Today

- Related rate example (water in cone)
- Implicit differentiation
  - Tangent line example
  - Power rule for fractional powers (next week)

Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.

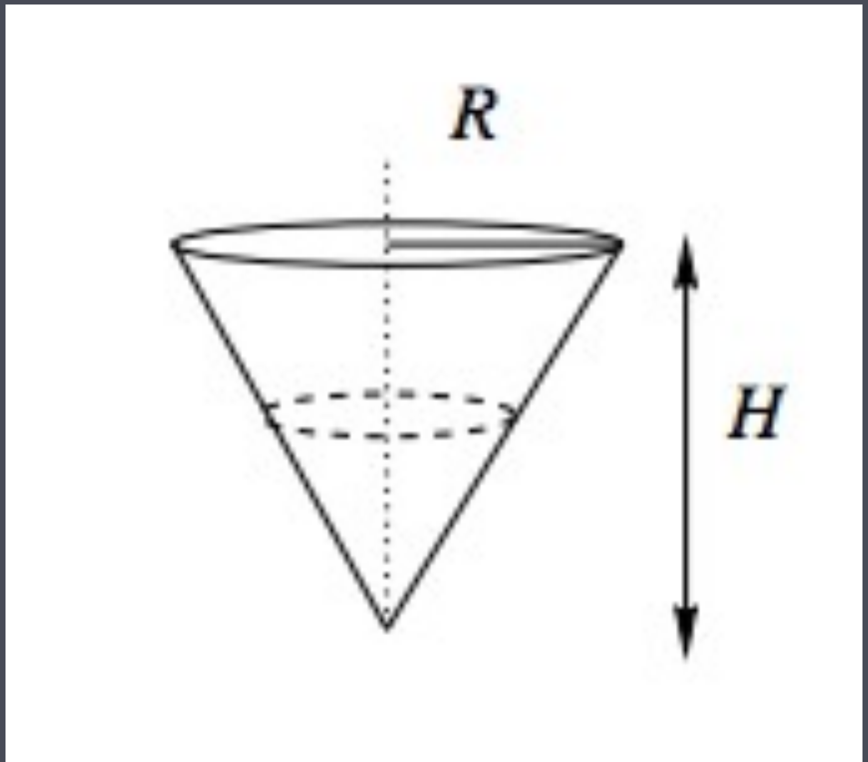
Which of the following matches your intuition for how these rates are related?

(A) h(t) decreases quickly at first and then slows down.

(B) h(t) increases quickly at first and then slows down.

(C) h(t) decreases slowly at first and then speeds up.

(D) h(t) increases slowly at first and then speeds up.



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Which is the relevant equation relating the quantities when the water is at height h (not rates of change yet)?

(A) $V = 1/3\ \pi\ R^2 H$

(B) $V = 1/3\ \pi\ (R^2/H^2)\ h$

(C) $V = 1/3\ \pi\ (R^2/H^2)\ h^3$

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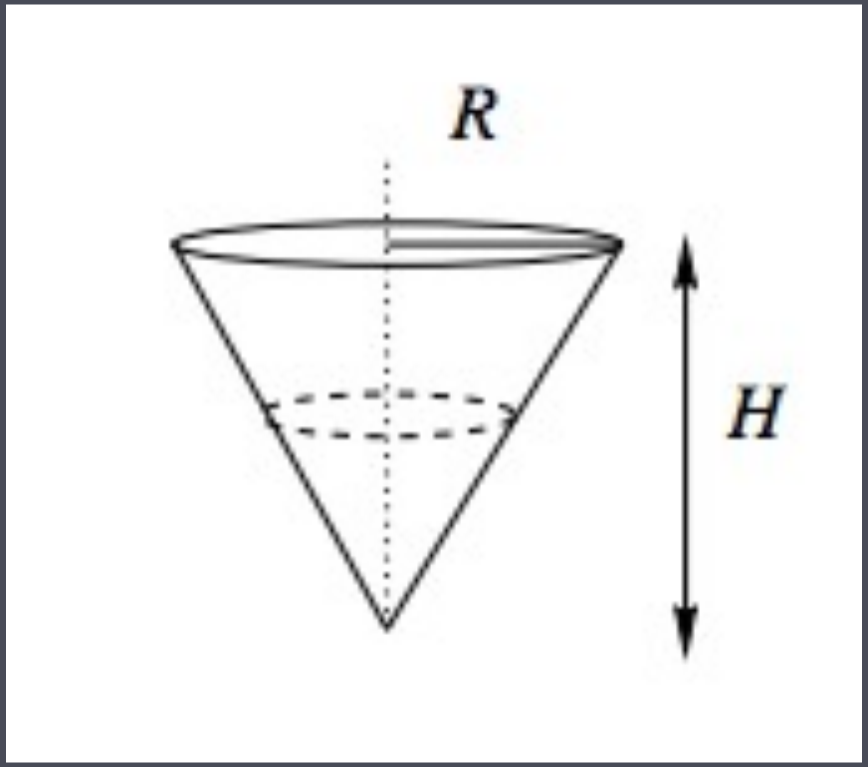

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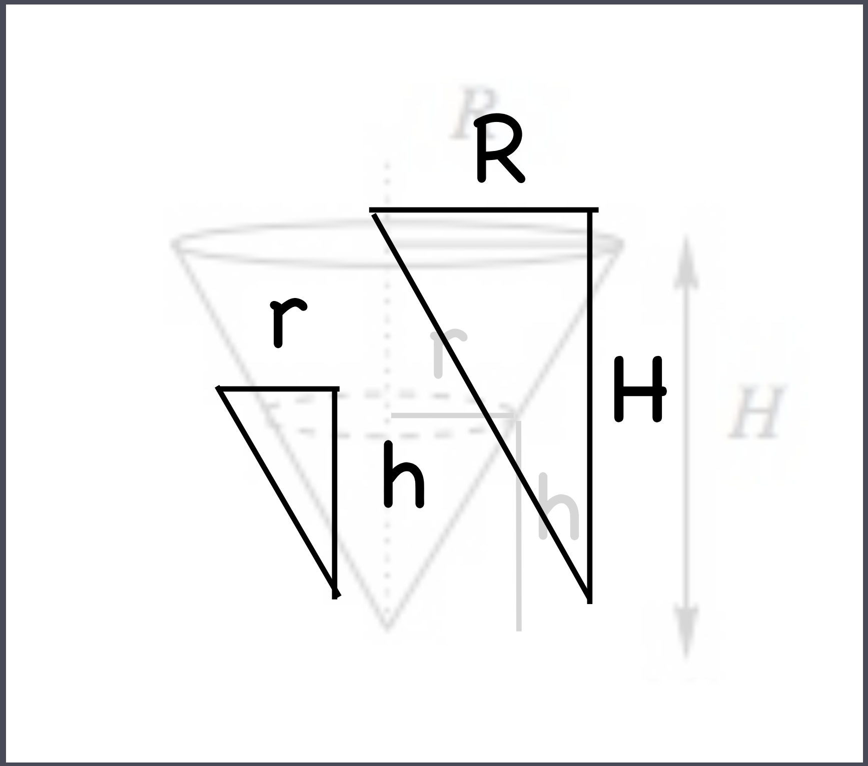

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Relate V to h. R and H are const. By similar ▲s, r=hR/H.

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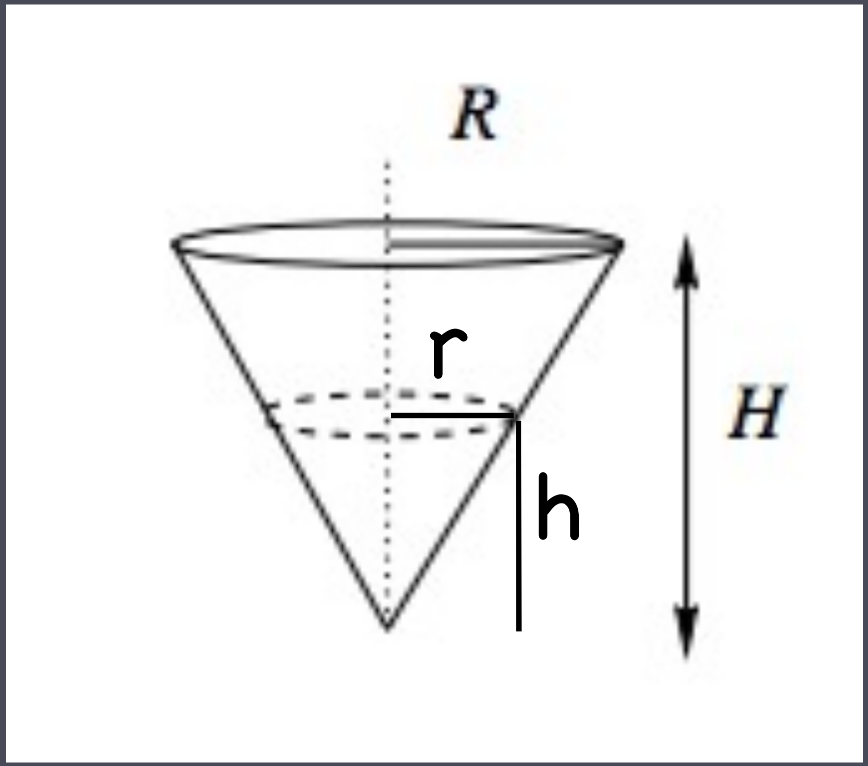

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Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.

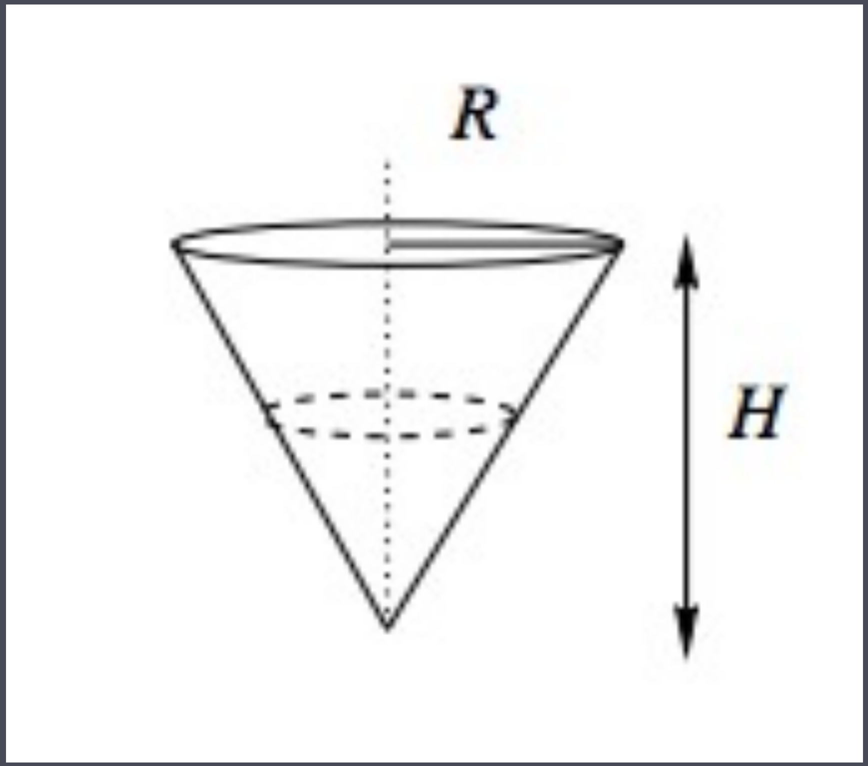
Which is the relevant equation relating the rates of change?

(A) $-k = 1/3\ \pi\ (R^2/H^2)\ h'$

(B) $V' = \pi\ (R^2/H^2)\ h^2\ k$

(C) $-k = \pi\ (R^2/H^2)\ h^2\ h'$

(D) $V' = 1/3\ \pi\ (2rr'\ h + r^2\ h')$



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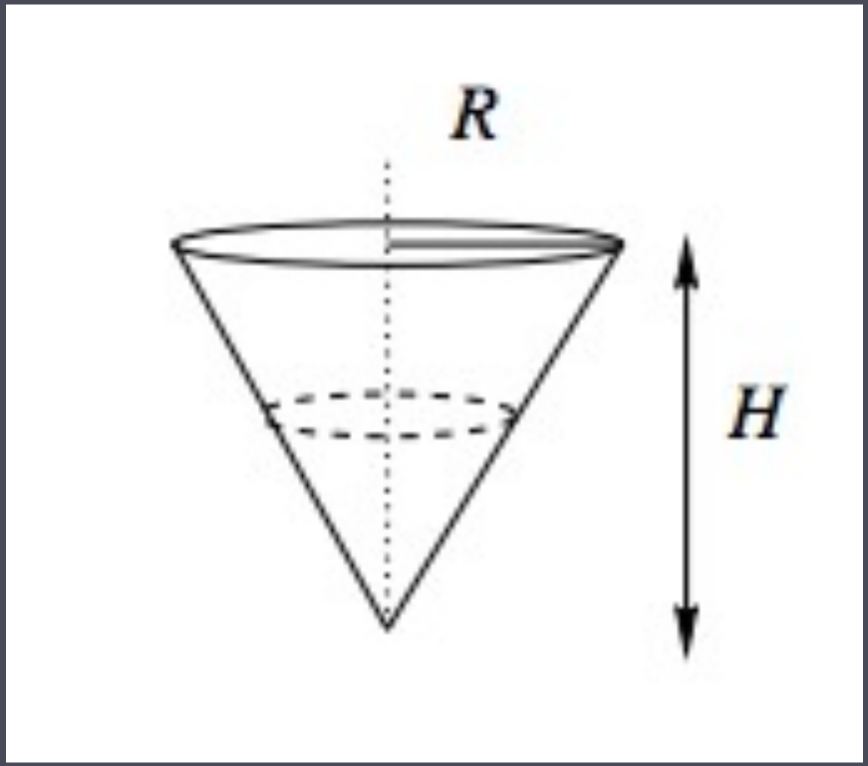

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$V(t) = 1/3\ \pi\ (R^2/H^2)\ h(t)^3$

$V'(t) = \pi\ (R^2/H^2)\ h(t)^2\ h'(t)$

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$V(h) = 1/3\ \pi\ (R^2/H^2)\ h^3$

$v(t) = V(h(t))$

$$\frac{dv}{dt} = \frac{dV}{dh}\frac{dh}{dt}$$

$$= \pi\ (R^2/H^2)\ h(t)^2\ \frac{dh}{dt}$$

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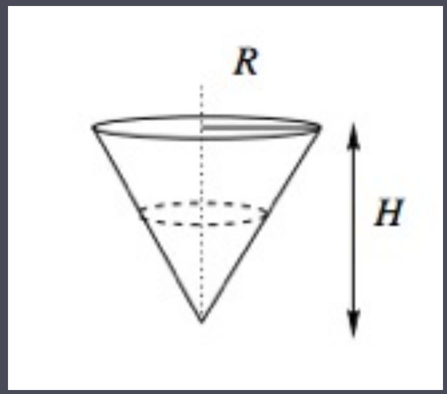
$$h'(t) = -k \,/\, (\pi\,(R^2/H^2)\,h^2)$$



Let's check this answer against our intuition:

(A) h(t) decreases quickly at first and then slows down.

(B) h(t) increases quickly at first and then slows down.

(C) h(t) decreases slowly at first and then speeds up.

(D) h(t) increases slowly at first and then speeds up.

(A) My intuition was right.

(B) My intuition was wrong.

(C) I don't know how to tell.

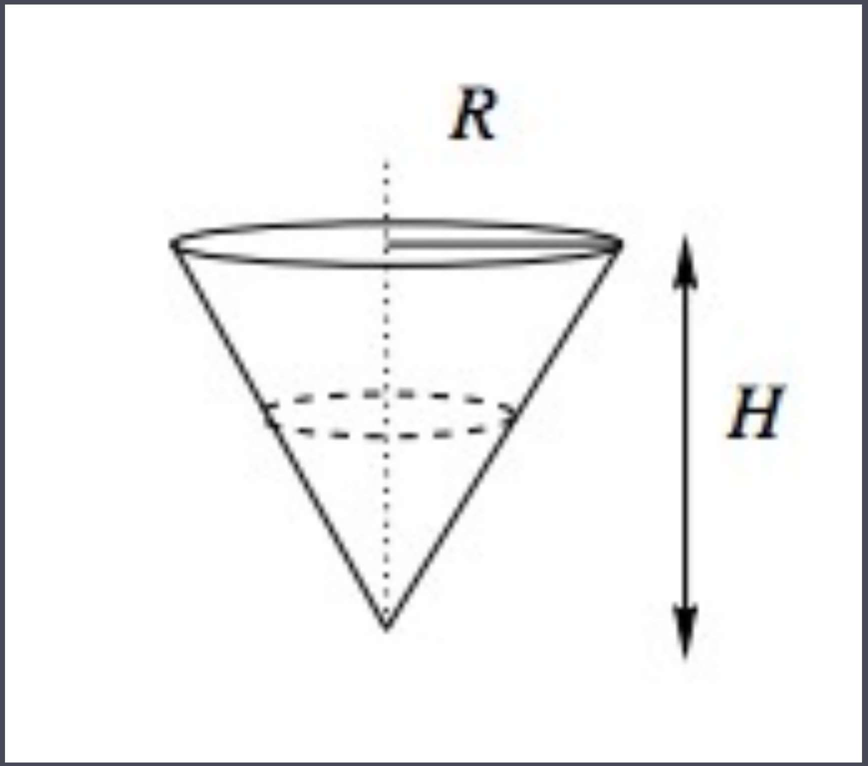
Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.

(A) $-k = \pi (R^2/H^2) h^2 h'$

(B) $V' = \pi (R^2/H^2) h^2 k$

(C) $h' = -k H^2/(\pi R^2 h^2)$

(D) $h' = -k/(\pi R^2)$



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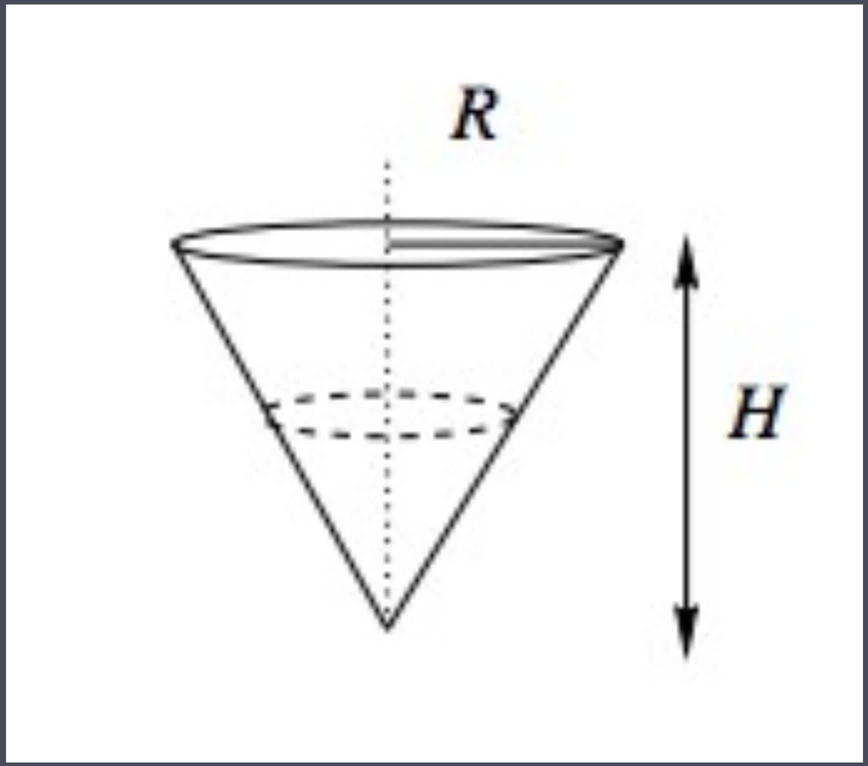


Plug h=H into $h' = -k / (\pi (R^2/H^2) h^2)$

Procedure

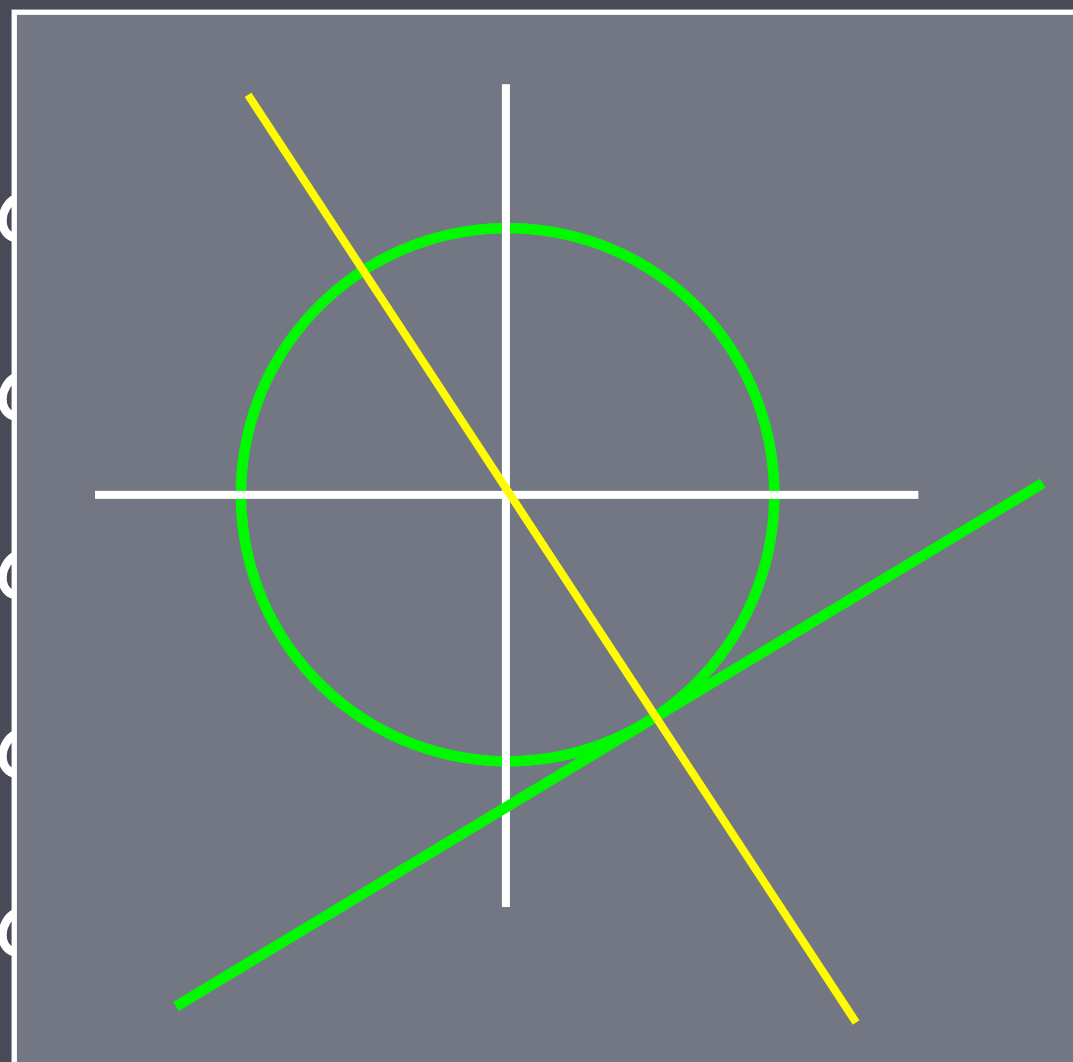
- Establish expectation(s) based on sketch or otherwise.
- Find equation relating Q_1 and Q_2.
- Take derivatives on both sides.
- Finally, plug in specific values.
- Reality check - compare answer against expectation.

Implicit differentiation

- Sometimes you don't want to or can't isolate a function whose derivative is required.
- e.g. Find tangent line to $x^2+y^2=25$ at (x_0,y_0).
- e.g. What is the highest point on the ellipse $x^2 + 3y^2 - xy = 1$?
- Let $y=y(x)$ and take "implicit derivative" of e.g. $x^2+y(x)^2=25$ ------>

Find the tangent line to the curve defined by $x^2+y^2=25$ at (3,-4).

What can you predict about the answer without calculus?



(A) The sl vill be positive.

(B) The sl vill be negative.

(C) The sl vill be 4/3.

(D) The sl vill be 3/4.

(E) The sl vill be -3/4.

Find the tangent line to the curve defined by $x^2+y^2=25$ at $(3,-4)$.

The derivative of each side of this equation must also be equal. That means...

(A) $2xx' + 2yy' = 25.$

(B) $2xx' + 2y = 25.$

(C) $2xx' + 2yy' = 0.$

(D) $2x + 2yy' = 0.$

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(D) $2x + 2yy' = 0.$

Assume that $y=y(x)$. If by ′ we mean d/dx, then (C) is technically ok but $dx/dx=1$.

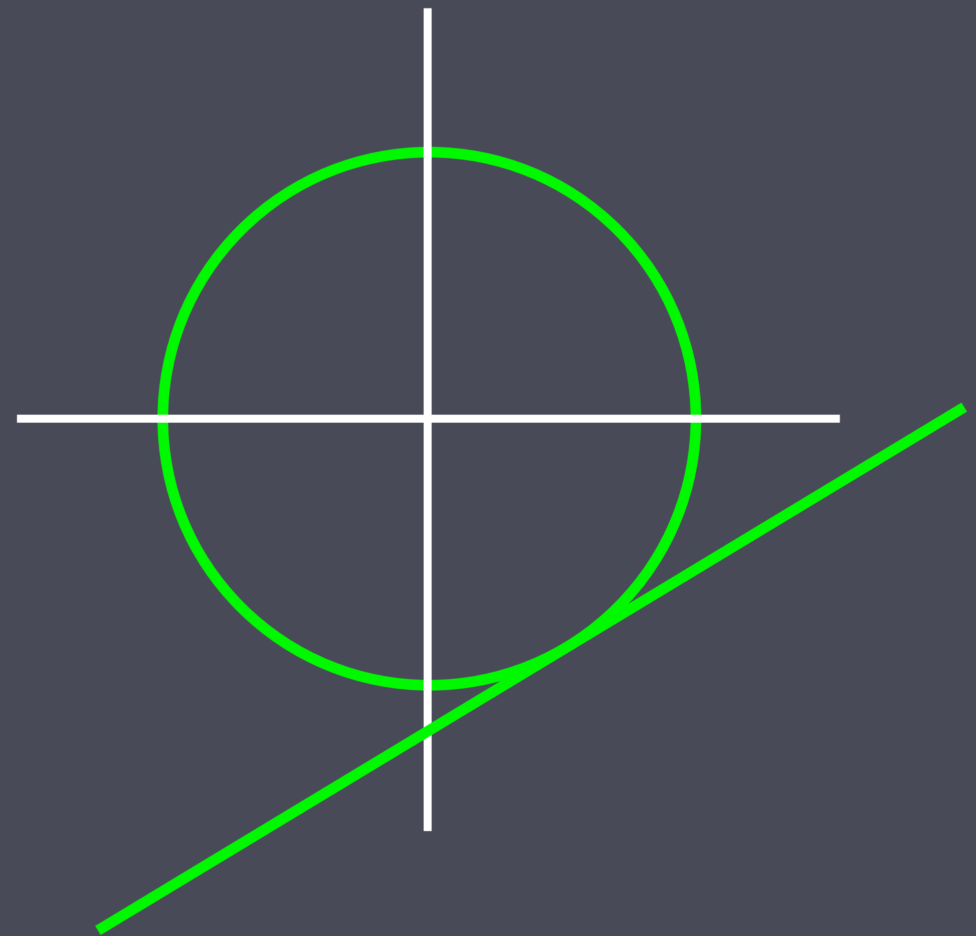
Find the tangent line to the curve defined by $x^2+y^2=25$ at (3,-4).

(A) $y' = -2x / 2y$

(B) $y = 3/4\ (x-3) - 4$

(C) $2x + 2yy' = 0$

(D) $y' = 3/4$



Find the tangent line to the curve defined by $x^2+y^2=25$ at (3,-4).

(A) $y' = -2x / 2y$ <--- correct formula for slope at the point (x,y).

(B) $y = 3/4 (x-3) - 4$

(C) $2x + 2yy' = 0$ <--- correct ans from previous slide.

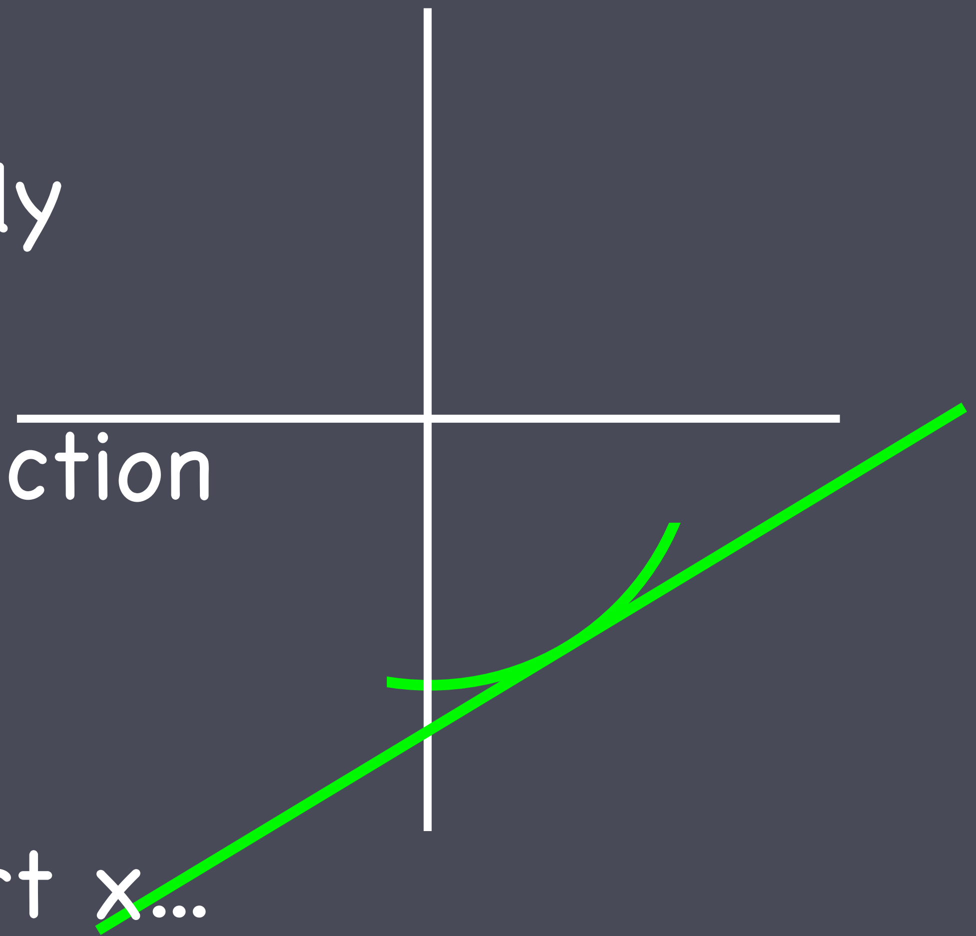
(D) $y' = 3/4$ <--- correct slope at (3,-4)

Find the tangent line to the curve defined by $x^2+y^2=25$ at (3,-4).

Comments on this problem:

- This is not even a function!
- Whose derivative are we actually taking?
- Near the point (3,-4), y is a function of x.

$$x^2+y(x)^2=25$$



- Take derivative of both sides wrt x...
- This doesn't work at (1,0)! (how might you deal with this?)