Today

- Trig derivatives
- Related rates with trig
  - Zebra Danio
  - Hands on a clock
- Reminders:
  - Friday is the last day of classes :o
  - Exam: Dec 6 @ 3:30 pm - SRC ABC

Before taking Math 102, I was aware that mathematics can be applied to problems in the Life Sciences.

(A) Strongly agree.

(B) Agree.

(C) Neutral

(D) Disagree

(E) Strongly disagree

After taking Math 102, my awareness that mathematics can be applied to problems in the Life Sciences has increased.

(A) Strongly agree.

(B) Agree.

(C) Neutral

(D) Disagree

(E) Strongly disagree

Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to0} (f(x+h) - f(x))/ h$

$= \lim_{h\to0} (\sin(x+h) - \sin(x)) /h$

$= \lim_{h\to0} (\sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x)) /h$

$= \lim_{h\to0} (\sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

$= \sin(x) \lim_{h\to0} (\cos(h)-1)/h$

$+ \cos(x) \lim_{h\to0} \sin(h) /h$

See what h=0.0001 gives...

$= \sin(x) \times 0 + \cos(x) \times 1 = \cos(x).$

Note: this last step requires a bunch of work to show.

Derivative of $g(x)=\cos(x)$

- $g'(x) = -\sin(x)$
- See last lecture's posted slides.

Other trig functions

The derivative of cot(x) is

(A) csc(x)cot(x)

(B) -csc(x)cot(x)

(C) $csc^2(x)$

(D) $-csc^2(x)$

(E) $sec^2(x)$

Other trig functions

The derivative of cot(x) is

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(B) -csc(x)cot(x)

(C) $\csc^2(x)$

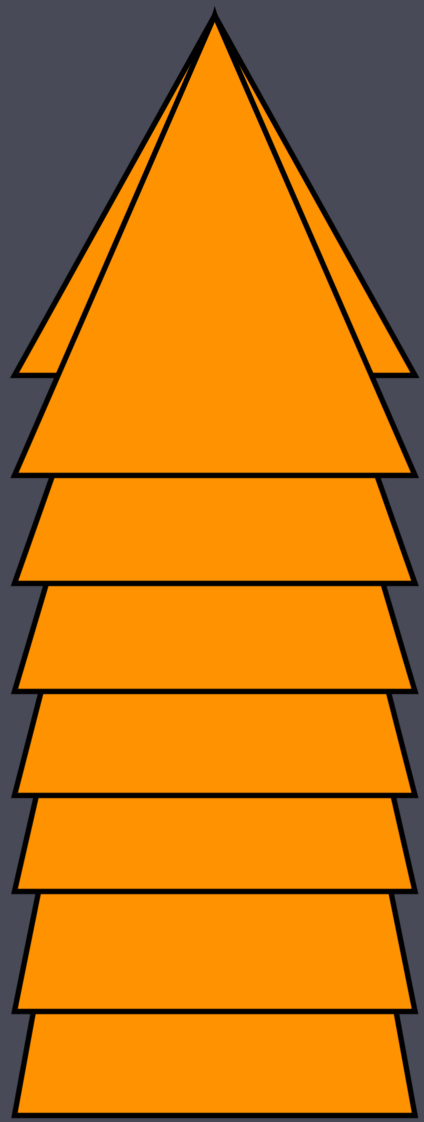
(D) $-\csc^2(x)$

(E) $\sec^2(x)$

Rewrite
cot(x) = cos(x)/sin(x)
and use quotient
rule.

Trig-related rates

- These usually come down to a triangle that changes in time. For example...



If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

Relate the two changing quantities (h and θ):

(A) $\sin(\theta) = 2/h$

(B) $\sin(\theta/2) = 1/h$

(C) $\sin(\theta/2) = 1/\mathrm{sqrt}(1+h^2)$

(D) $\tan(\theta) = 2/h$

(E) $\tan(\theta/2) = 1/h$

If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

Relate the two changing quantities (h and θ):

(A) sin(θ) = 2/h

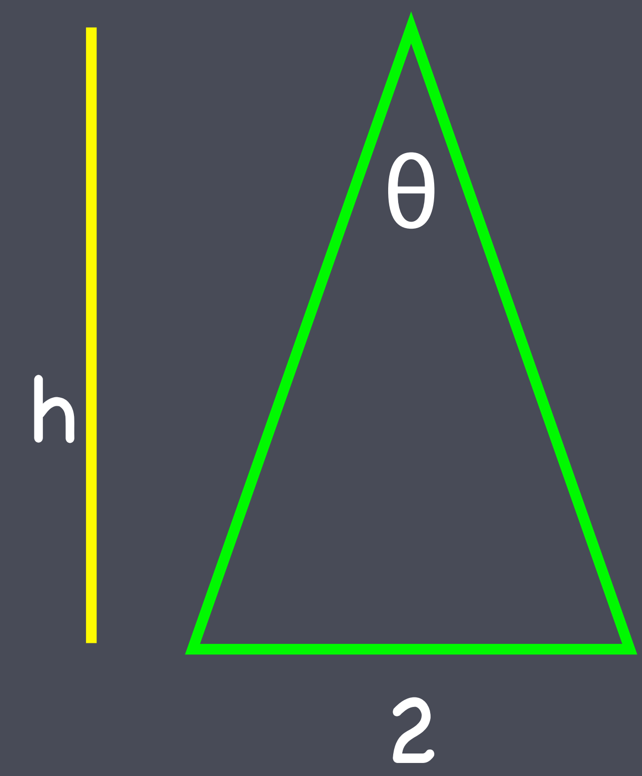
(B) sin(θ/2) = 1/h

(C) sin(θ/2) = 1/sqrt(1+h²)

This will get messy.

(D) tan(θ) = 2/h

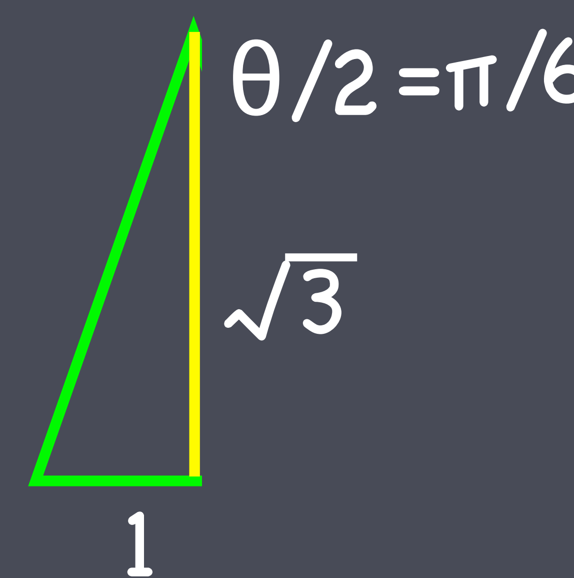
(E) tan(θ/2) = 1/h



If the height of an isosceles triangle with base 2m changes at a rate h'=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

- Take derivatives to relate their rates of change (h' and θ'):
  - $\tan(\theta/2) = 1/h$
  - $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$
  - $\theta' = -2\ h'/(h^2 \sec^2(\theta/2)) = -2\ h' \cos^2(\theta/2)\ /h^2$

    $= -2 \cos^2(\theta/2) \quad = -3/2$ radians/s



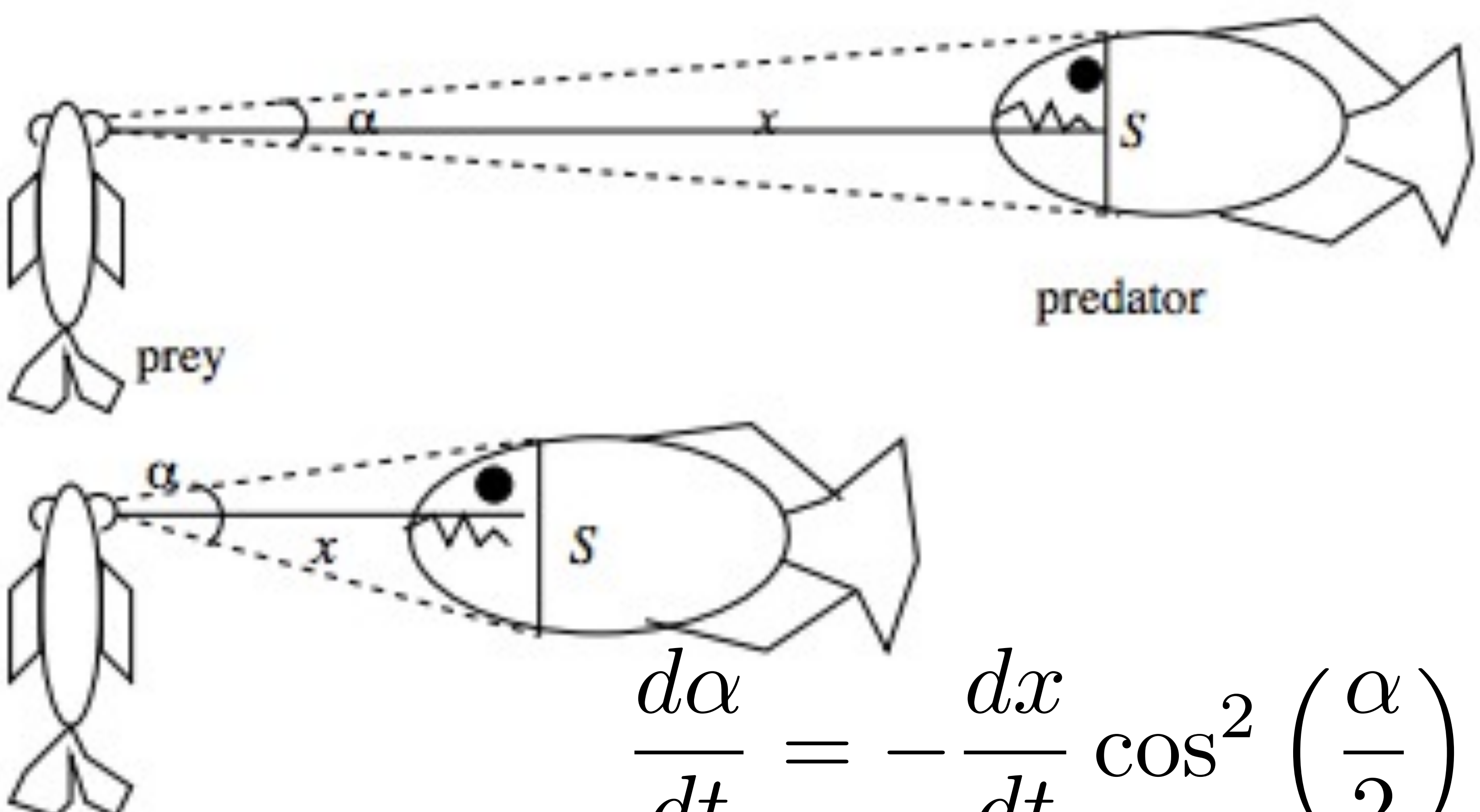
θ = ... (A) π/6 (B) π/4 (C) π/3 (D) 2π/3 (E) π.

Zebra Danio escape response

http://en.wikipedia.org/wiki/File:Zebrafisch.jpg

Zebra Danio escape response

10.9.1 The Zebra danio and its escape response



$$\frac{d\alpha}{dt} = -\frac{dx}{dt}\cos^2\left(\frac{\alpha}{2}\right)\frac{S}{x^2}$$

ZD tries to escape when α′ is above a threshold value.

What is $\cos^2(a)$ when $\tan(a)=p/q$?

(A) $(p^2+q^2) / q^2$

(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) p^2/q^2

What is $\cos^2(a)$ when $\tan(a)=p/q$?

(A) $(p^2+q^2) / q^2$

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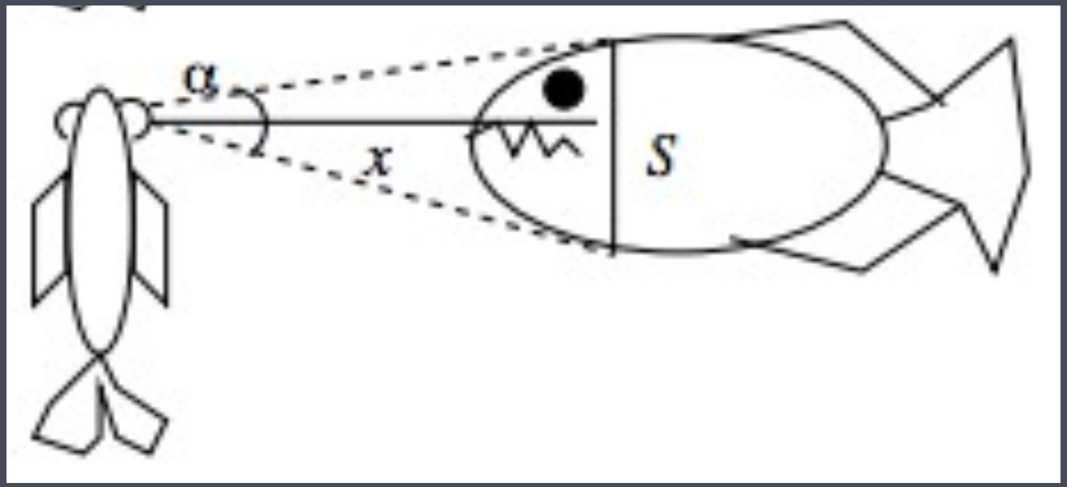
(D) $q^2 / (p^2+q^2)$

(E) p^2/q^2



$$\frac{d\alpha}{dt} = -\frac{dx}{dt}\cos^2\left(\frac{\alpha}{2}\right)\frac{S}{x^2}$$

$$= -\frac{dx}{dt}\frac{x^2}{x^2+\frac{S^2}{4}}\frac{S}{x^2}$$

$$= -\frac{dx}{dt}\frac{S}{x^2+\frac{S^2}{4}} = v\frac{S}{x^2+\frac{S^2}{4}}$$

Assuming the Zebra Danio reacts to a rapidly changing optical angle α, it will try to escape from...

(A) ...a very large predator (large S).

(B) ...a very small predator (small S).

(C) ...a predator that is far away (large x).

(D) ...a slow-moving predator (small v).

(E) ...a fast-moving predator (large v).

$$\frac{d\alpha}{dt} = v \frac{S}{x^2 + \frac{S^2}{4}}$$

Assuming the Zebra Danio reacts to a rapidly changing optical angle α, it will try to escape from...

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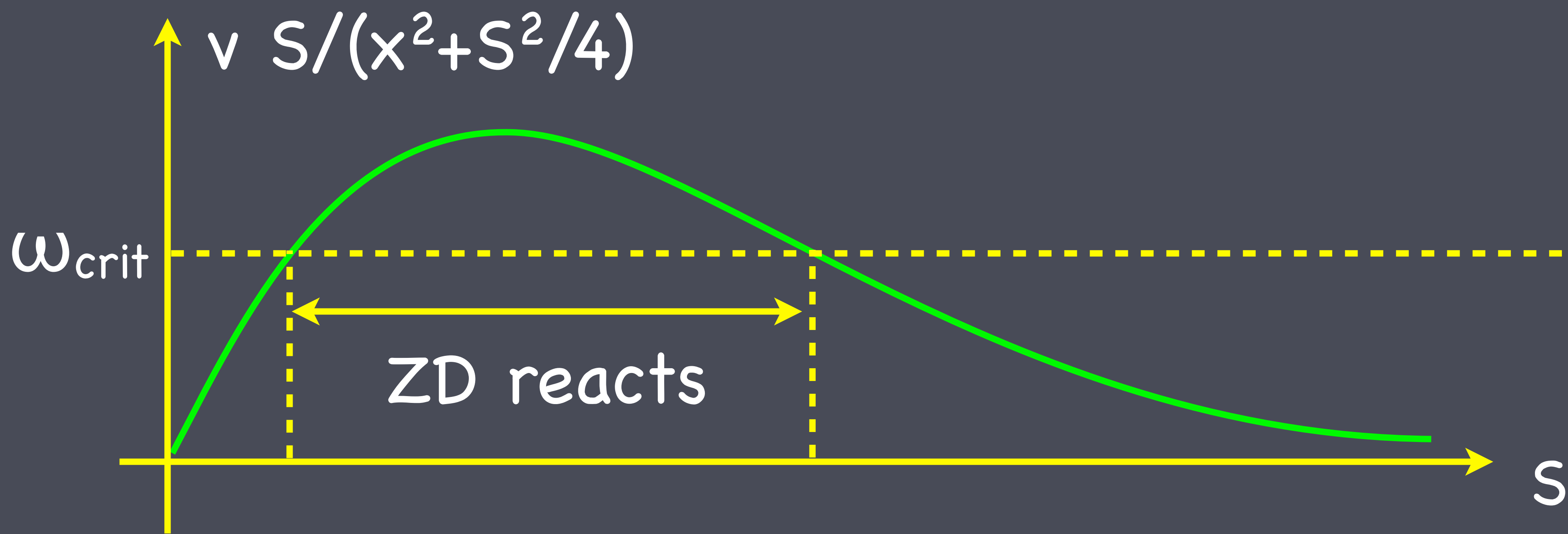
$$\frac{d\alpha}{dt} = v\frac{S}{x^2 + \frac{S^2}{4}}$$

If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator distance x constant, plot
$\alpha' = v\,S/(x^2+S^2/4)$ as function of S.

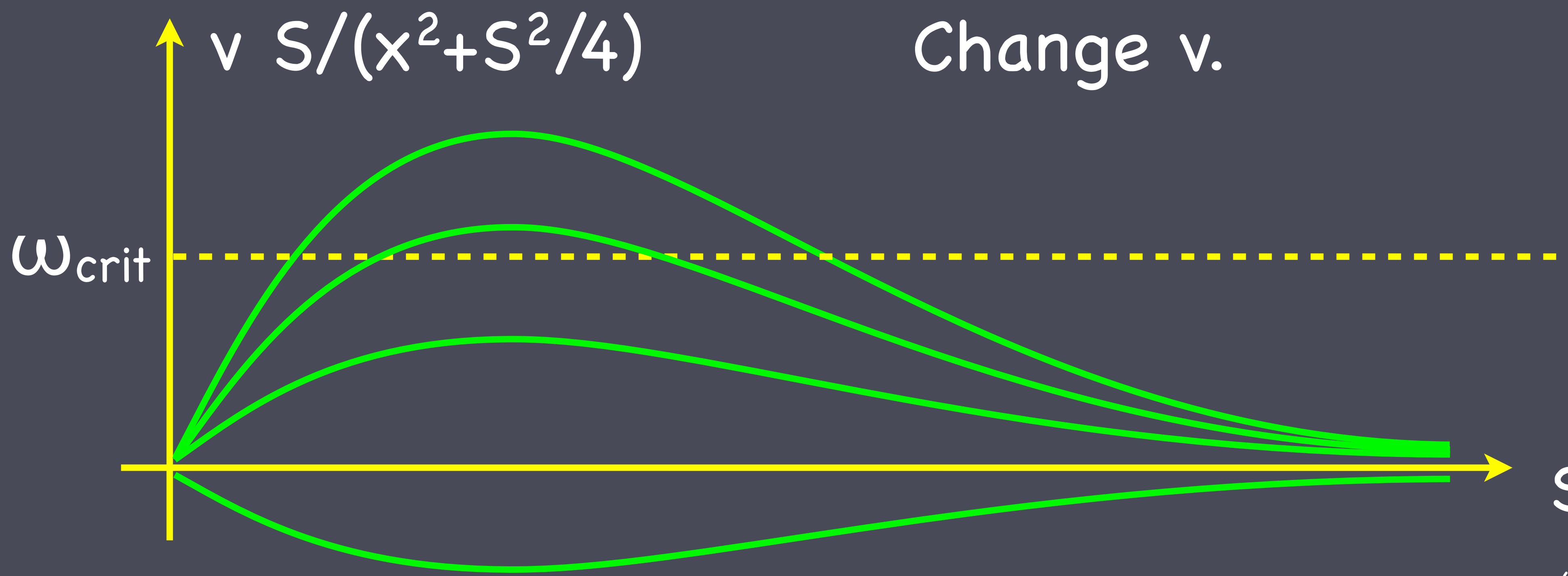


(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator distance x constant, plot $\alpha' = v\, S/(x^2+S^2/4)$ as function of S.

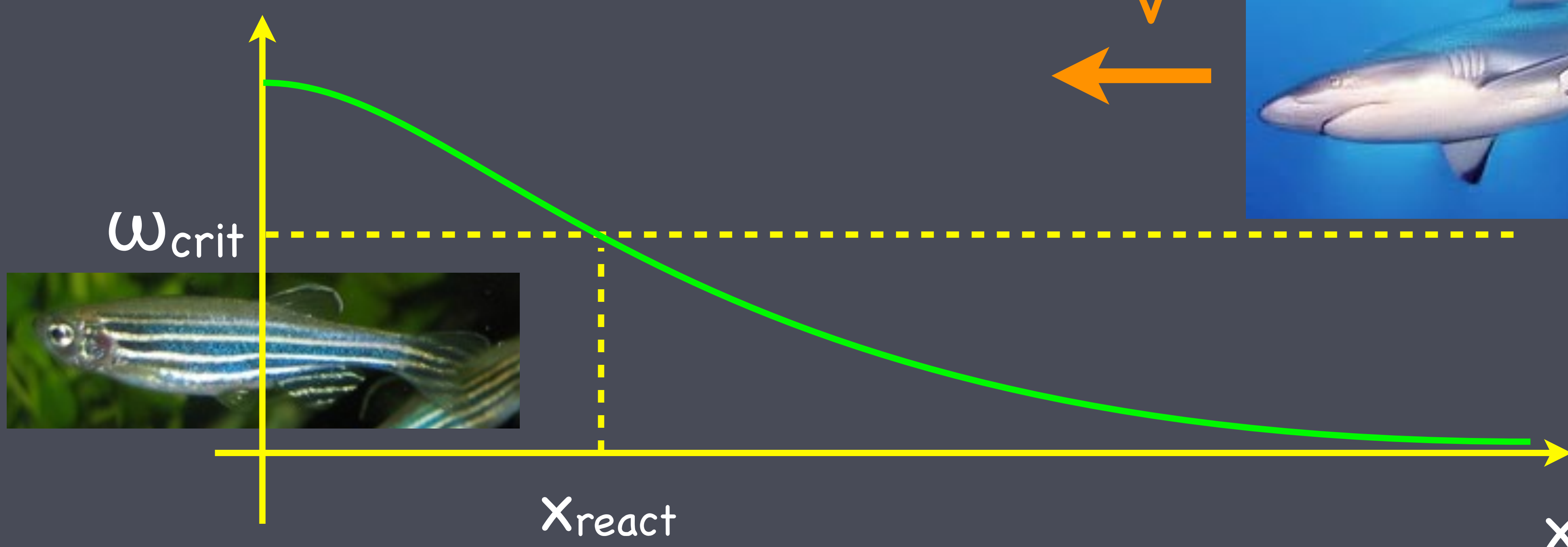
Change v.



(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away.
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator size S constant, plot
$\alpha' = v\, S/(x^2+S^2/4)$ as function of x.



(A) ...a very large predator.
(B) ...a very small predator.
(C) ...a predator that is far away
(D) ...a slow-moving predator.
(E) ...a fast-moving predator.

Shark image - http://en.wikipedia.org/wiki/File:Tibur%C3%B3n.jpg

Triangle with two sides of fixed length, angle between them changes.

Relate the two changing quantities:

(A) $a^2 = b^2 + c^2$

(B) $a^2 = b^2 + c^2 - 2bc\cos(\theta)$

(C) $a/\sin(A) = b/\sin(B)$

(D) $\sin(\theta) = a/b$