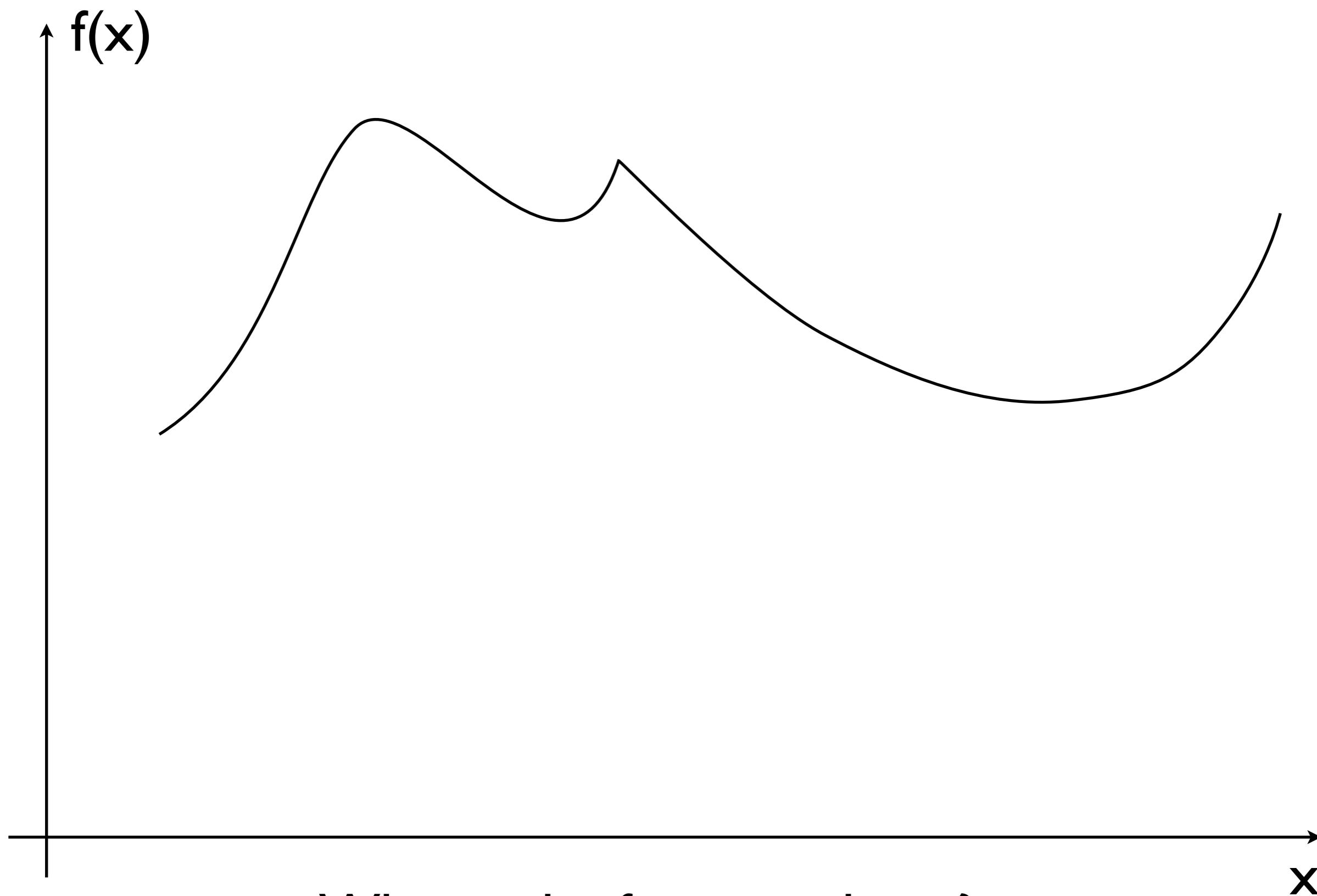


Today

- Putting it all together – using f , f' and f'' to sketch a graph.
- Absolute extrema
- Intro to optimization

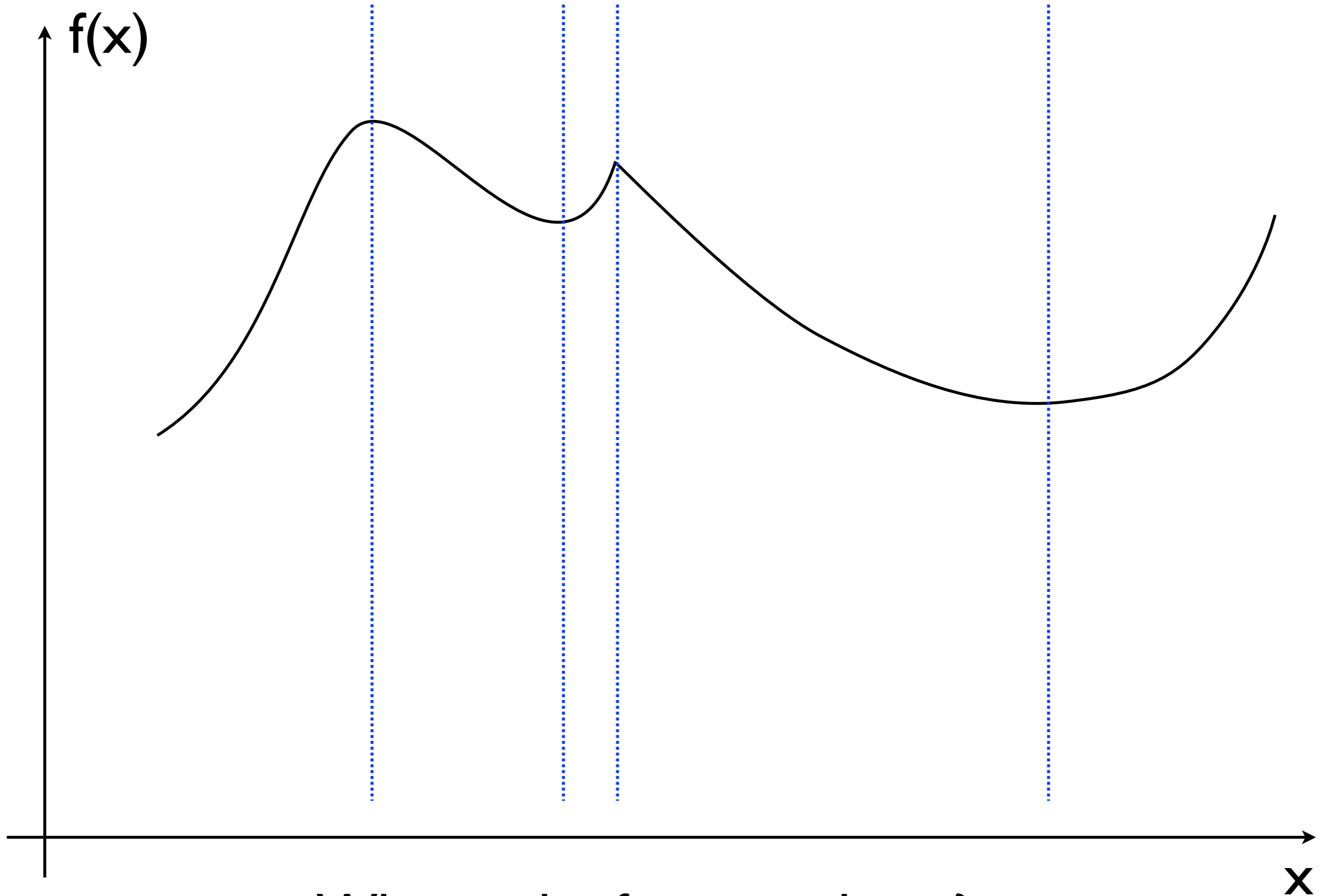
Using f , f' and f'' to
graph f

Annotating the graph of $f(x)$ with $f'(x)$ info



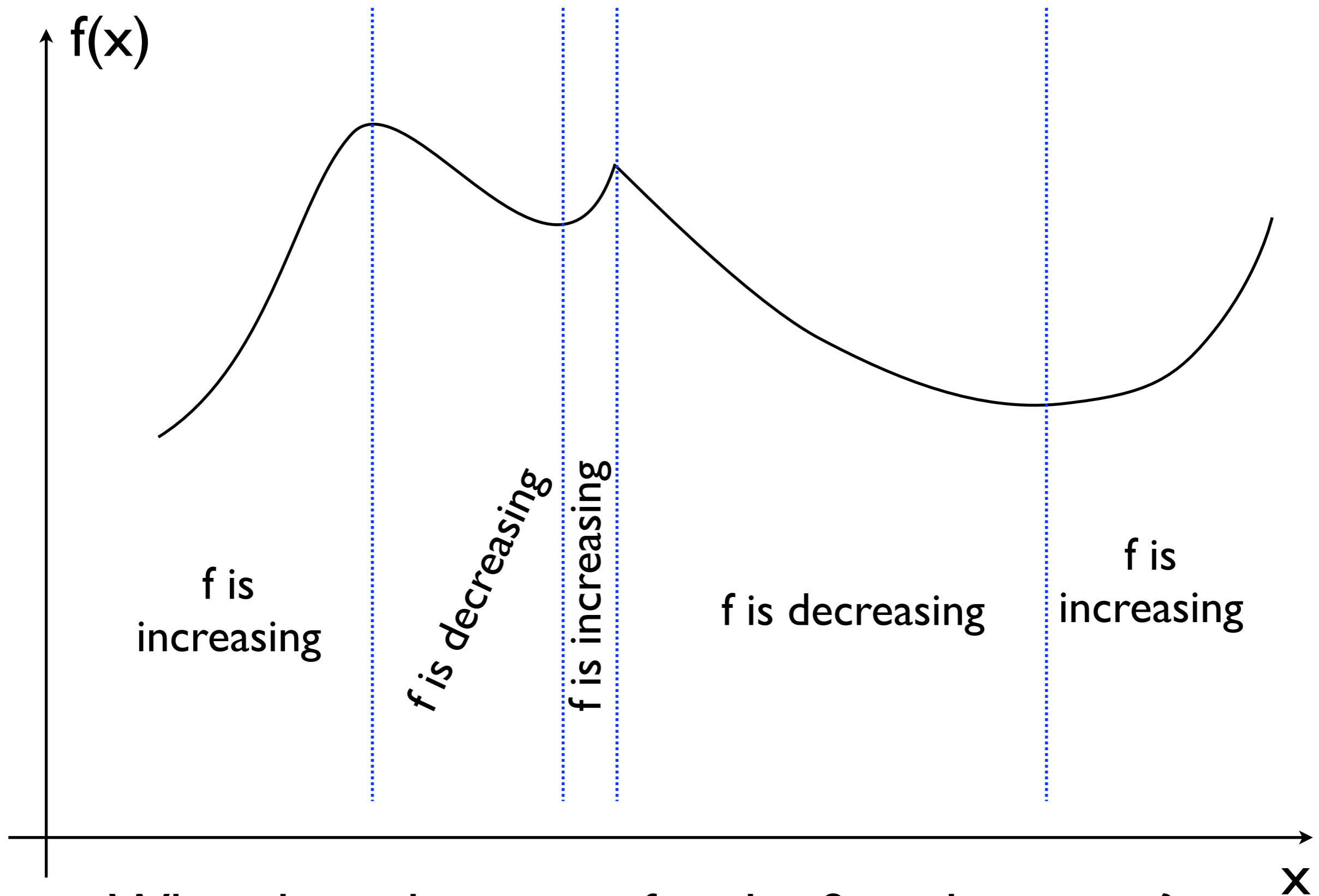
What is the function doing?

Annotating the graph of $f(x)$ with $f'(x)$ info



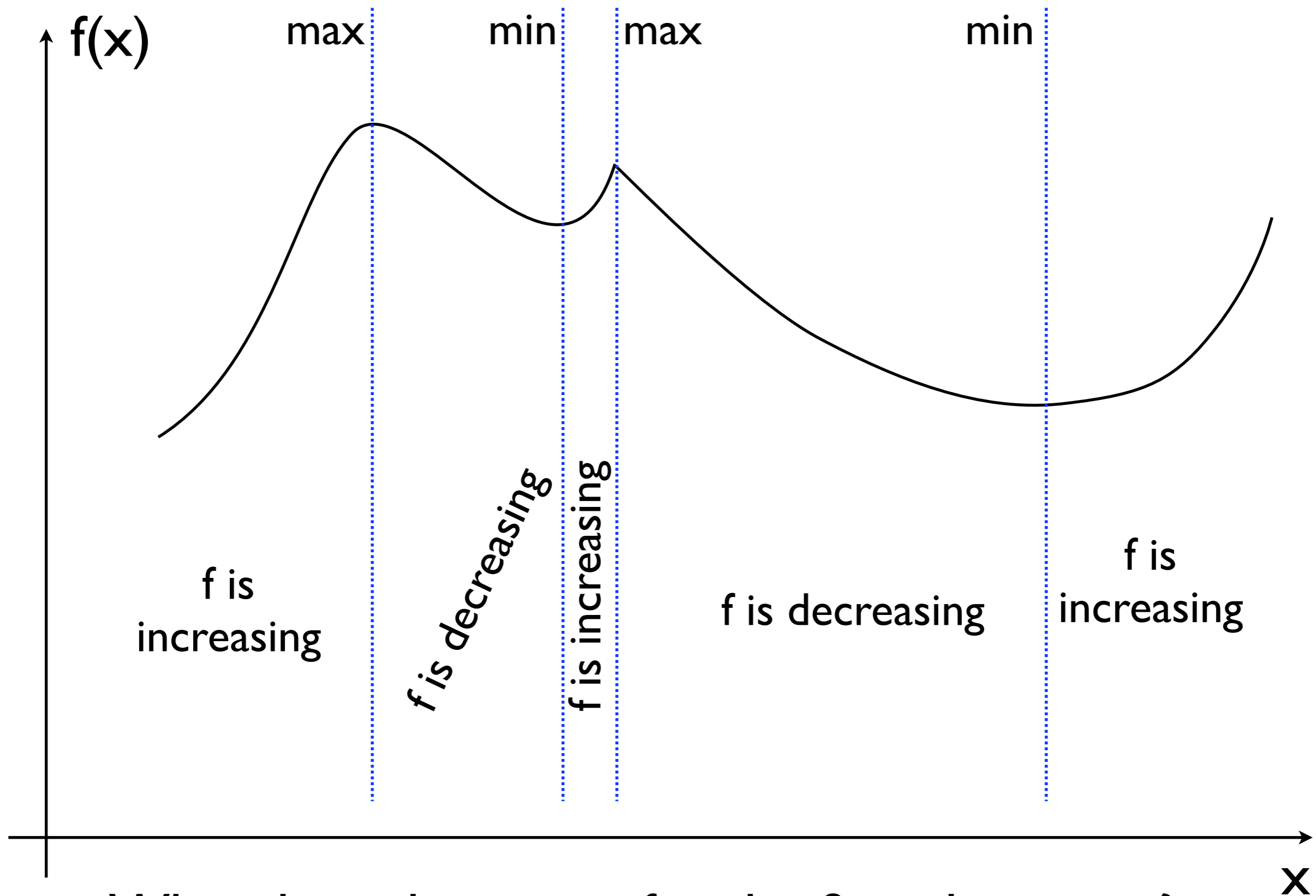
What is the function doing?

Annotating the graph of $f(x)$ with $f'(x)$ info



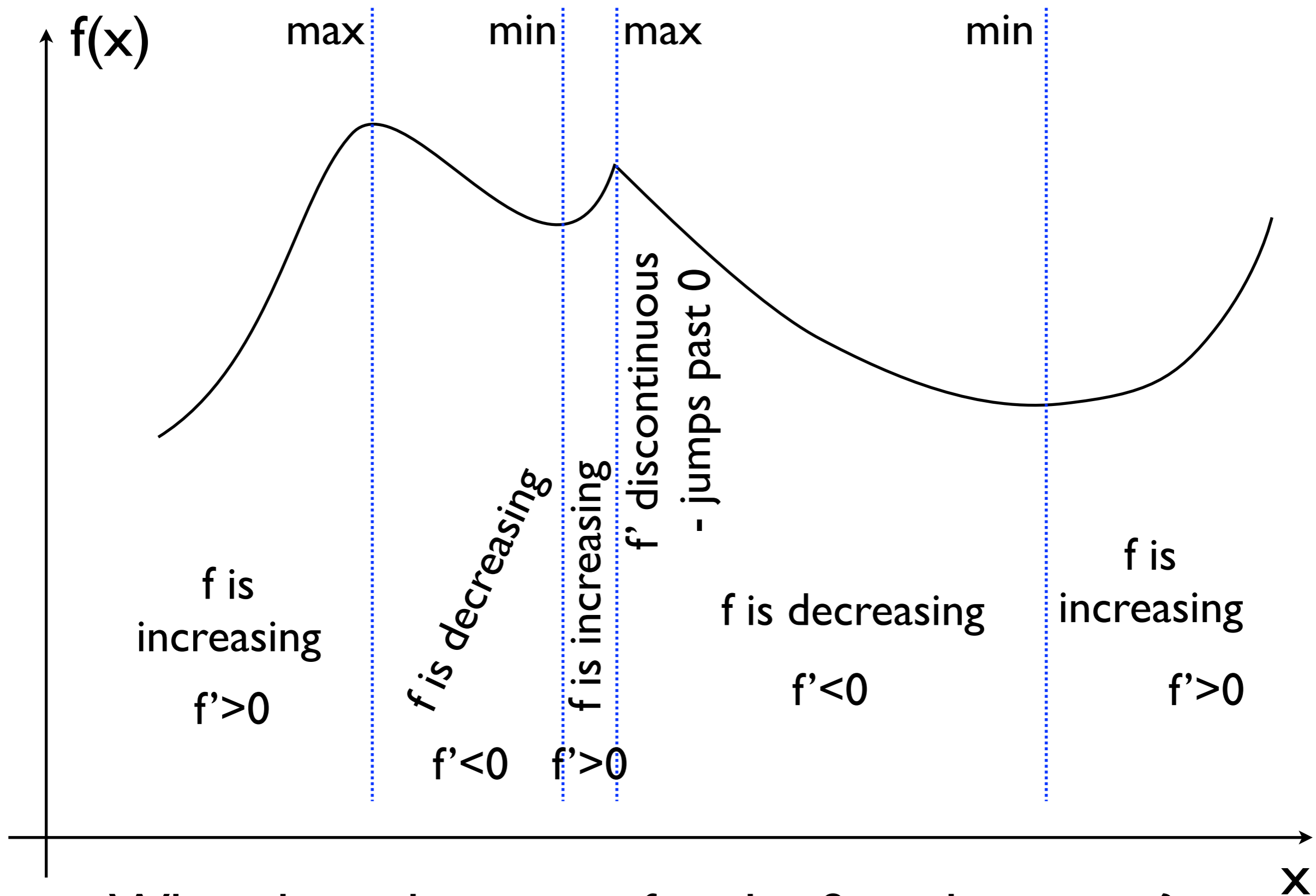
What does that mean for the first derivative?

Annotating the graph of $f(x)$ with $f'(x)$ info



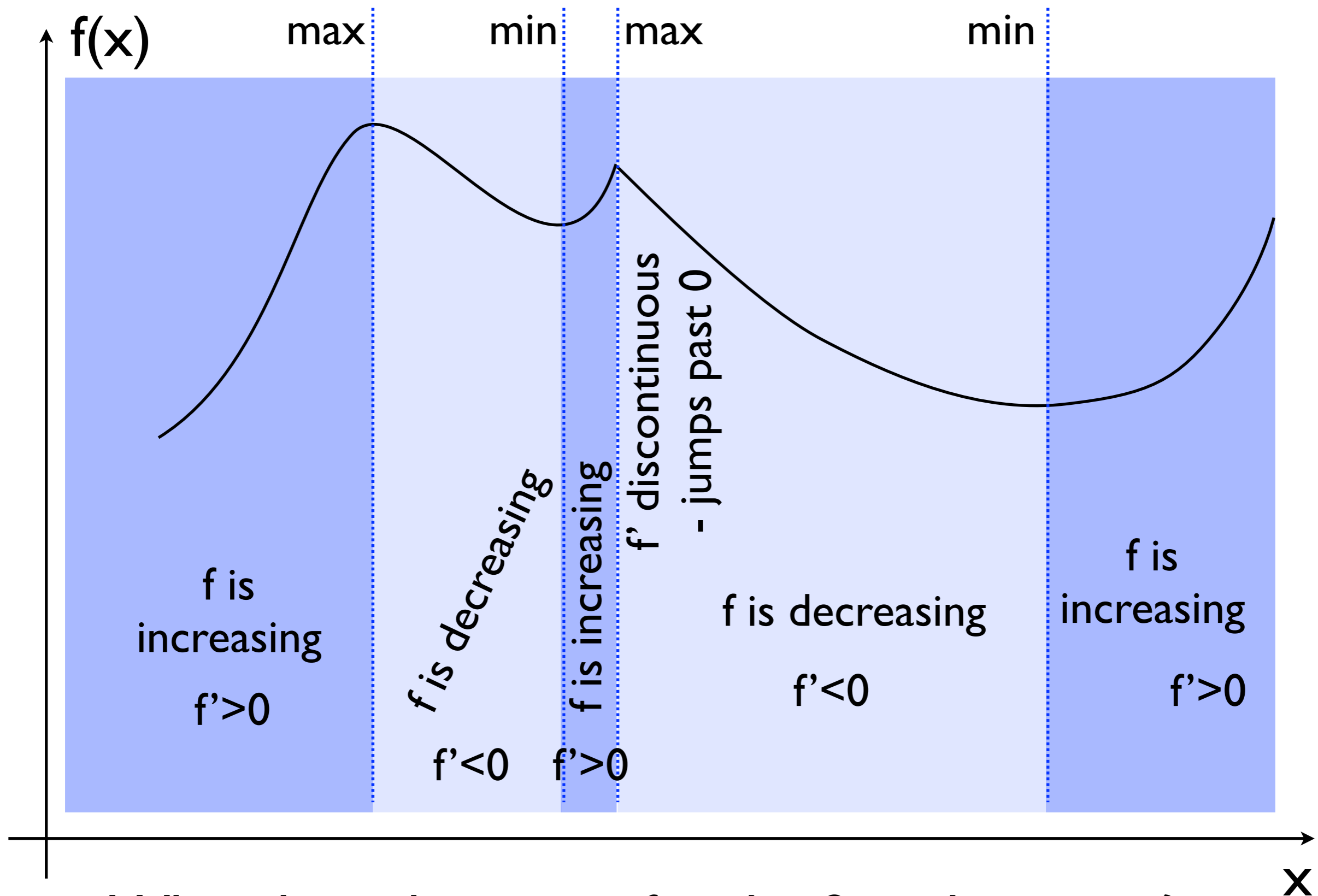
What does that mean for the first derivative?

Annotating the graph of $f(x)$ with $f'(x)$ info



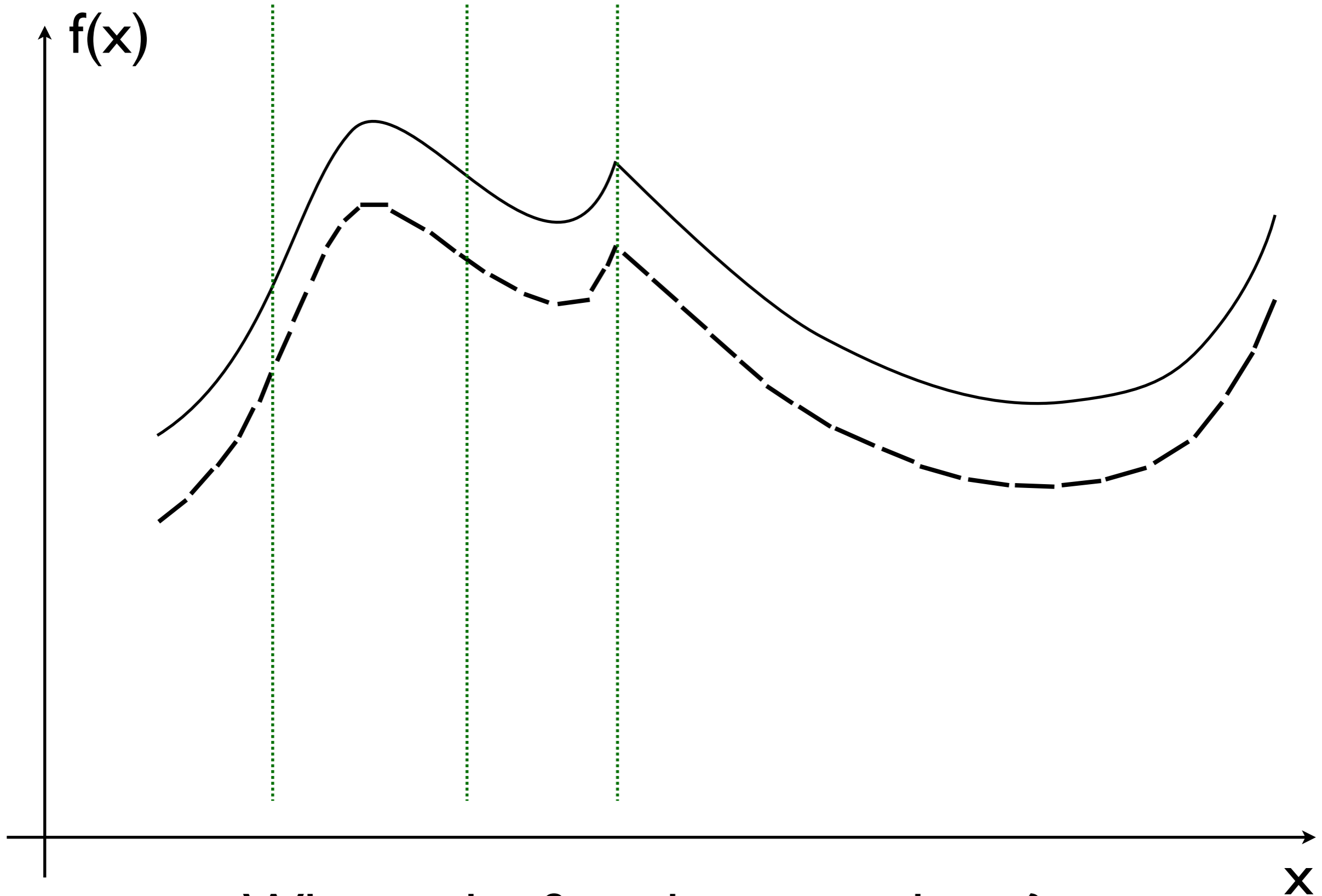
What does that mean for the first derivative?

Annotating the graph of $f(x)$ with $f'(x)$ info



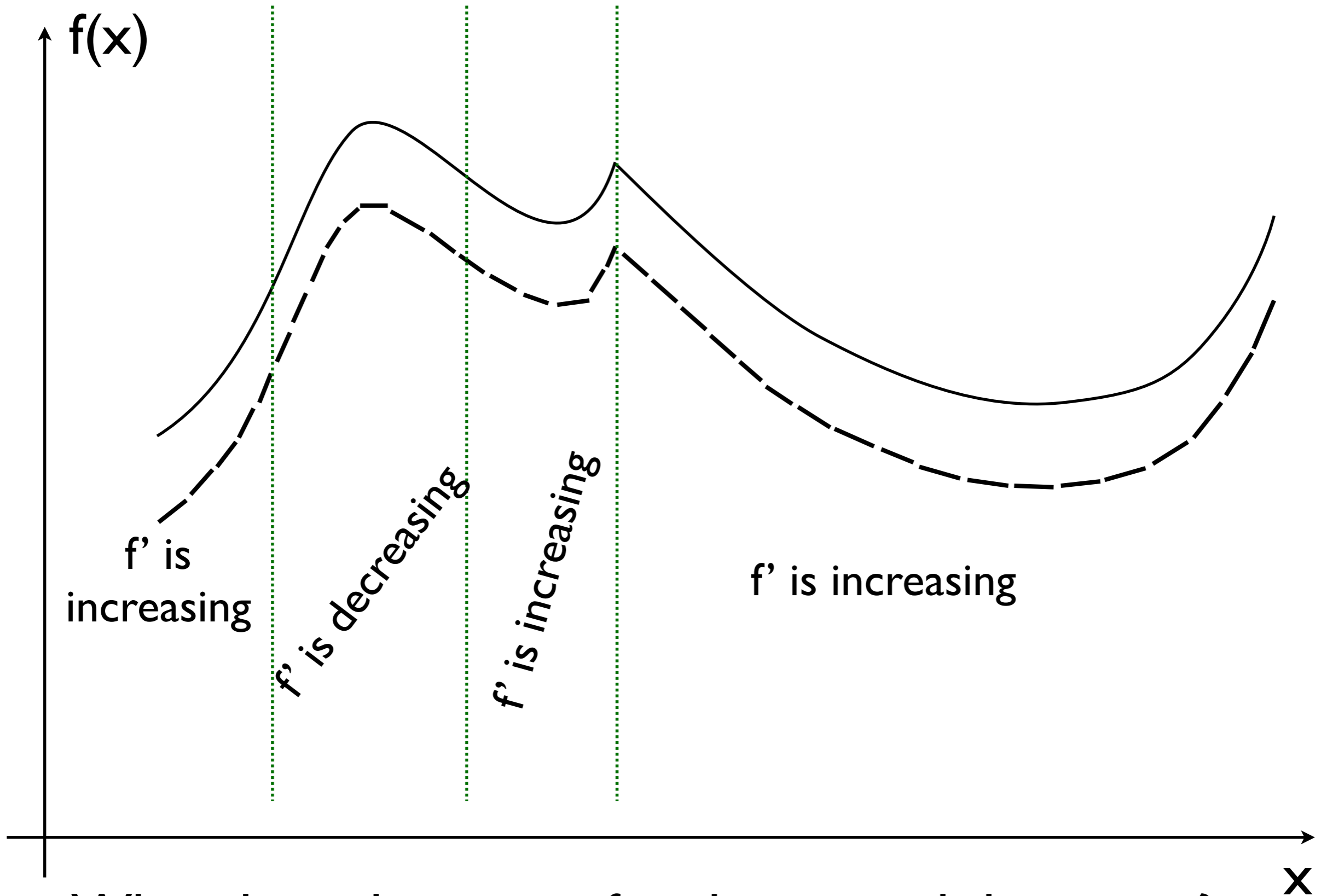
What does that mean for the first derivative?

Annotating the graph of $f(x)$ with $f''(x)$ info



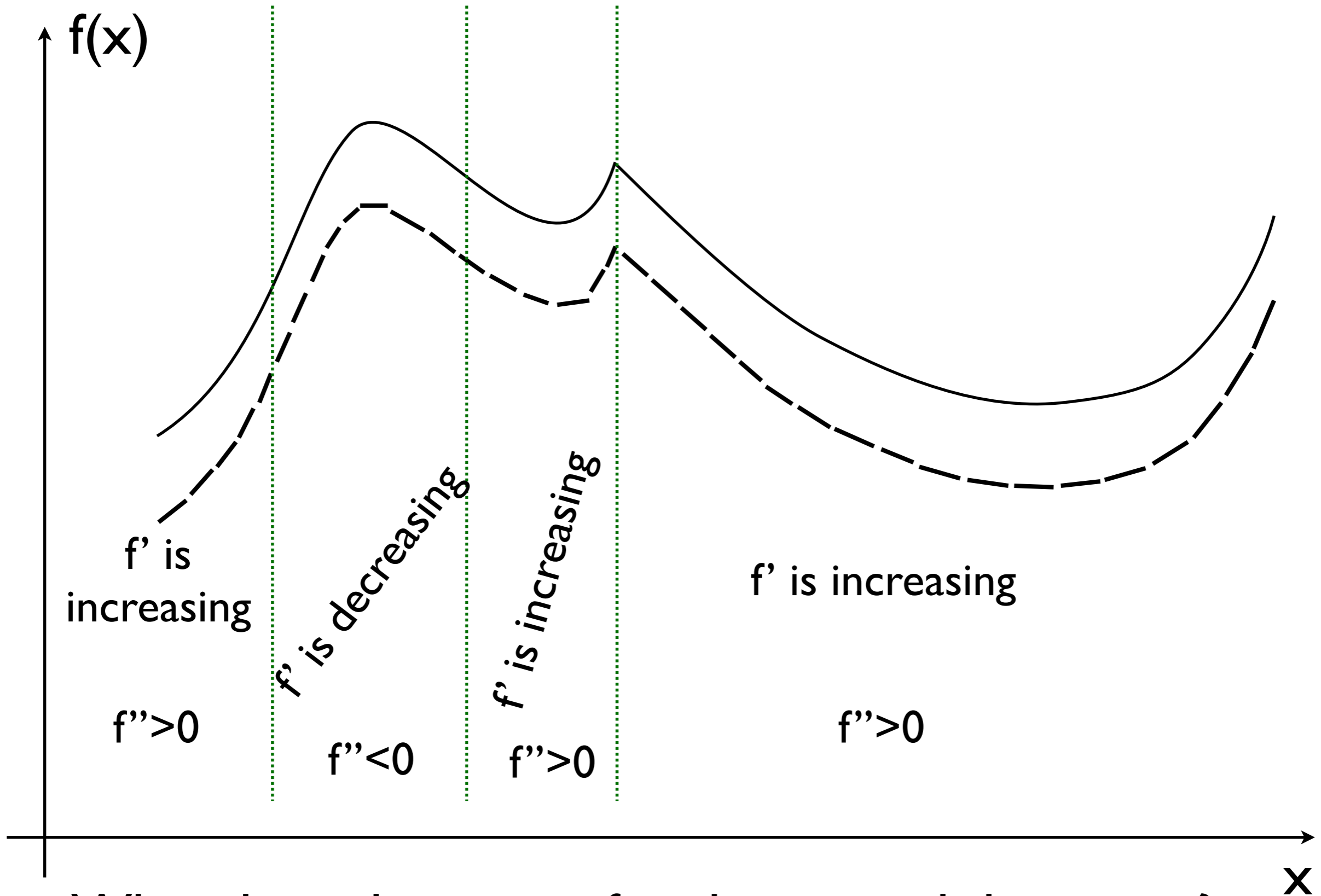
What is the first derivative doing?

Annotating the graph of $f(x)$ with $f''(x)$ info



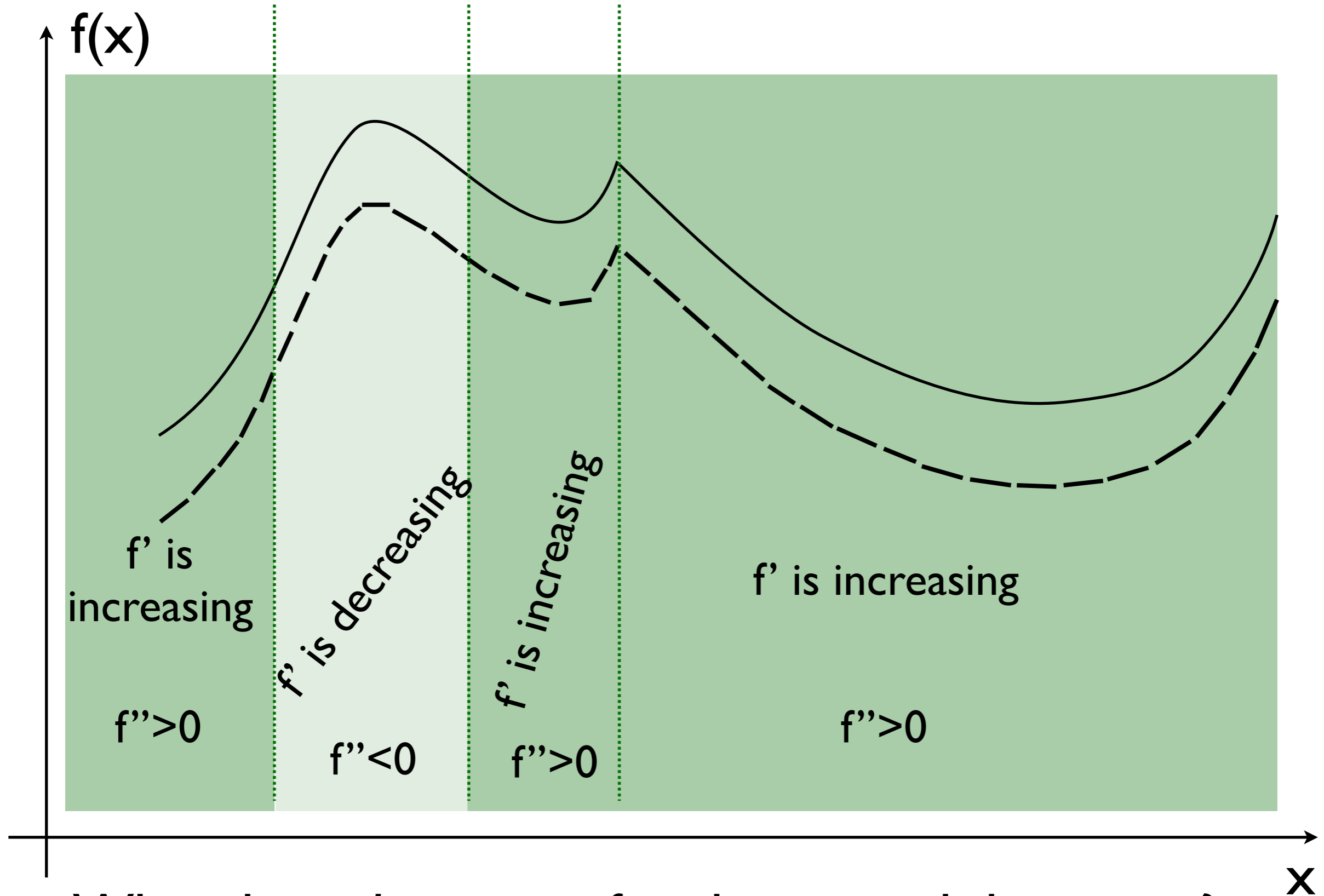
What does that mean for the second derivative?

Annotating the graph of $f(x)$ with $f''(x)$ info



What does that mean for the second derivative?

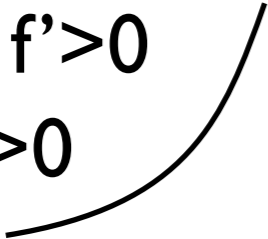
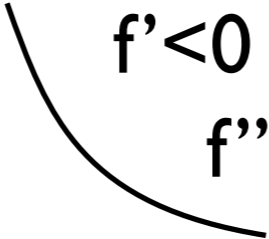
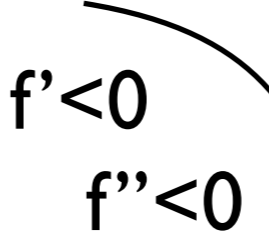
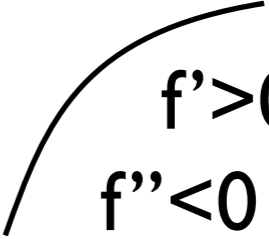




Annotating the graph of $f(x)$ with $f''(x)$ info



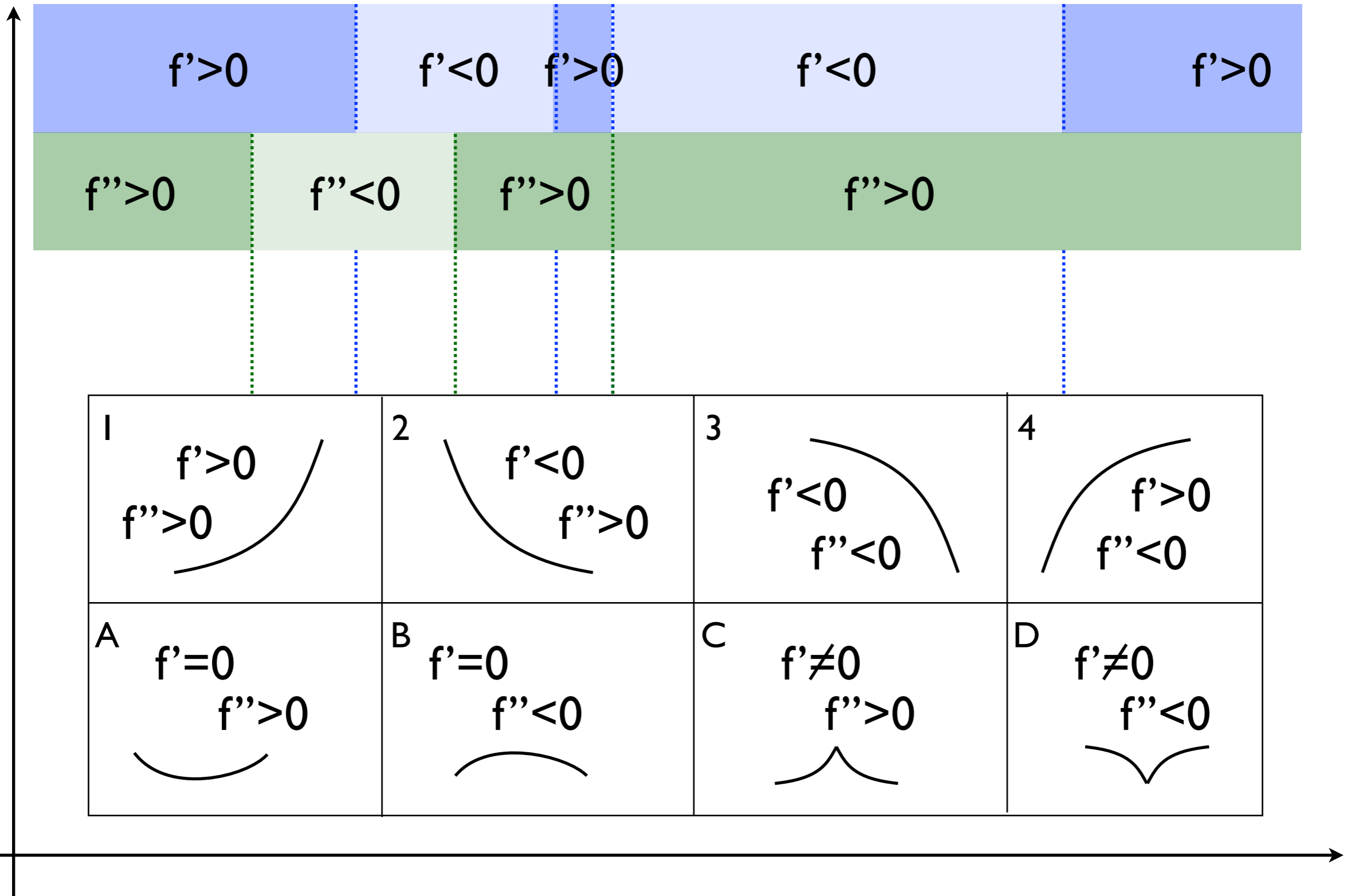
What does that mean for the second derivative?

What you have to do to graph it.

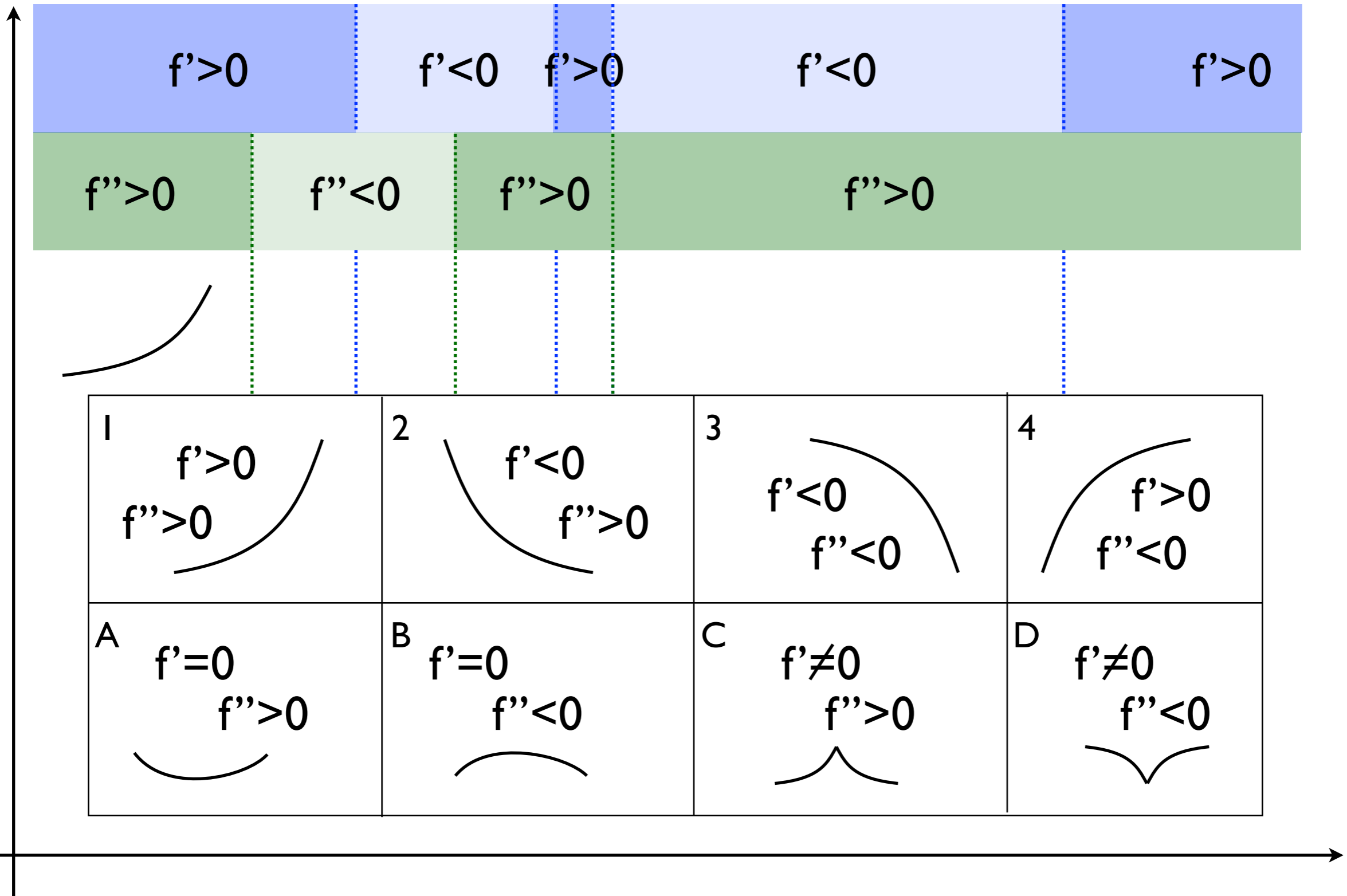
The parts list:

1 $f' > 0$ $f'' > 0$ 	2 $f' < 0$ $f'' > 0$ 	3 $f' < 0$ $f'' < 0$ 	4 $f' > 0$ $f'' < 0$ 
A $f' = 0$ $f'' > 0$ 	B $f' = 0$ $f'' < 0$ 	C $f' \neq 0$ $f'' > 0$ 	D $f' \neq 0$ $f'' < 0$ 

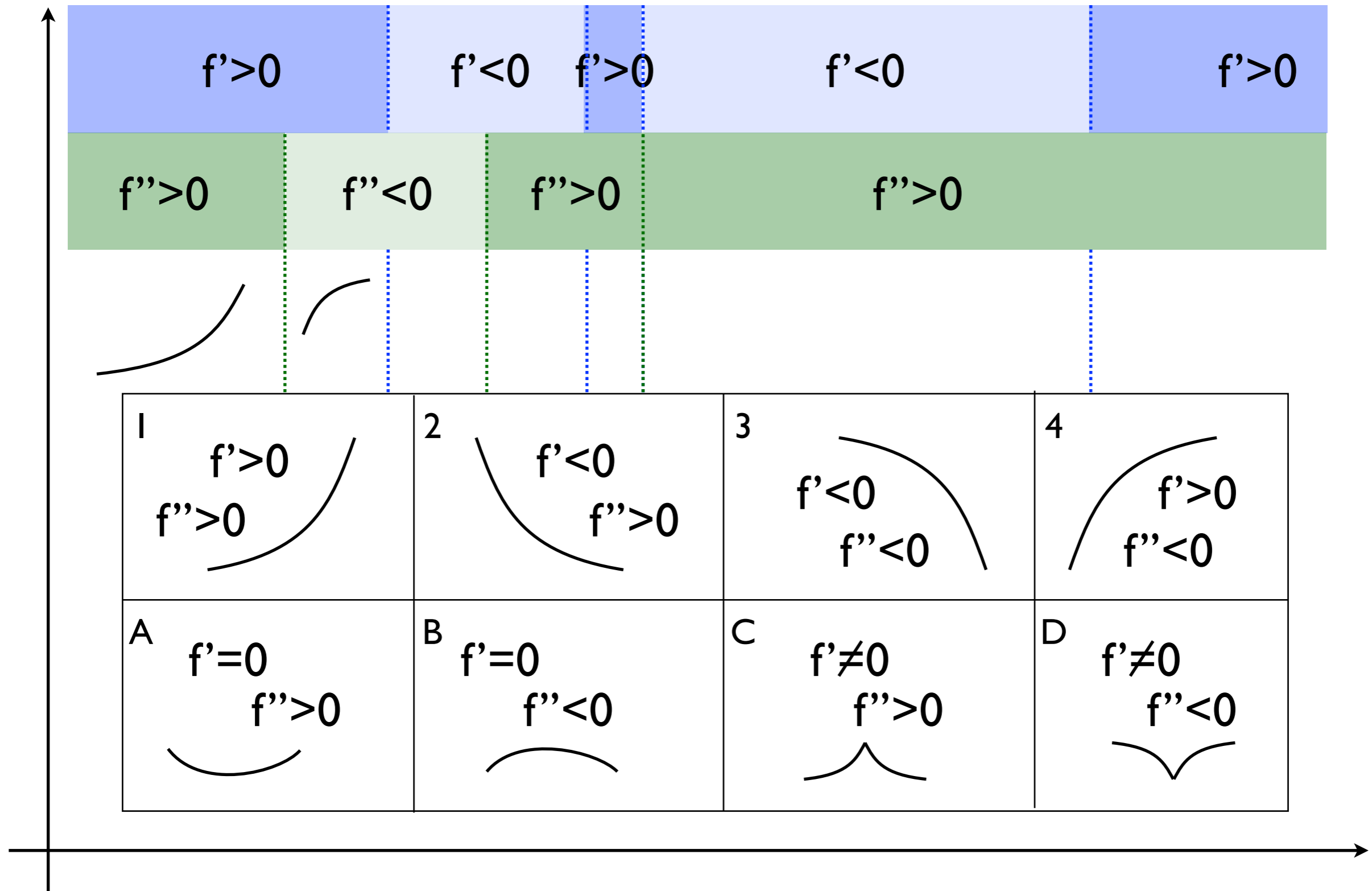
What you have to do to graph it.



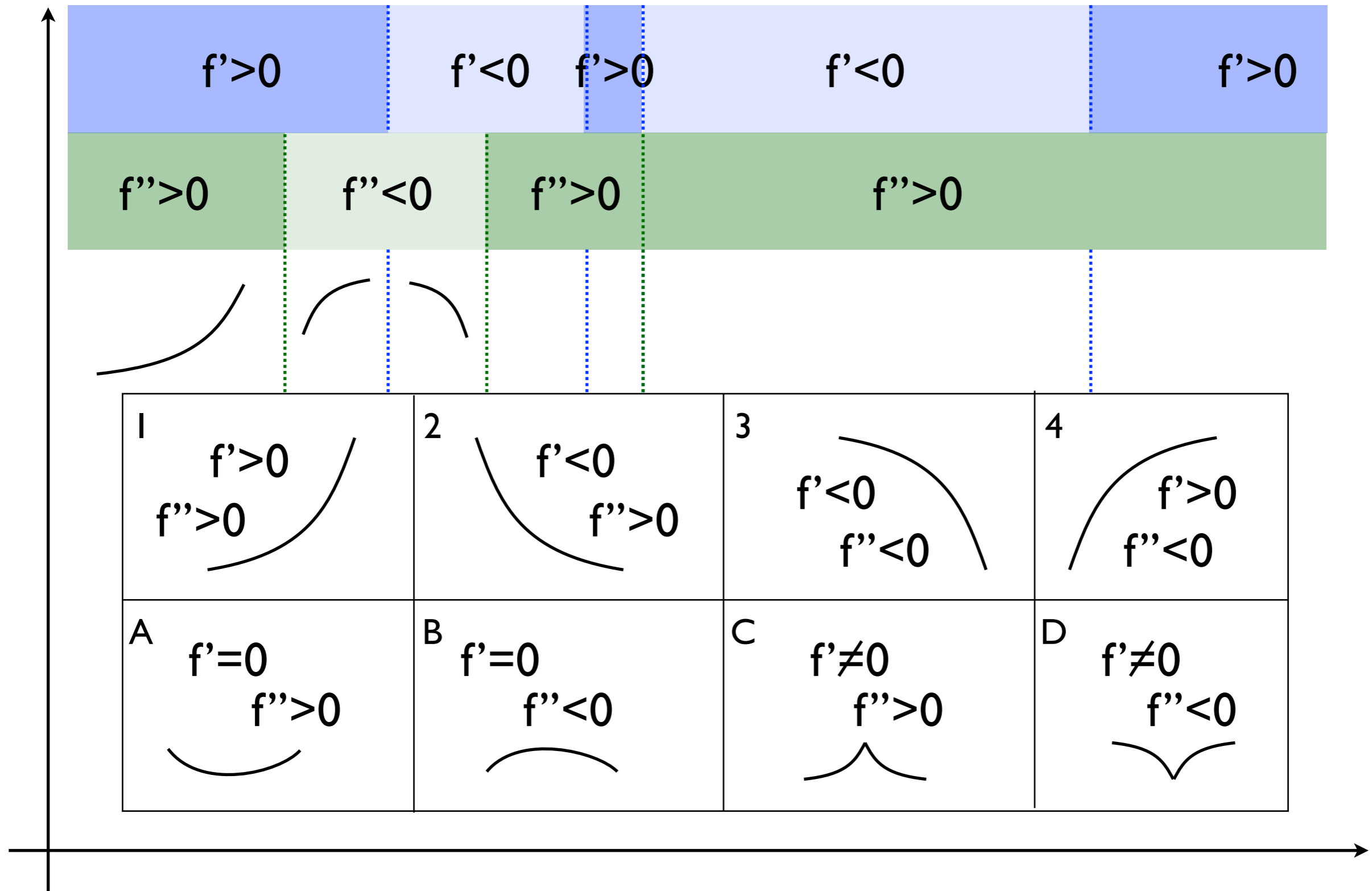
What you have to do to graph it.



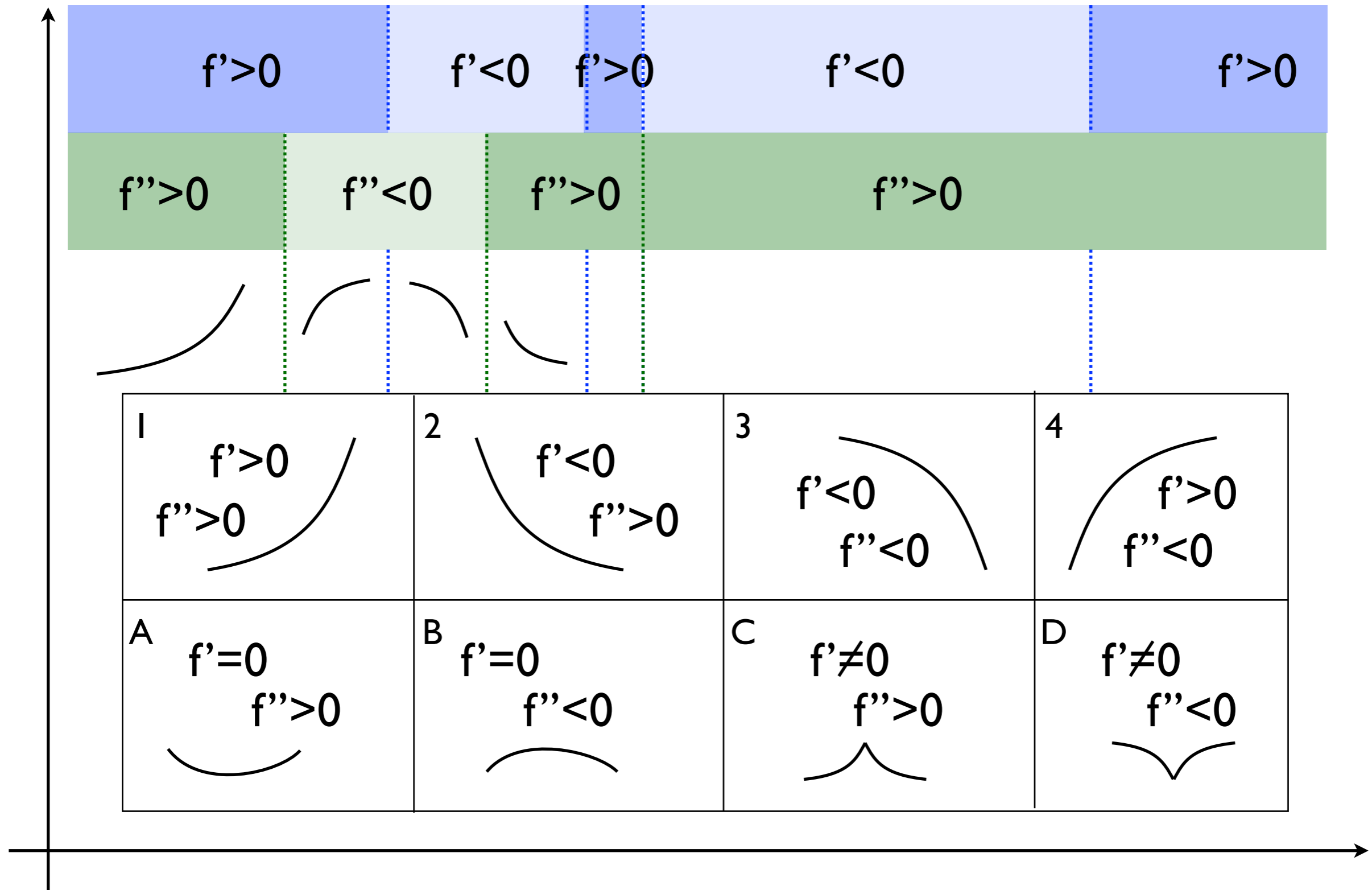
What you have to do to graph it.



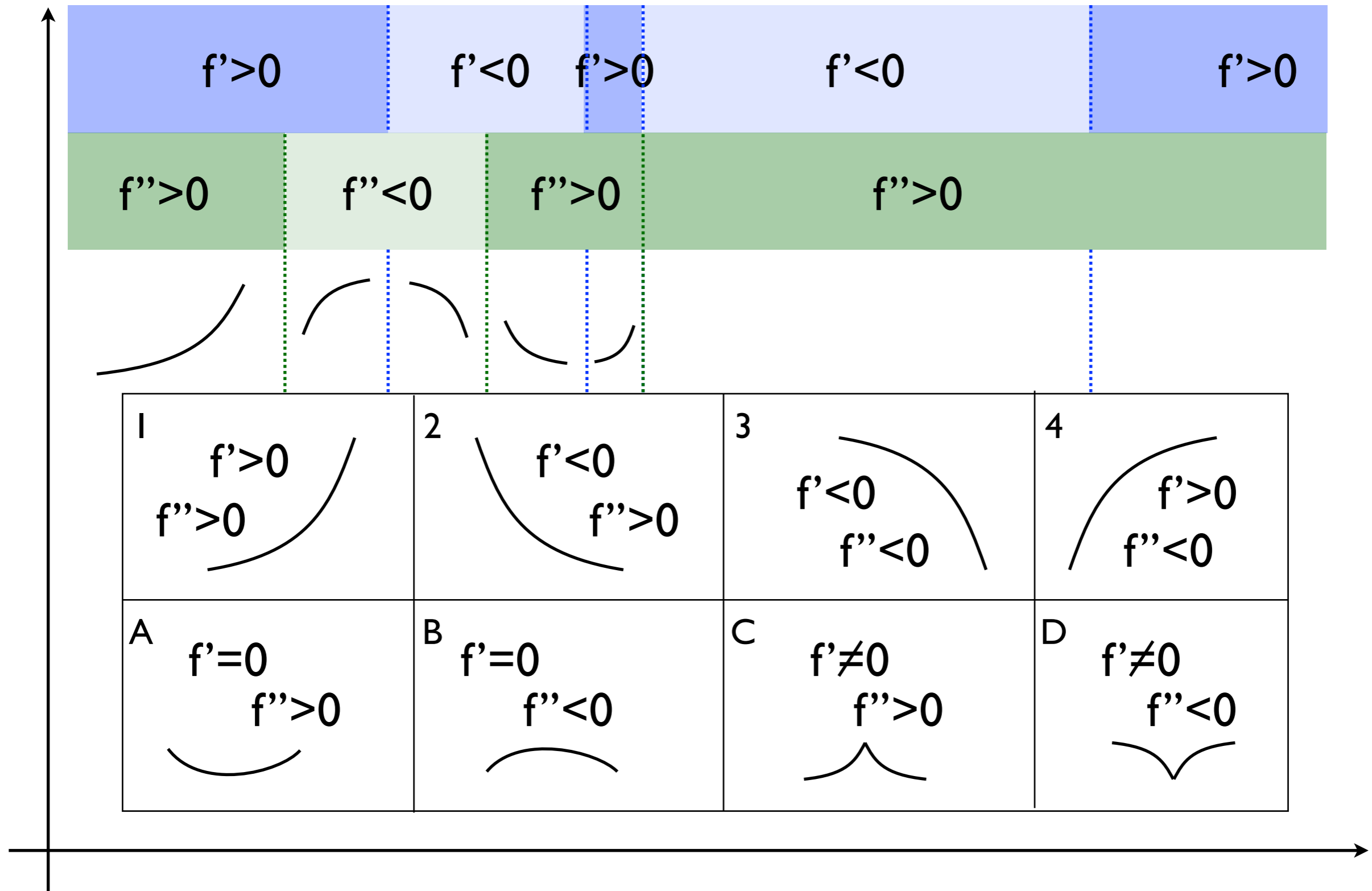
What you have to do to graph it.



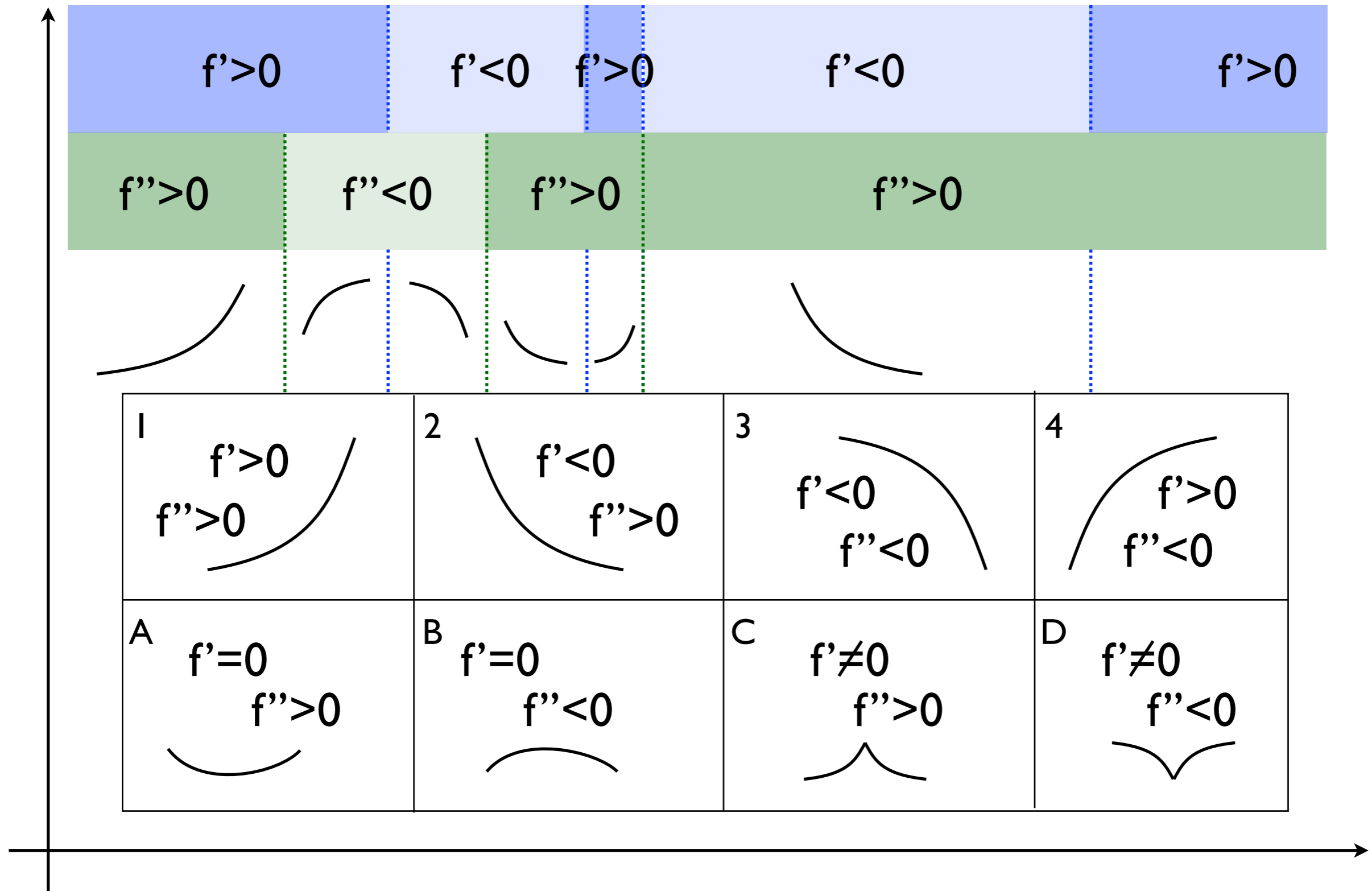
What you have to do to graph it.



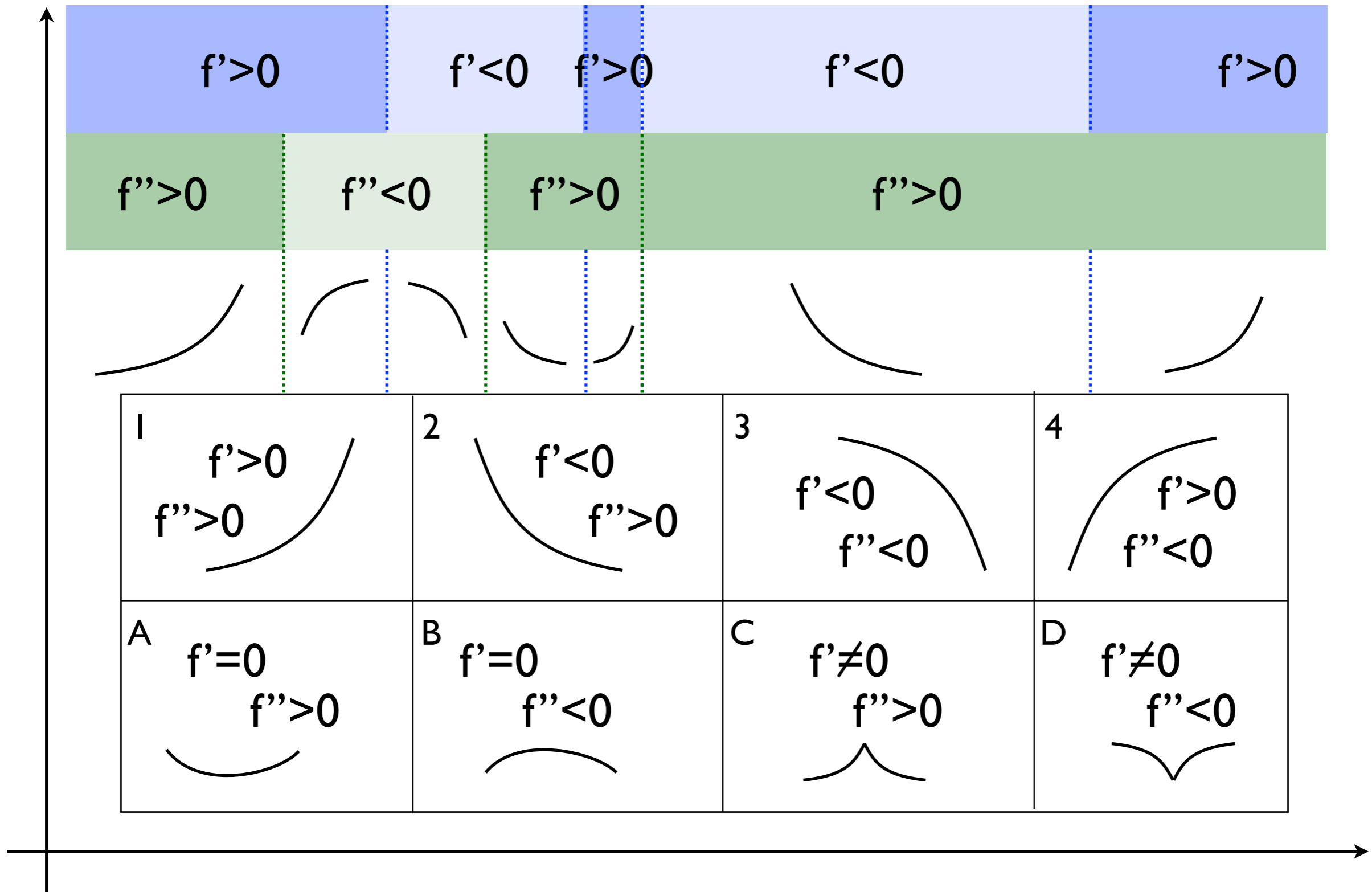
What you have to do to graph it.



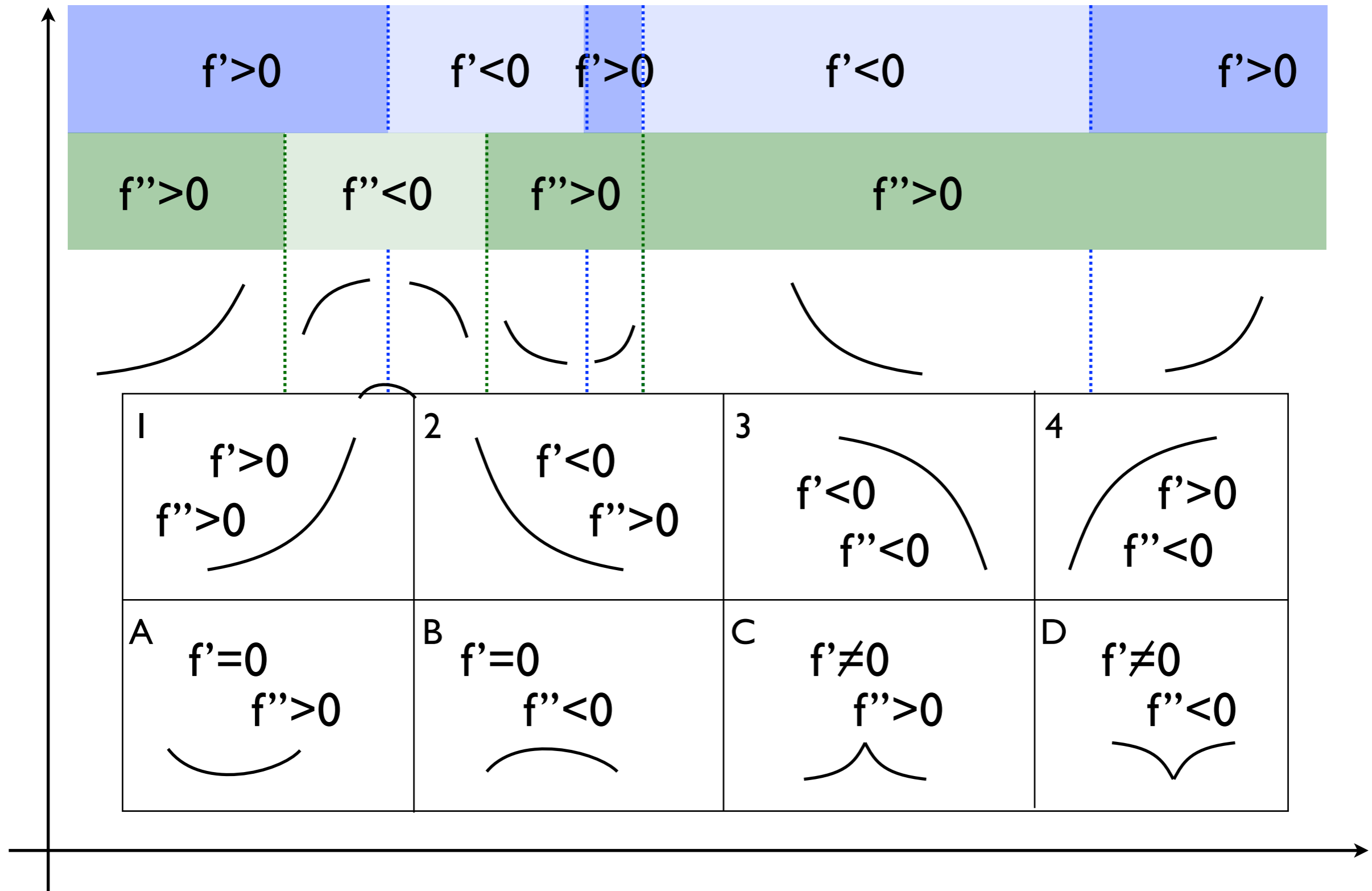
What you have to do to graph it.



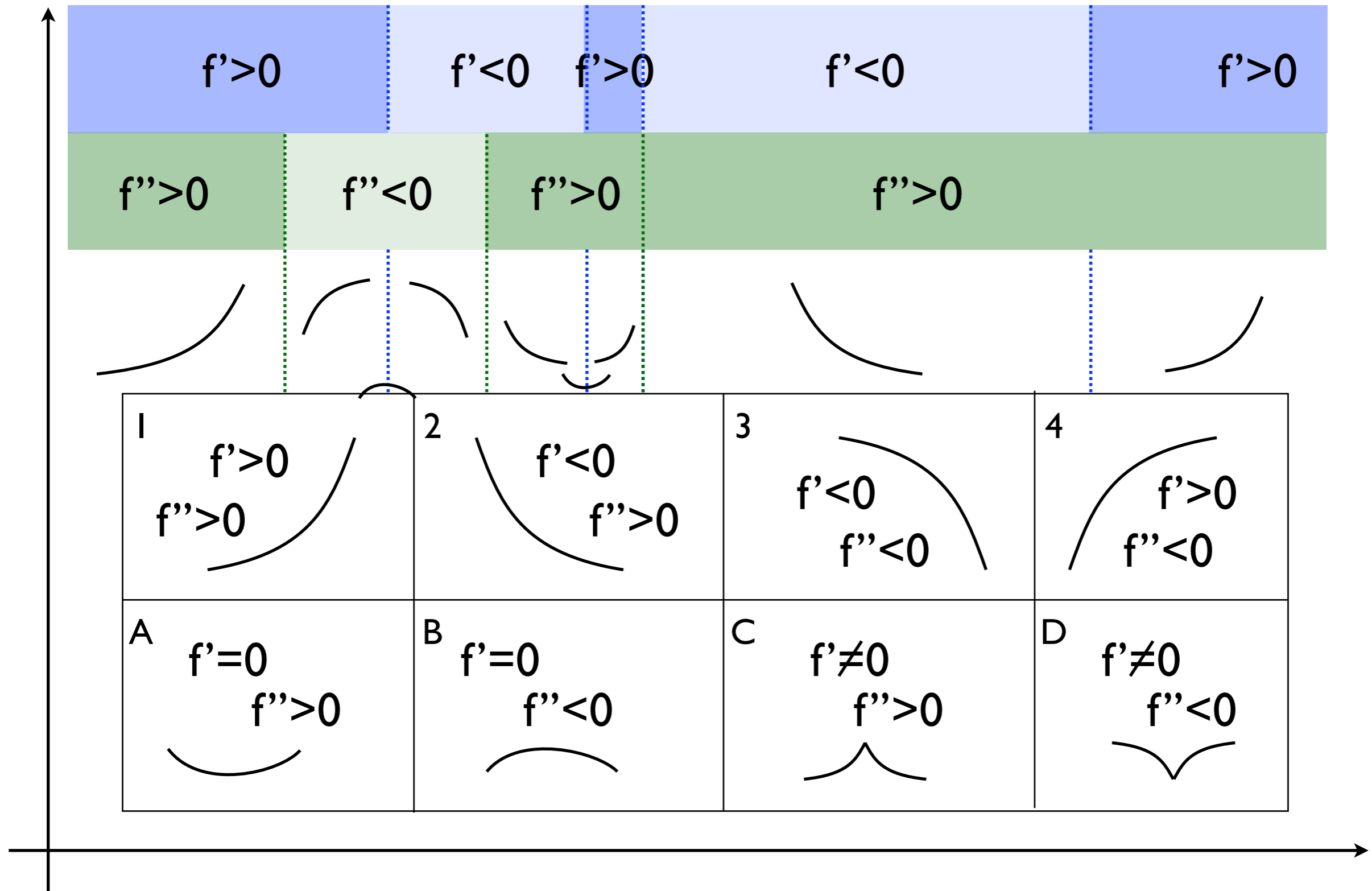
What you have to do to graph it.



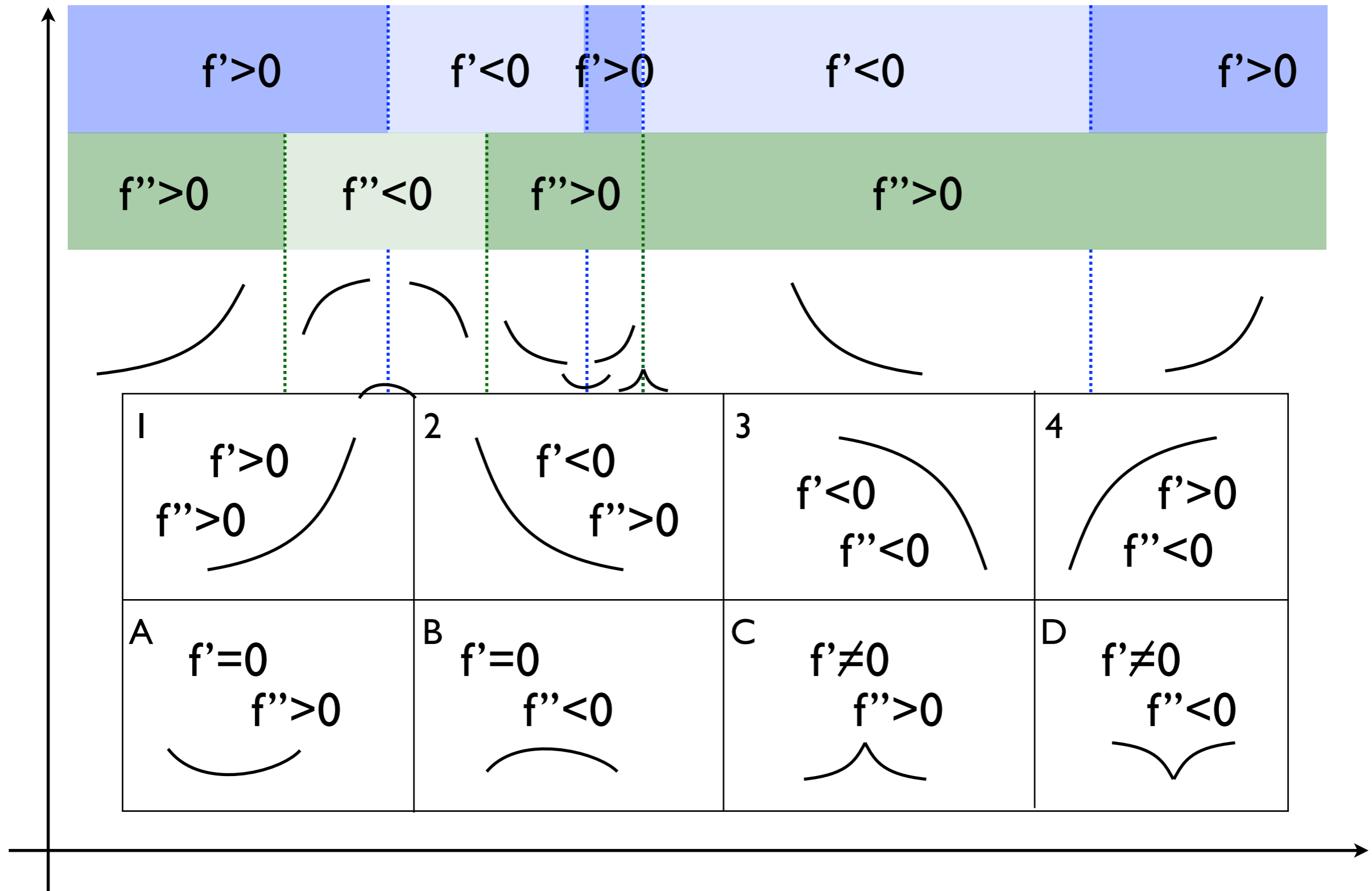
What you have to do to graph it.



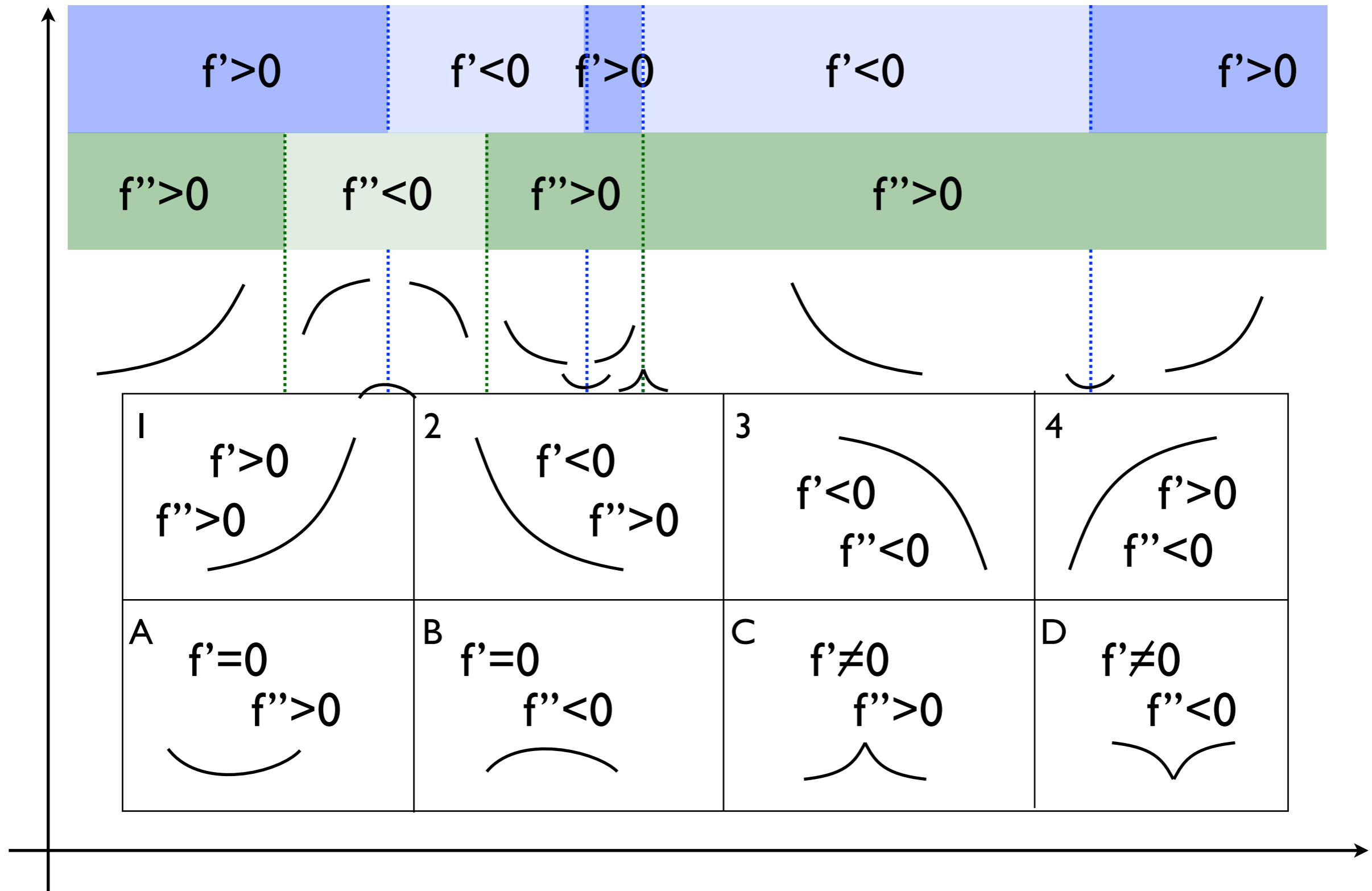
What you have to do to graph it.



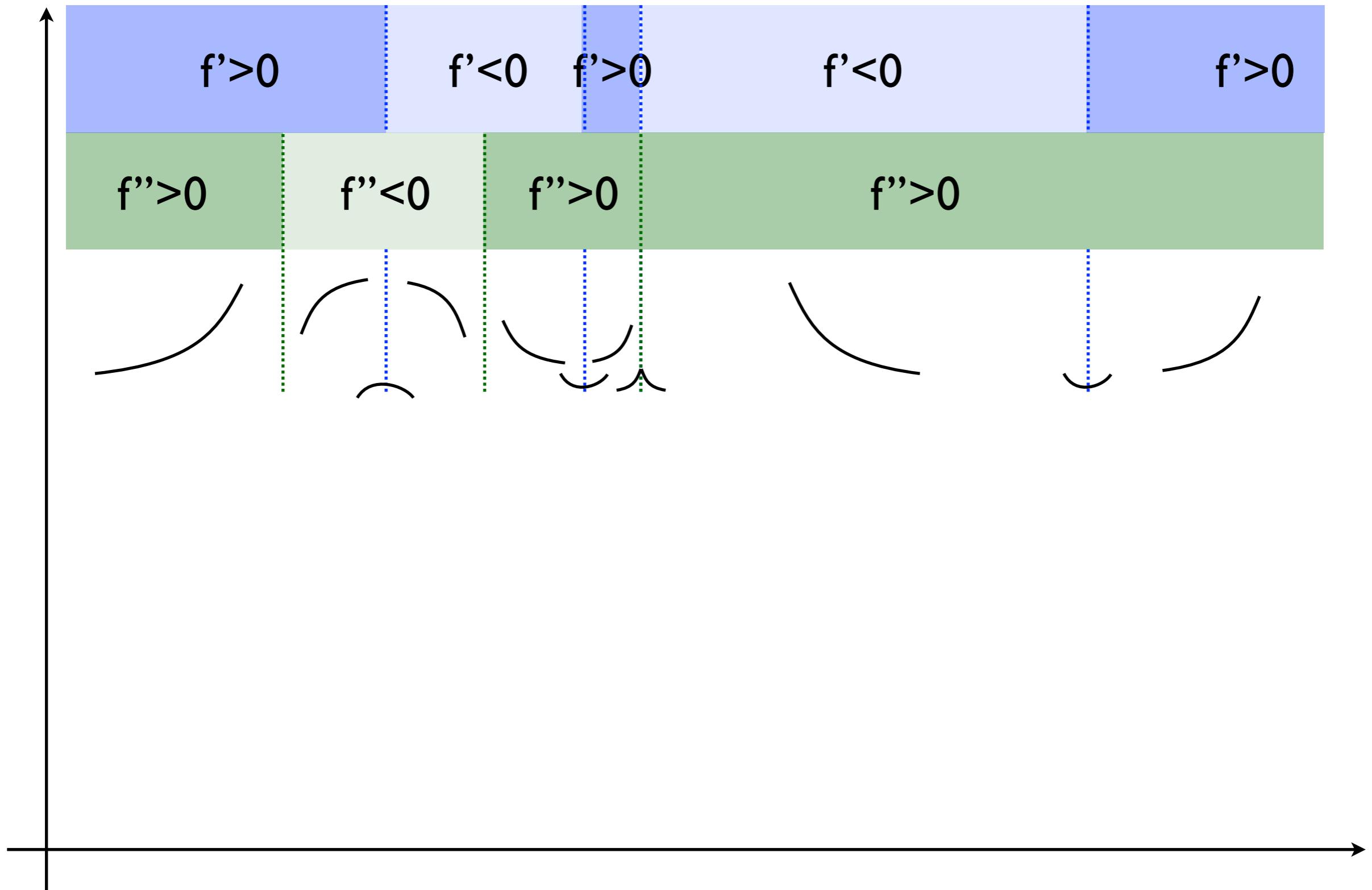
What you have to do to graph it.



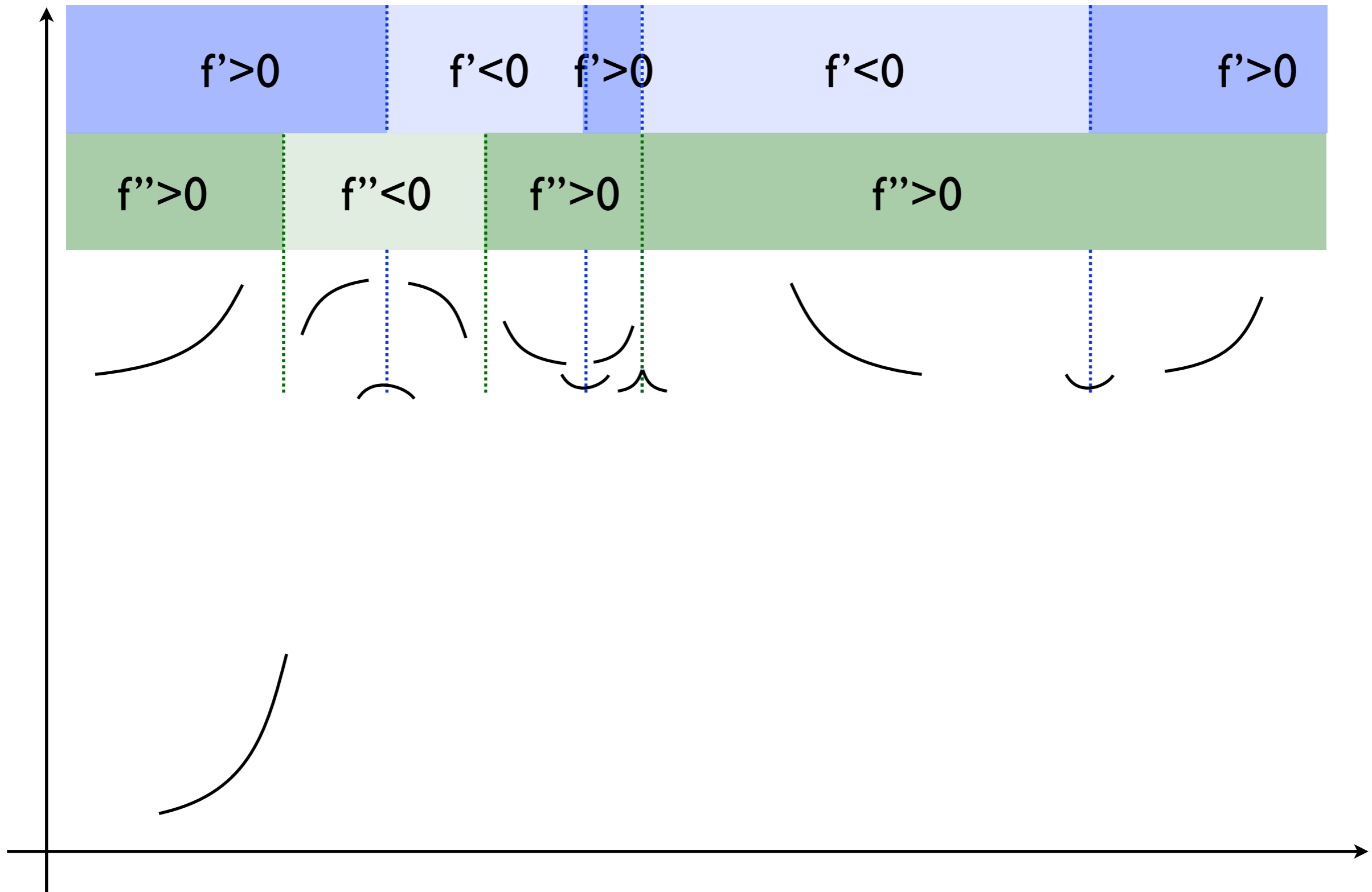
What you have to do to graph it.



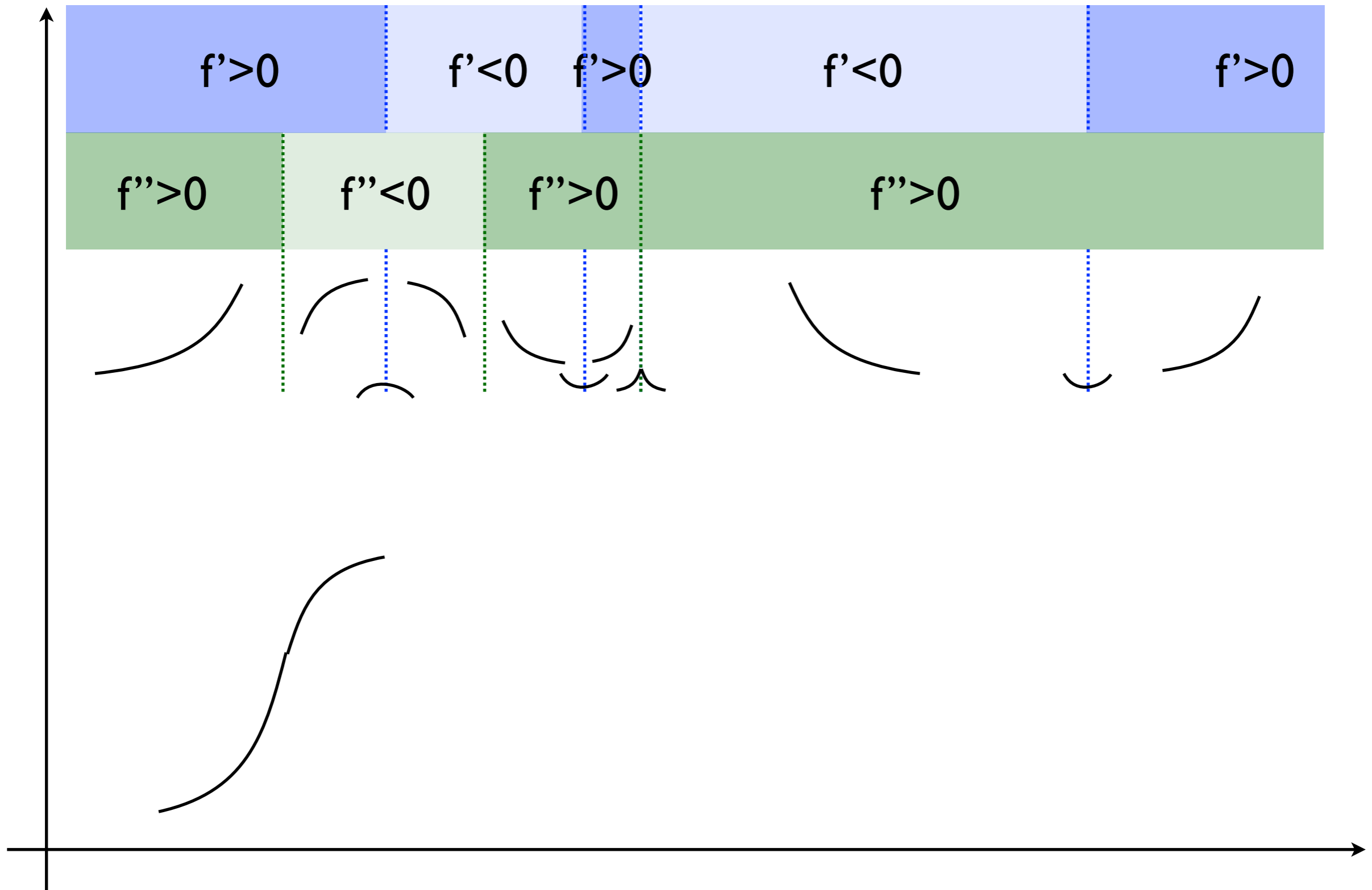
What you have to do to graph it.



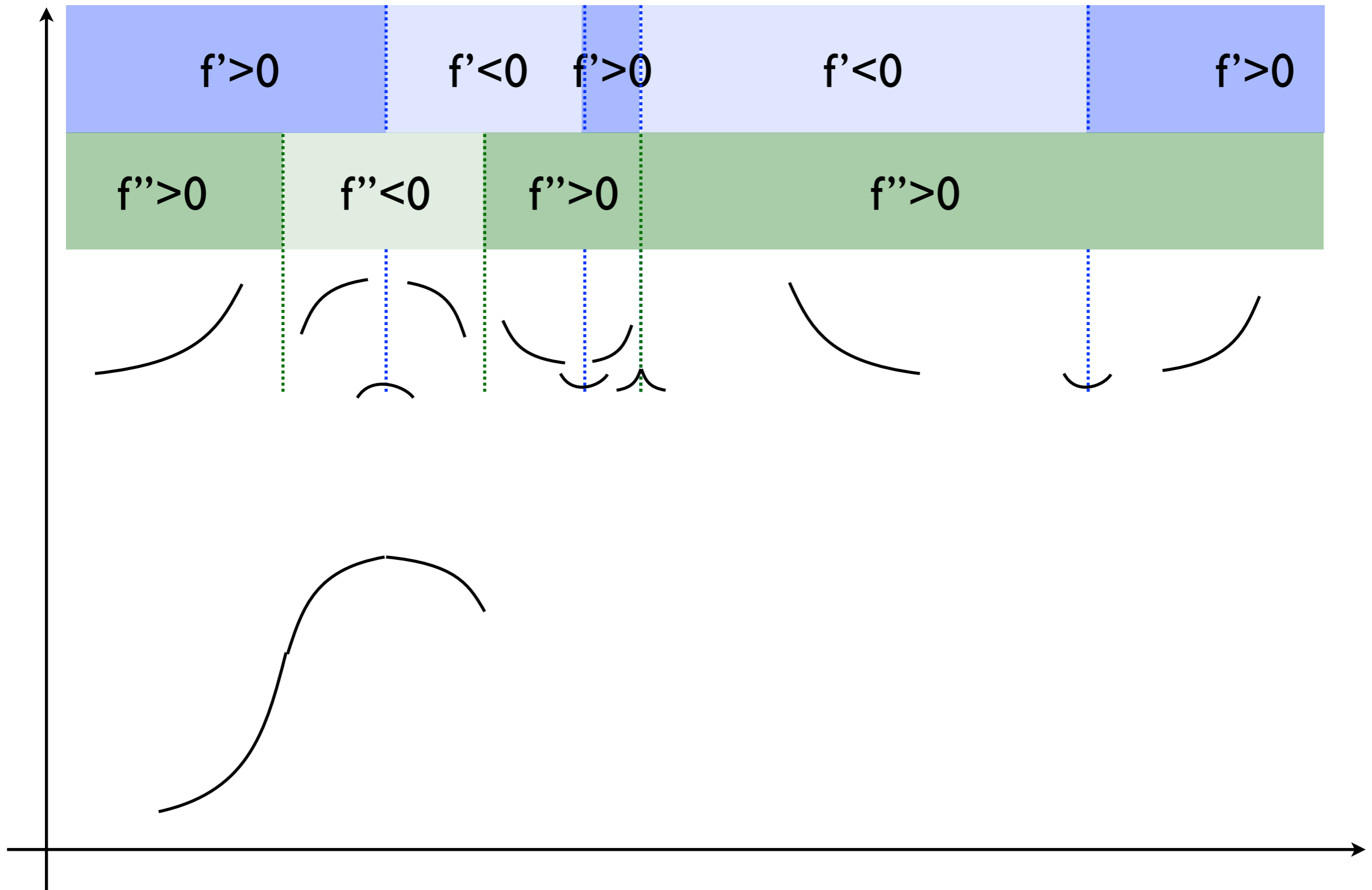
What you have to do to graph it.



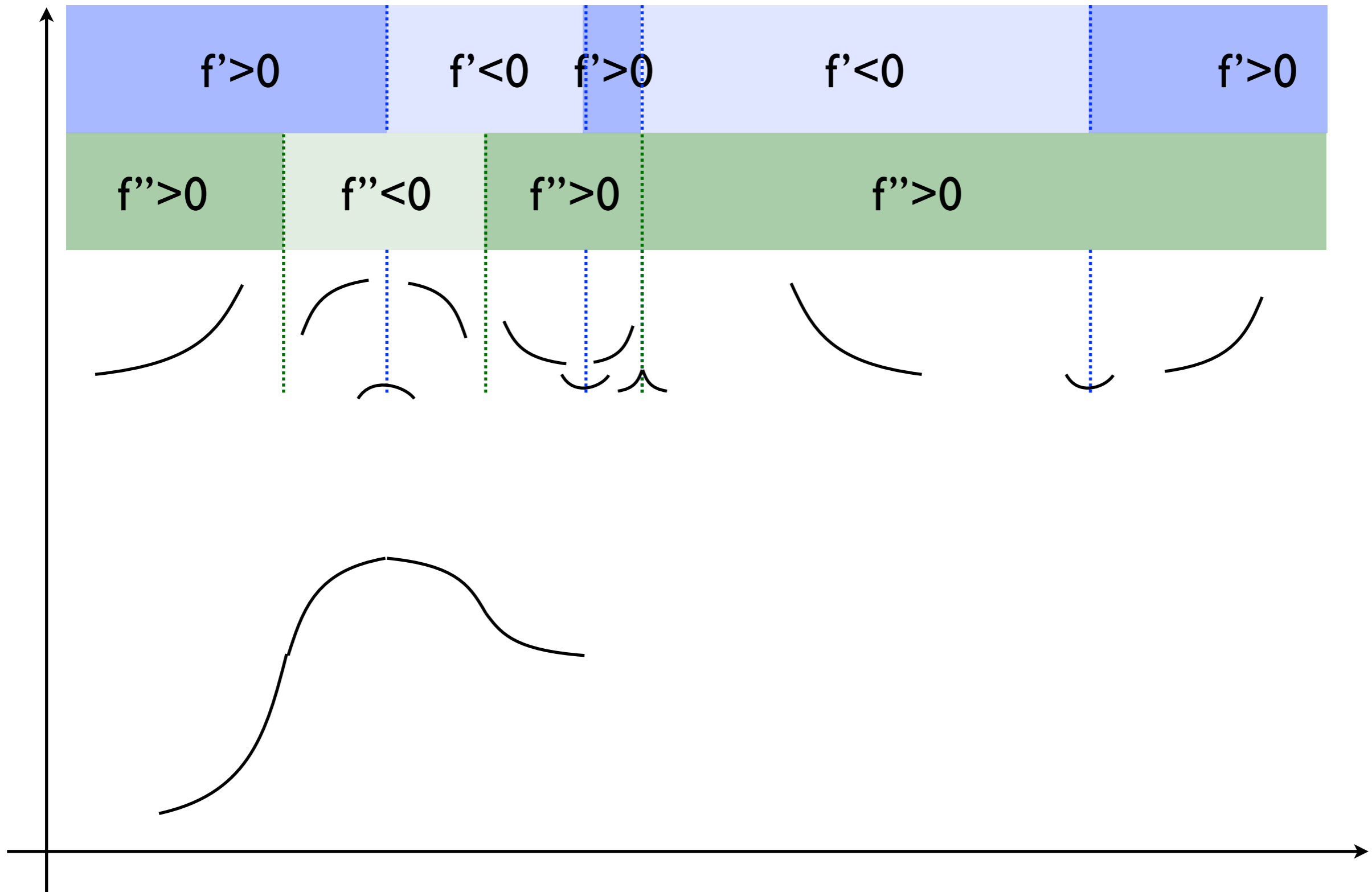
What you have to do to graph it.



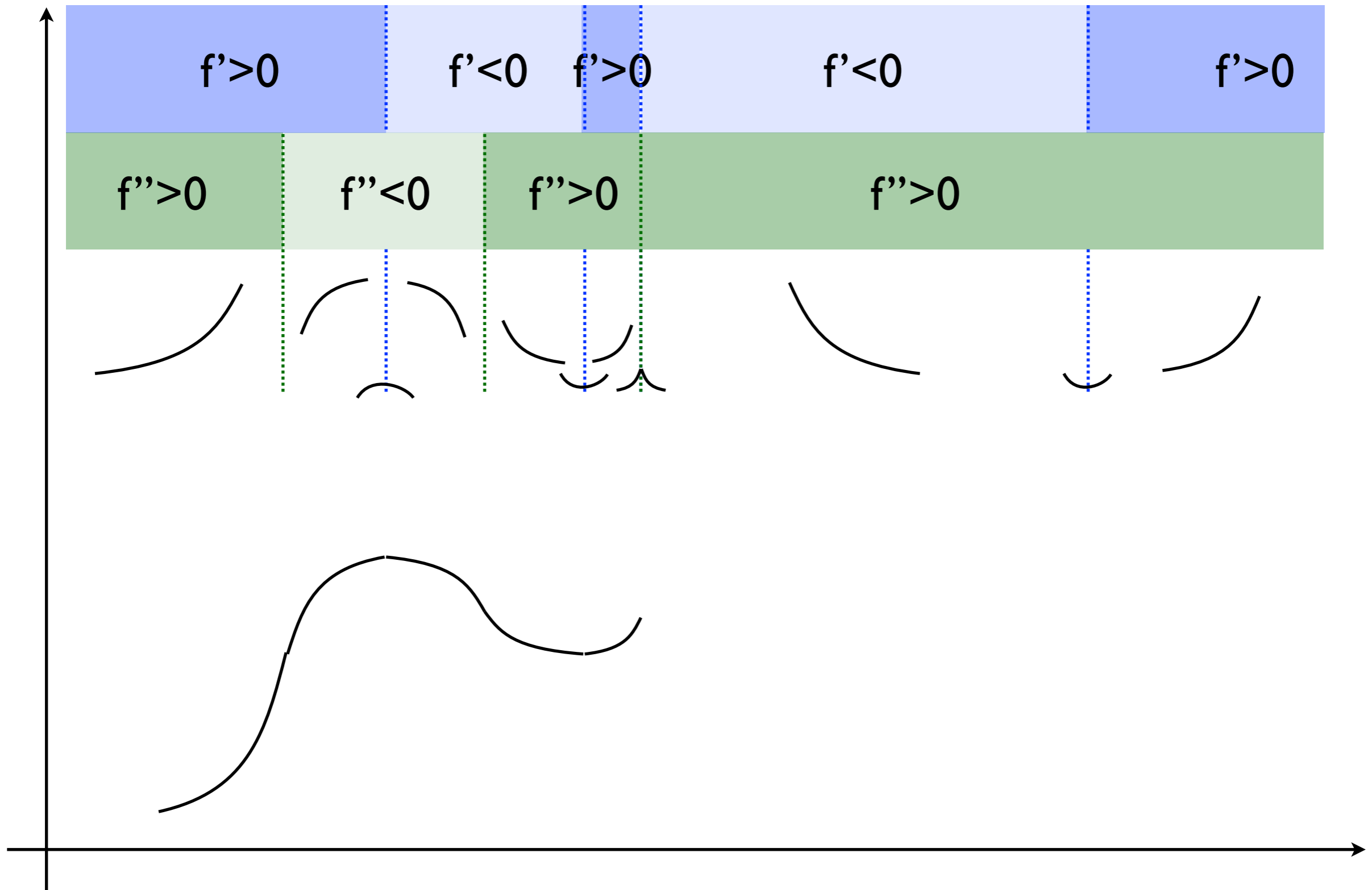
What you have to do to graph it.



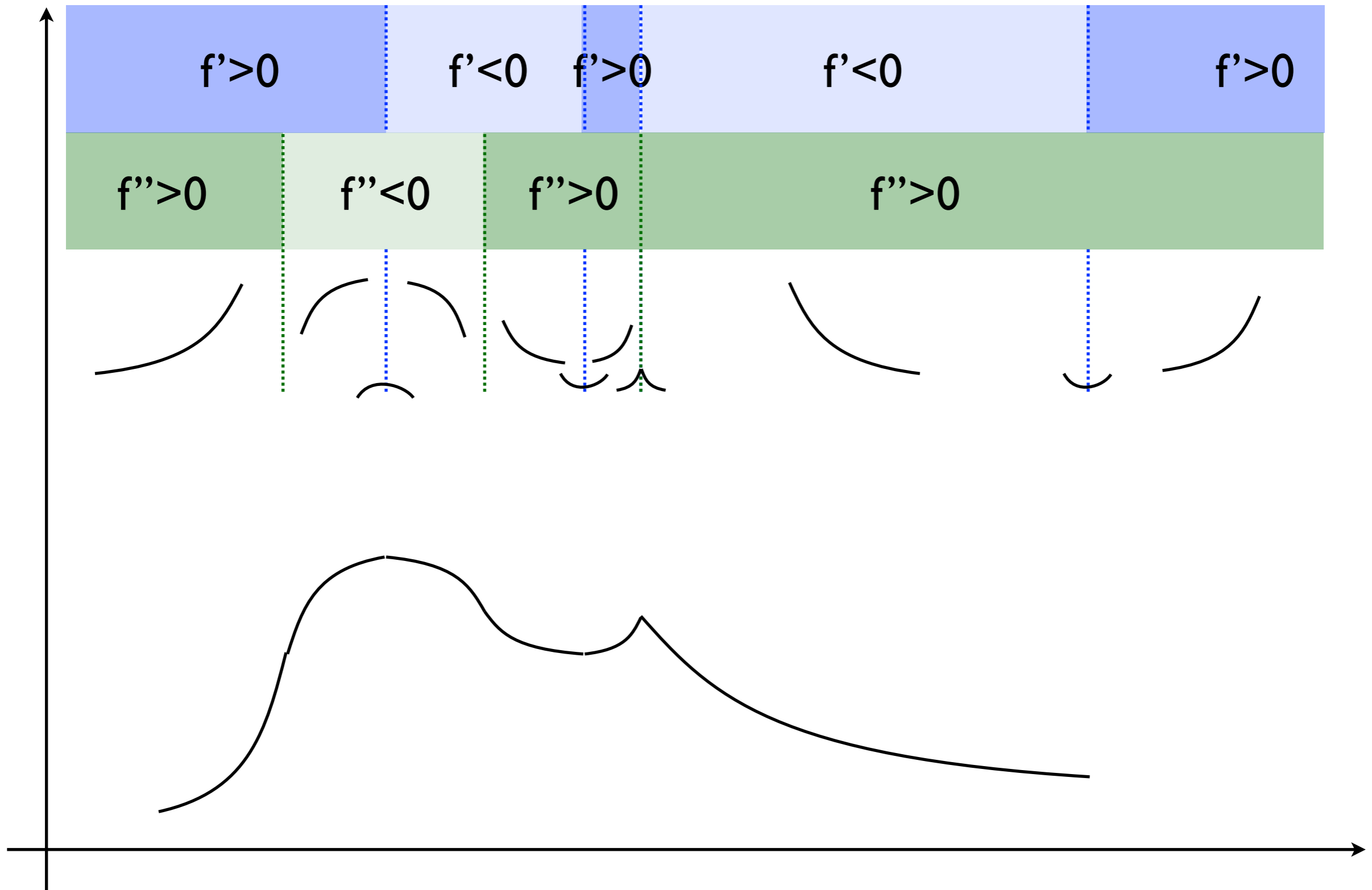
What you have to do to graph it.



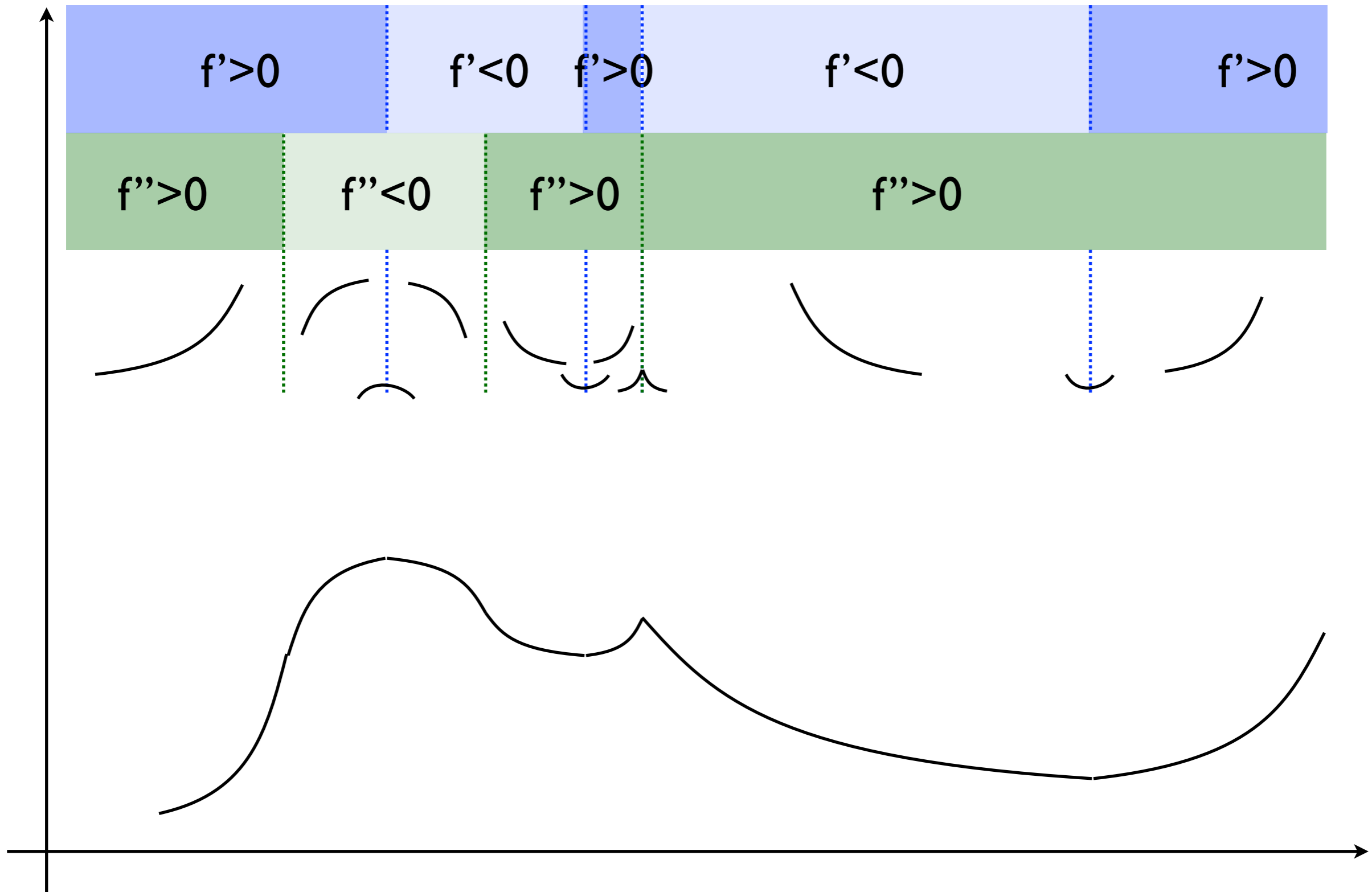
What you have to do to graph it.



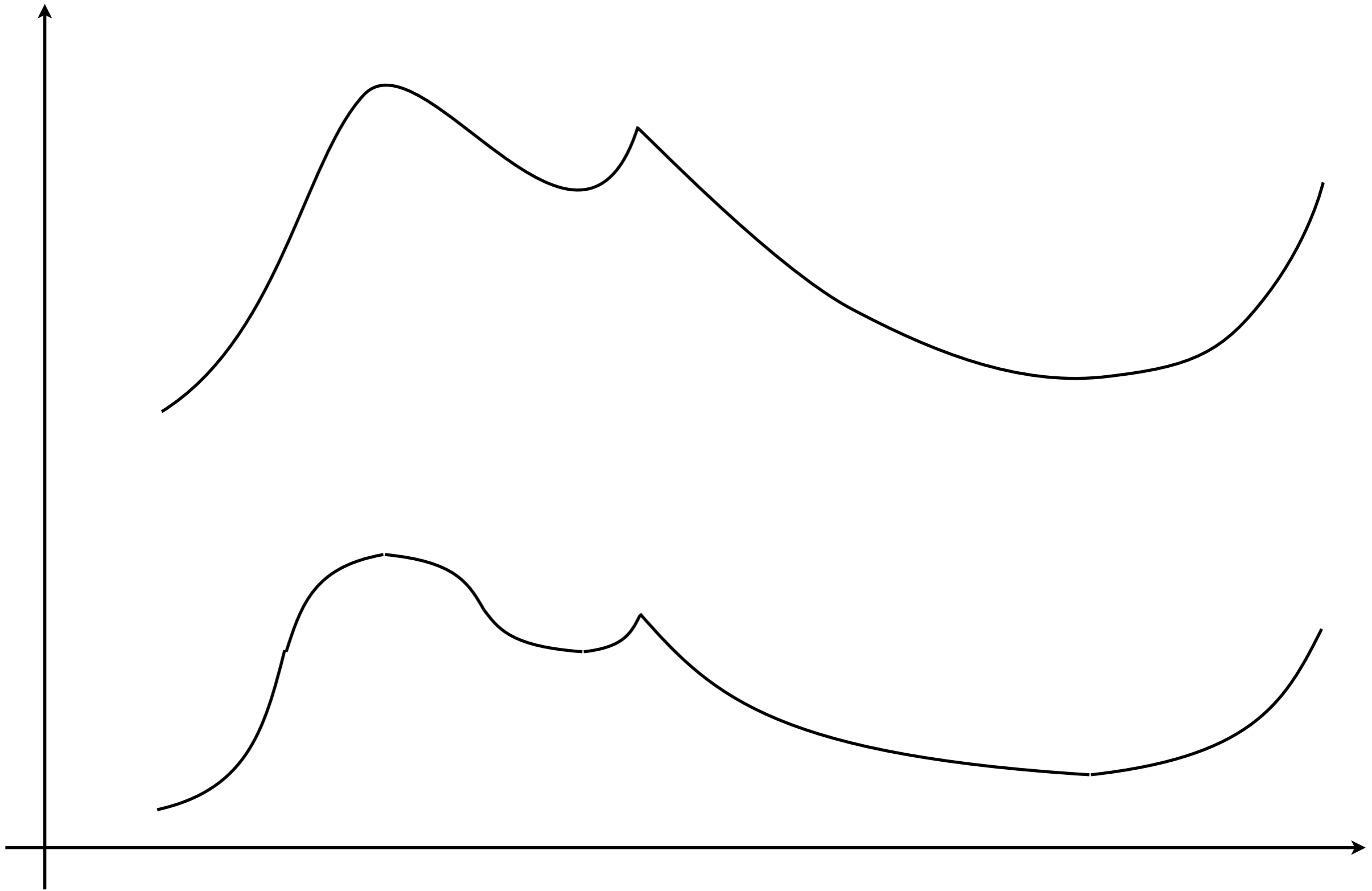
What you have to do to graph it.



What you have to do to graph it.



What you have to do to graph it.



Sketch the graph of

$$f(x) = 3x^4 - 4x^3$$

Sketch the graph of

$$f(x) = 3x^4 - 4x^3$$

x					
f(x)					

Sketch the graph of

$$f(x) = 3x^4 - 4x^3$$

x		0		$4/3$	
$f(x)$		0		0	

Sketch the graph of

$$f(x) = 3x^4 - 4x^3$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$		0		0	

Sketch the graph of

$$f(x) = 3x^4 - 4x^3$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	0	$+$

Sketch the graph of

$$f'(x) = 12(x^3 - x^2)$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	0	$+$

x					
$f'(x)$					

Sketch the graph of

$$f'(x) = 12(x^3 - x^2)$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	0	$+$

x		0		1	
$f'(x)$		0		0	

Sketch the graph of

$$f'(x) = 12(x^3 - x^2)$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	0	$+$

x	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$f'(x)$		0		0	

Sketch the graph of

$$f'(x) = 12(x^3 - x^2)$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	0	$+$

x	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$f'(x)$	$-$	0	$-$	0	$+$

Sketch the graph of

$$f''(x) = 12(3x^2 - 2x)$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	0	$+$

x	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$f'(x)$	$-$	0	$-$	0	$+$

x					
$f''(x)$					

Sketch the graph of

$$f''(x) = 12(3x^2 - 2x)$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	0	$+$

x	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$f'(x)$	$-$	0	$-$	0	$+$

x		0		$2/3$	
$f''(x)$		0		0	

Sketch the graph of

$$f''(x) = 12(3x^2 - 2x)$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	0	$+$

x	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$f'(x)$	$-$	0	$-$	0	$+$

x	$(-\infty, 0)$	0	$(0, 2/3)$	$2/3$	$(2/3, \infty)$
$f''(x)$		0		0	

Sketch the graph of

$$f''(x) = 12(3x^2 - 2x)$$

x	$(-\infty, 0)$	0	$(0, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	0	$+$

x	$(-\infty, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$f'(x)$	$-$	0	$-$	0	$+$

x	$(-\infty, 0)$	0	$(0, 2/3)$	$2/3$	$(2/3, \infty)$
$f''(x)$	$+$	0	$-$	0	$+$

The whole table

x	$(-\infty, 0)$	0	$(0, 2/3)$	$2/3$	$(2/3, 1)$	1	$(1, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	$-$	$-$	$-$	$-$	0	$+$
$f'(x)$	$-$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$f''(x)$	$+$	0	$-$	0	$+$	$+$	$+$	$+$	$+$

$$f(x) = 3x^4 - 4x^3$$

The whole table

x	$(-\infty, 0)$	0	$(0, 2/3)$	$2/3$	$(2/3, 1)$	1	$(1, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	$-$	$-$	$-$	$-$	0	$+$
$f'(x)$	$-$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$f''(x)$	$+$	0	$-$	0	$+$	$+$	$+$	$+$	$+$

Not a min/max

$$f(x) = 3x^4 - 4x^3$$

The whole table

x	$(-\infty, 0)$	0	$(0, 2/3)$	$2/3$	$(2/3, 1)$	1	$(1, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	$-$	$-$	$-$	$-$	0	$+$
$f'(x)$	$-$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$f''(x)$	$+$	0	$-$	0	$+$	$+$	$+$	$+$	$+$

Not a min/max

minimum

$$f(x) = 3x^4 - 4x^3$$

The whole table

x	$(-\infty, 0)$	0	$(0, 2/3)$	$2/3$	$(2/3, 1)$	1	$(1, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	+	0	-	-	-	-	-	0	+
$f'(x)$	-	0	-	-	-	0	+	+	+
$f''(x)$	+	0	-	0	+	+	+	+	+

inflection point

$$f''(x) = 3x^4 - 4x^3$$

The whole table

x	$(-\infty, 0)$	0	$(0, 2/3)$	$2/3$	$(2/3, 1)$	1	$(1, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	$-$	$-$	$-$	$-$	0	$+$
$f'(x)$	$-$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$f''(x)$	$+$	0	$-$	0	$+$	$+$	$+$	$+$	$+$

$f''(x) = -4x^3$
 inflection point

The whole table

x	$(-\infty, 0)$	0	$(0, 2/3)$	$2/3$	$(2/3, 1)$	1	$(1, 4/3)$	$4/3$	$(4/3, \infty)$
$f(x)$	$+$	0	$-$	$-$	$-$	$-$	$-$	0	$+$
$f'(x)$	$-$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$f''(x)$	$+$	0	$-$	0	$+$	$+$	$+$	$+$	$+$

$$f(x) = 3x^4 - 4x^3$$