Today

- Qualitative analysis examples.

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This question assumes that $f(x)$ is a smooth function for all x.

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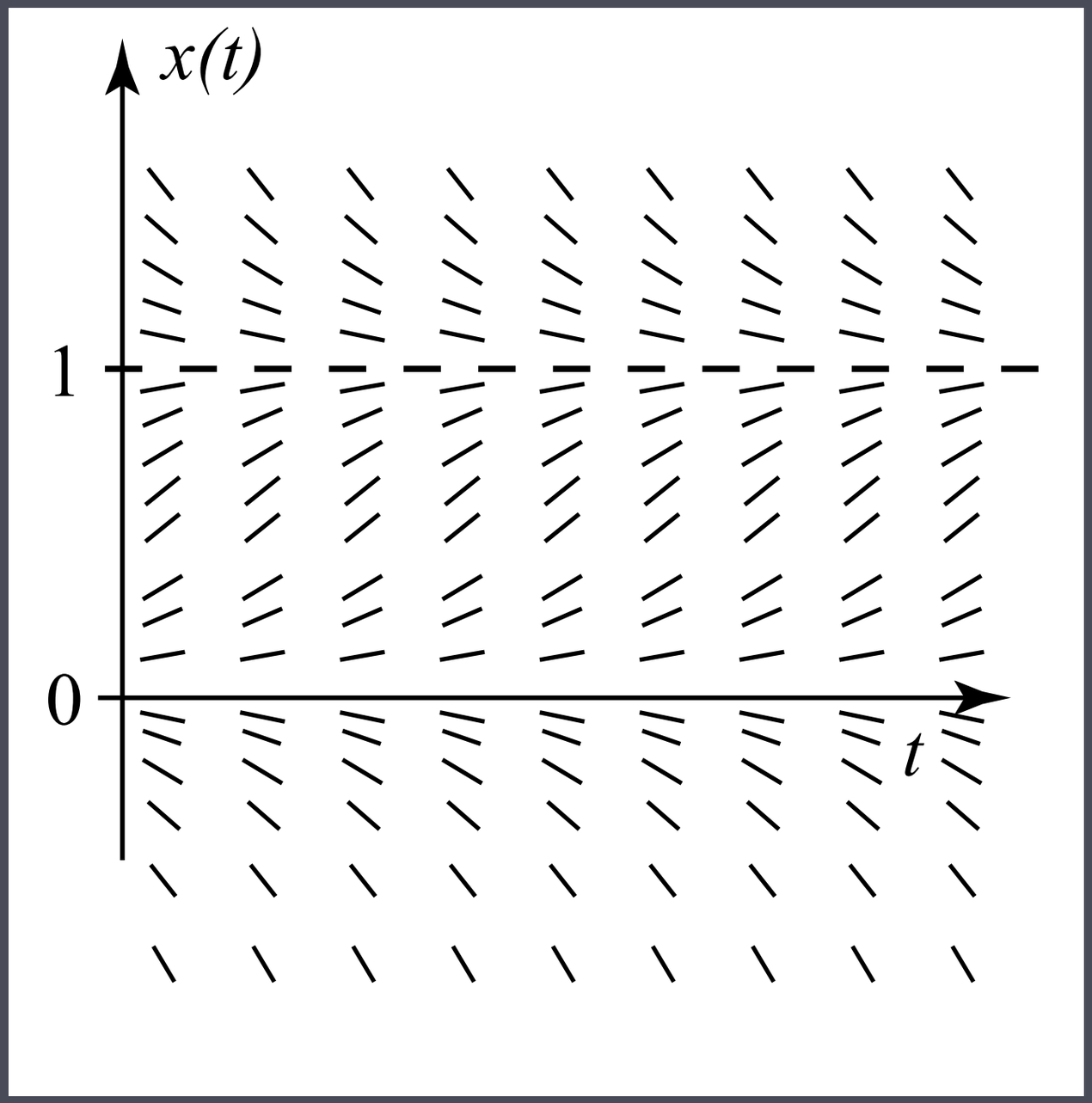

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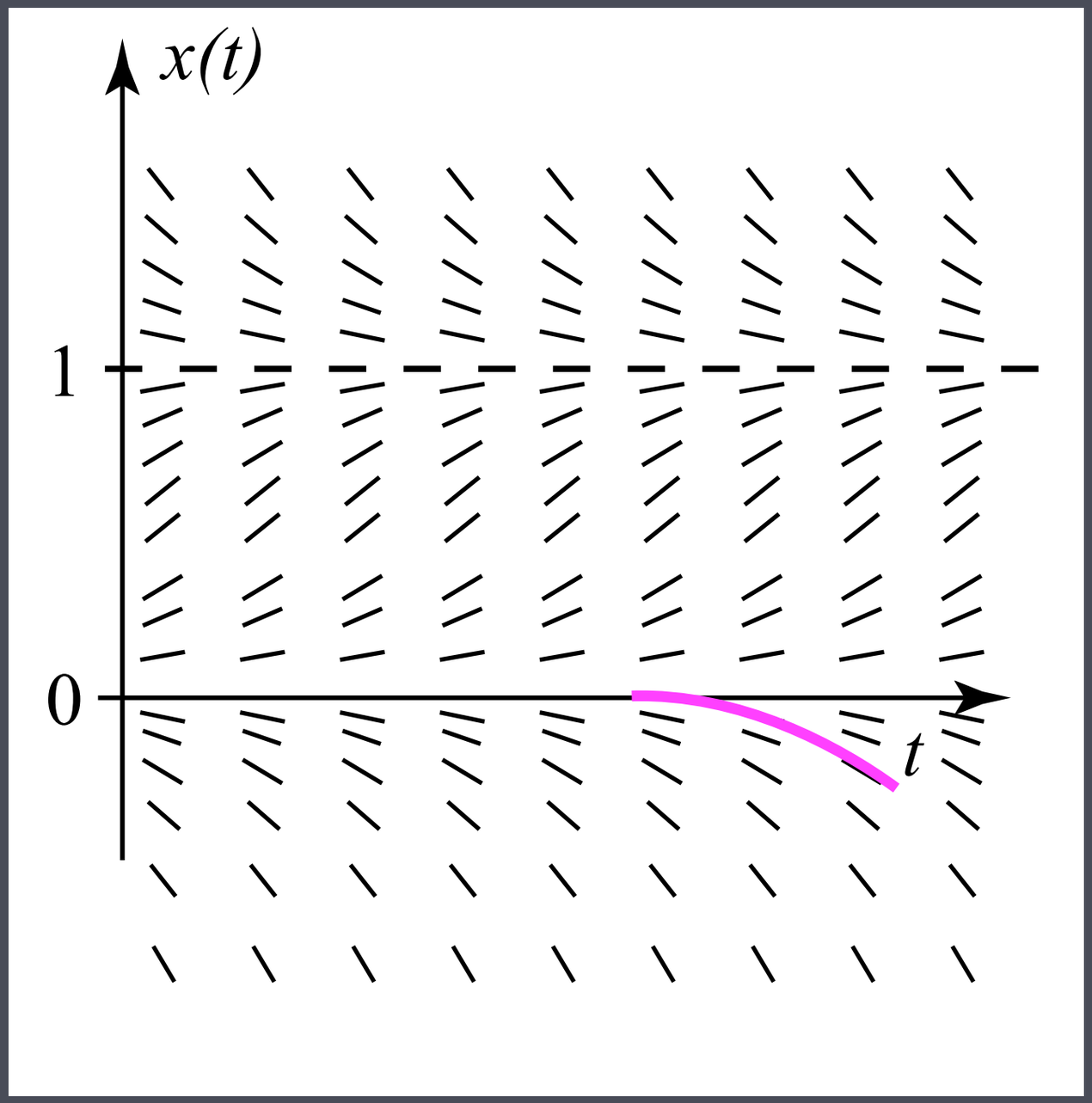

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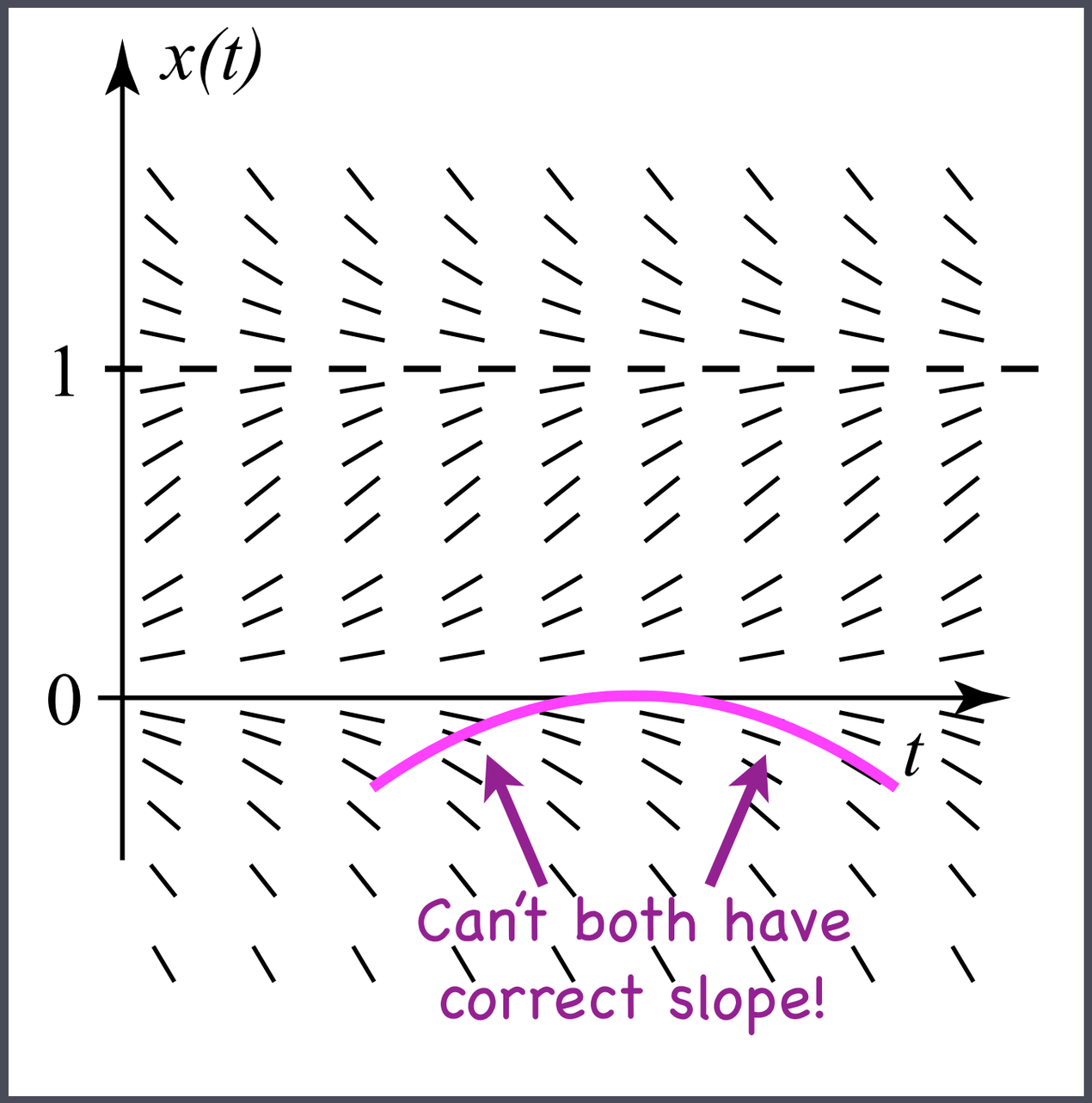

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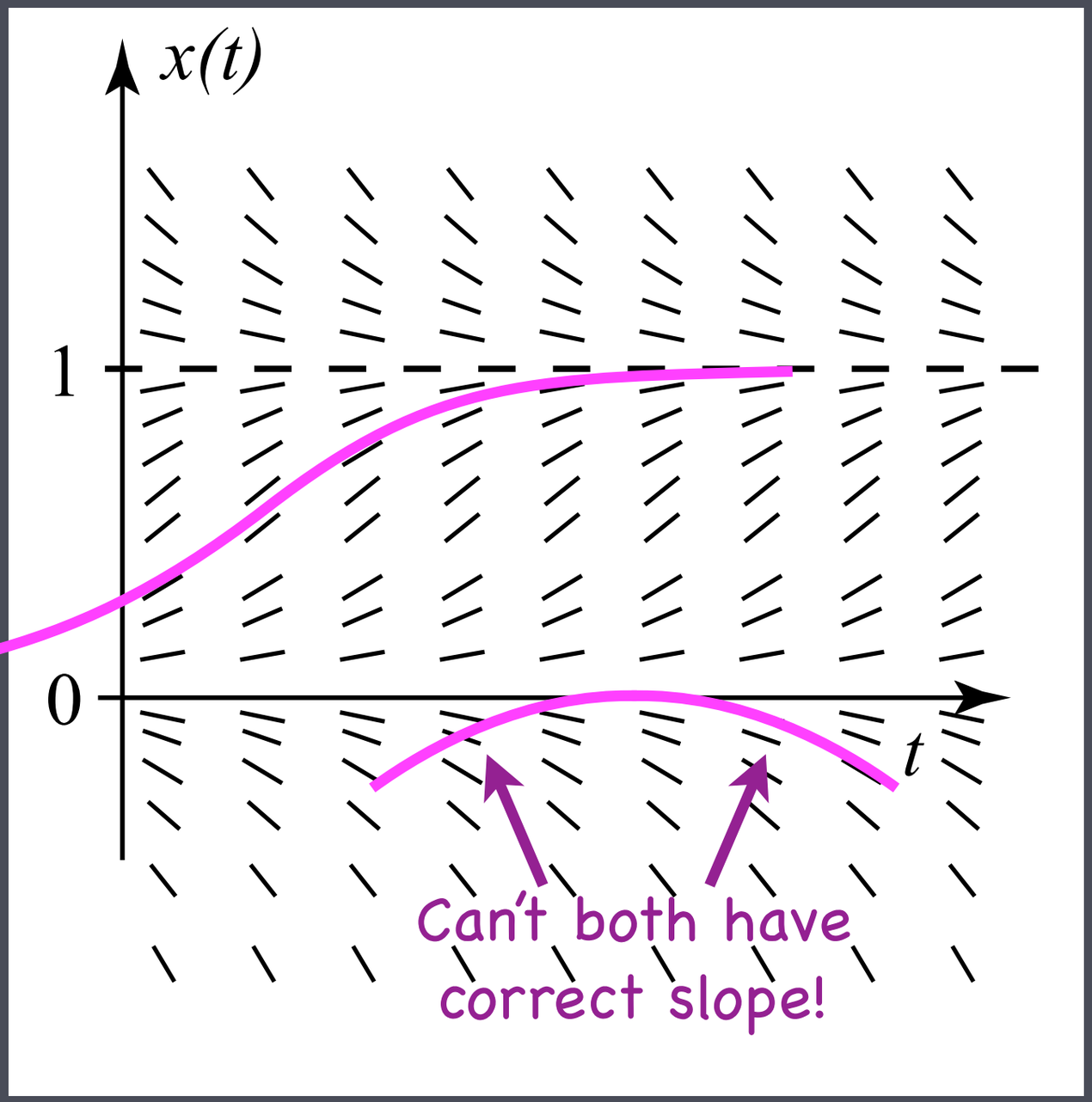

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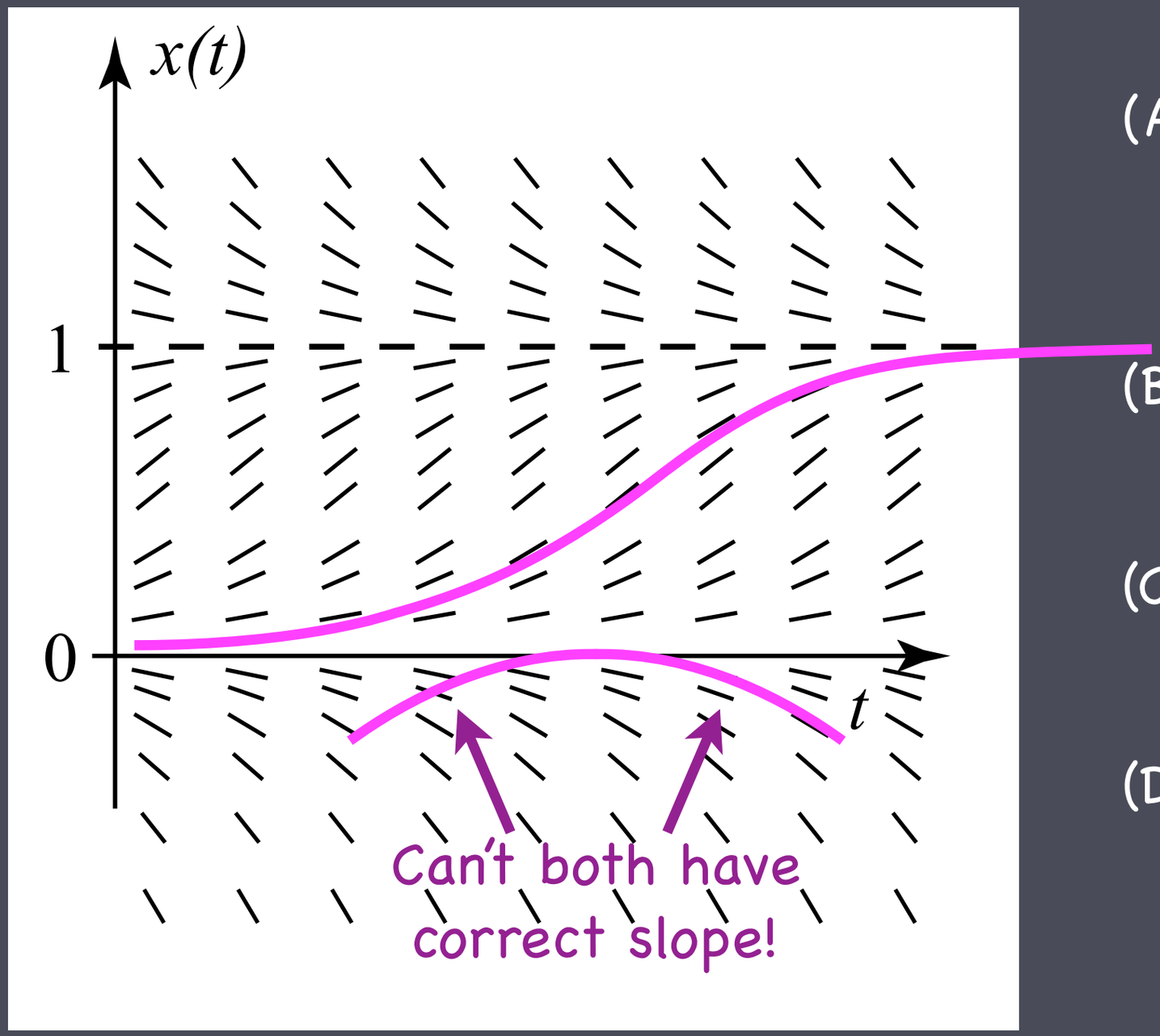

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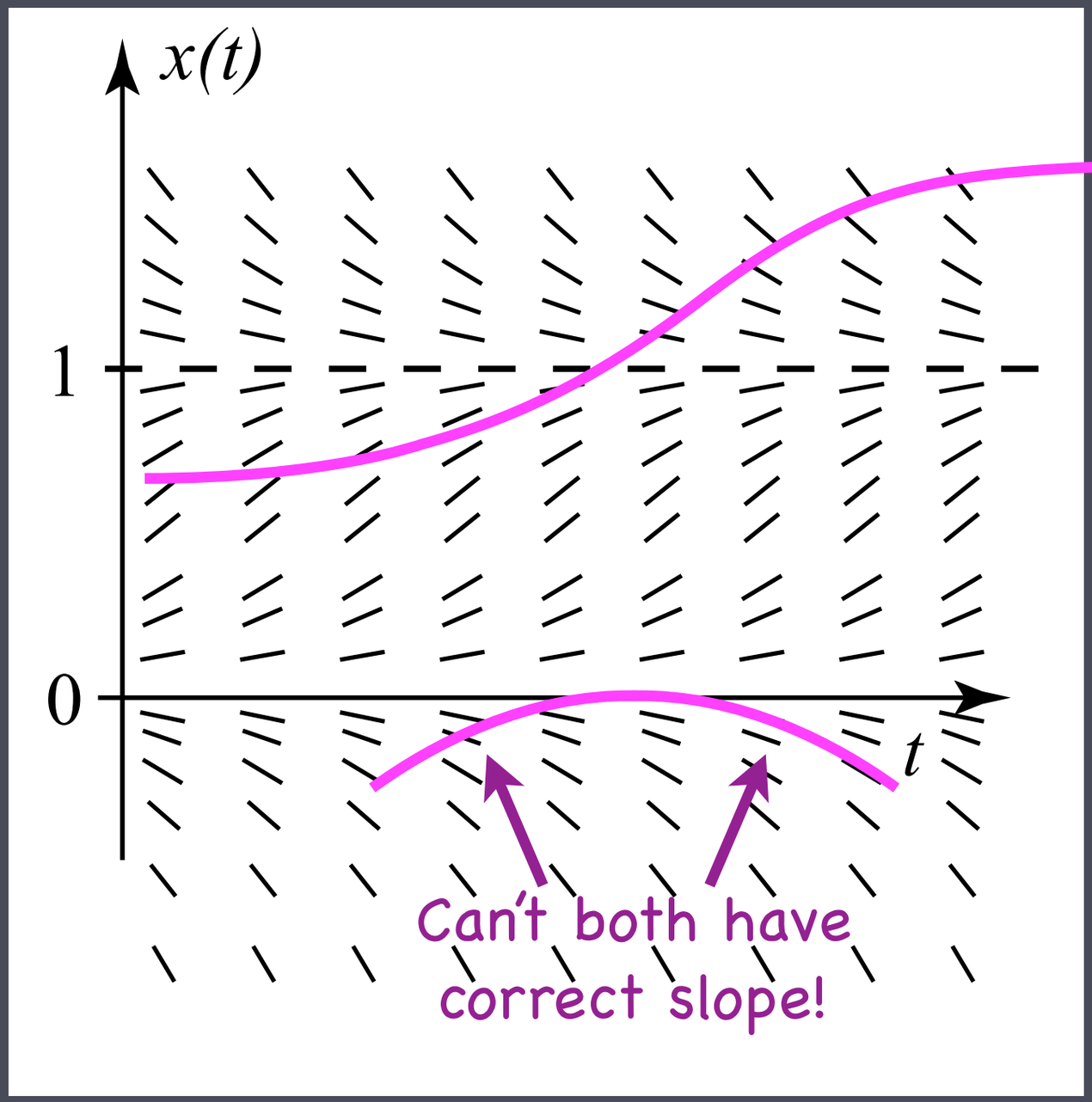

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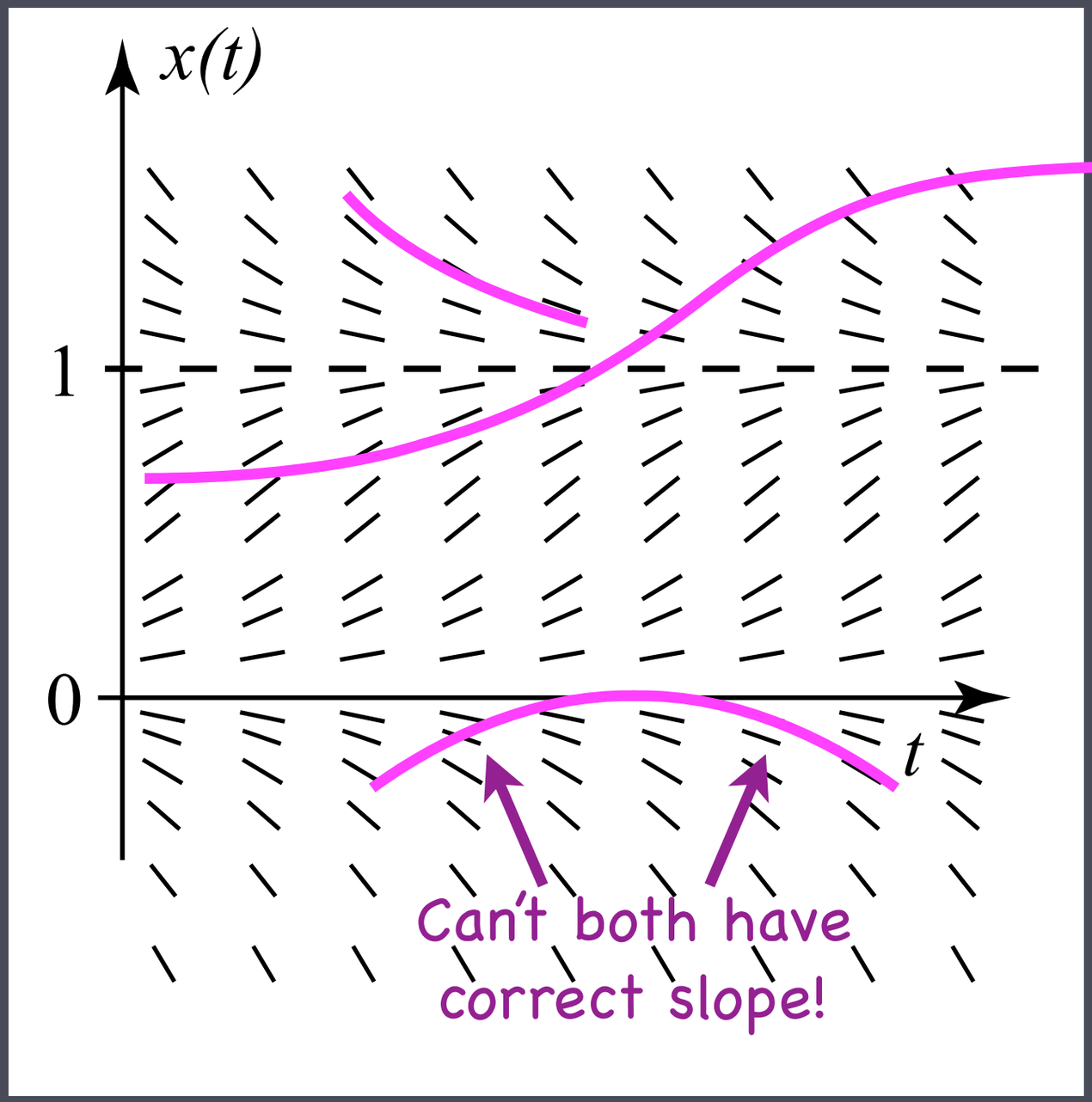

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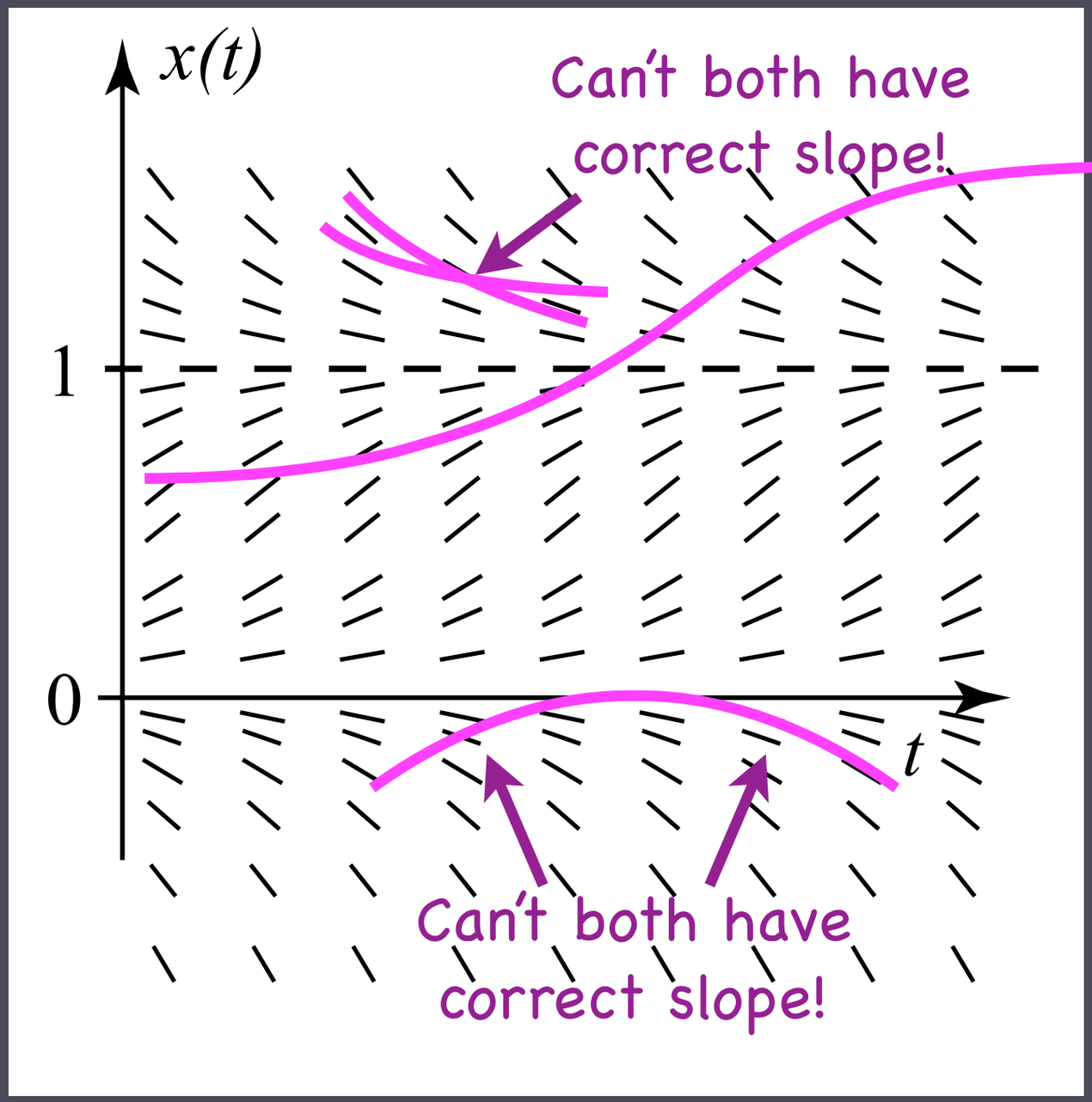

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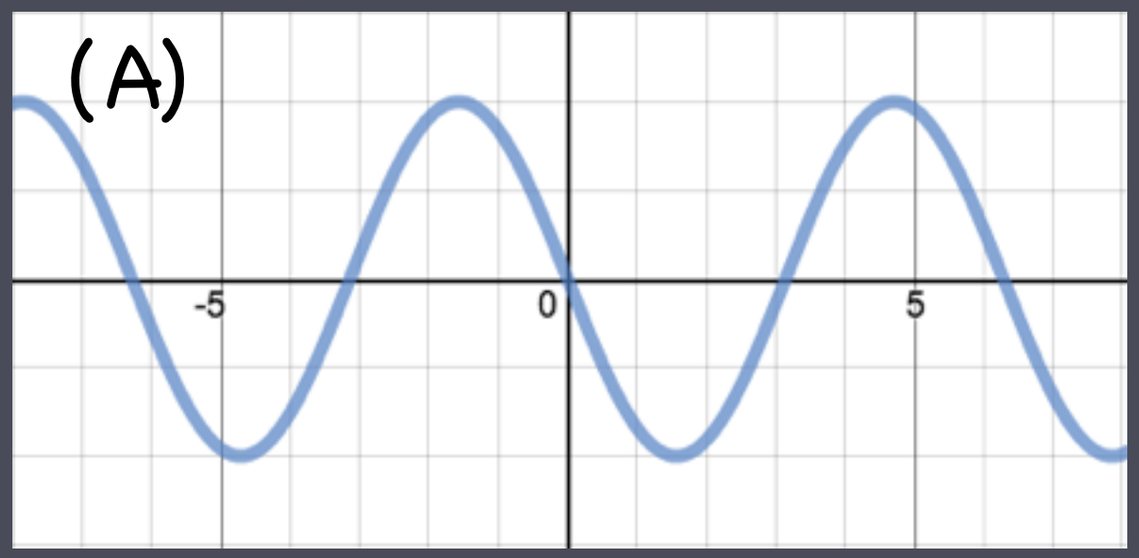

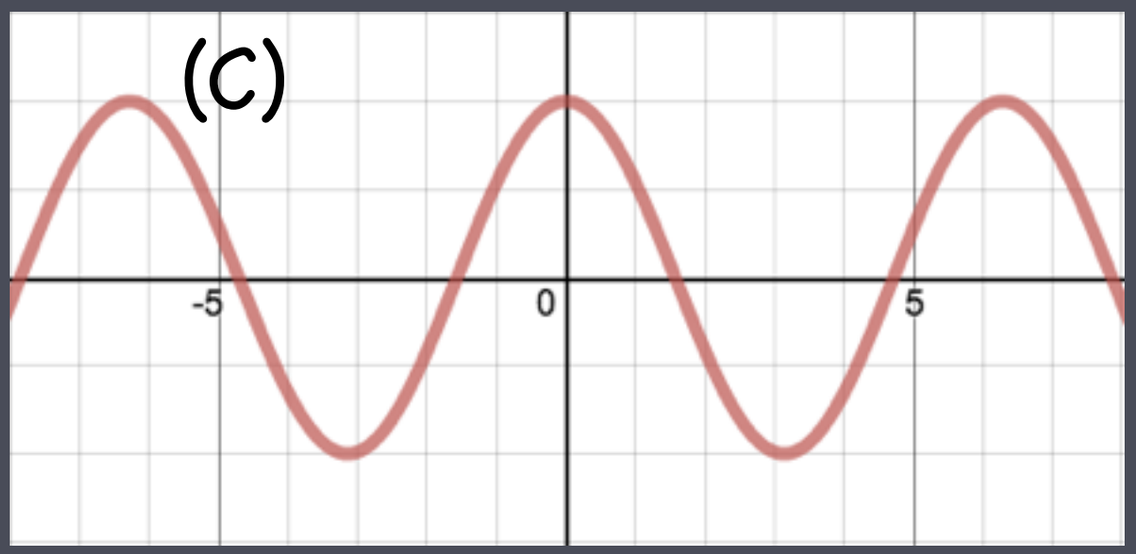

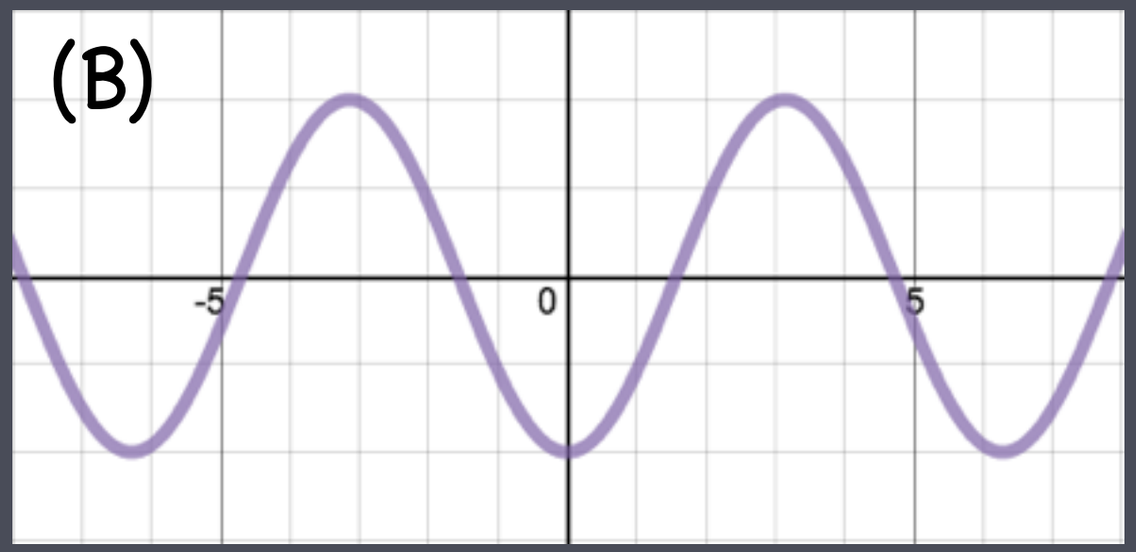

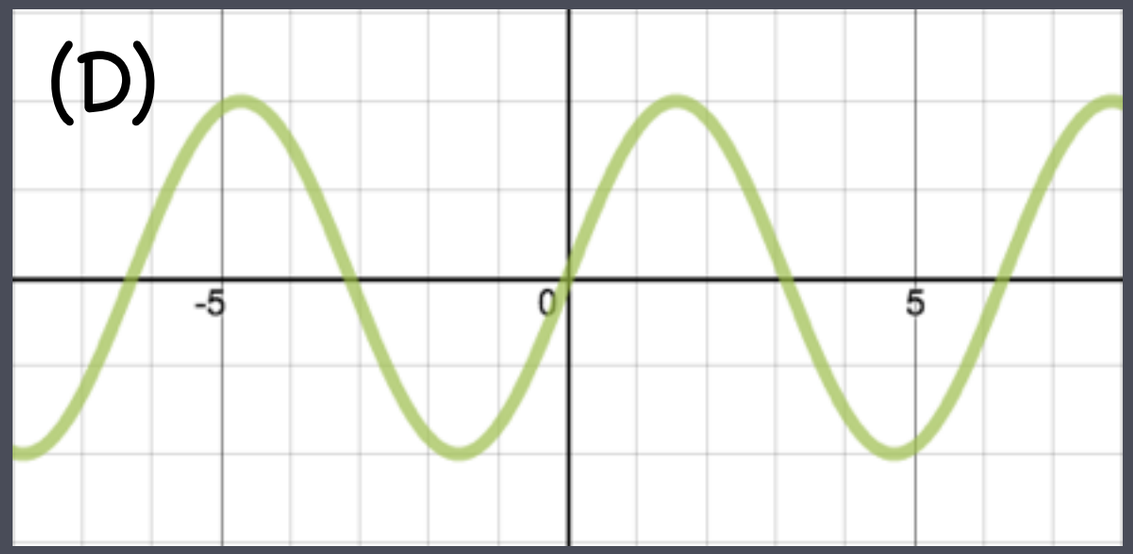

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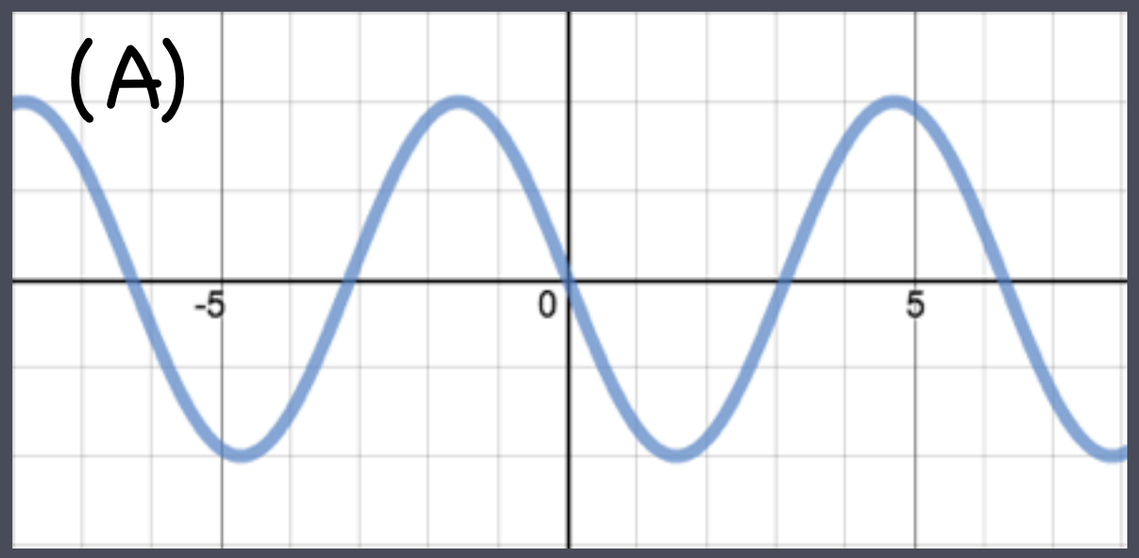

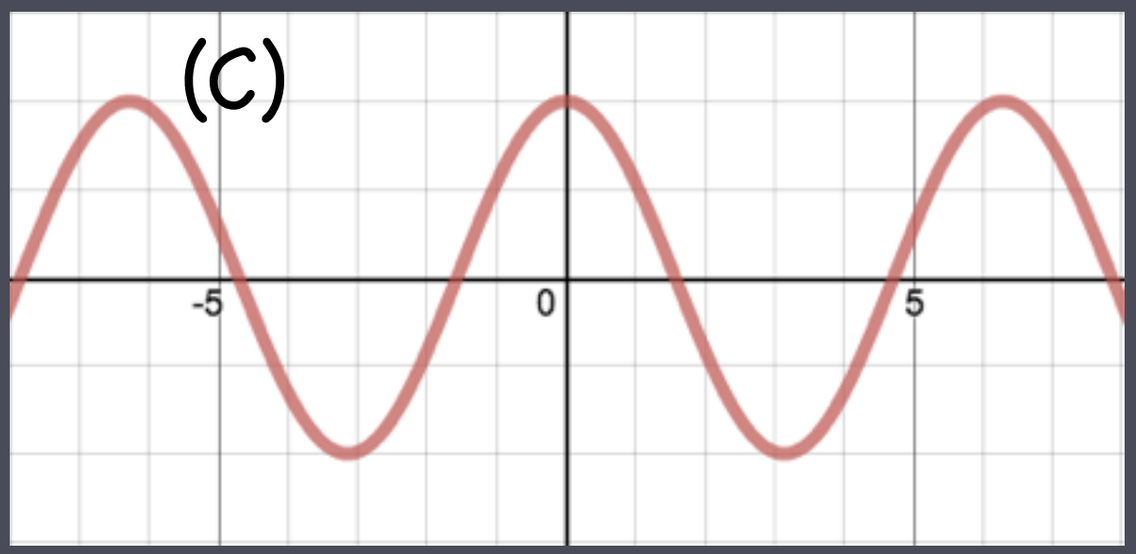

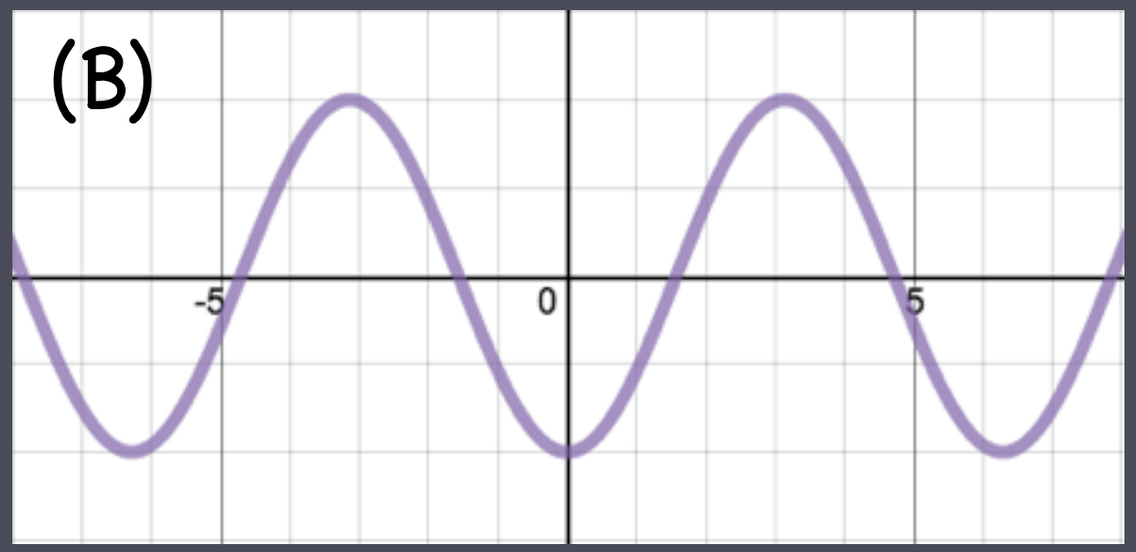

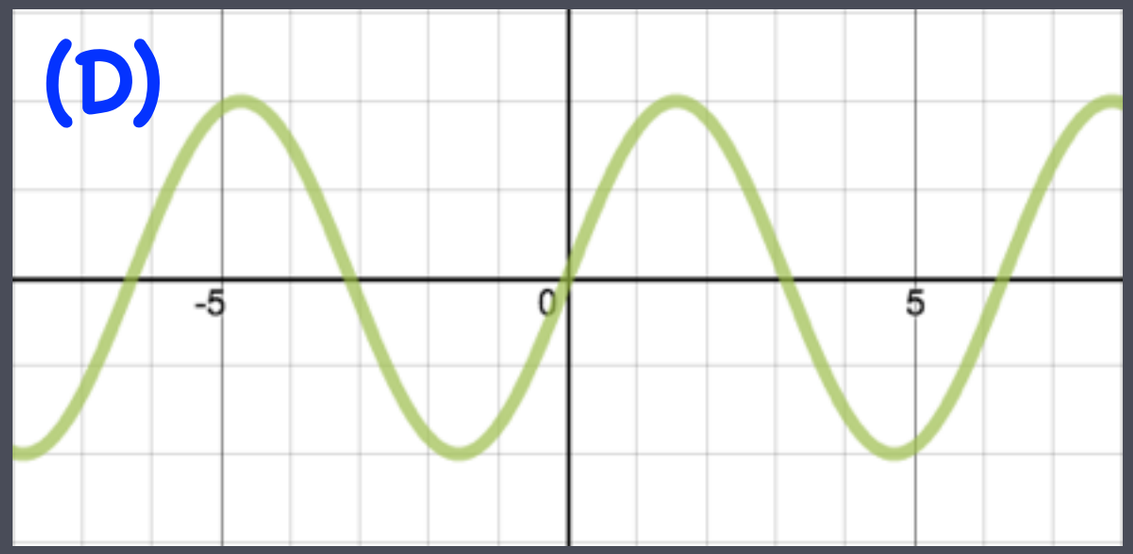

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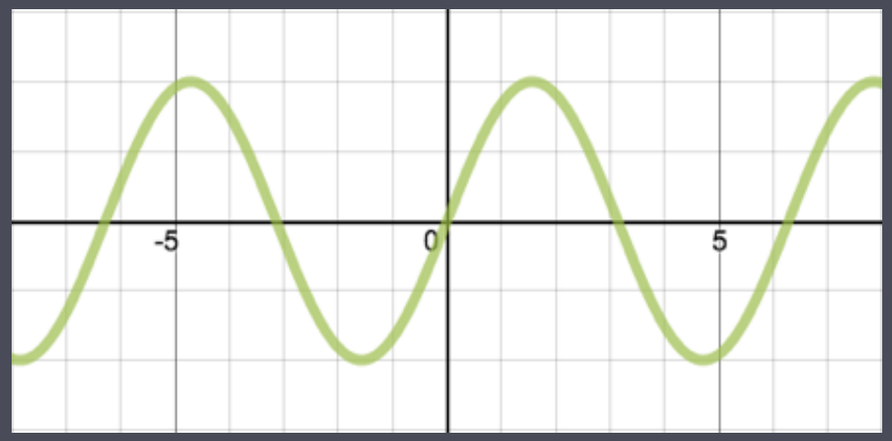

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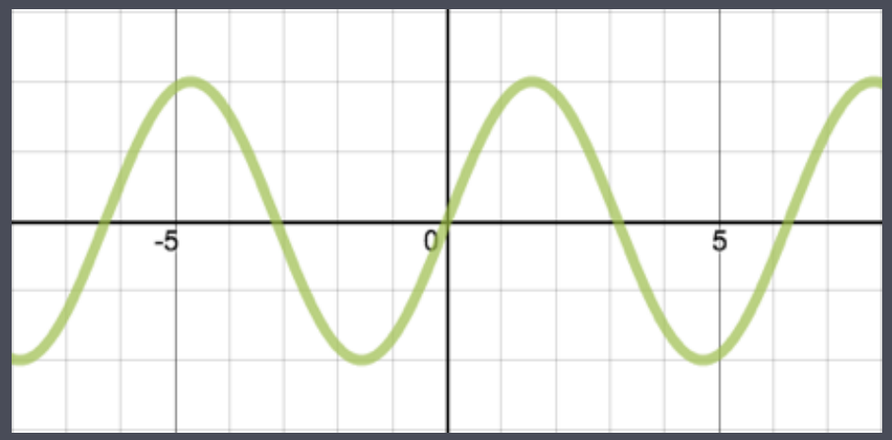

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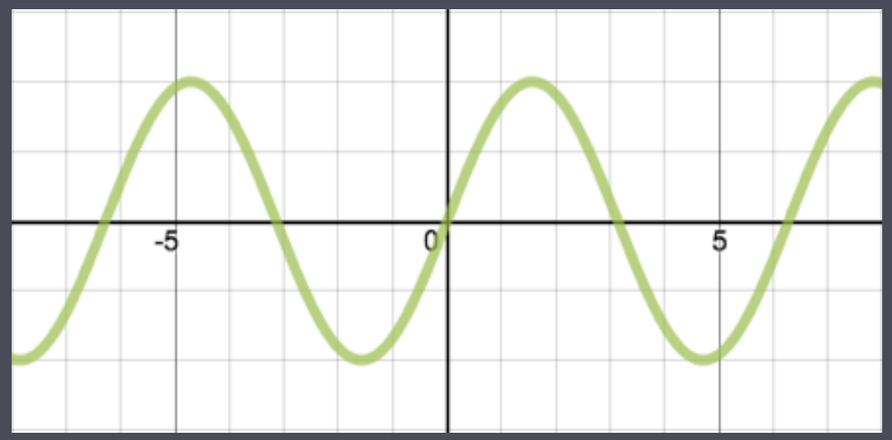

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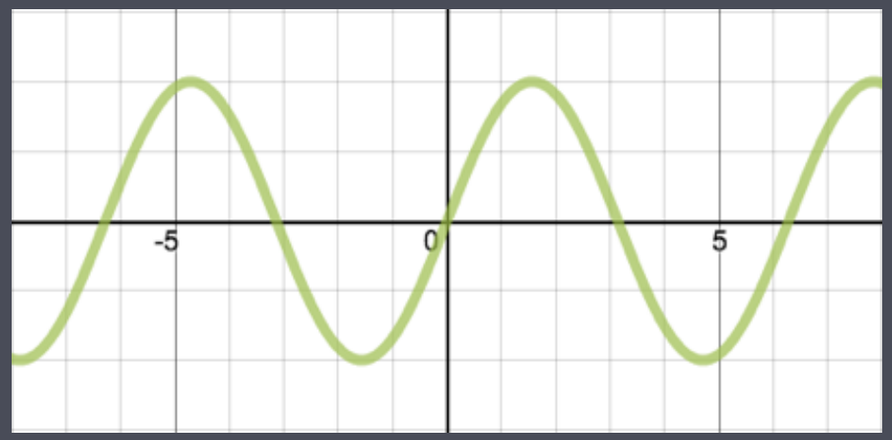

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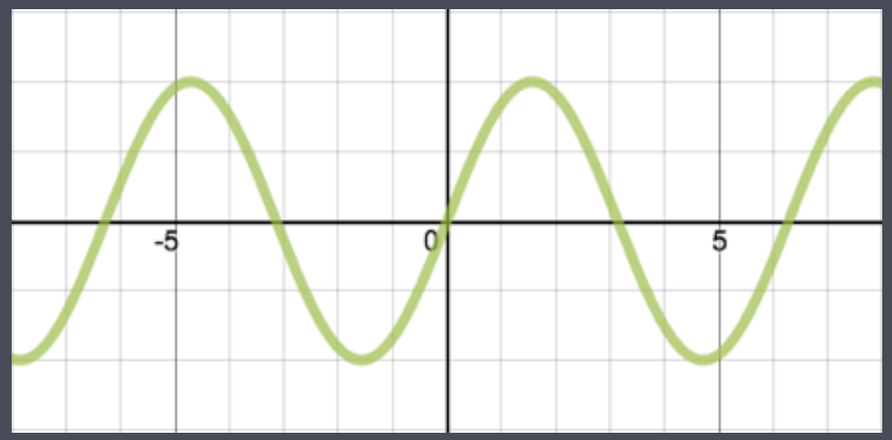

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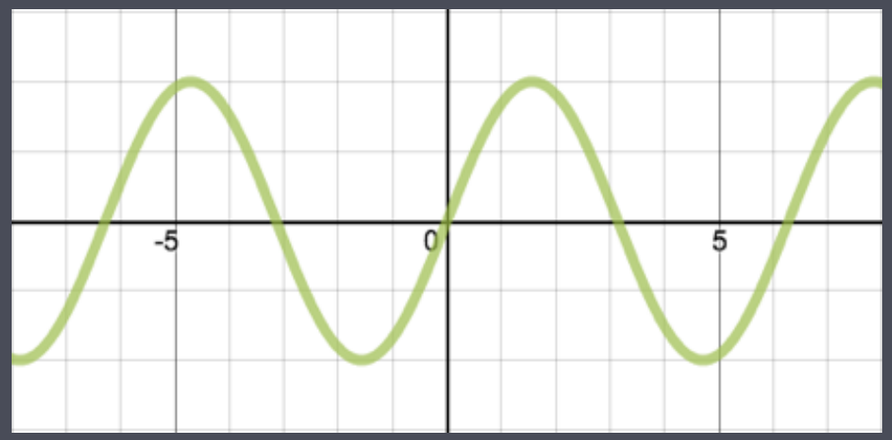

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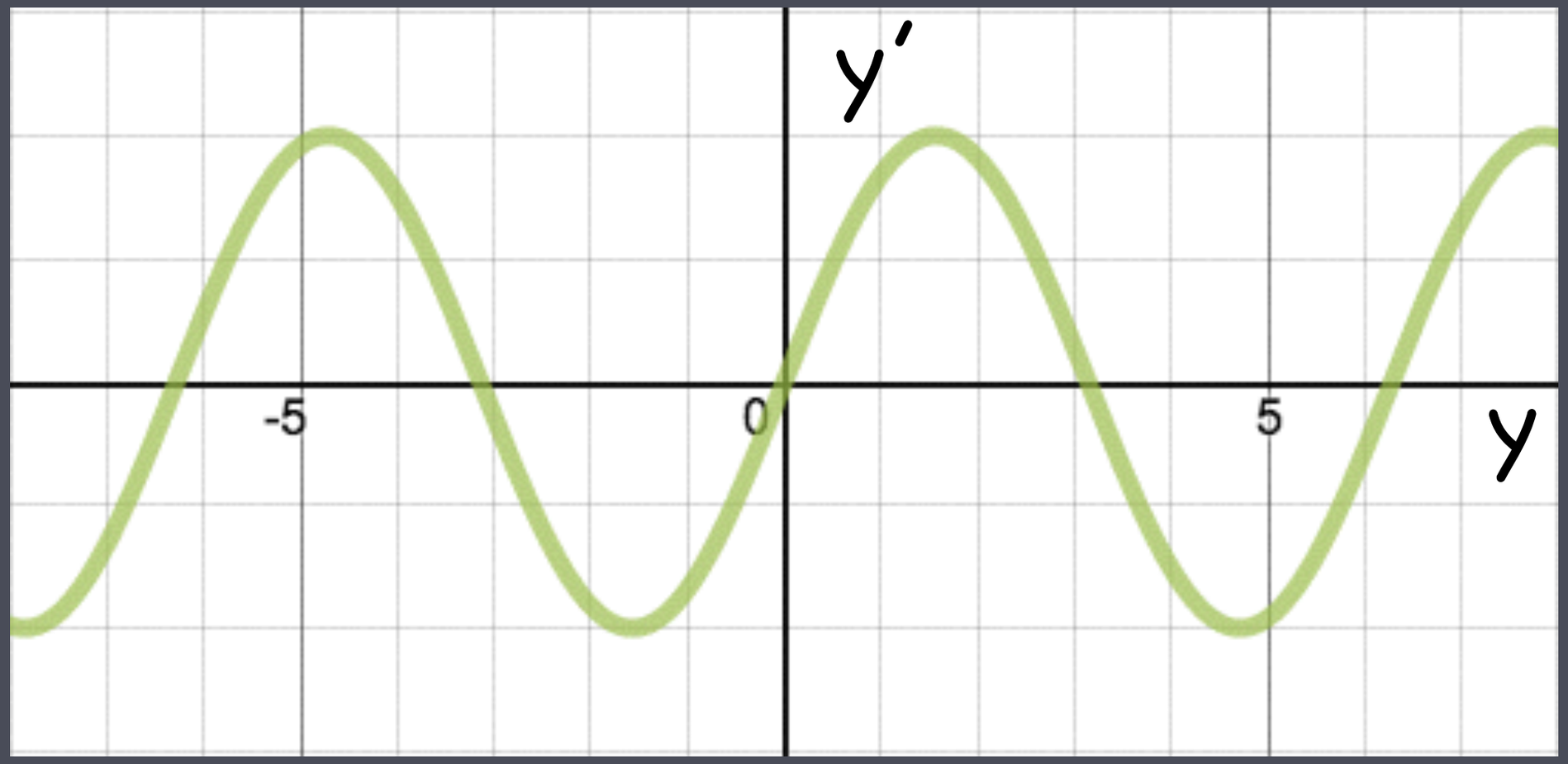
Fill in the arrows and steady states on the phase line.



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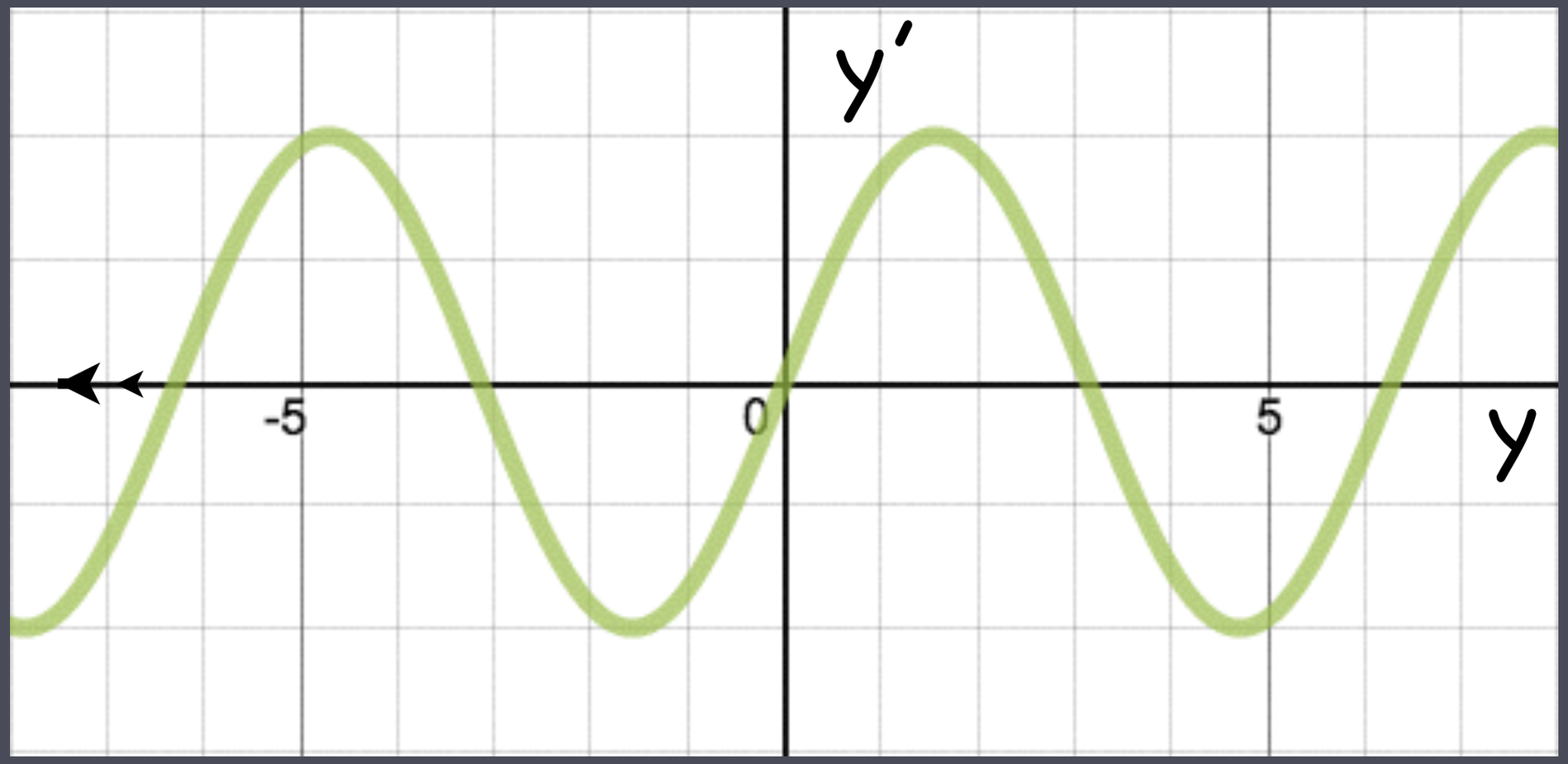

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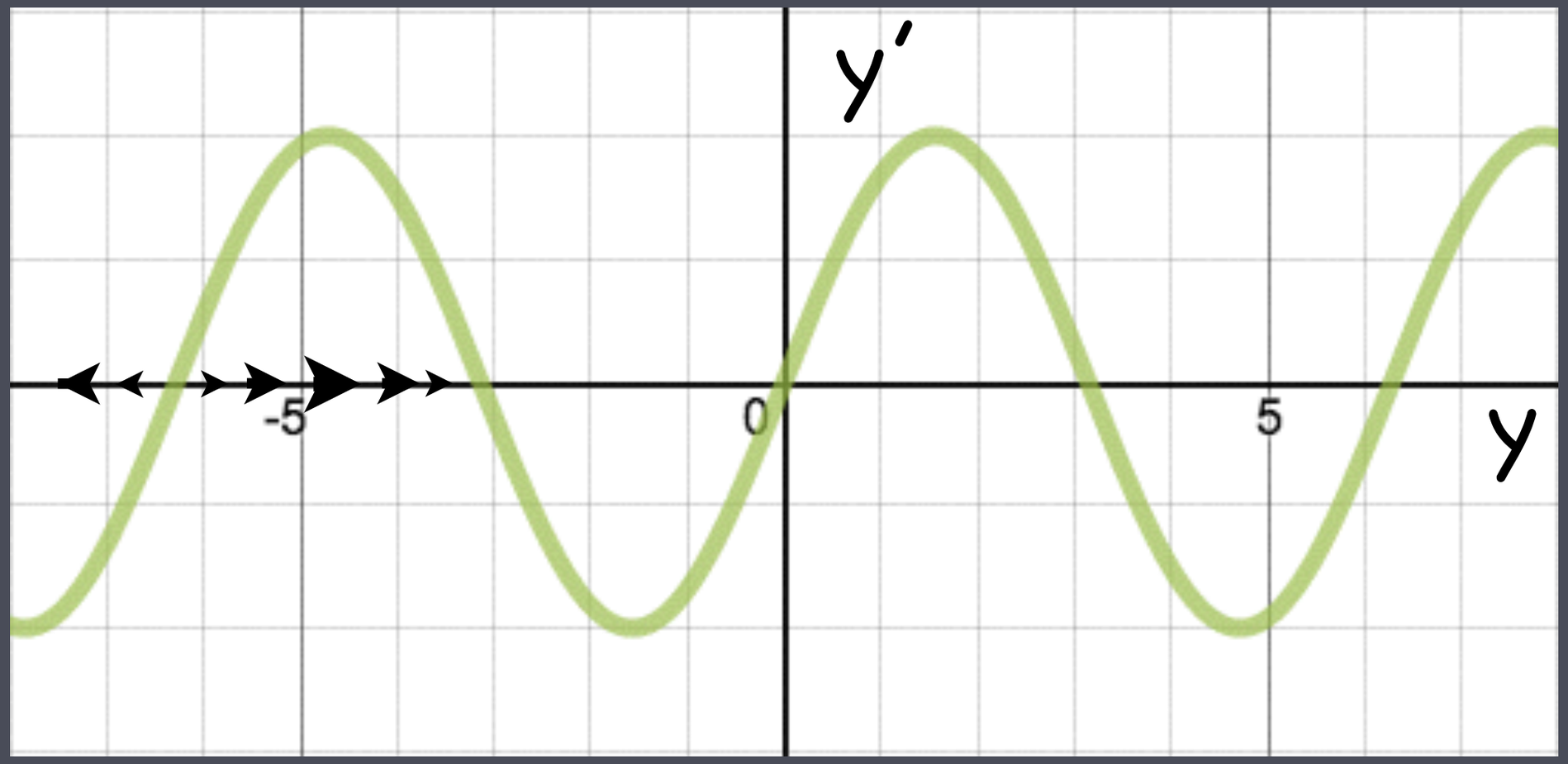

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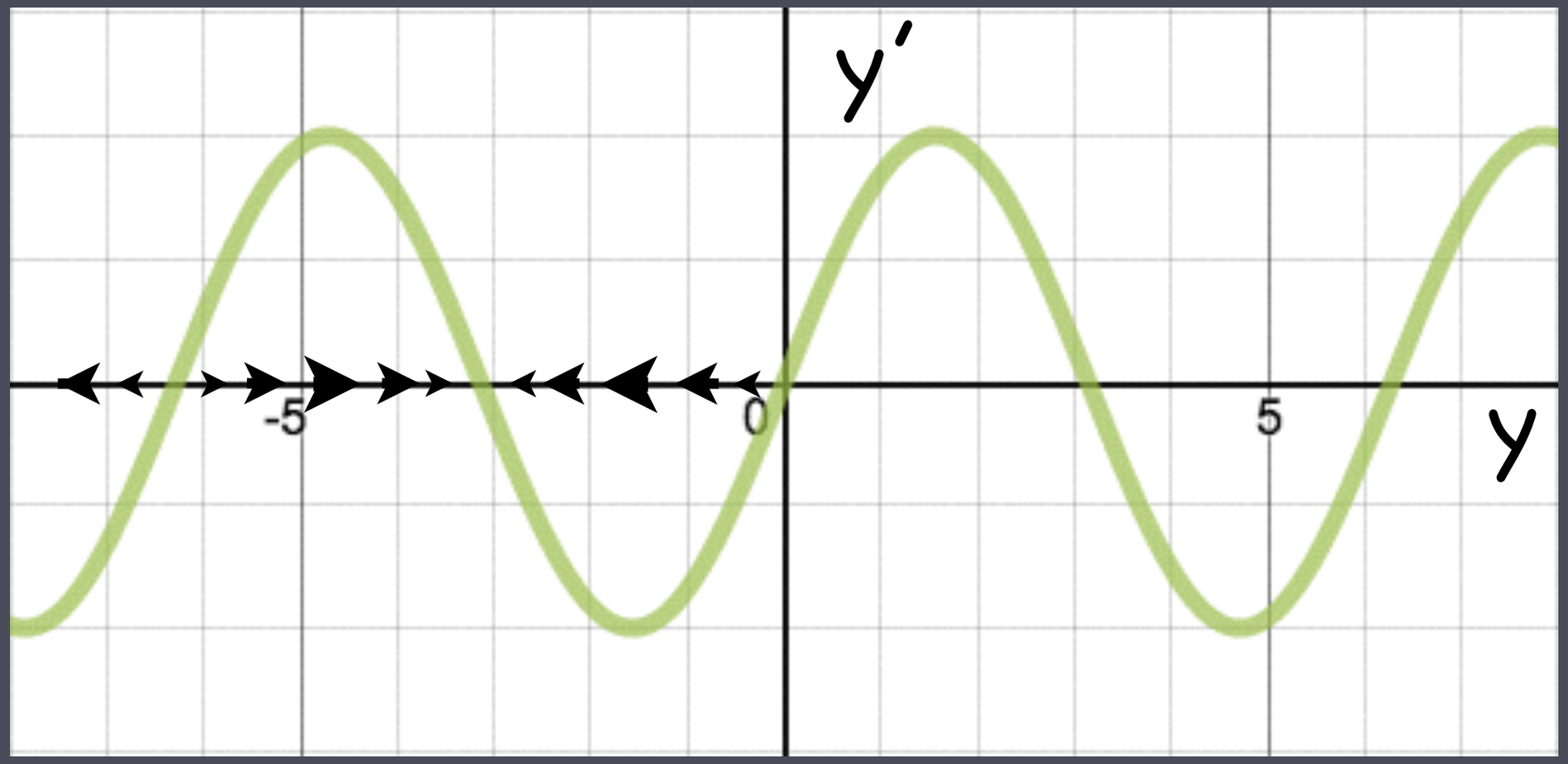

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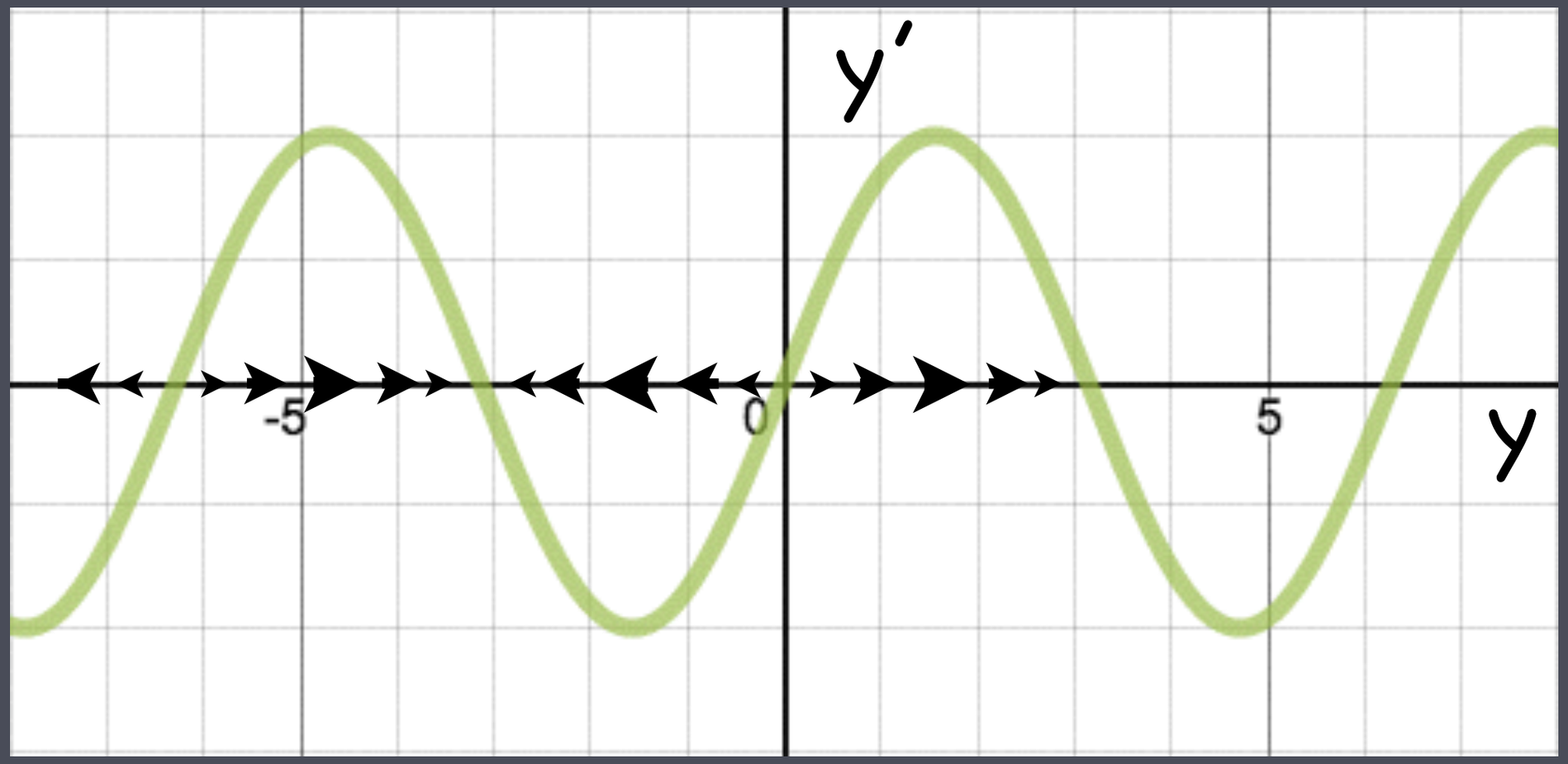

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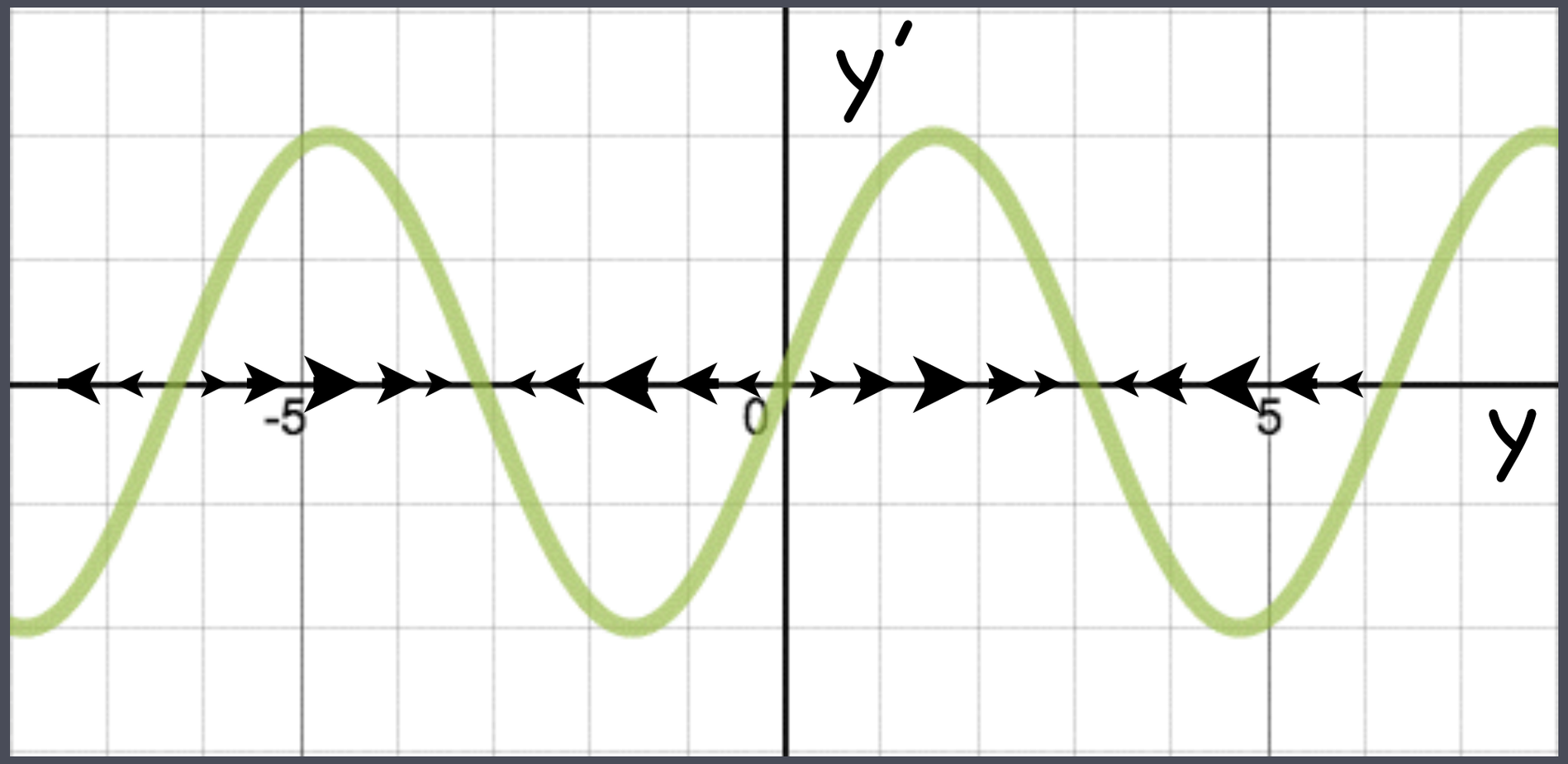

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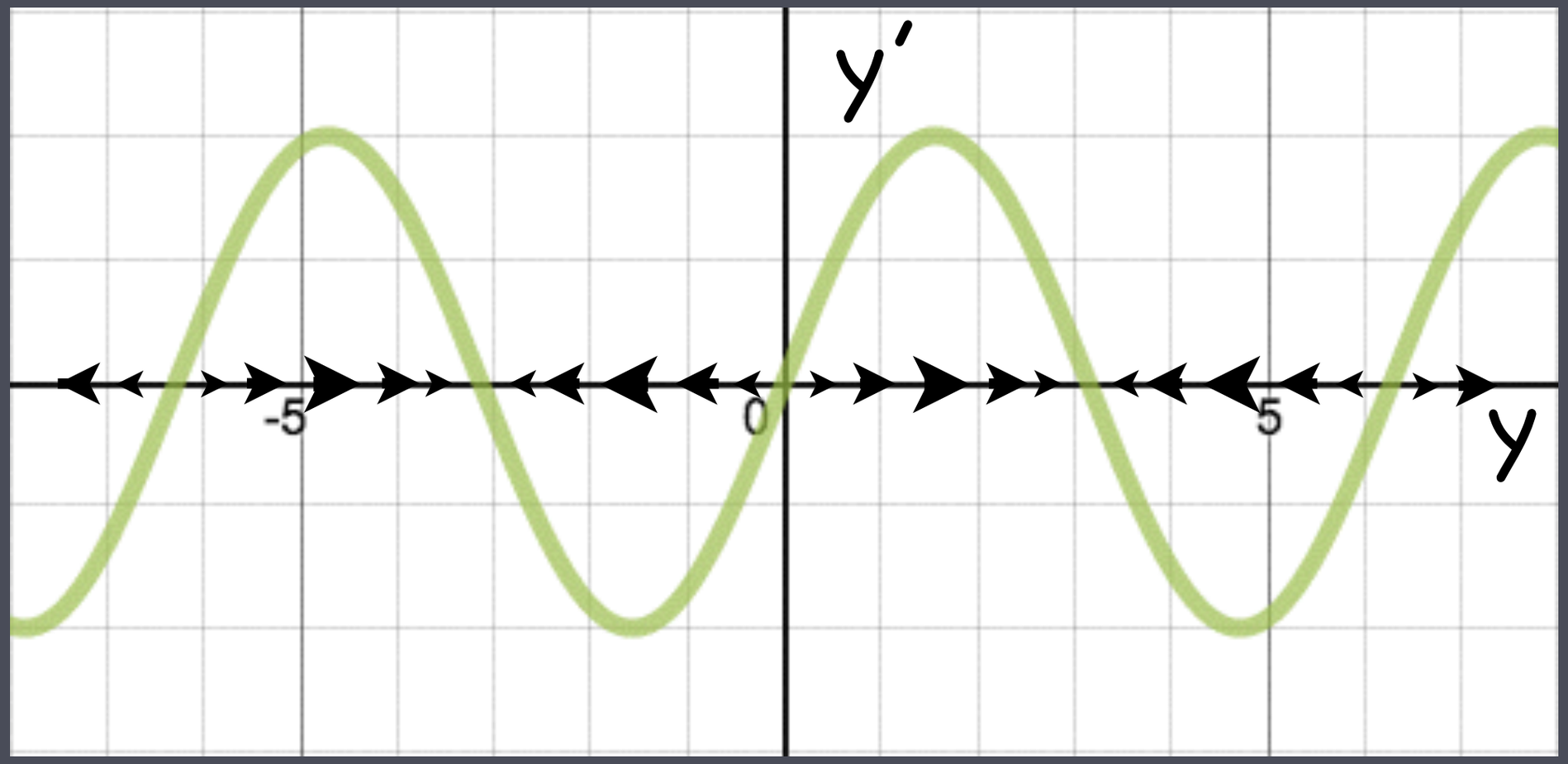

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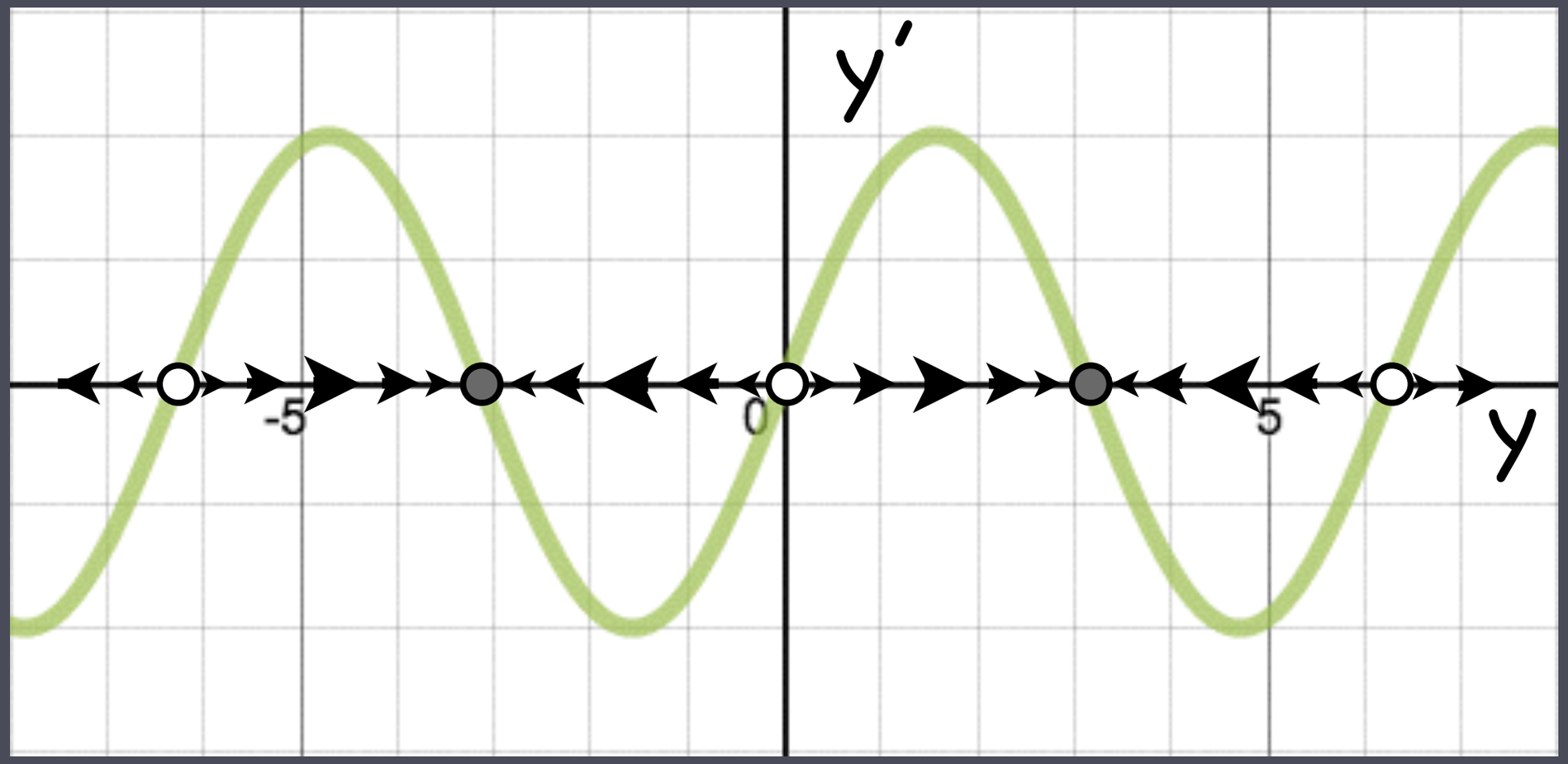

Filled circle ● - stable steady state

Empty circle ○ - unstable steady state

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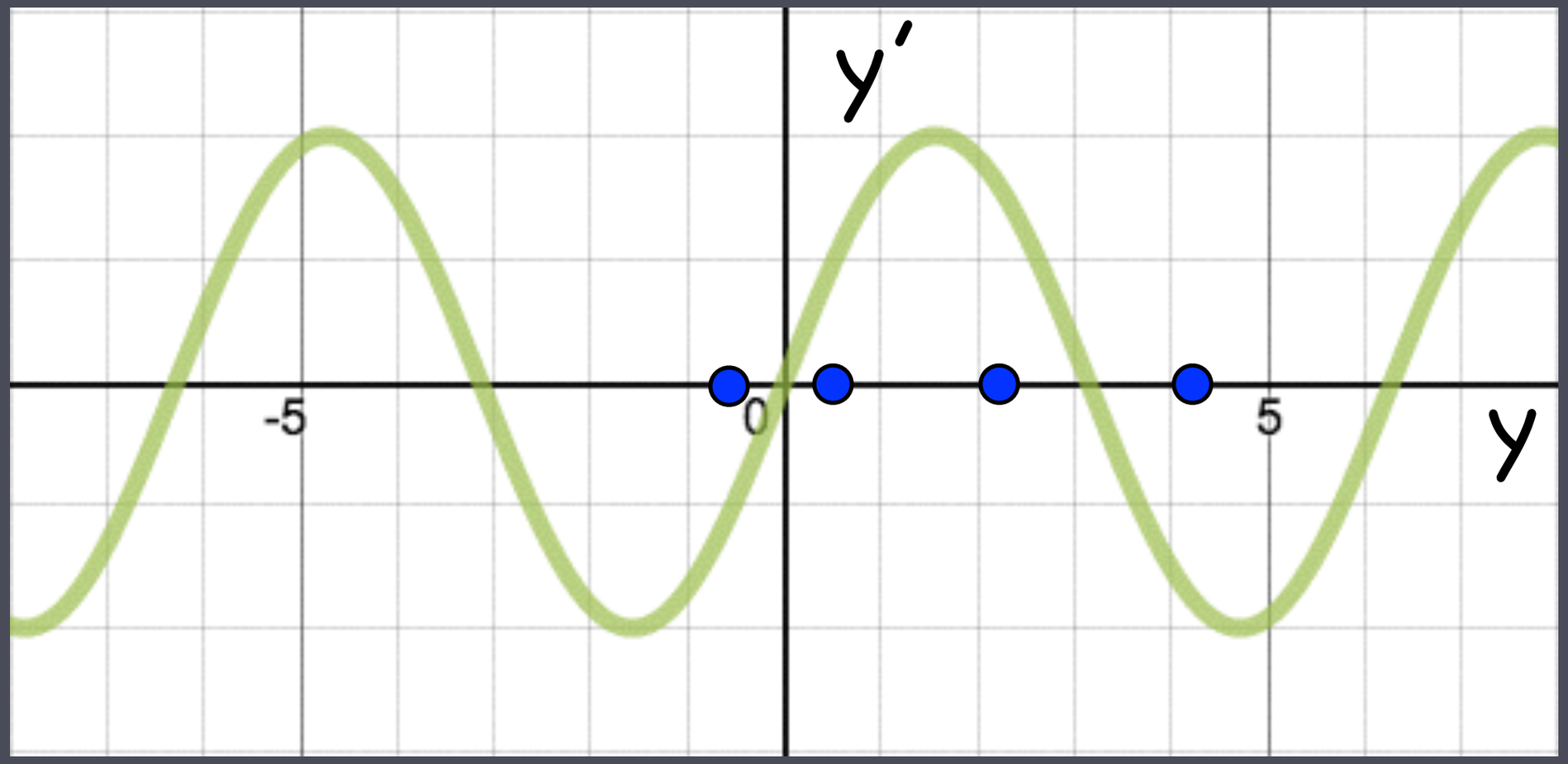
Sketch a few solutions y(t).



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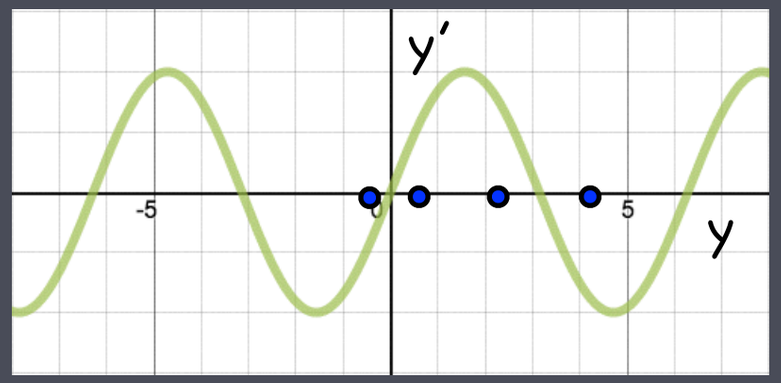

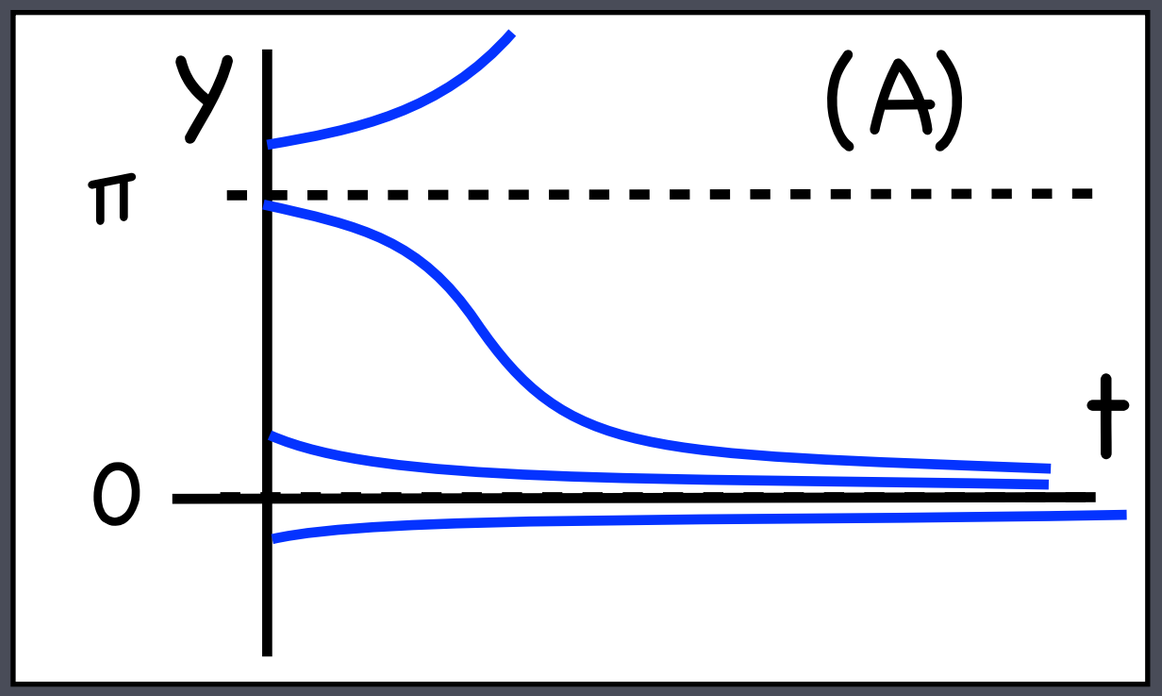

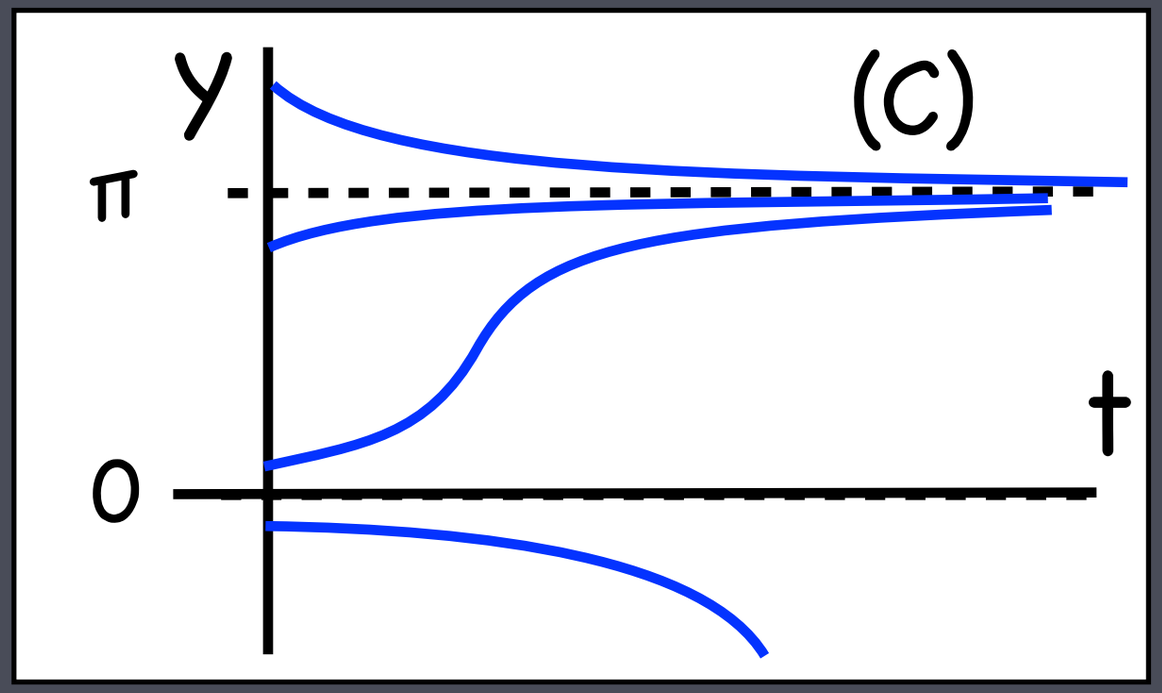

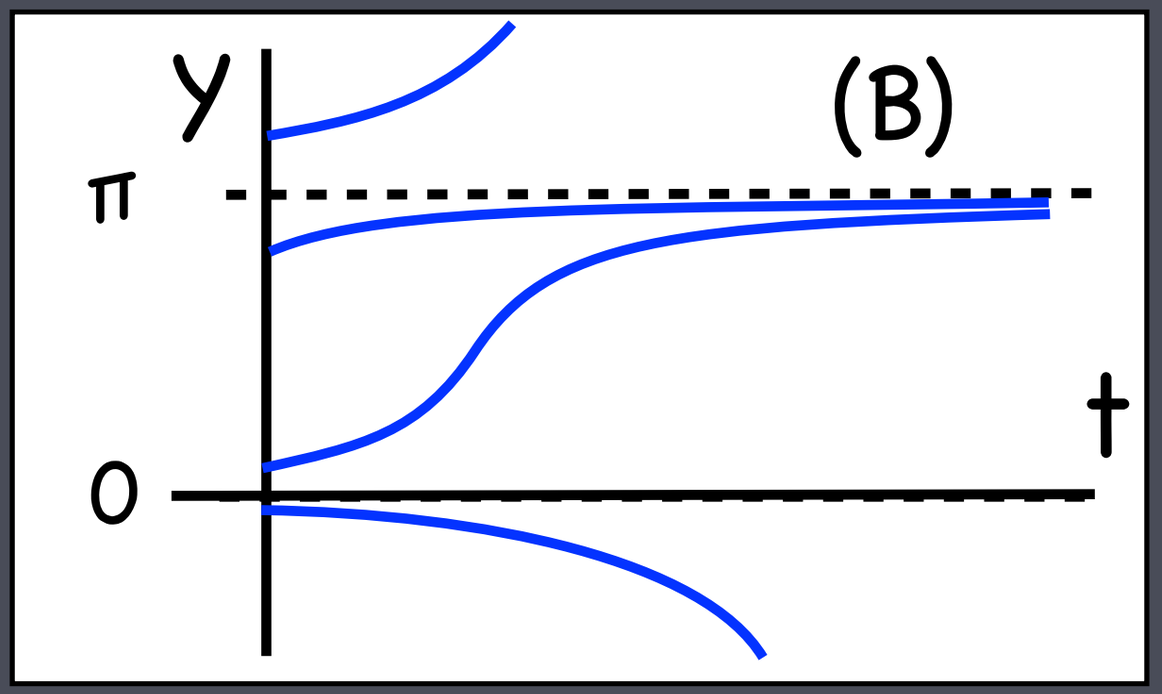

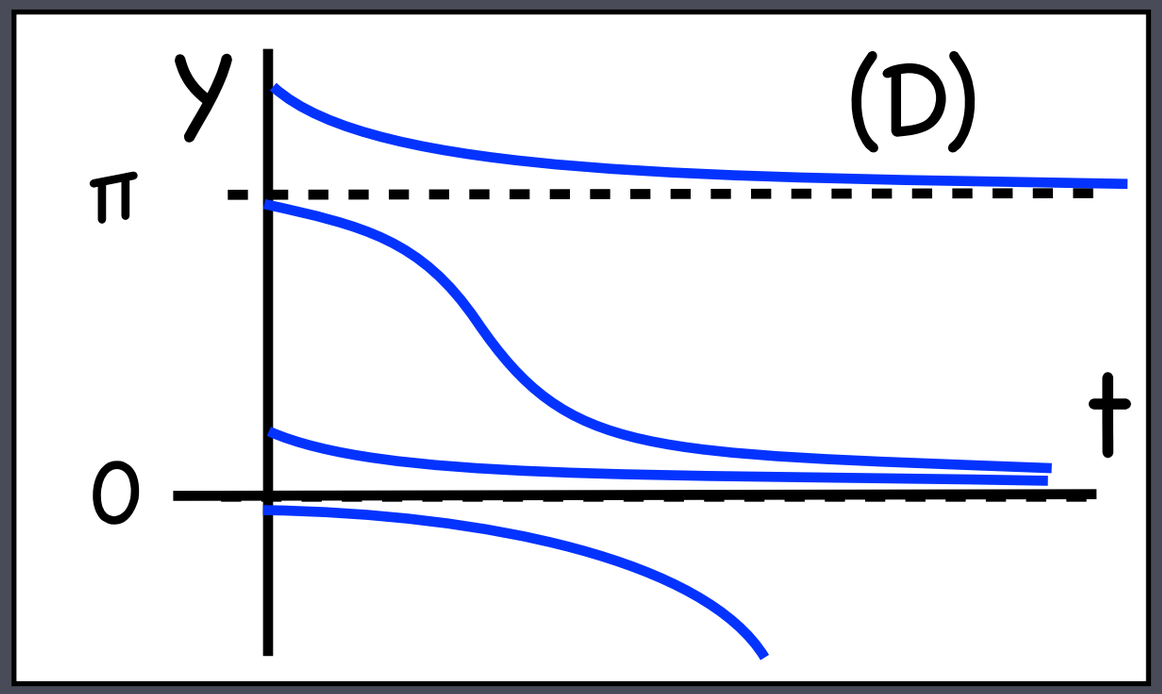

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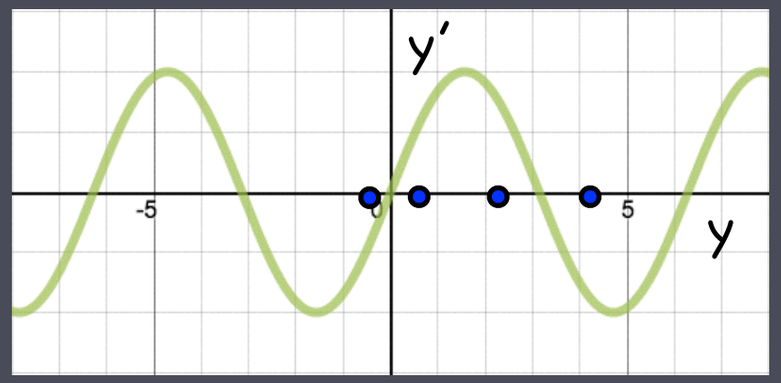

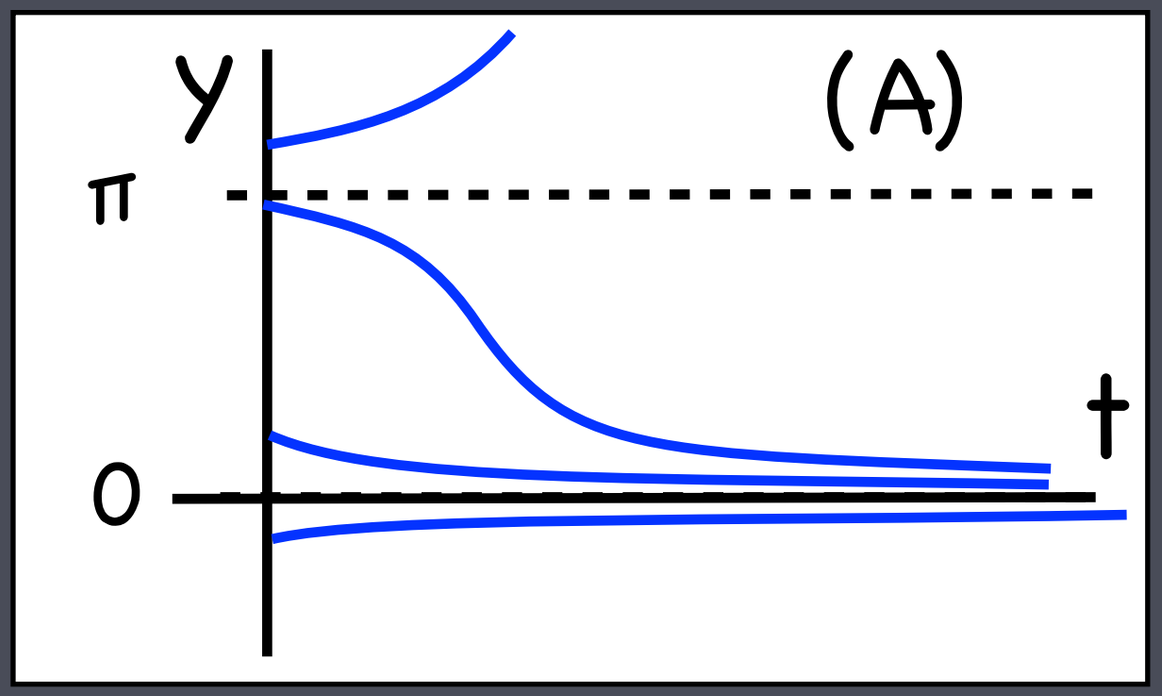

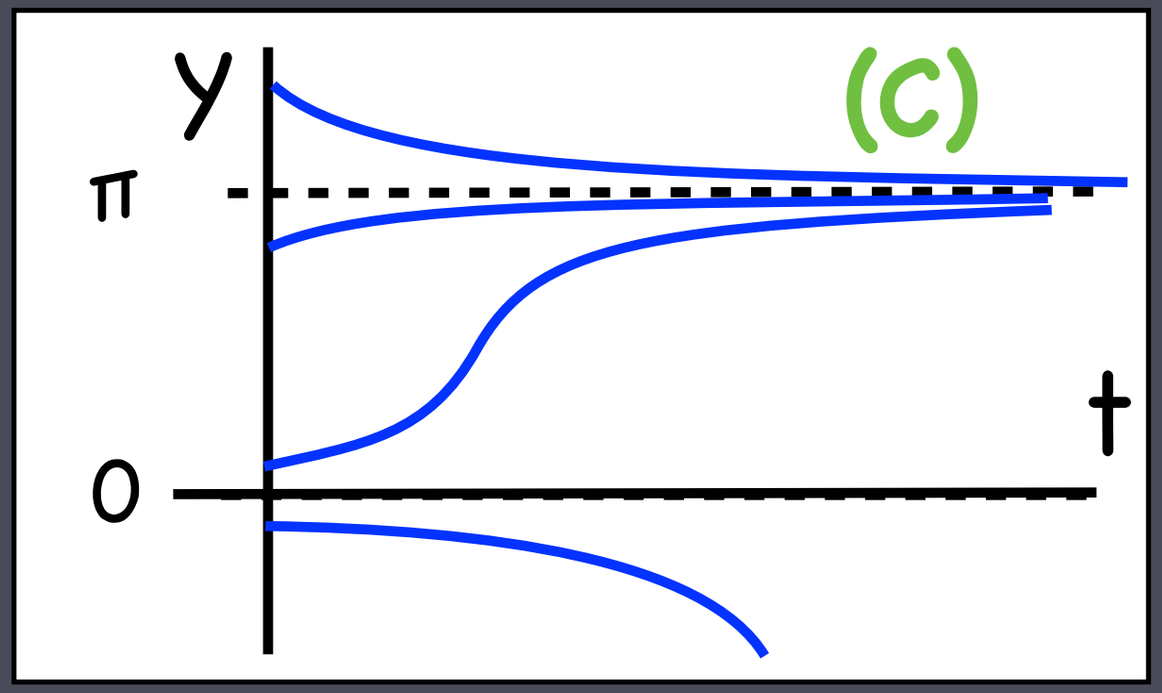

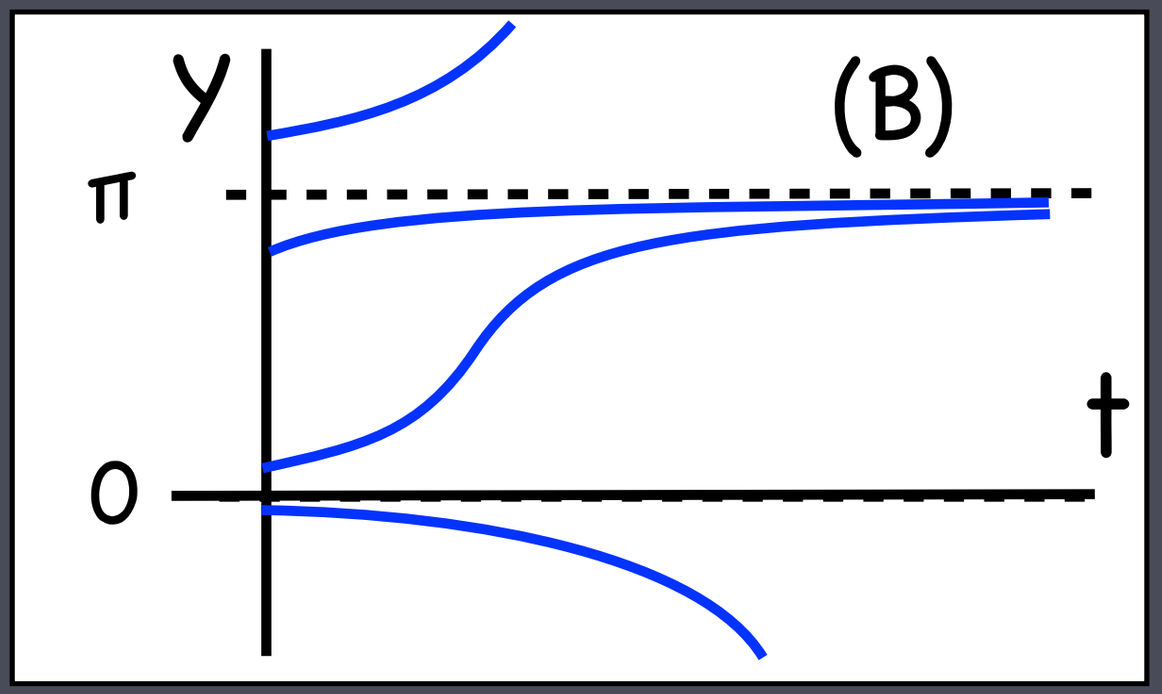

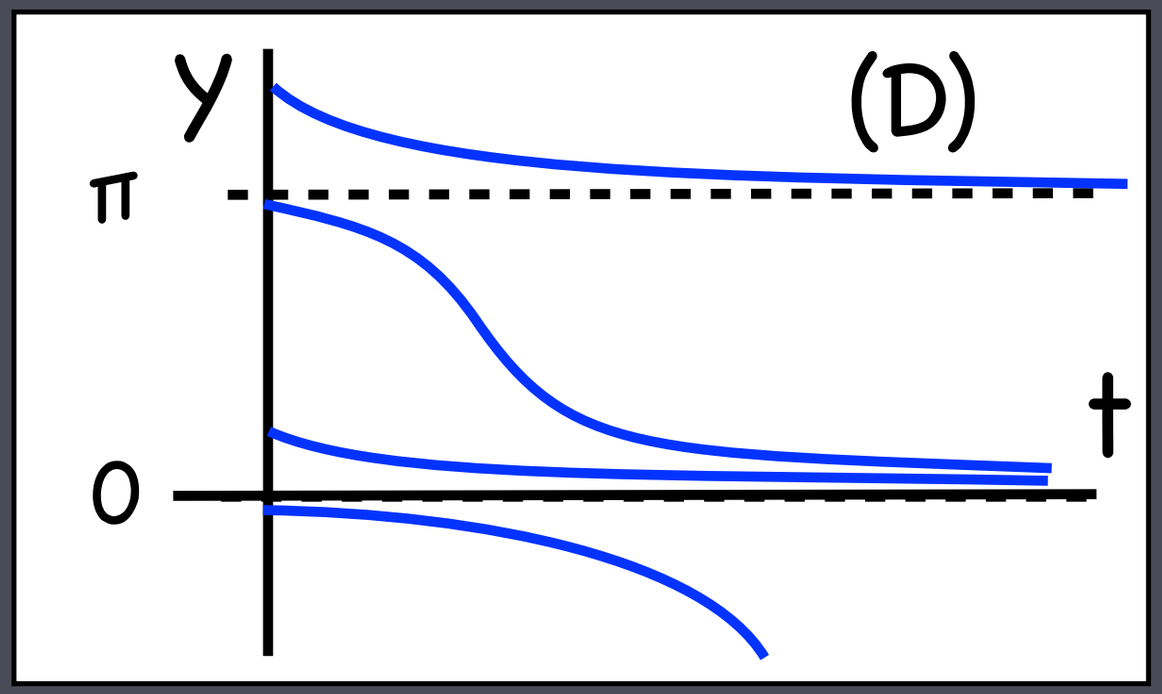

What you should be able to do:

- Identify steady states for a DE.
- Draw/interpret the phase line for a DE.
- Draw/interpret a slope field for a DE.
- Determine stability of steady states.
- Determine long-term behaviour of solutions.
- Sketch the graphs of solutions using phase line and/or slope fields (slopes, concavity, IPs, h-asymptotes).

Some biological examples

- Allee effect: $P' = rP\,(1-P/K)\,(P/T-1)$ where $T<K$
- Lac operon: $c' = c^2/(k^2+c^2) - ac$