Today

Qualitative analysis examples.
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What are the steady states of this equation?

Draw the slope fields for this equation.
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- What value does a solution starting at $y(0)=0.2$ approach for large $t$?
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Draw the slope fields for this equation.

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What value does a solution starting at \( y(0) = 0.2 \) approach for large \( t \)?

Does that solution have any inflection points?
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Sketch it.
Given that position tells you velocity, i.e. \( x' = f(x) \), which of the following is false?

(A) A solution \( x(t) \) cannot have a local max (as a function of \( t \)).

(B) If \( x(t) \) is a solution then so is \( y(t) = x(t-c) \).

(C) If \( x(t) \) is a solution then so is \( y(t) = x(t)+C \).

(D) If \( x(t) \) and \( y(t) \) are two different solutions, they cannot cross.

This question assumes that \( f(x) \) is a smooth function for all \( x \).
Given that position tells you velocity, i.e. \( x' = f(x) \), which of the following is false?

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Can't both have correct slope!
\[ f(y) = \sin(y) \]
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\[ y' = \sin(y) \]

A solution satisfying the initial condition \( y(0) = y_0 \) will approach \( y^* \) as \( t \to \infty \). Which \( y_0 \) and \( y^* \) pair is correct?

(A) \( y_0 = -\frac{3\pi}{2}, y^* = -2\pi \).

(B) \( y_0 = -\frac{\pi}{2}, y^* = -\frac{\pi}{2} \).

(C) \( y_0 = \frac{\pi}{4}, y^* = \frac{\pi}{2} \).

(D) \( y_0 = \frac{3\pi}{4}, y^* = \pi \).

(E) \( y_0 = \pi, y^* = 0 \).
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A solution satisfying the initial condition $y(0) = y_0$ will approach $y^*$ as $t \to \infty$.

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(A) $y_0 = -\frac{3\pi}{2}, y^* = -2\pi. \text{X} \quad \Rightarrow \quad y^* = -\pi$

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\[ y' = \sin(y) \]

Fill in the arrows and steady states on the phase line.
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Filled circle \( \bigcirc \) - stable steady state

Empty circle \( \bigcirc \) - unstable steady state
$y' = \sin(y)$

Sketch a few solutions $y(t)$. 
\[ y' = \sin(y) \]
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What you should be able to do:

- Identify steady states for a DE.
- Draw/interpret the phase line for a DE.
- Draw/interpret a slope field for a DE.
- Determine stability of steady states.
- Determine long-term behaviour of solutions.
- Sketch the graphs of solutions using phase line and/or slope fields (slopes, concavity, IPs, h-asymptotes).
Some biological examples

- **Allee effect**: \( P' = rP \left(1 - \frac{P}{K}\right) \left(\frac{P}{T} - 1\right) \) where \( T < K \)

- **Lac operon**: \( c' = \frac{c^2}{(k^2 + c^2)} - ac \)