Today

- log graph and semi-log plots
- Exponential derivative (what is C_a?)
- Bacterial growth example
- Doubling time, half life, characteristic time
- Exponential behaviour as solution to DE

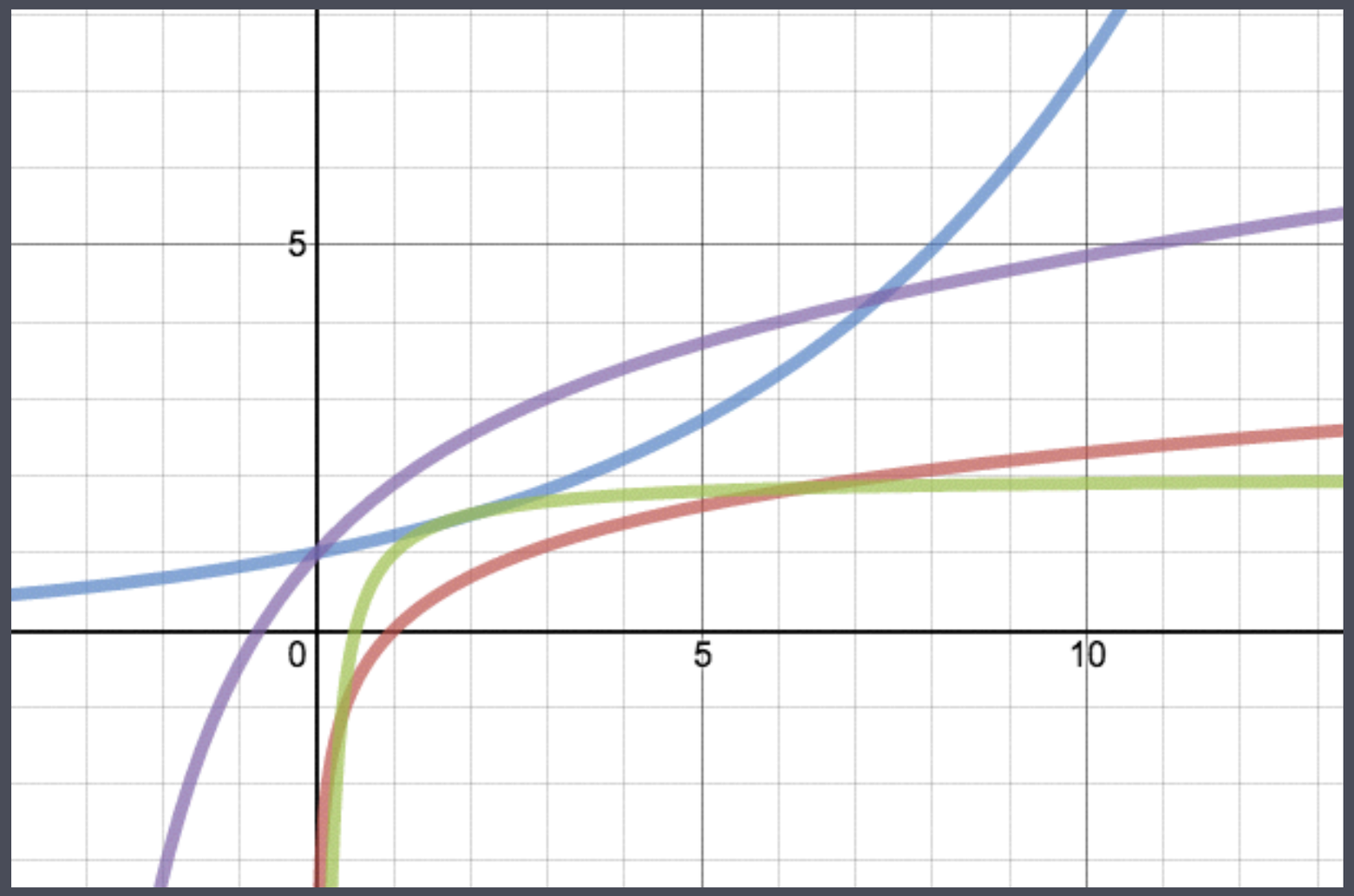
Which of following is the graph of ln(x)?

(A)

(B)

(C)

(D)



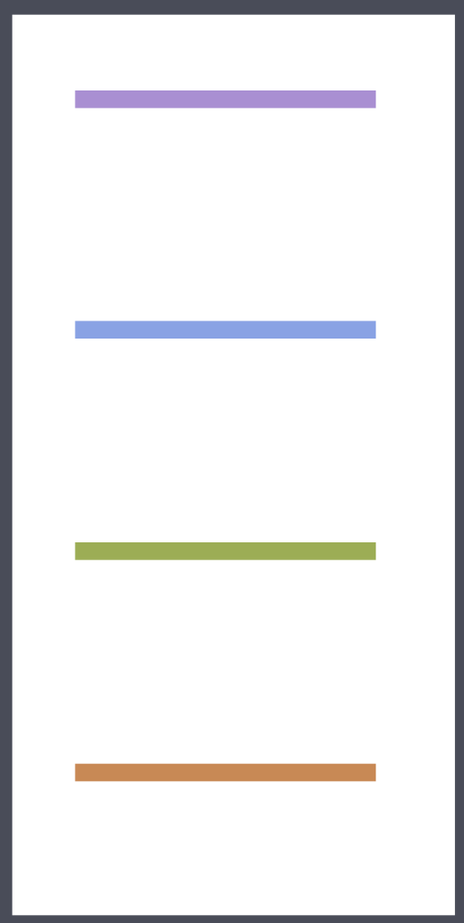
Which of following is the graph of ln(x)?

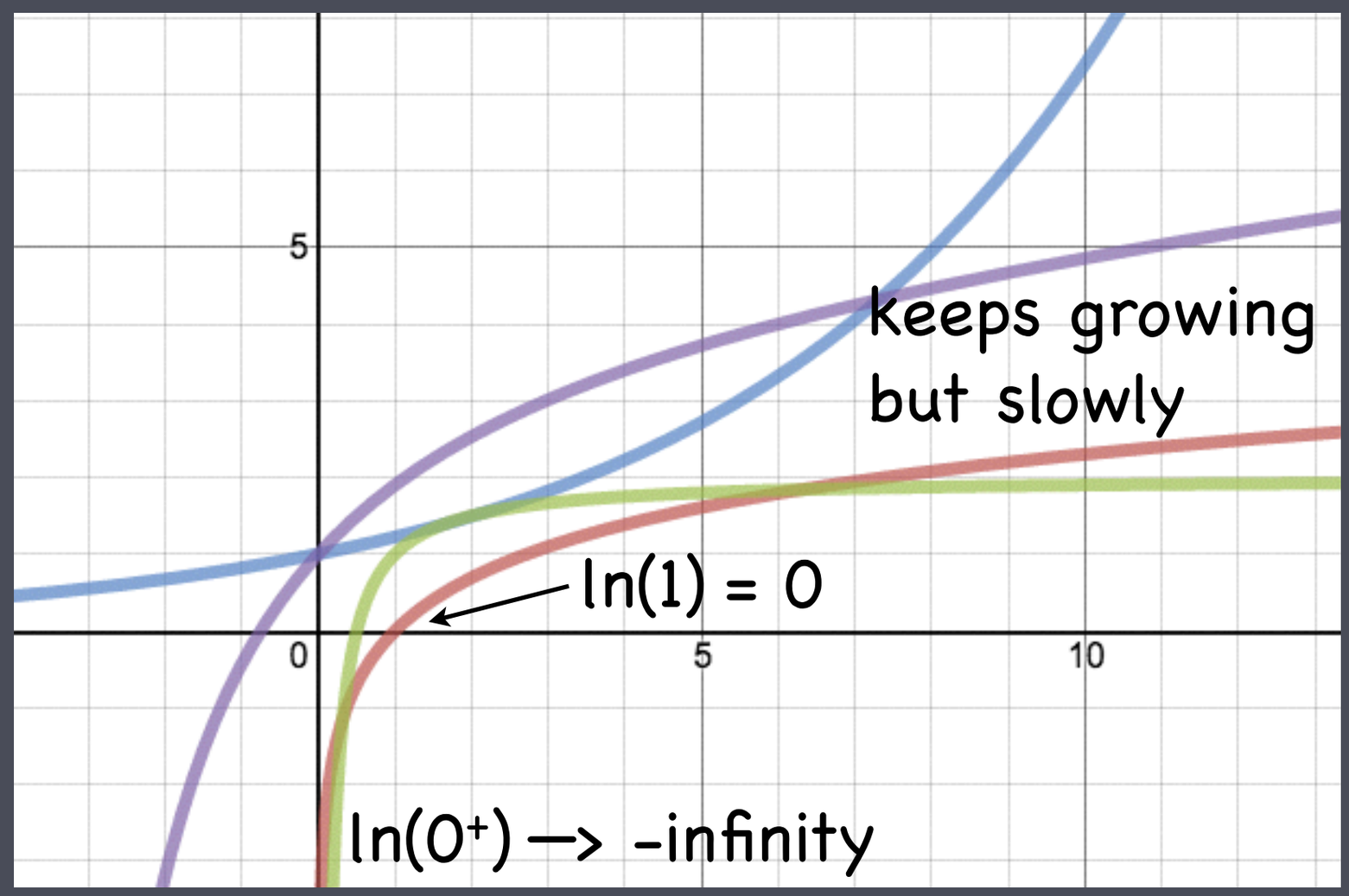
(A)

(B)

(C)

(D)





Log-log and semi-log plots

- A log-log plot is a plot on which you plot log(y) versus log(x) instead of y versus x.
- A semi-log plot is a plot on which you plot log(y) versus x instead of y versus x.

<---this is what OSH 5 asks you to use

Semi-log plot of exponential function

- Suppose $y = ae^{kx}$. a and k are constants.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ae^{kx}) = \ln(a) + kx$.
- $V = A + kx$ where $A=\ln(a)$.
- On a semi-log plot, $y= ae^{kx}$ looks linear.

Log-log plot of power function

- Suppose $y = ax^p$.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ax^p) = \ln(a) + p \ln(x)$.
- $V = A + pU$ where $A=\ln(a)$, $U=\ln(x)$.
- On a log-log plot, $y=ax^p$ looks linear.

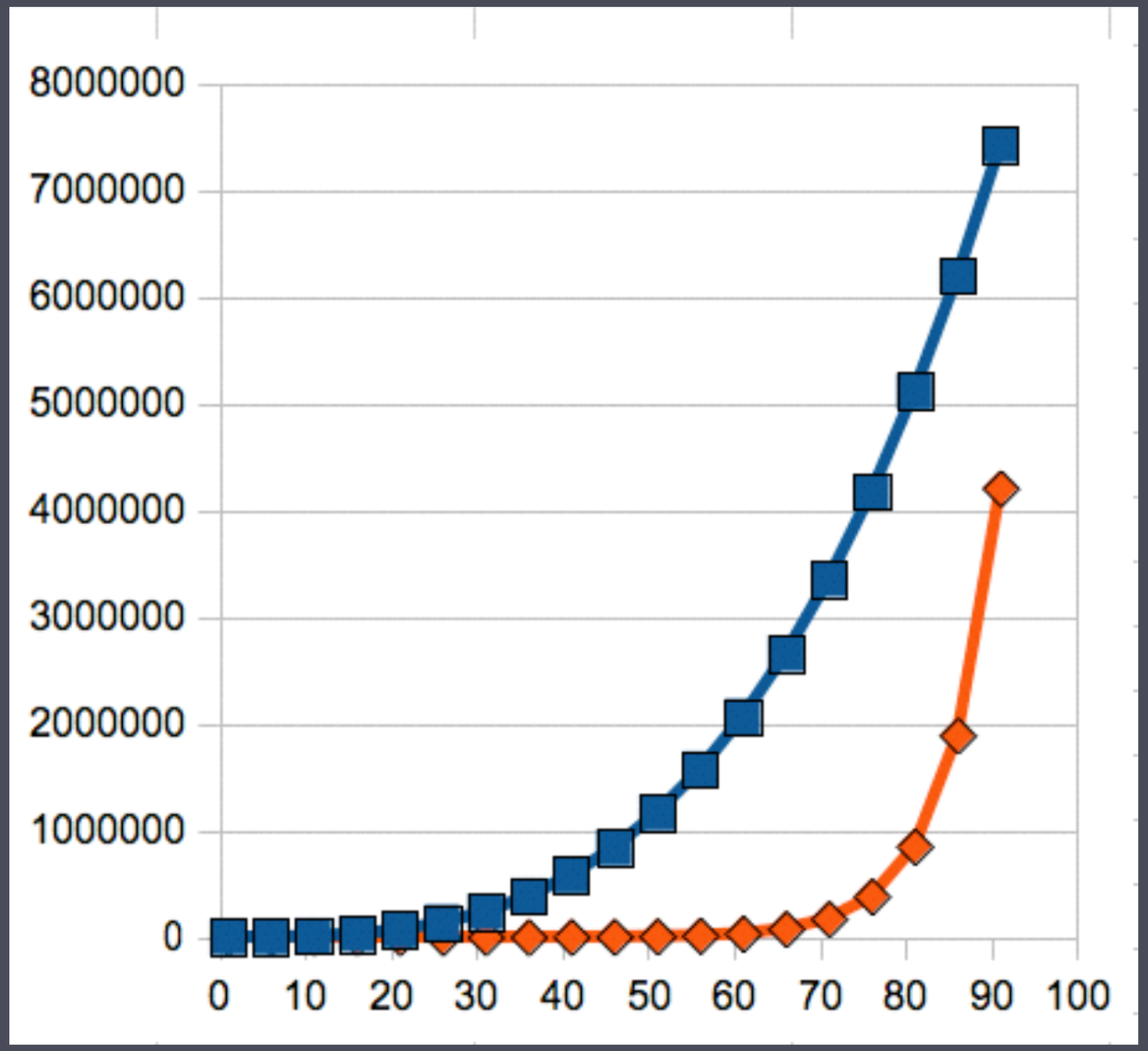
Slide not shown in class but include for interest.

Regular, log-log and semi-log plots

Two data sets.

Power function?

Exponential function?



Regular x-y plot.

Slide not shown in class but include for interest.

Plot $Y_i = \ln(y_i)$ versus x_i.

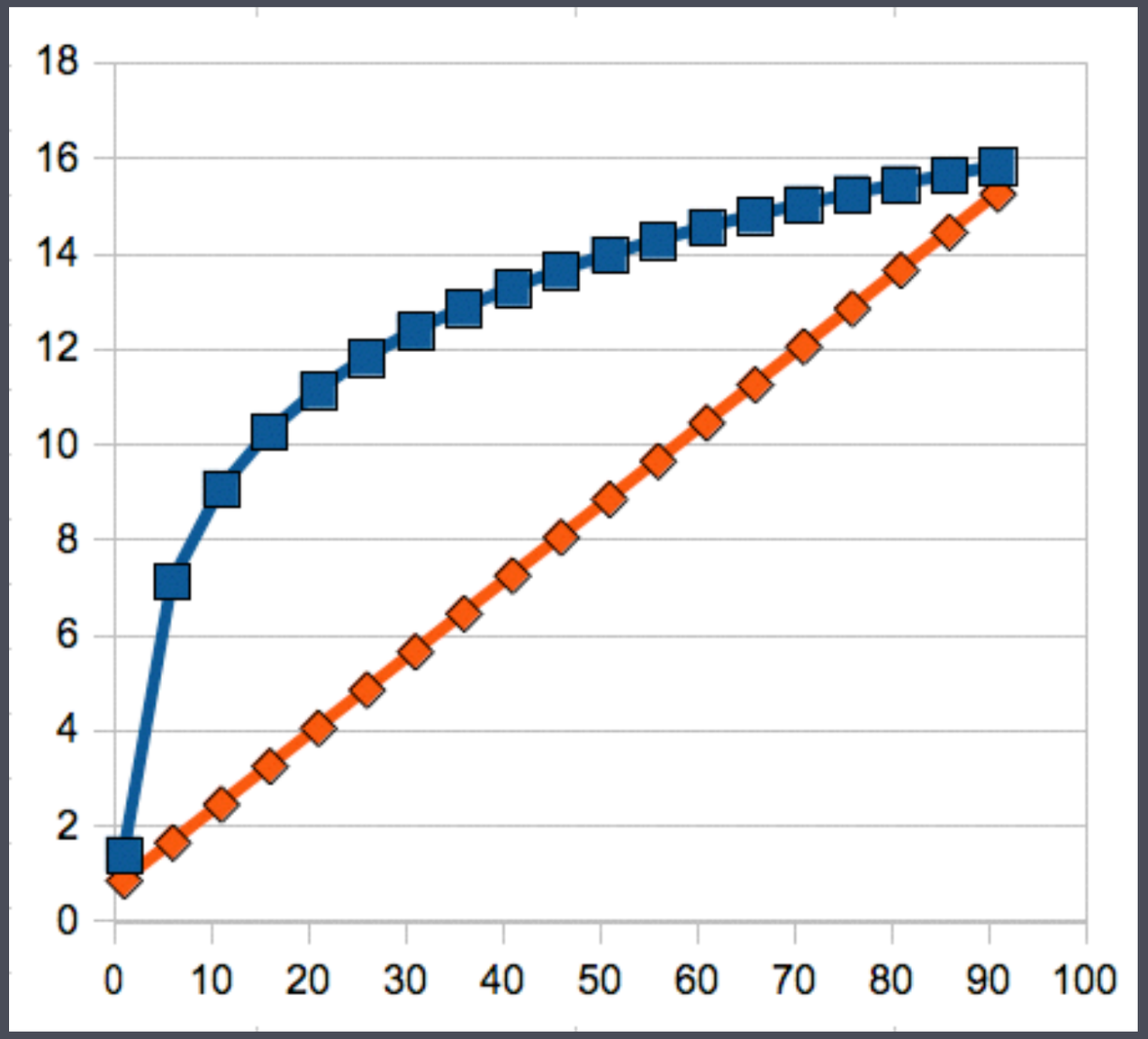
Conclude that:

(A) Blue is power function.

(B) Blue is exponential.

(C) Orange is power function.

(D) Orange is exponential.



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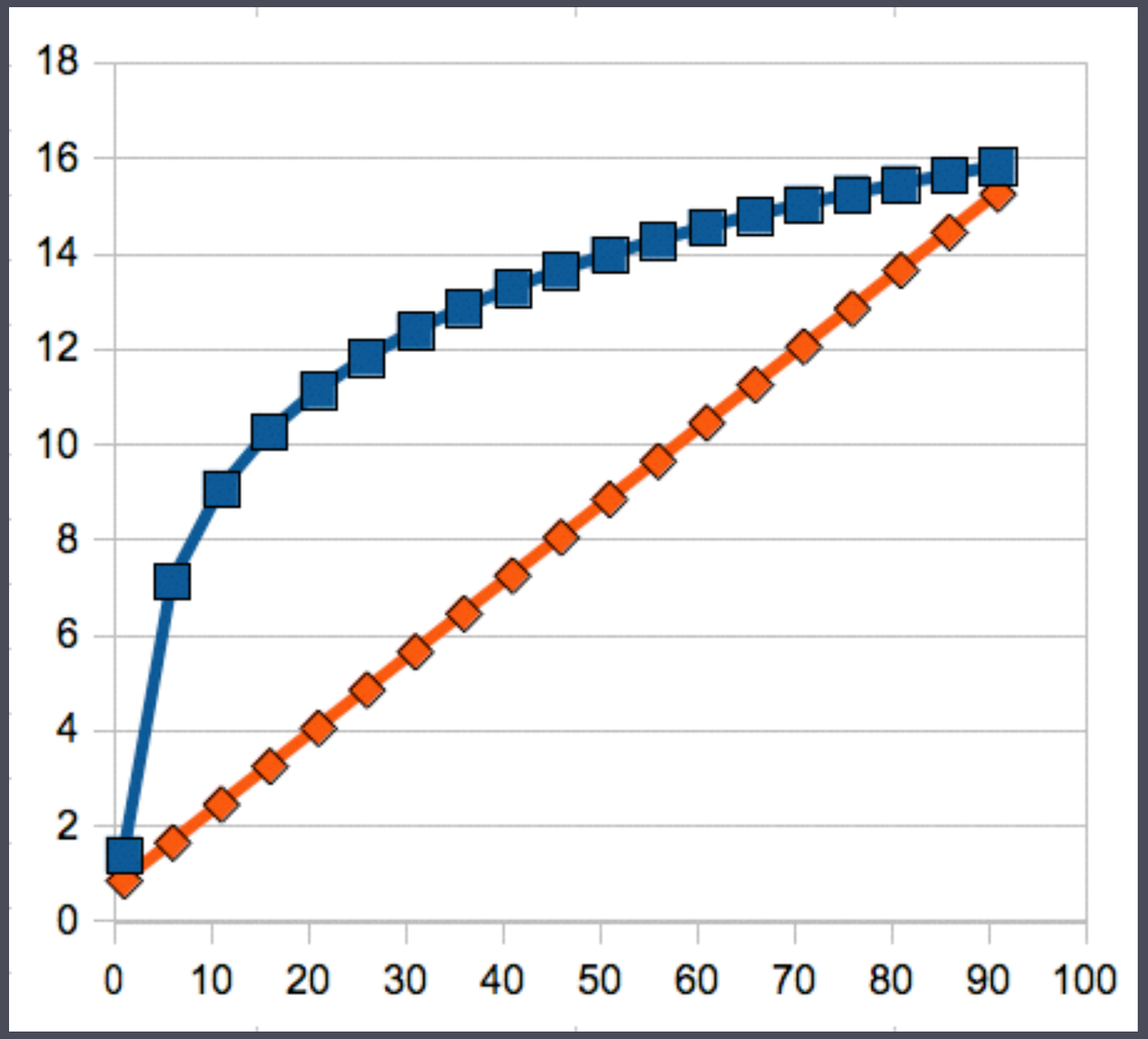

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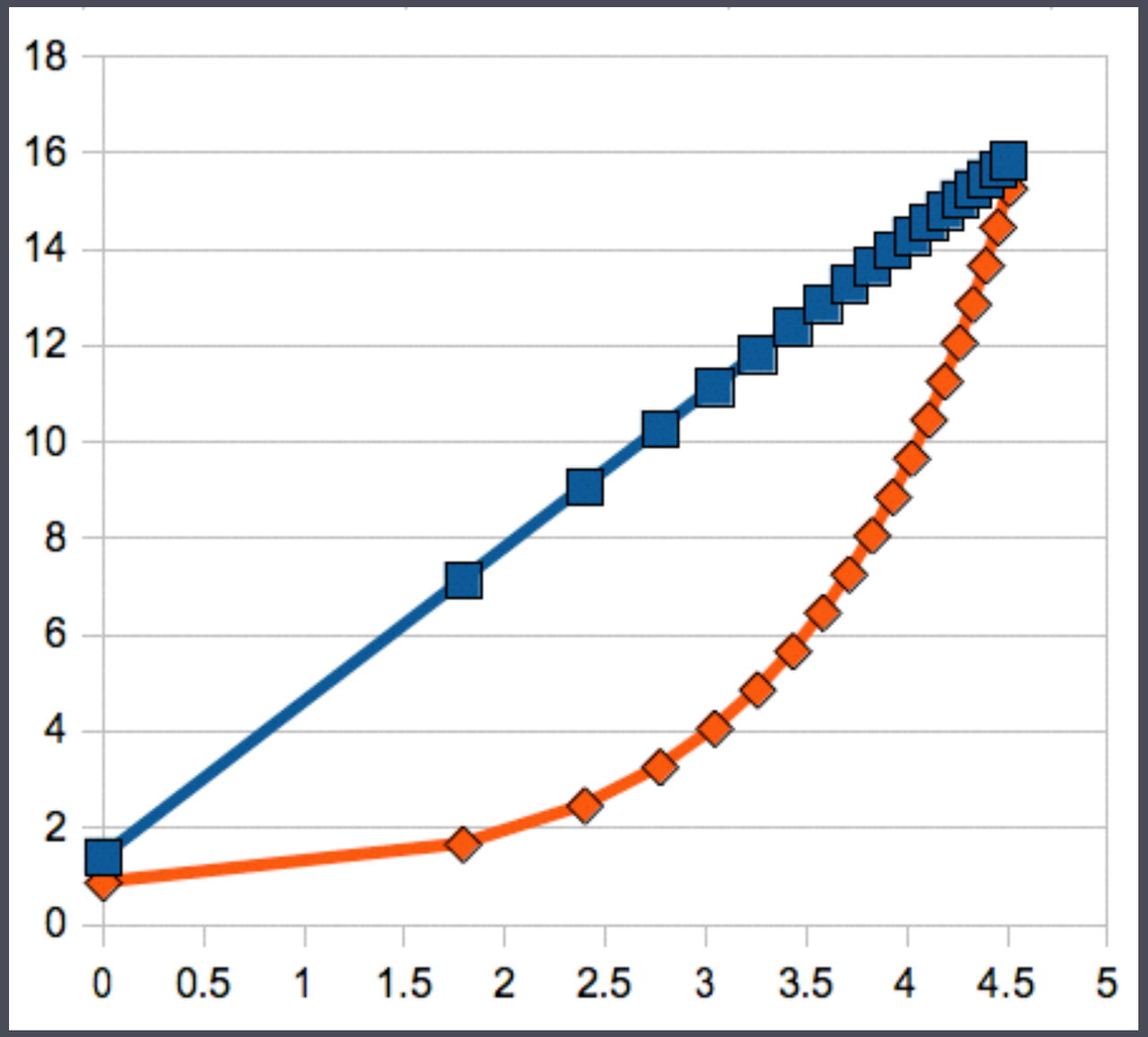

Log-log plot.

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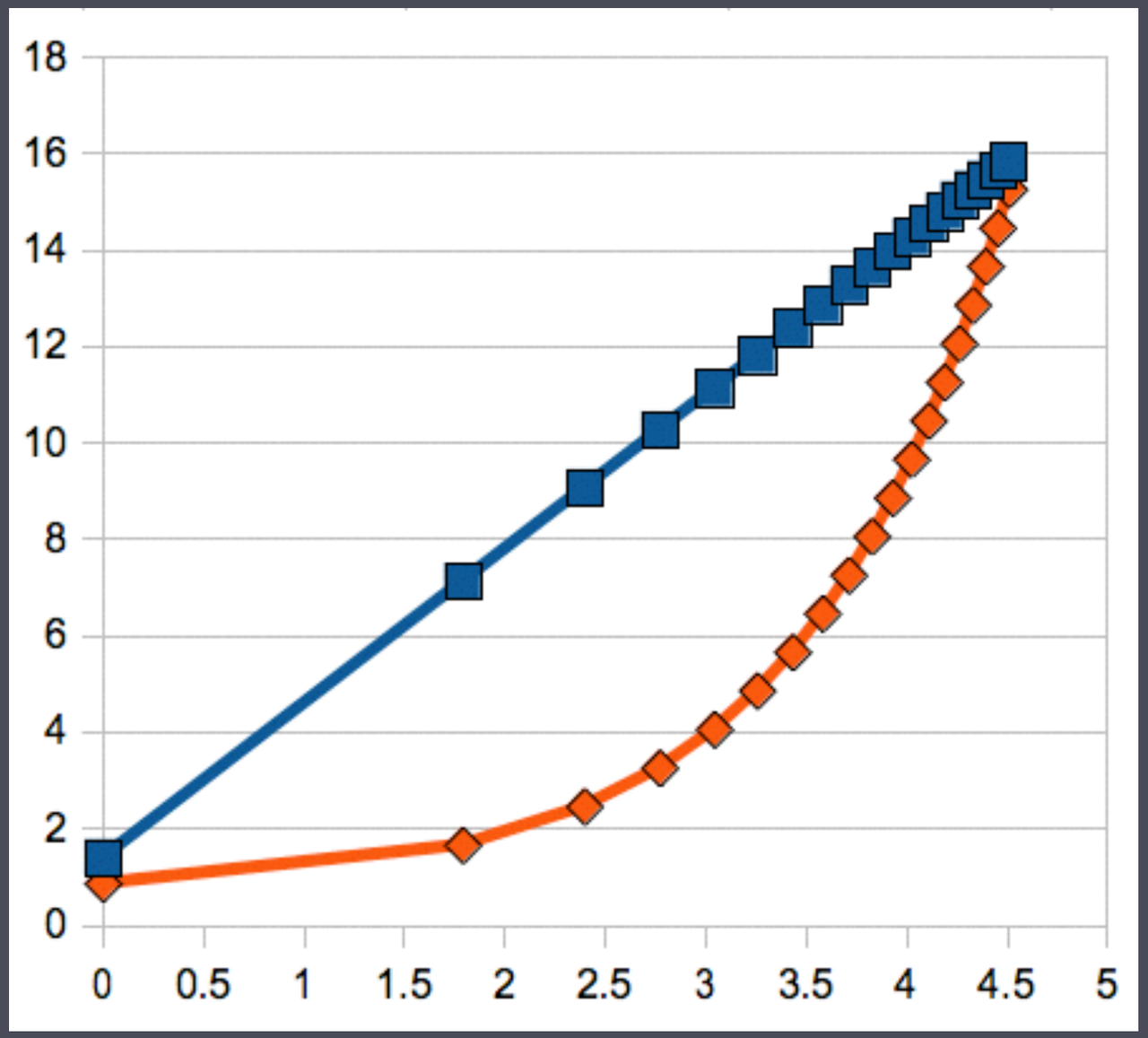

Log-log plot.

Slide not shown in class but include for interest.

$f(x)=a^x$. $f'(x)=C_a a^x$. $C_a=??$

- Recall that we got stuck on this derivative.
- Time to get unstuck...

$$f(x) = e^{\ln(2)x}.$$

(A) $f'(x) = e^{\ln(2)x}$.

(B) $f'(x) = \ln(2)e^{\ln(2)x}$.

(C) $f'(x) = \ln(2)\cdot 1/2\cdot e^{\ln(2)x}$.

(D) $f'(x) = \ln(2)xe^{\ln(2)x-1}$.

$$f(x) = e^{\ln(2)x}.$$

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(D) $f'(x) = \ln(2)xe^{\ln(2)x-1}$.

$$f(x) = e^{\ln(2)x}.$$

(A) $f(x) = 2x.$

(B) $f(x) = (e^{\ln(2)})^x = 2^x.$

(C) $f(x) = e^{\ln(2)}\, e^x = 2e^x.$

(D) $f(x) = e^{\ln(x^2)} = x^2.$

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(A) $f(x) = 2x.$

(B) $f(x) = (e^{\ln(2)})^x = 2^x.$

(C) $f(x) = e^{\ln(2)}\, e^x = 2e^x.$

(D) $f(x) = e^{\ln(x^2)} = x^2.$

From the last two clicker Qs...

- $f(x) = e^{\ln(2)x}$ --> $f'(x) = \ln(2)e^{\ln(2)x}$.
- $f(x) = e^{\ln(2)x}$ --> $f(x) = 2^x$.
- So $f(x) = 2^x$ --> $f'(x) = 2^x \ln(2)$.
- In general, $f(x) = a^x$ --> $f'(x) = a^x \ln(a)$.

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

What value of k makes $a^x = e^{kx}$?

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(E) $k=\ln(-a)$

$$a^x = (e^k)^x$$

$$a = e^k$$

$$\ln(a) = \ln(e^k)$$

$$\ln(a) = k\ \ln(e)$$

$$\ln(a) = k$$

$$f(x) = a^x = e^{\ln(a)x}$$

$$\text{-->}\ f'(x) = a^x \ln(a).$$

A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

Rough estimate (no calcs, just intuition):

(A) ~1 week.

(B) ~2 weeks.

(C) ~1 month.

(D) ~1 year.

(E) ~10^4 days ≈ 27 years.

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A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $p(t) = e^{t/24}$.

(B) $p(t) = 100{,}000\ 2^{t/24}$.

(C) $p(t) = e^{\ln(2)t}$.

(D) $p(t) = 2^{-t/24}$.

(E) $p(t) = 2^{t/24}$.

A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $p(t) = e^{t/24}$.

(B) $p(t) = 100{,}000\ 2^{t/24}$.

(C) $p(t) = e^{\ln(2)t}$. $= 2^t$ <-- t measured in days.

(D) $p(t) = 2^{-t/24}$.

(E) $p(t) = 2^{t/24}$. <-- t measured in hours.

A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $t=\ln(10^5)/\ln(2)$

(B) $t=10^5/\ln(2)$

(C) $t=\ln(10^5)/2$

(D) $t=100{,}000/24$ days

A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $t=\ln(10^5)/\ln(2)$

(B) $t=10^5/\ln(2)$

(C) $t=\ln(10^5)/2$

(D) $t=100{,}000/24$ days

$$p(t) = 2^t$$

$$10^5 = 2^t$$

$$\ln(10^5) = \ln(2^t)$$

$$\ln(10^5) = t\ln(2)$$

$$t \approx 16.6 \text{ days}$$

Doubling time

- Let $c(t) = c_0 e^{kt}$. At $t=0$, $c(0) = c_0 e^0 = c_0$.
- If $k>0$, $c(t)$ is increasing and doubles when $c_0 e^{kt} = 2c_0$.
- That is when $t=\ln(2)/k$.
- This is called the doubling time.

Half-life

- Let $c(t) = c_0e^{kt}$. At $t=0$, $c(0) = c_0e^0 = c_0$.
- If $k<0$, $c(t)$ is decreasing and halves when $c_0e^{kt} = c_0/2$.
- That is when $t=-\ln(2)/k$.
- If written $c(t) = c_0e^{-kt}$ with $k>0$ then $t=\ln(2)/k$ (same as doubling time).
- This is called the half-life.

Characteristic time / mean life

- Let $c(t) = c_0 e^{-kt}$. At $t=0$, $c(0) = c_0 e^0 = c_0$.
- If $k>0$, $c(t)$ is decreasing and reaches $1/e$ its original value when $c_0 e^{kt} = c_0/e$.
- That is when $t=1/k$.
- This is called the characteristic time or mean life. Just like half-life but replace 2 with e (could be called 1/e-life).