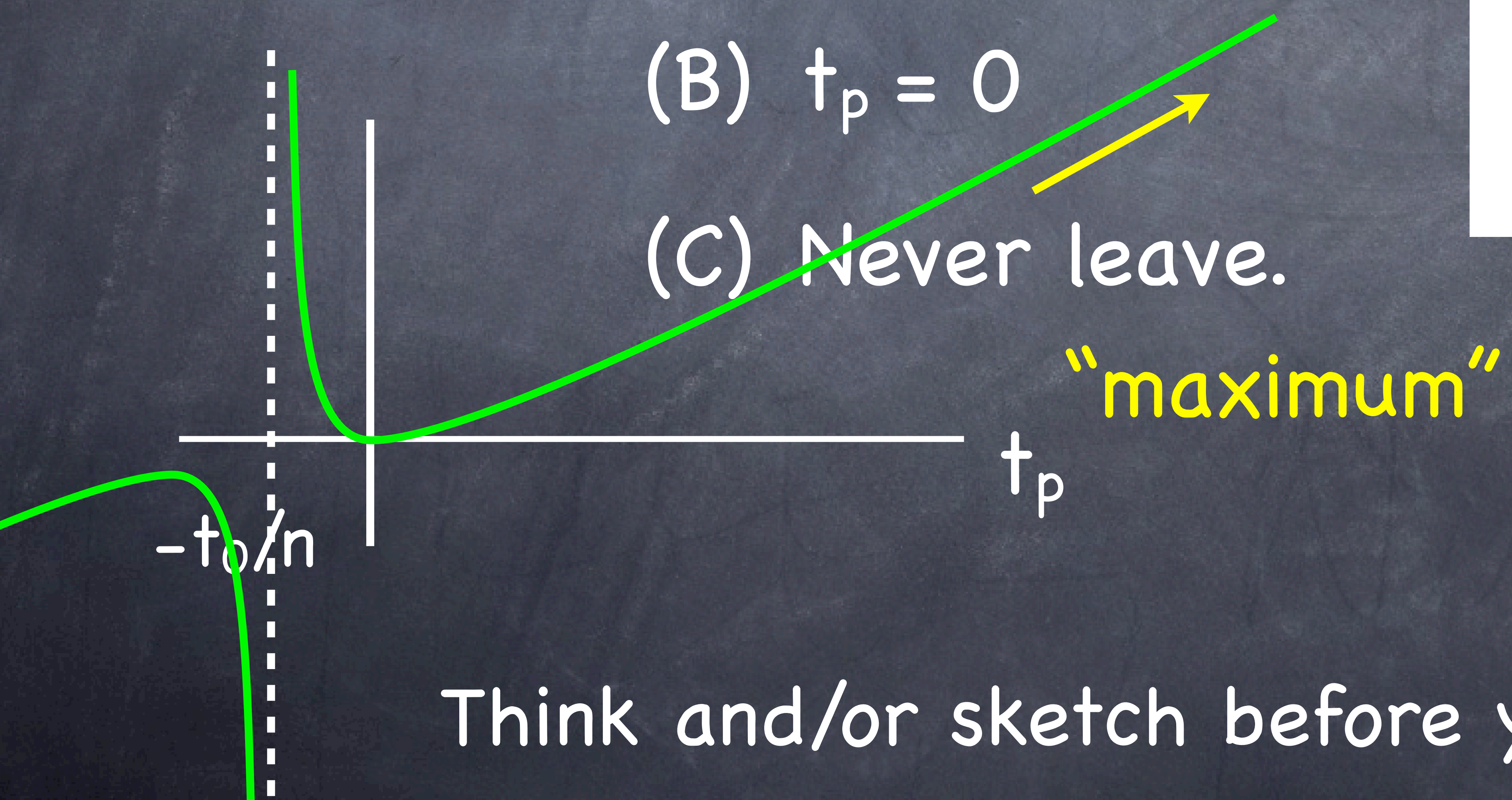

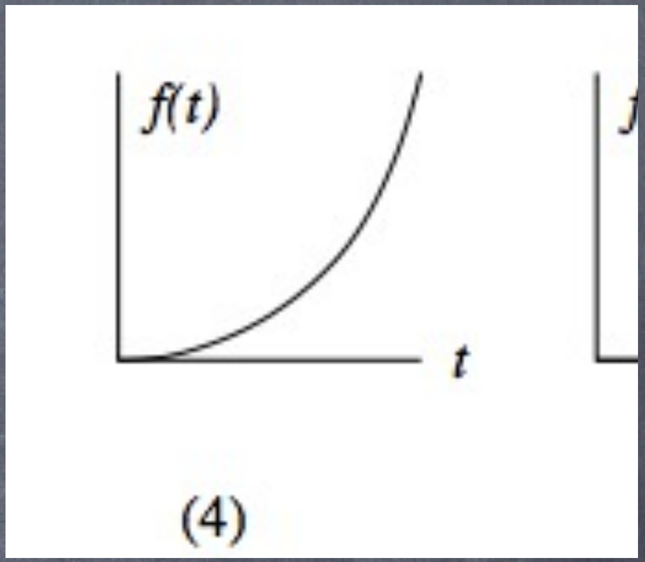

Think and/or sketch before you calculate.

Composition of functions

If $f(x) = 2x+3$ and $g(x) = -4x+2$,

(A) $h(x) = f(g(x)) = -8x+7$

(B) $h(x) = f(g(x)) = -8x-10$

(C) $h(x) = f(g(x)) = -8x^2-8x+6$

(D) $h(x) = f(g(x)) =-8x+5$

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Composition of functions

If $h(x) = f(g(x))$, then

(A) $h'(x) = f'(x)g'(x)$

(B) $h'(x) = f'(x)g(x) + f(x)g'(x)$

(C) $h'(x) = f'(g'(x))$

(D) $h'(x) = f'(g(x))g'(x)$

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Composition of functions

If $h(x) = f(g(x))$, then

For $f(x) = 2x+3$ and $g(x) = -4x+2$, $h'(x) = -8$! But...

(A) $h'(x) = f'(x)g'(x)$

(B) $h'(x) = f'(x)g(x) + f(x)g'(x)$

(C) $h'(x) = f'(g'(x))$

(D) $h'(x) = f'(g(x))g'(x)$

Composition of functions

For $f(x) = 2x+3$ and
$g(x) = -4x+2$,
$h'(x)=-8$! But...

If $h(x)= f(g(x))$, then

(A) $h'(x) = f'(x)g'(x)$ ----------------> $h'(x) = 2(-4)$

(B) $h'(x) = f'(x)g(x) + f(x)g'(x)$

(C) $h'(x) = f'(g'(x))$

(D) $h'(x) = f'(g(x))g'(x)$

Composition of functions

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(A) $h'(x) = f'(x)g'(x)$ ----------------> $h'(x) = 2(-4)$

(B) $h'(x) = f'(x)g(x) + f(x)g'(x)$ ---> $[\, h'(x) = 2(-4x+2) + (2x+3)(-4)$

(C) $h'(x) = f'(g'(x))$

(D) $h'(x) = f'(g(x))g'(x)$

Composition of functions

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(A) $h'(x) = f'(x)g'(x)$ ----------------> $h'(x) = 2(-4)$

(B) $h'(x) = f'(x)g(x) + f(x)g'(x)$ ---> [$h'(x) = 2(-4x+2) + (2x+3)(-4)$

(C) $h'(x) = f'(g'(x))$ ---------------------> $h'(x) = 2$

(D) $h'(x) = f'(g(x))g'(x)$

Composition of functions

For $f(x) = 2x+3$ and $g(x) = -4x+2$, $h'(x)=-8$! But...

If $h(x)= f(g(x))$, then

(A) $h'(x) = f'(x)g'(x)$ -----------------> $h'(x) = 2(-4)$

(B) $h'(x) = f'(x)g(x) + f(x)g'(x)$ ---> $\left[h'(x) = 2(-4x+2) + (2x+3)(-4) \right.$

(C) $h'(x) = f'(g'(x))$ ---------------------> $h'(x) = 2$

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Come up with an f and g that violate (A)!

Related rates

- Quantity 1 depends on quantity 2 and quantity 2 depends on a third quantity (e.g. time): $Q_1(Q_2(t))$
- Given the rate of change of one of them, find the rate of change of the other.

The radius of a spherical tumor grows at a constant rate, k. Determine the rate of growth of the volume of the tumor when the radius is one centimeter.

Which is the relevant equation relating the quantities (not rates of change yet)?

(A) $V = 4/3\ \pi\ r^3$

(B) $V' = 4\ \pi\ r^2\ k$

(C) $V' = 4\ \pi\ k^2$

(D) $V = 4/3\ \pi\ k^3$

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(D) $V' = 4/3 \pi k^3$

Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.

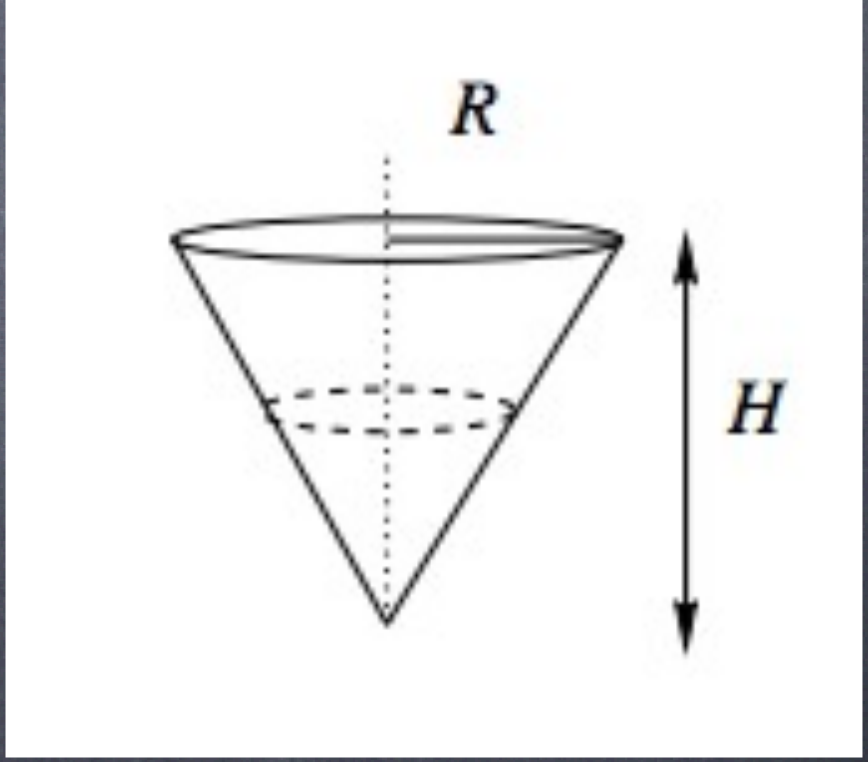
Which is the relevant equation relating the quantities (not rates of change yet)?

(A) $V = 1/3\ \pi\ R^2 H$

(B) $V = 1/3\ \pi\ (R^2/H^2)\ h$

(C) $V = 1/3\ \pi\ (R^2/H^2)\ h^3$

(D) $V = 1/3\ \pi\ r^2 h$



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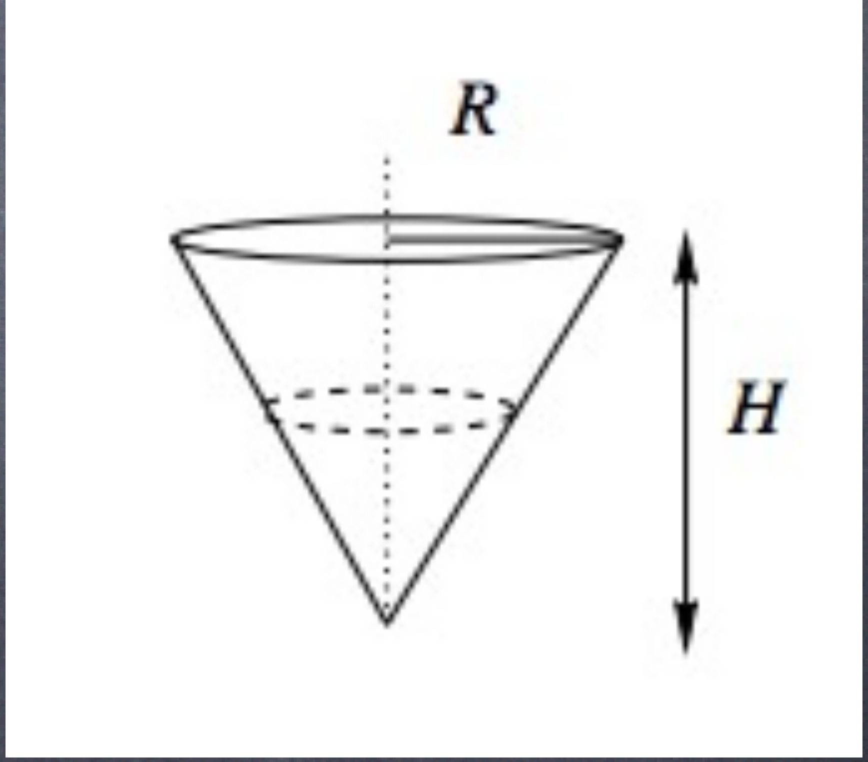

Continued next lecture...