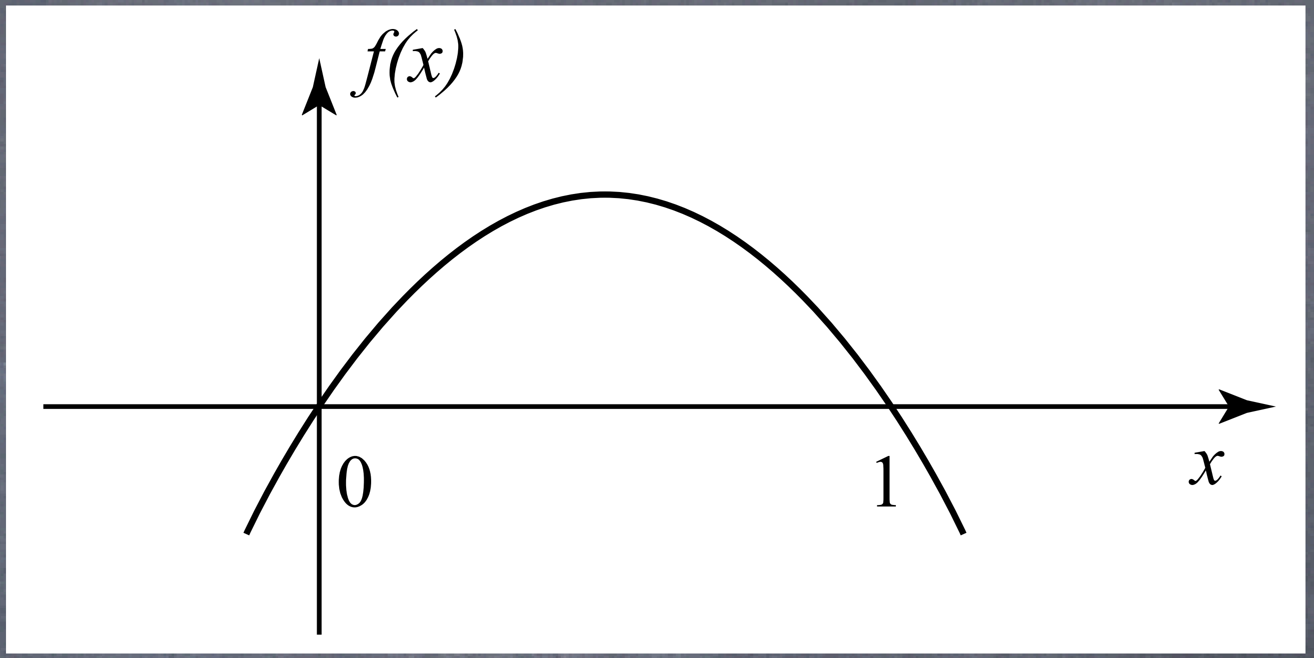
Today

- More on qualitative analysis of DEs
- More on slope fields
- A Newton's method example (spreadsheet)
- A few DE examples

Determine stability

$$x' = x(1 - x)$$



- If you start at x(0)=-0.01, the solution

(A) increases

(B) decreases

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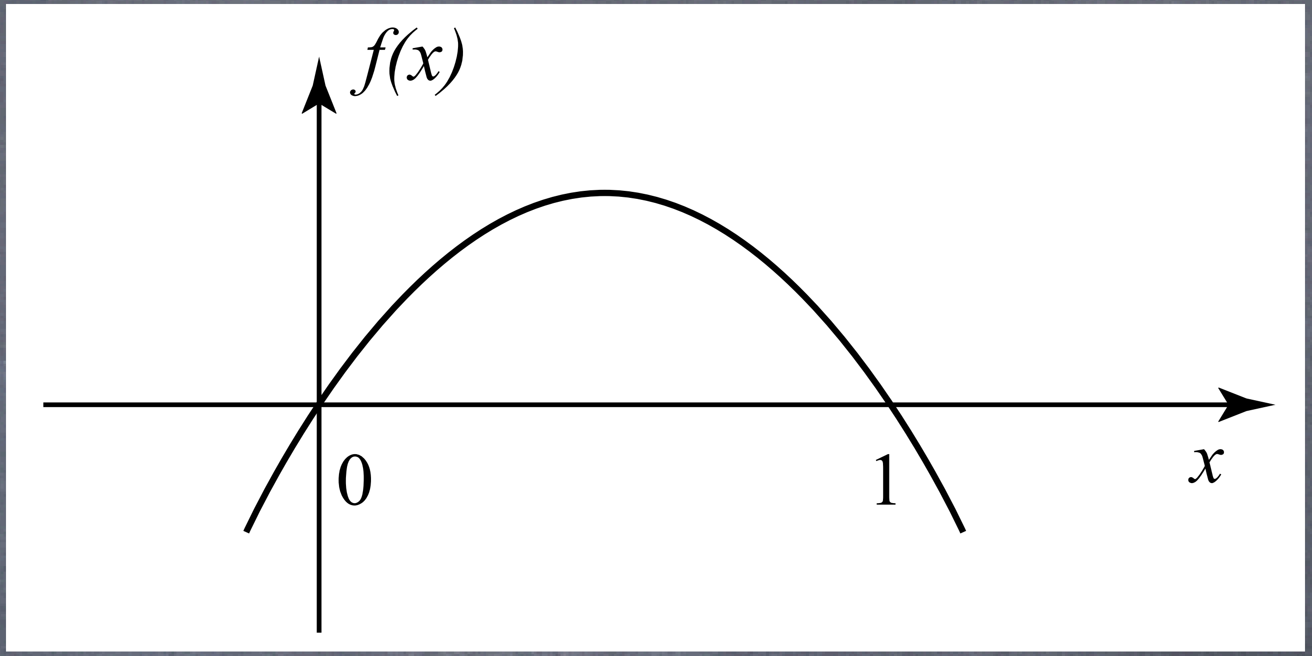

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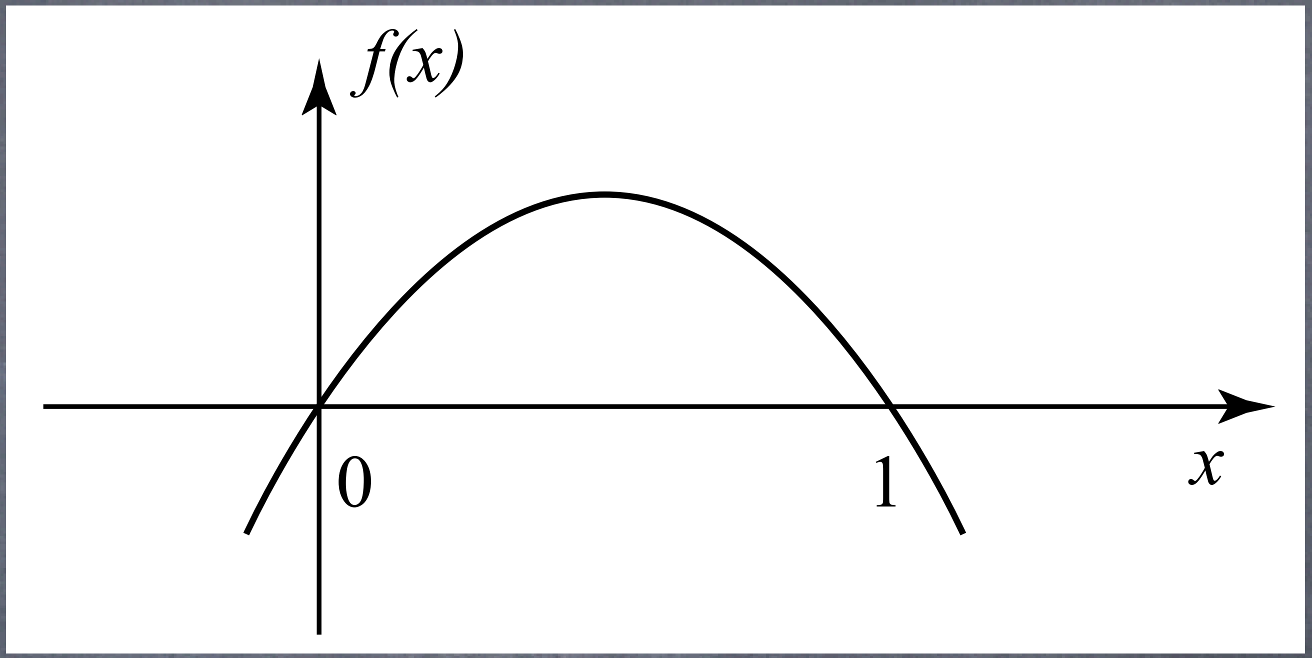

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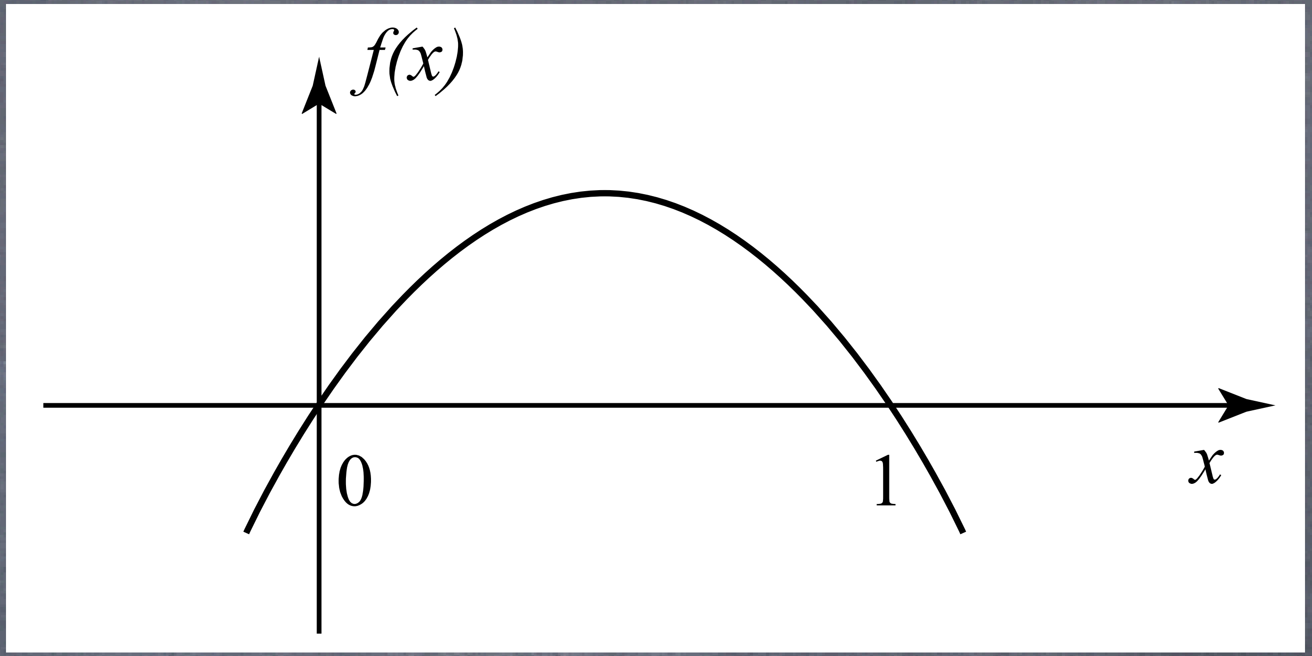

- If you start at x(0)=1.01, the solution

(A) increases

(B) decreases

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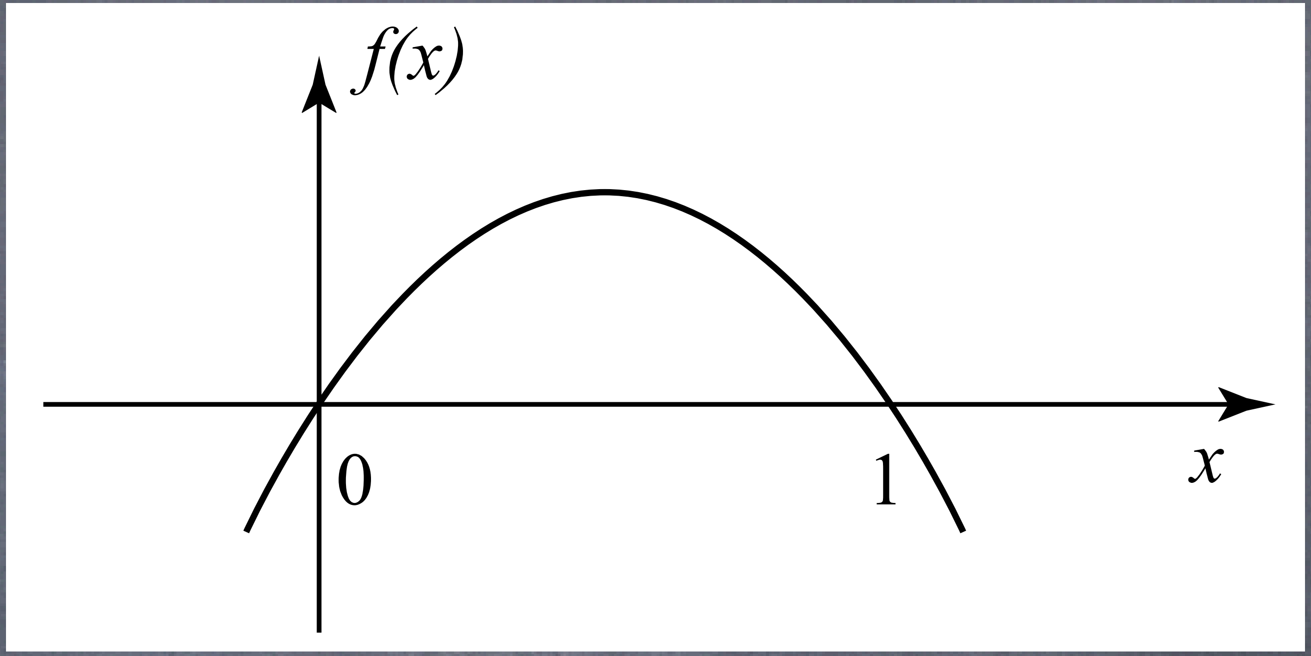

(A) Both x(t)=0 and x(t)=1 are stable steady states.

(B) x(t)=0 is stable and x(t)=1 is unstable.

(C) x(t)=0 is unstable and x(t)=1 is stable.

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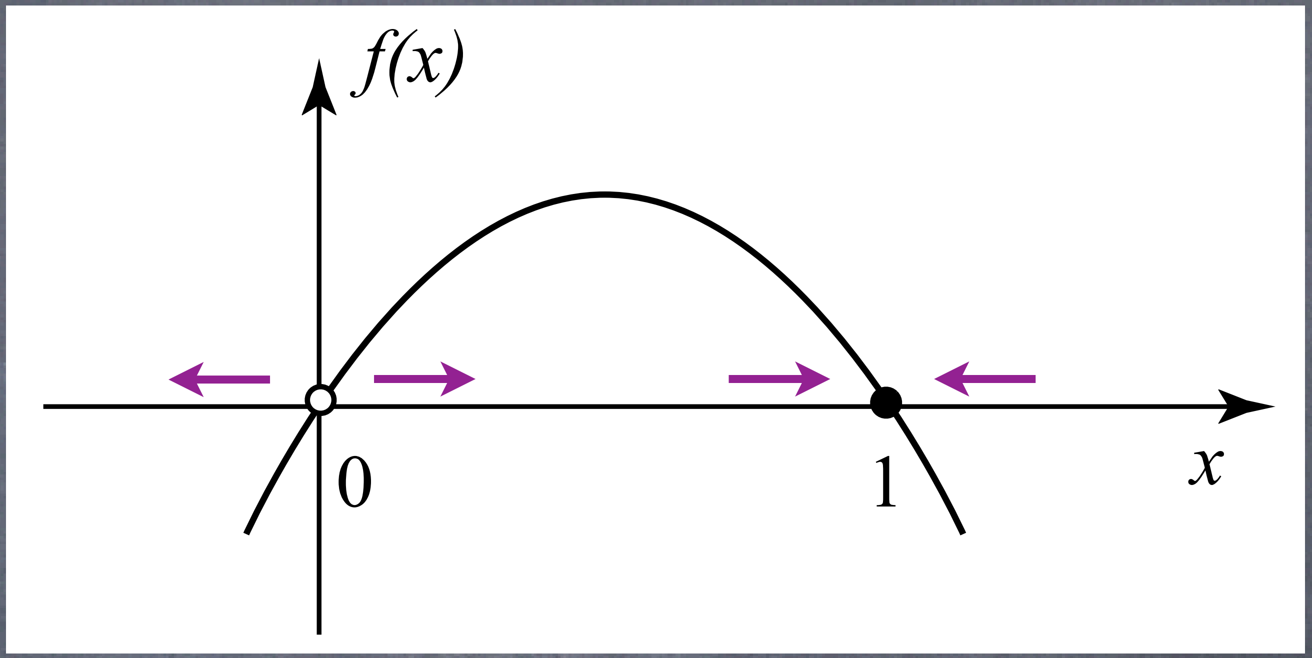

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Stable - solid dot. Unstable - empty dot.

Given that position tells you velocity, i.e. $x' = f(x)$, which of the following is false?

(A) A solution $x(t)$ cannot have a local max (as a function of t).

(B) If $x(t)$ is a solution then so is $y(t) = x(t-c)$.

(C) If $x(t)$ is a solution then so is $y(t) = x(t) + C$.

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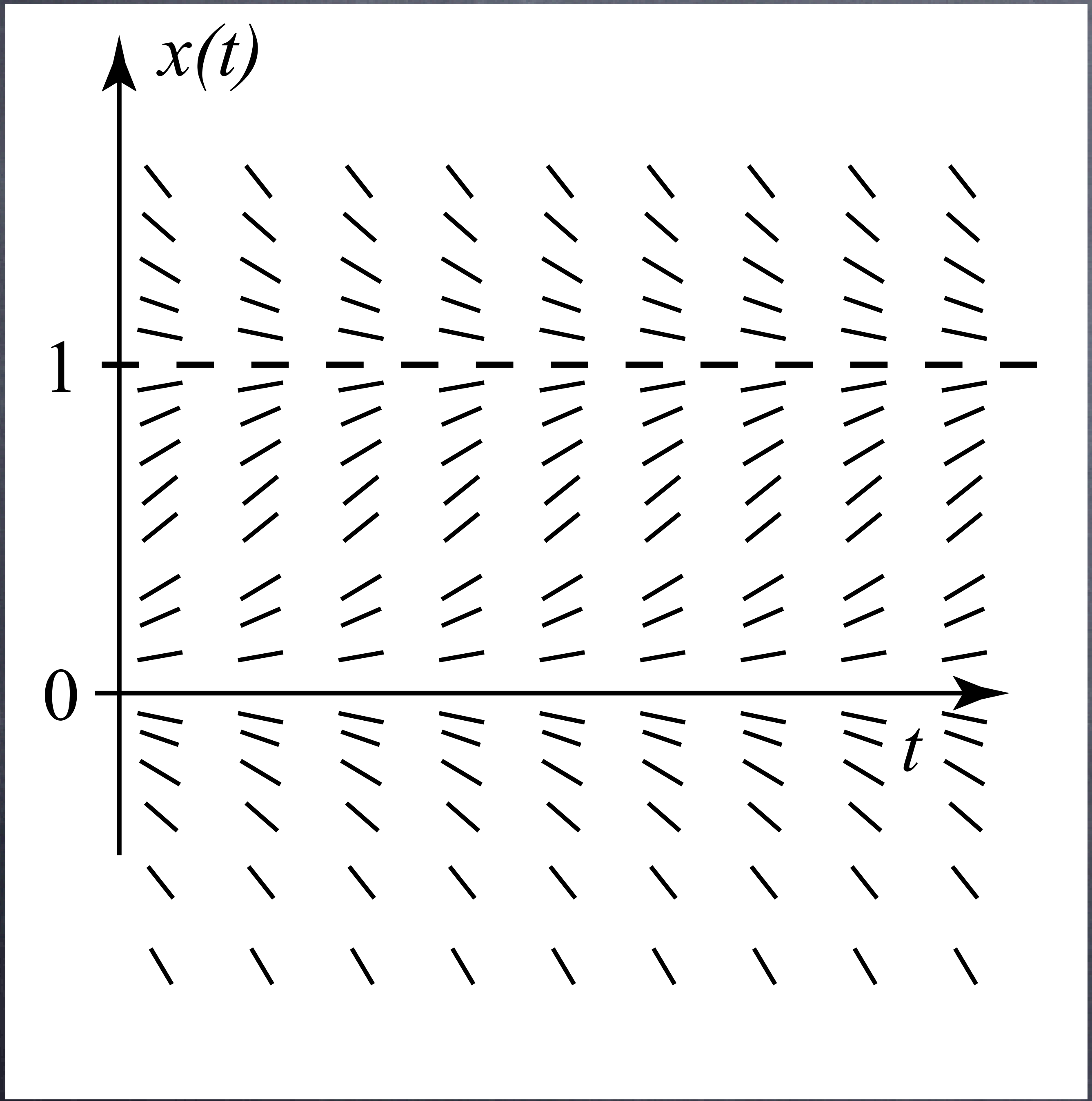

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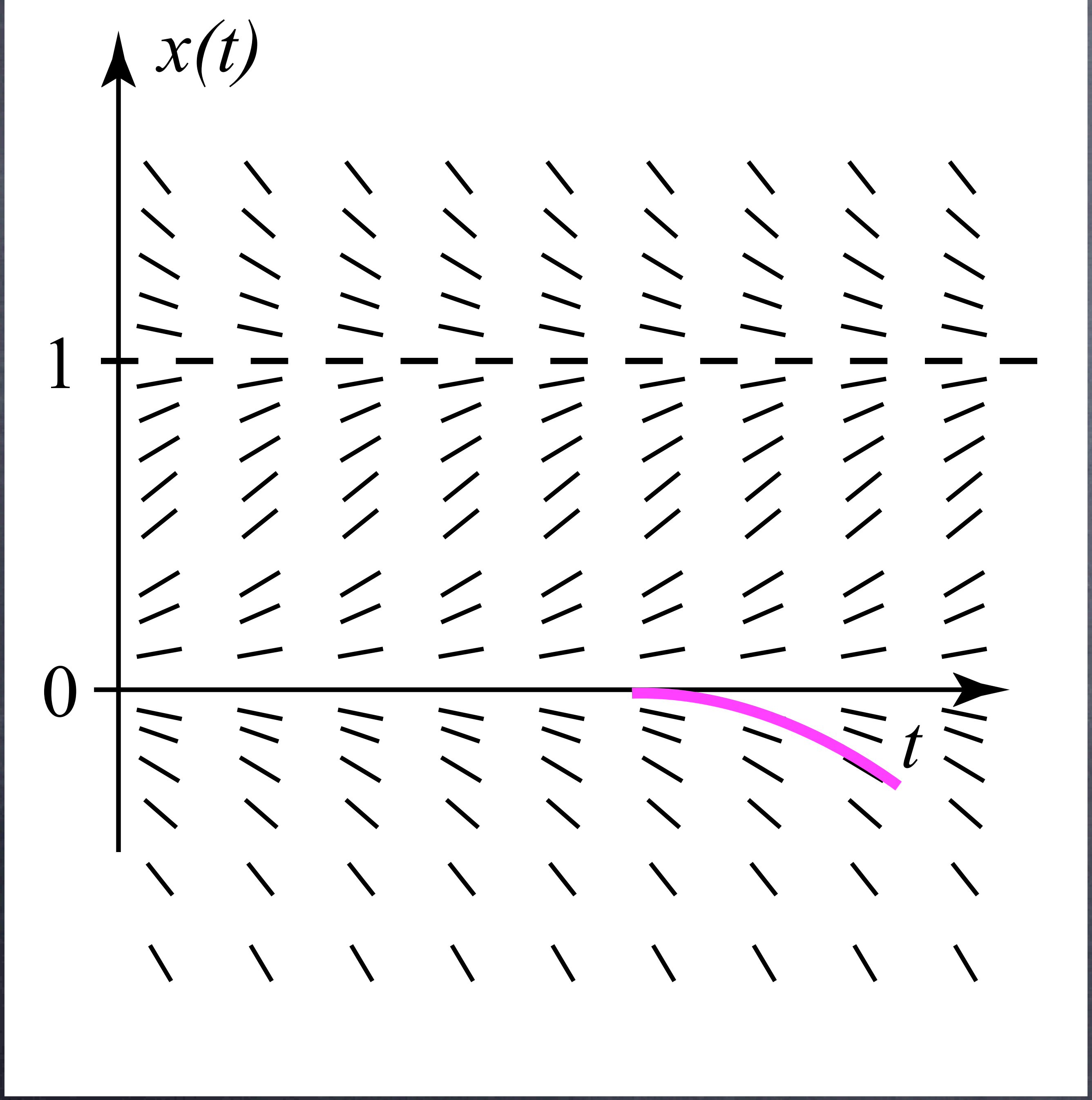

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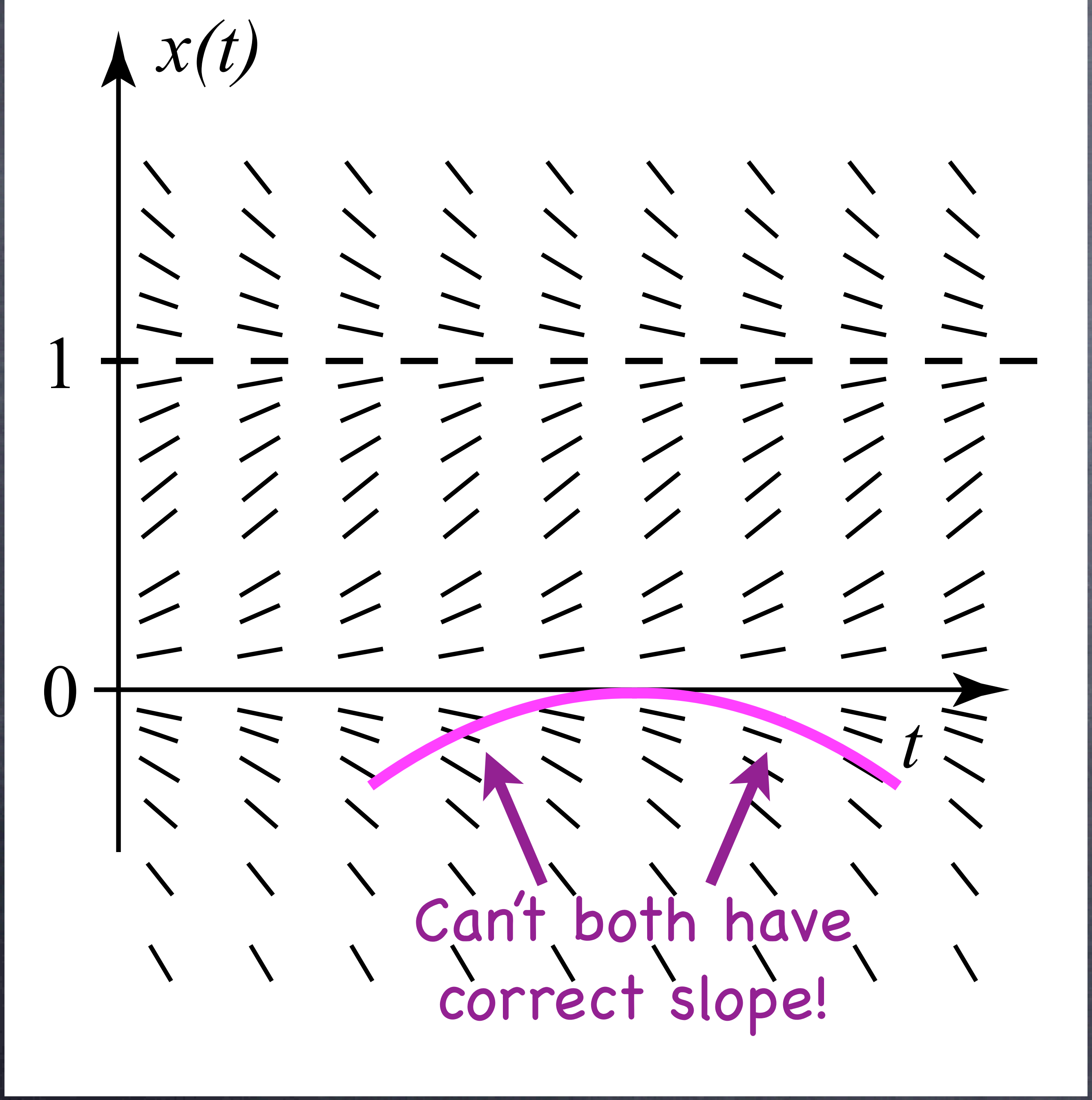

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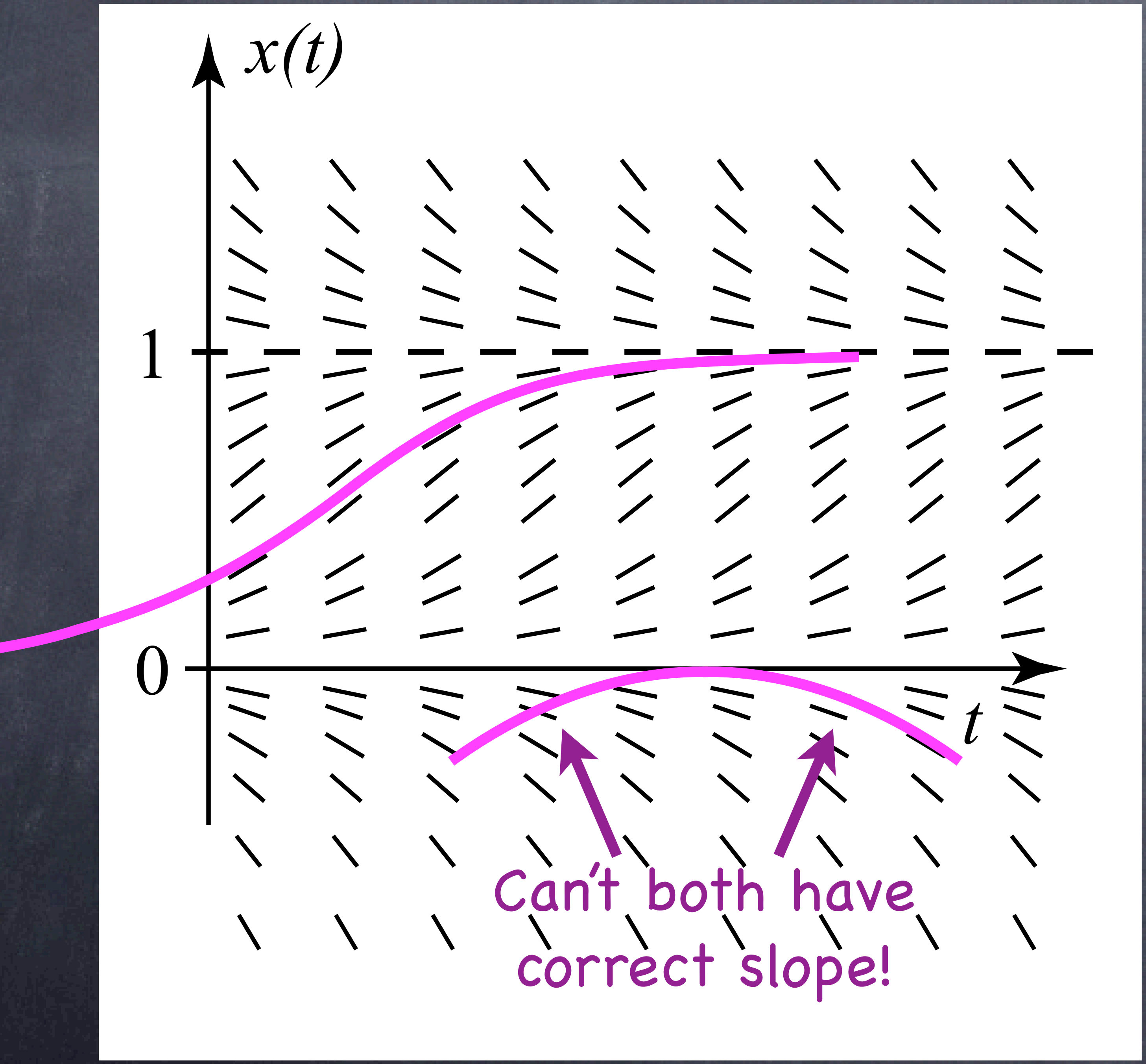

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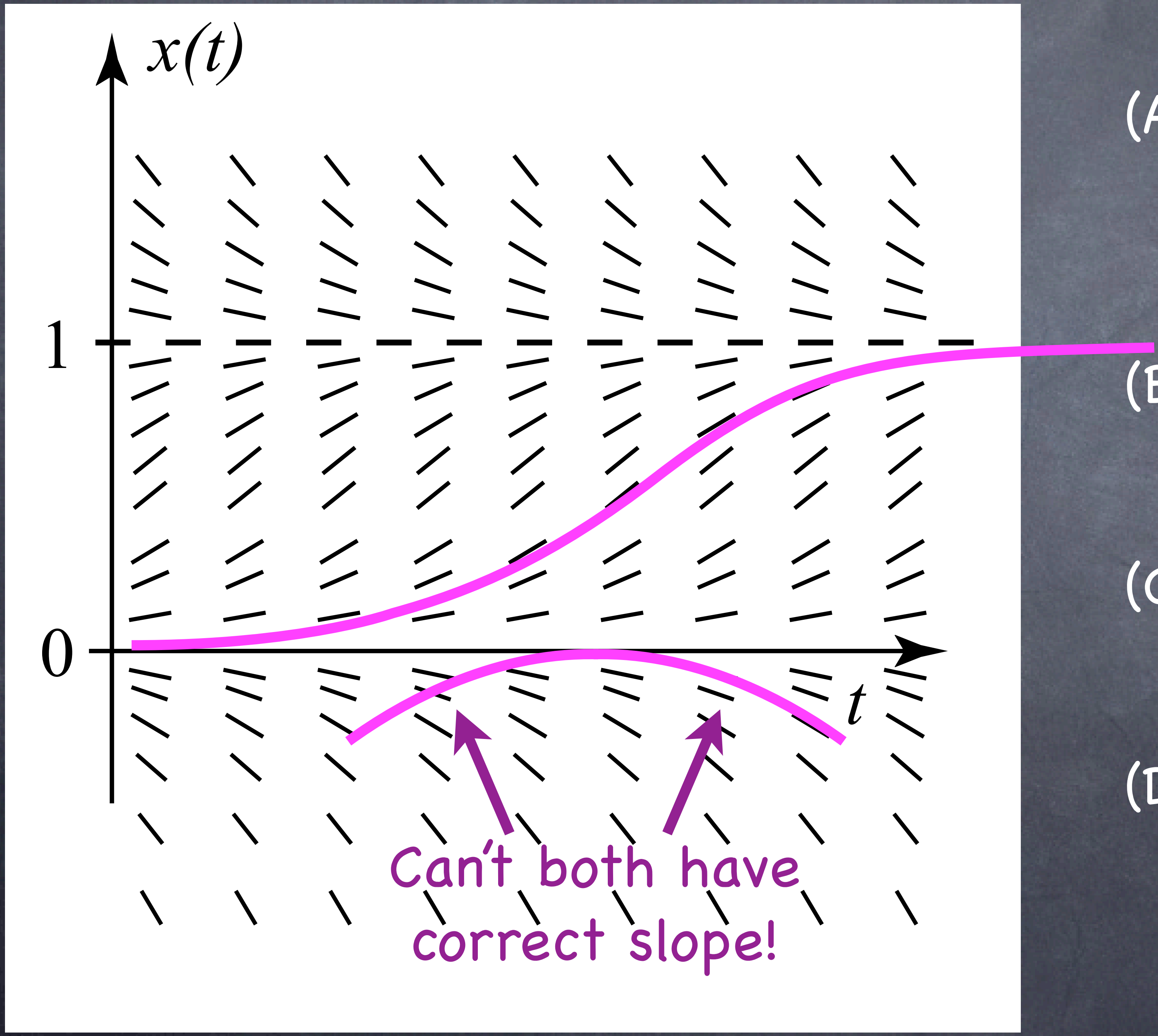

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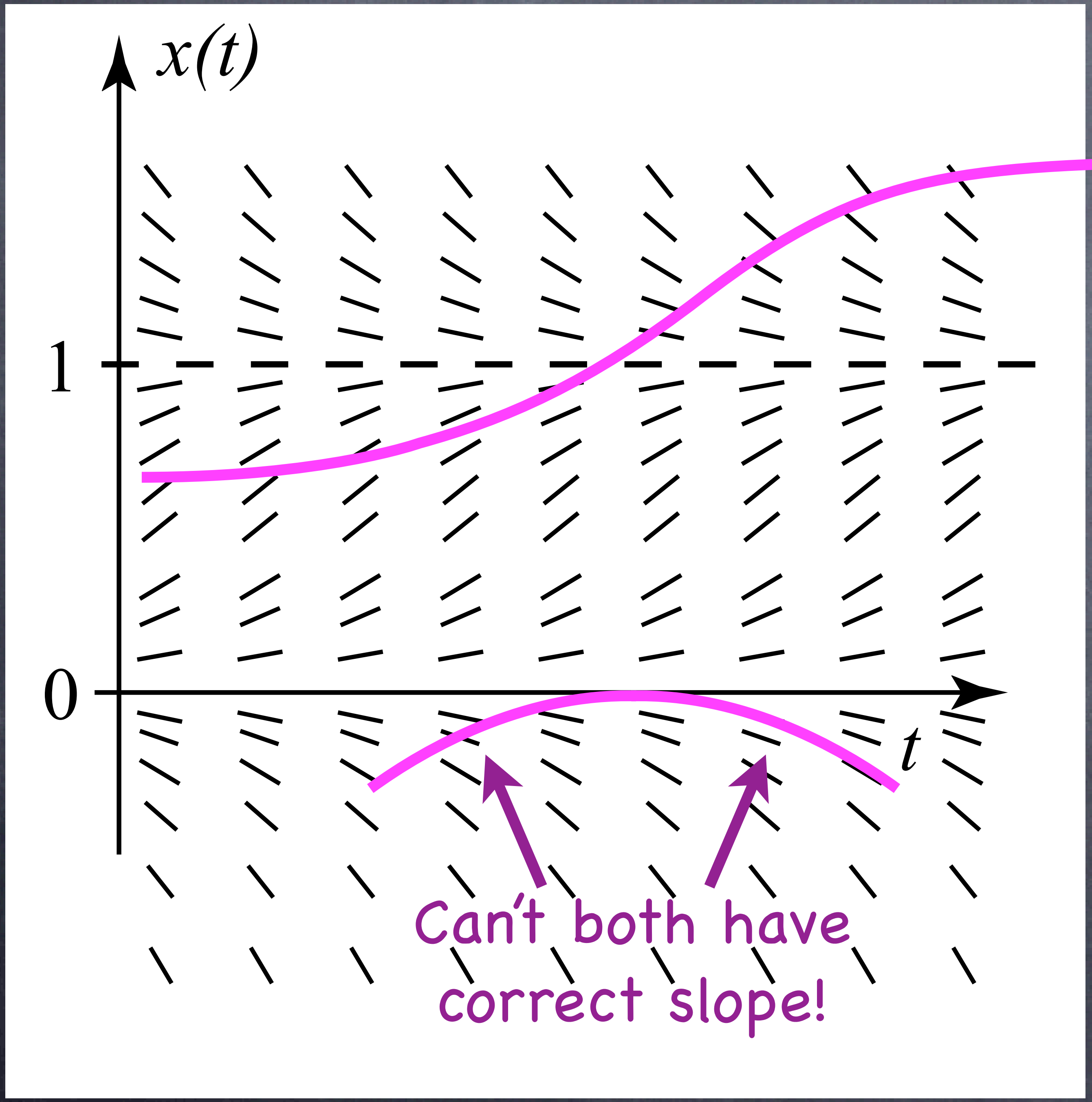

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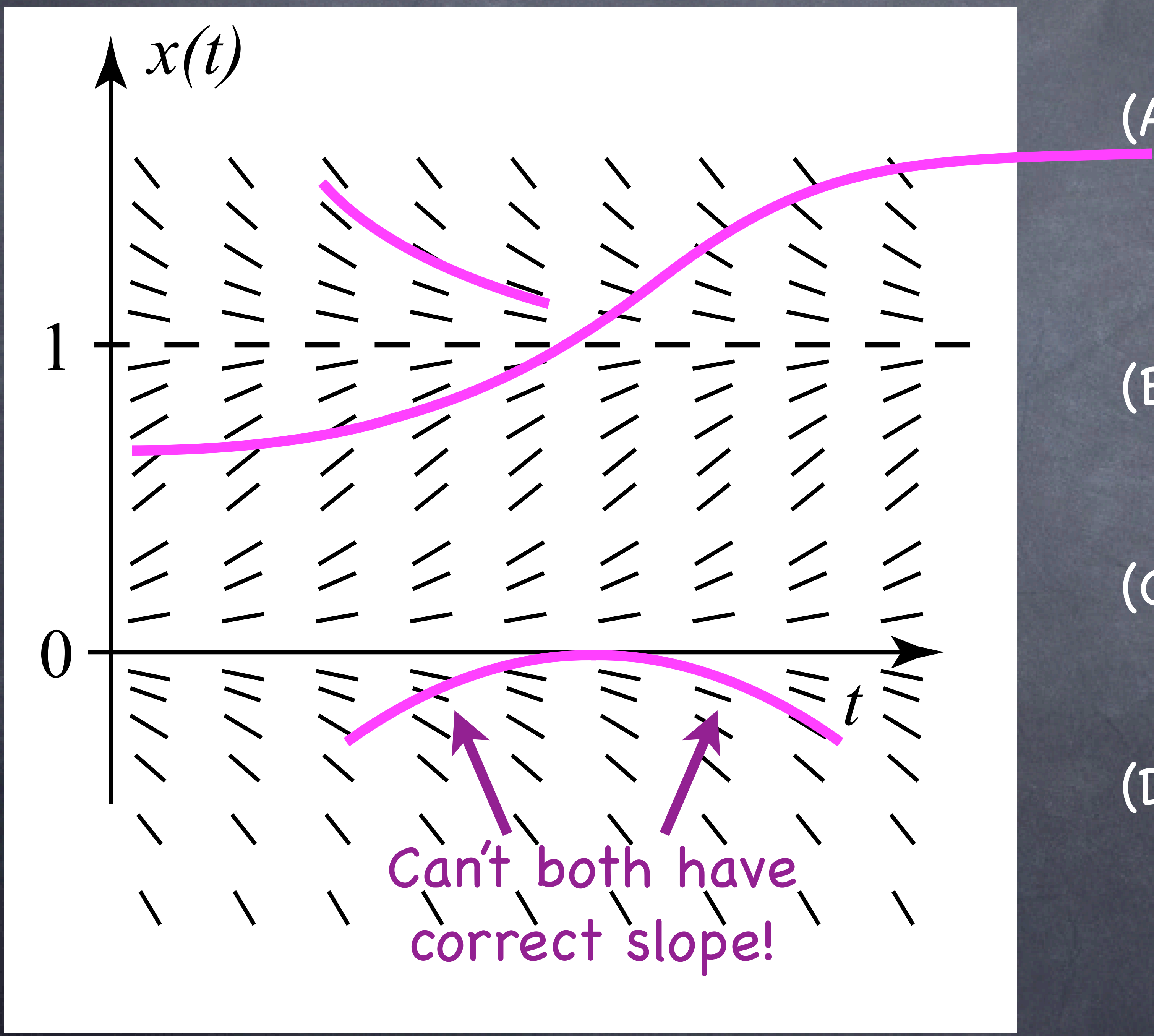

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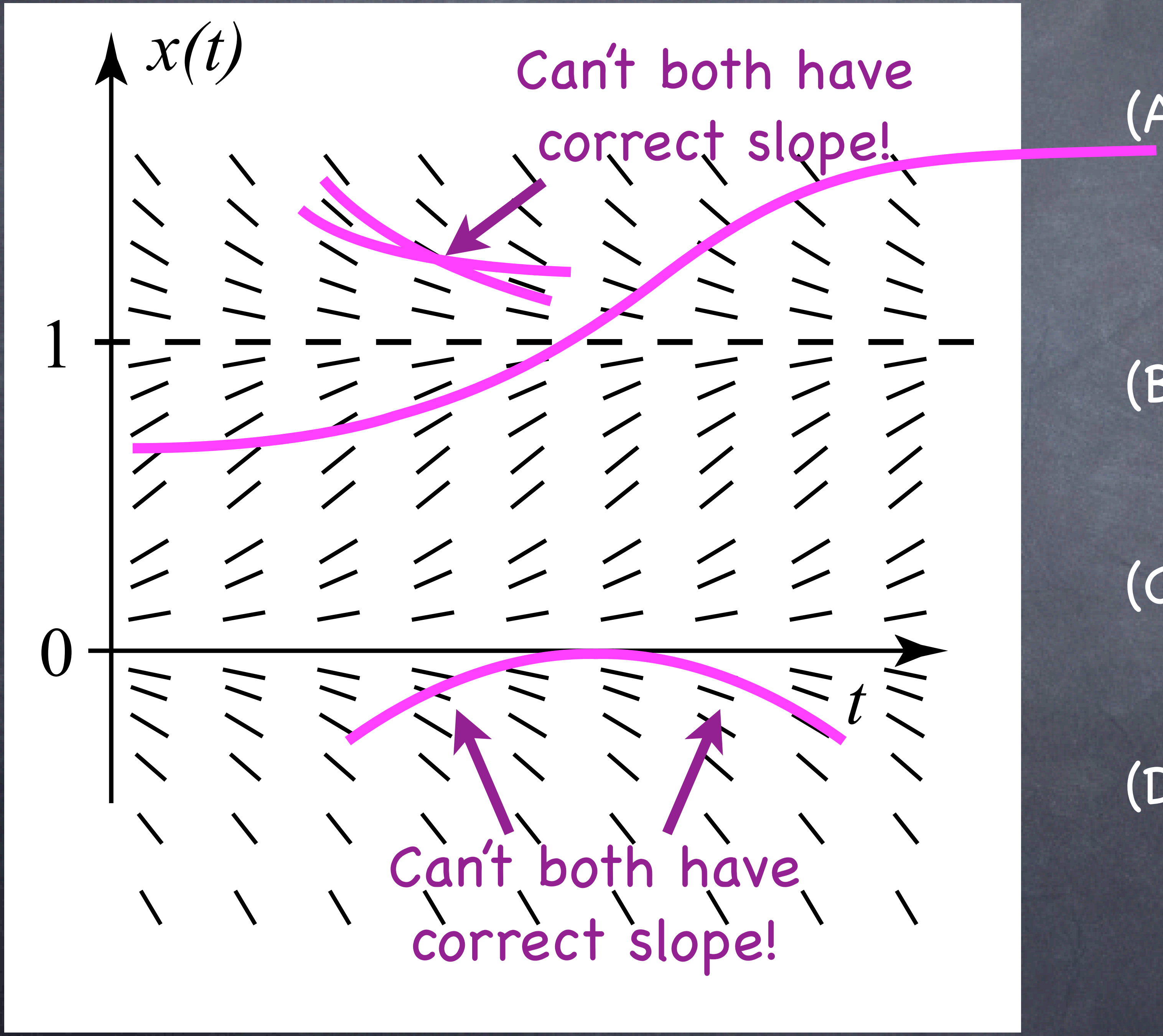

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More general slope fields

- For $x' = x(1-x)$, slope depended only on position, not time.
- General case: $$\frac{dy}{dt} = f(y, t)$$ ← t appears explicitly!

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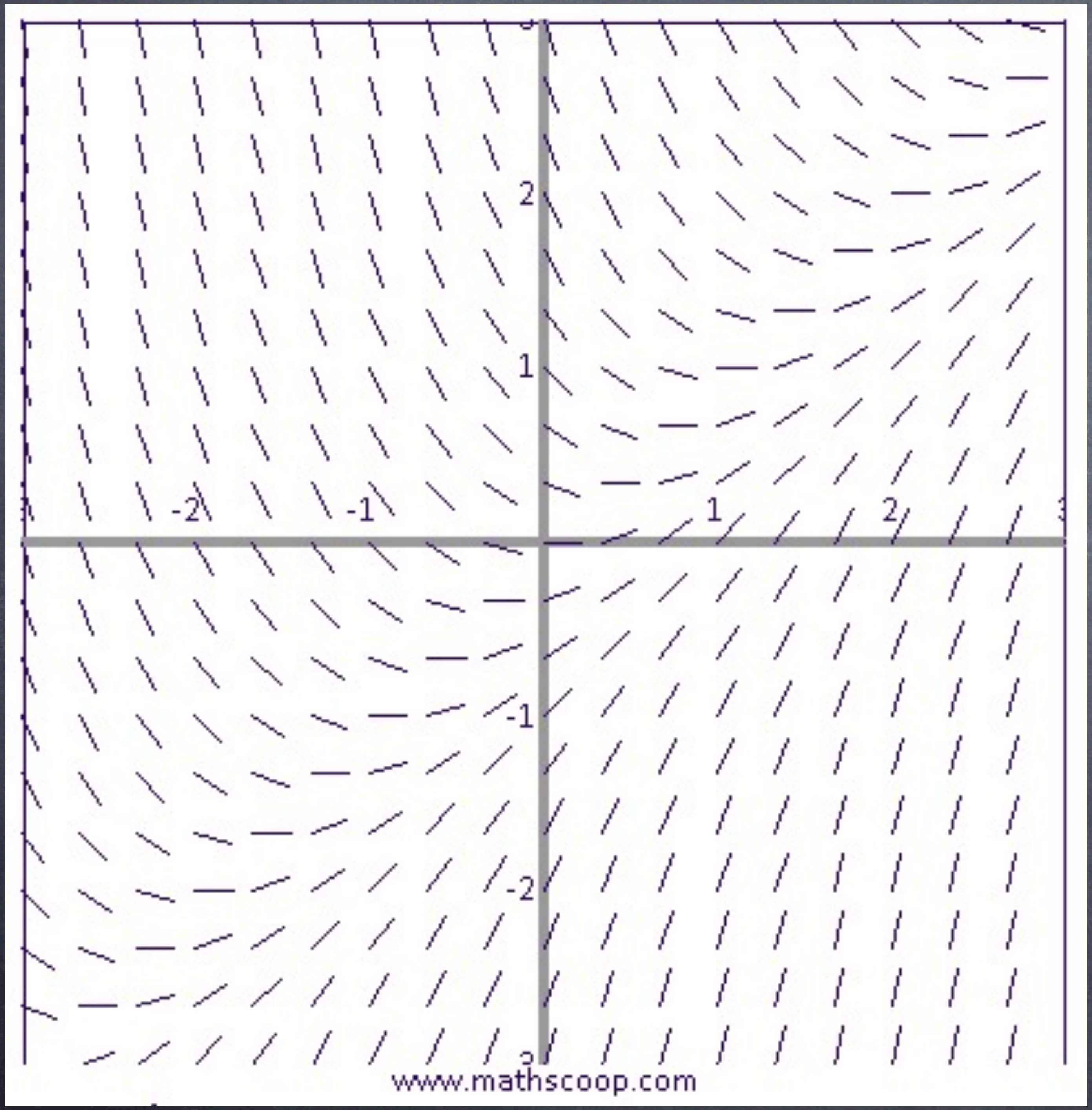
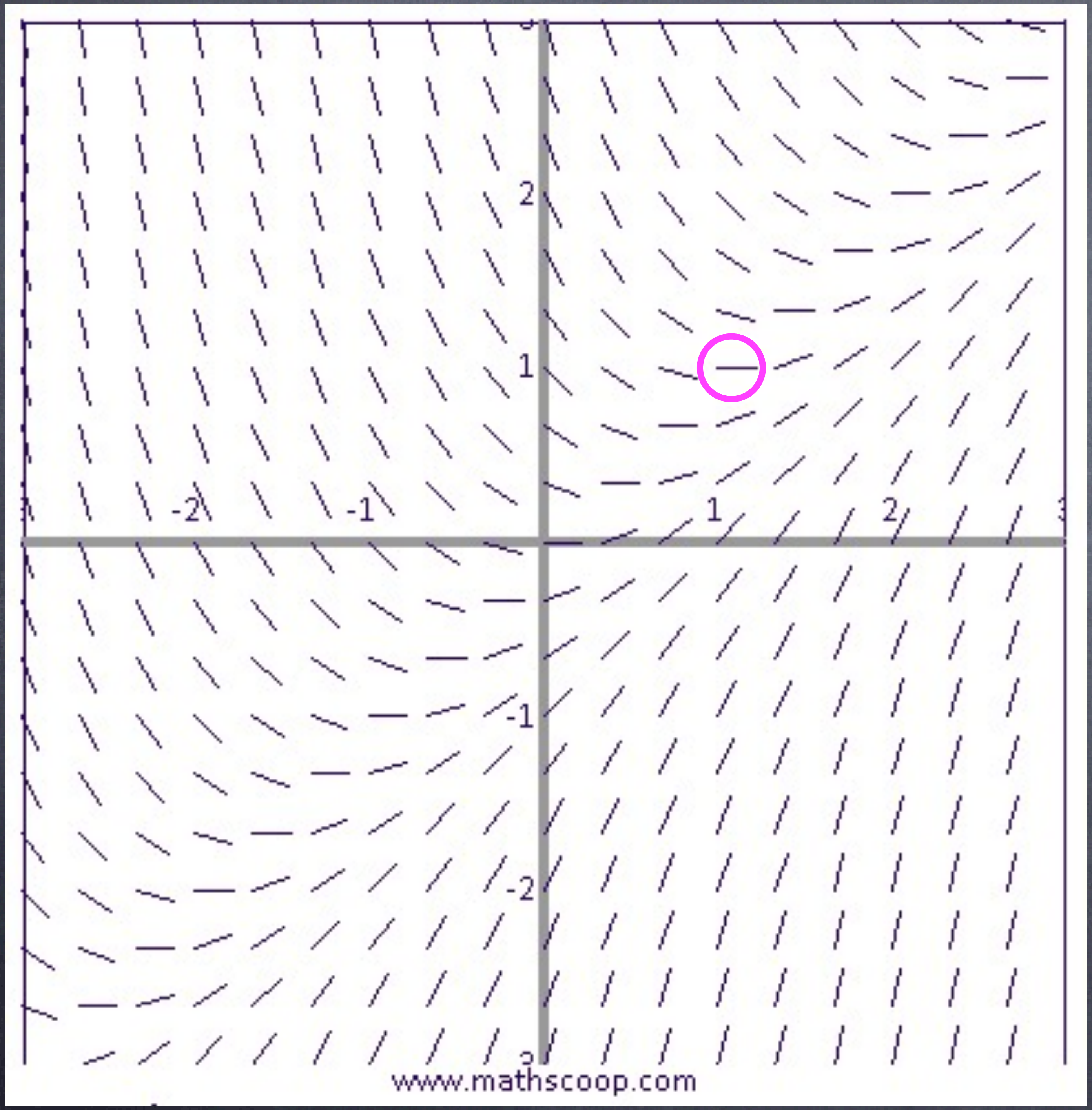
- General case: $$\frac{dy}{dt} = f(y, t)$$ ↖ t appears explicitly!
- Example: $$\frac{dy}{dt} = t - y$$

Slope field for y'=t-y

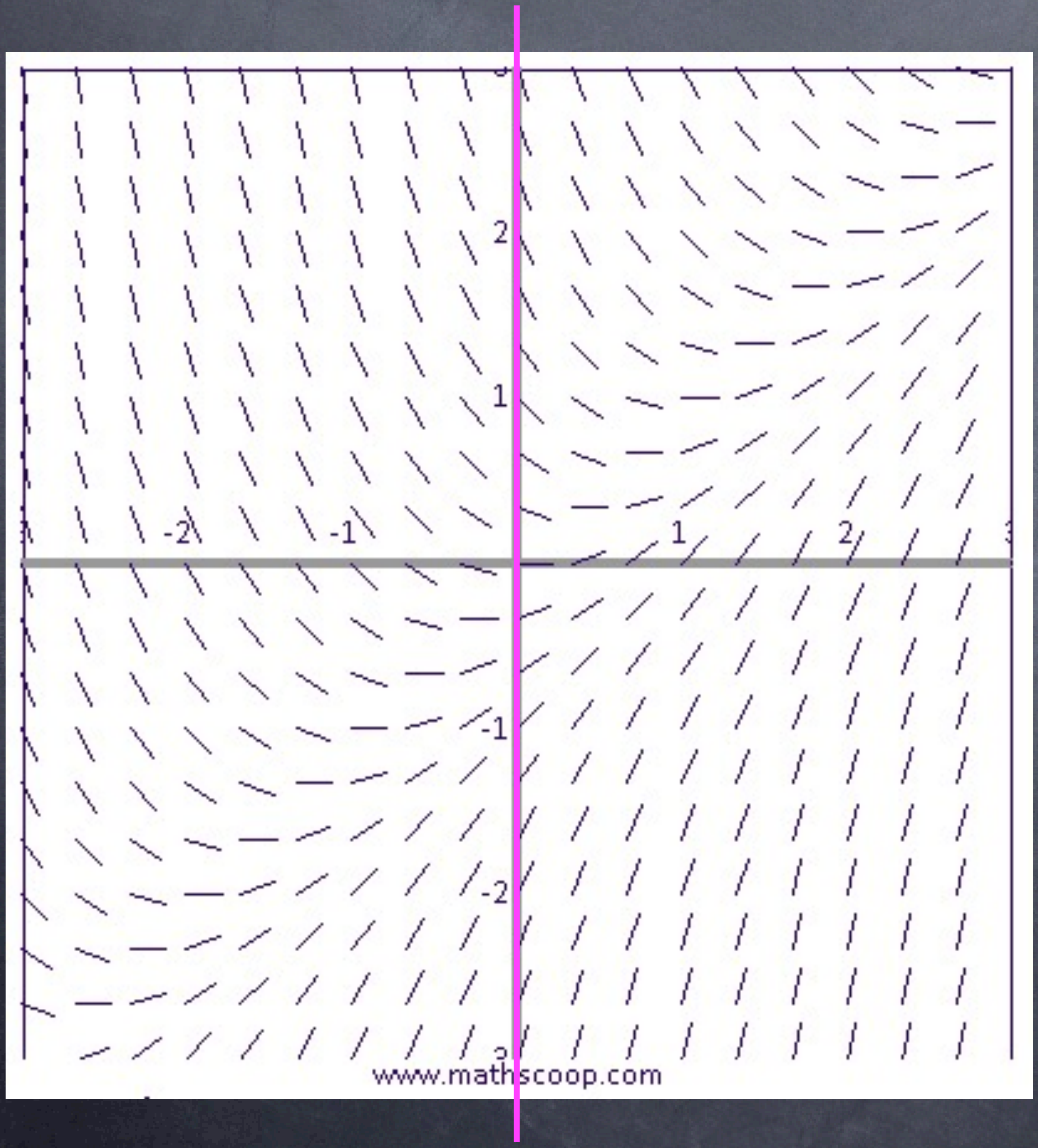


Slope field for y′=t–y



At (1,1), y′ = 0.

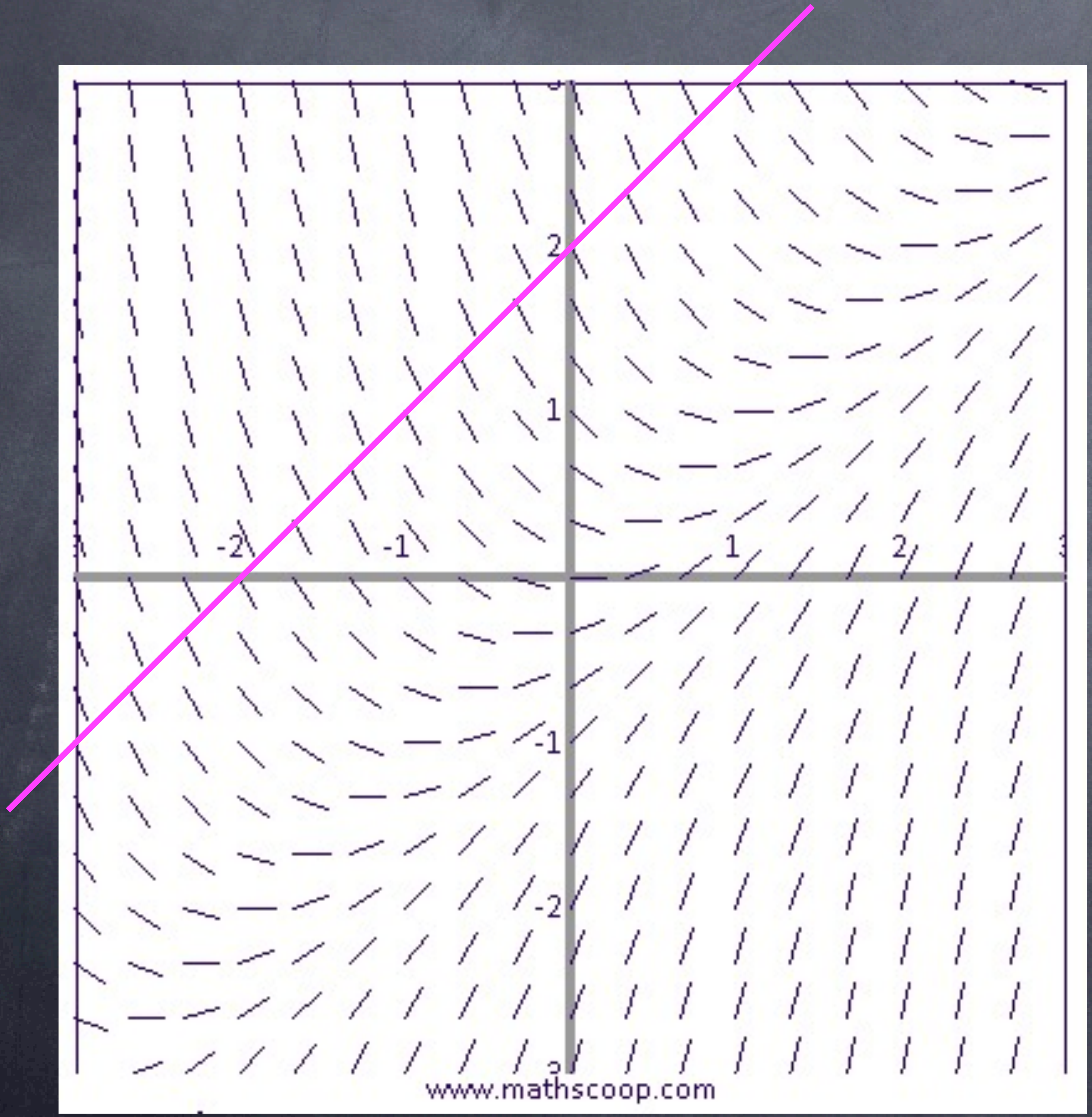
Slope field for y′=t–y



At (1,1), y′ = 0.

At (0,a), y′ = –a.

Slope field for y′=t−y

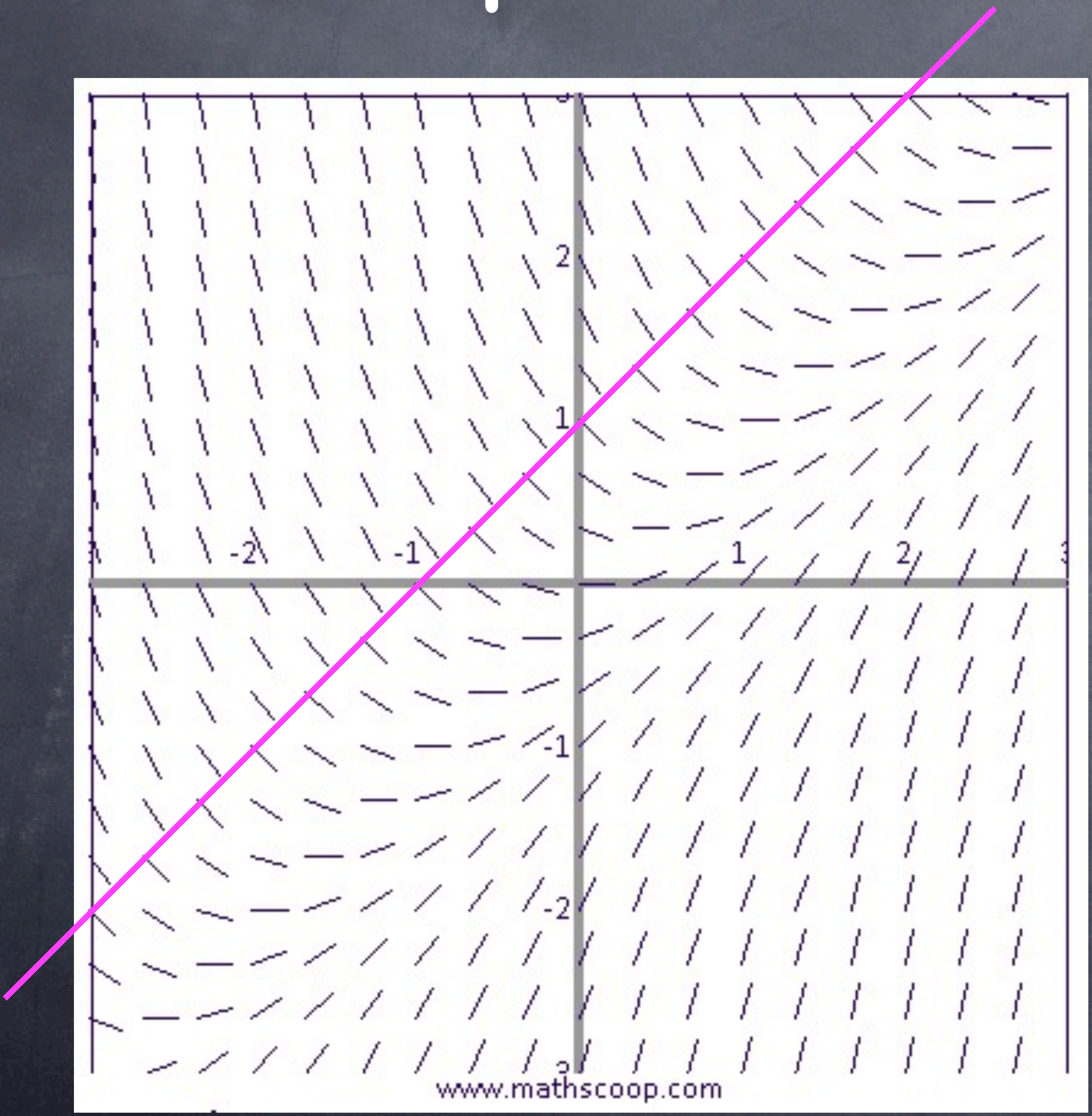


At (1,1), y′ = 0.

At (0,a), y′ = −a.

Along the line y=t+2,
y′ = t−(t+2) = −2.

Slope field for y′=t–y



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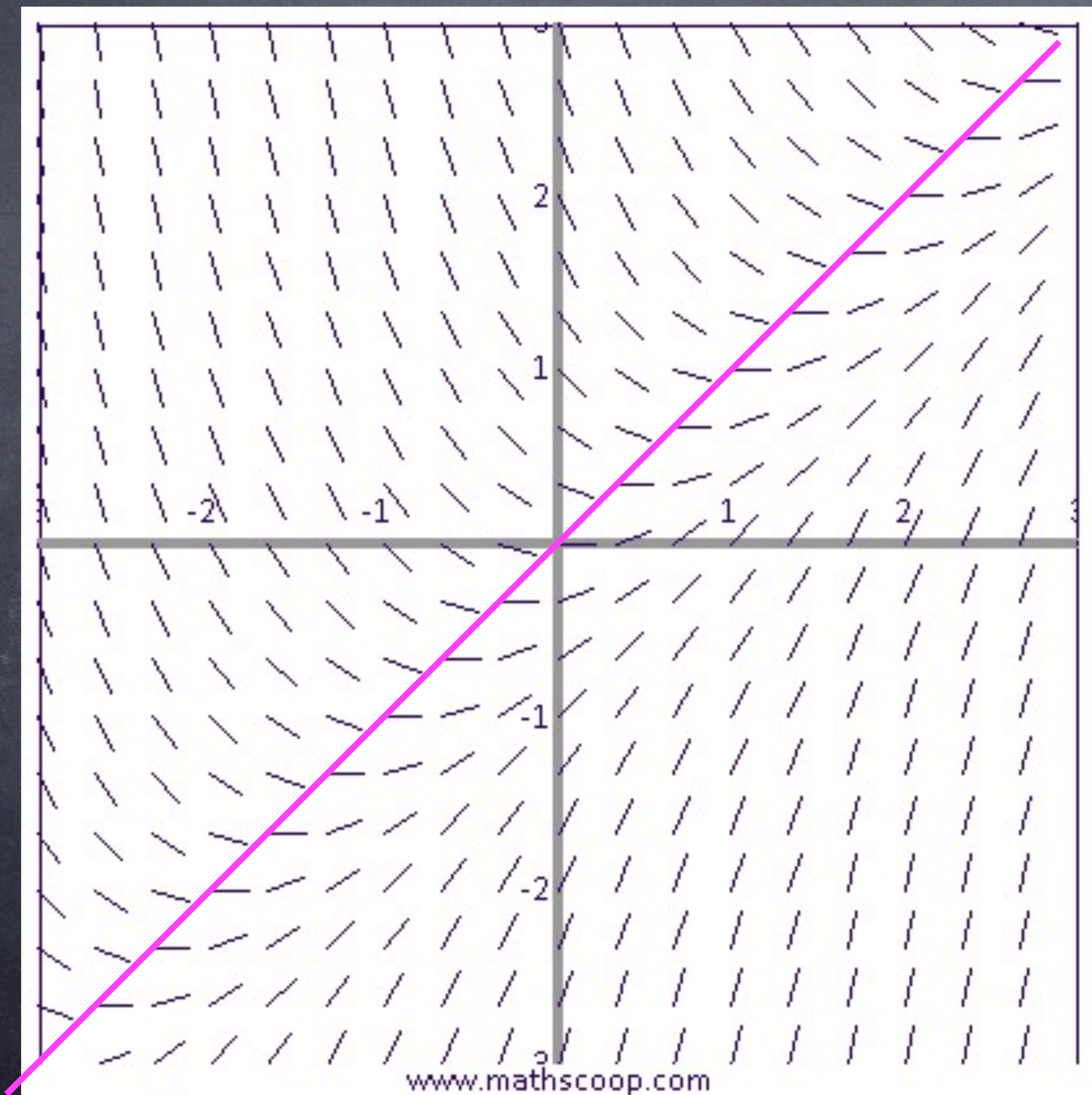

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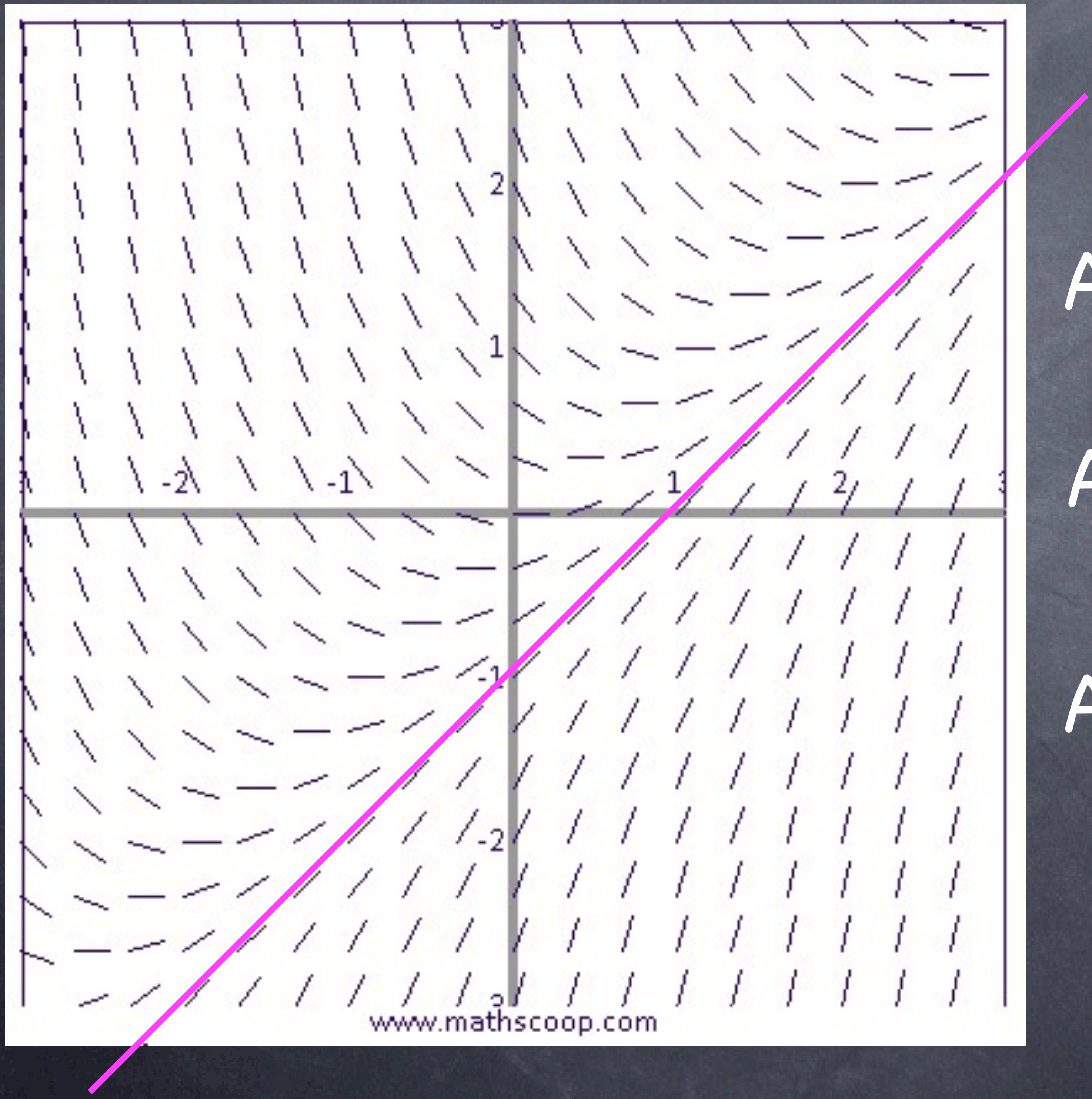
Slope field for y′=t–y

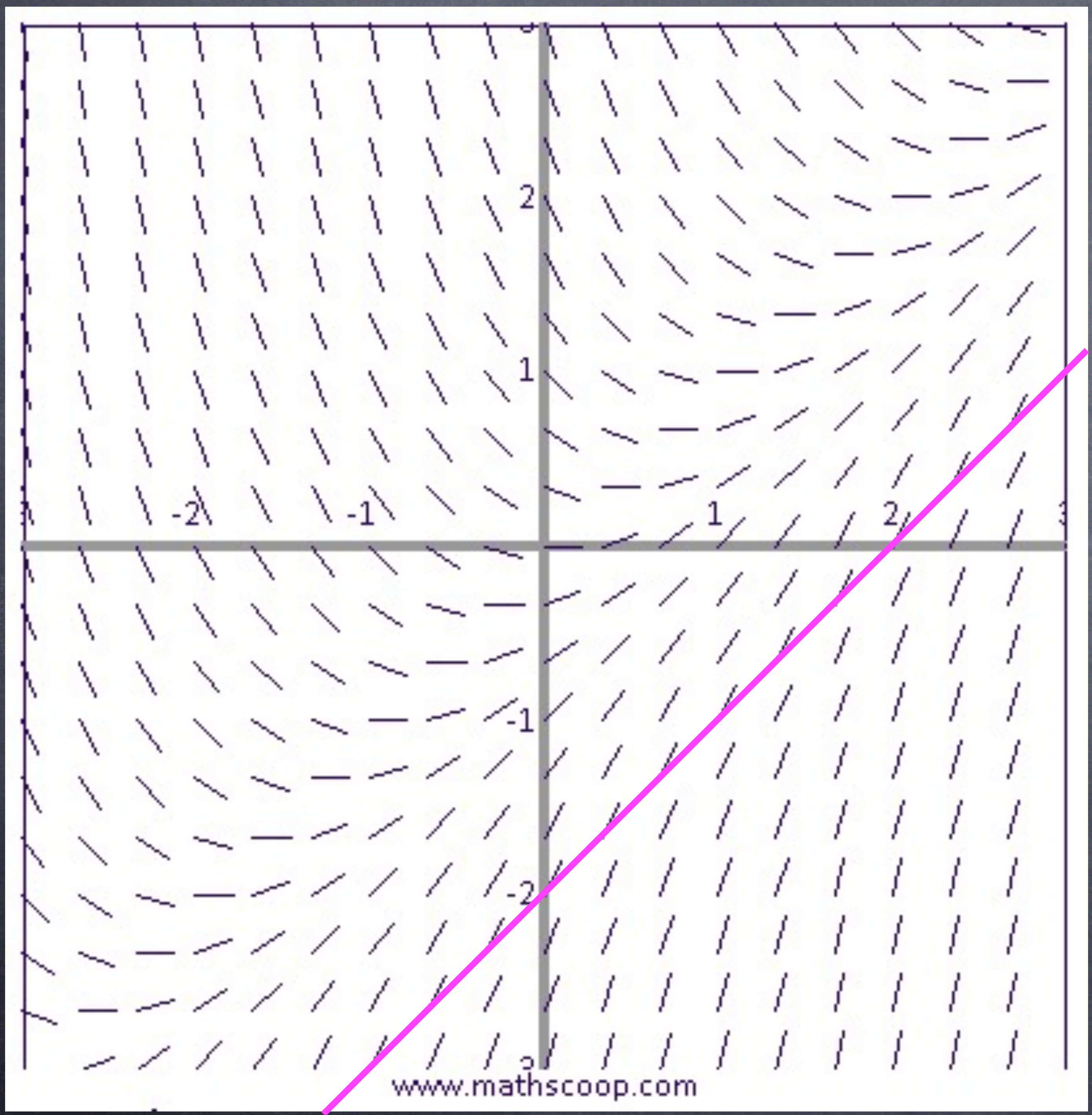


At (1,1), y′ = 0.

At (0,a), y′ = –a.

Along the line y=t–1,
y′ = t–(t–1) = 1.

Slope field for y′=t−y



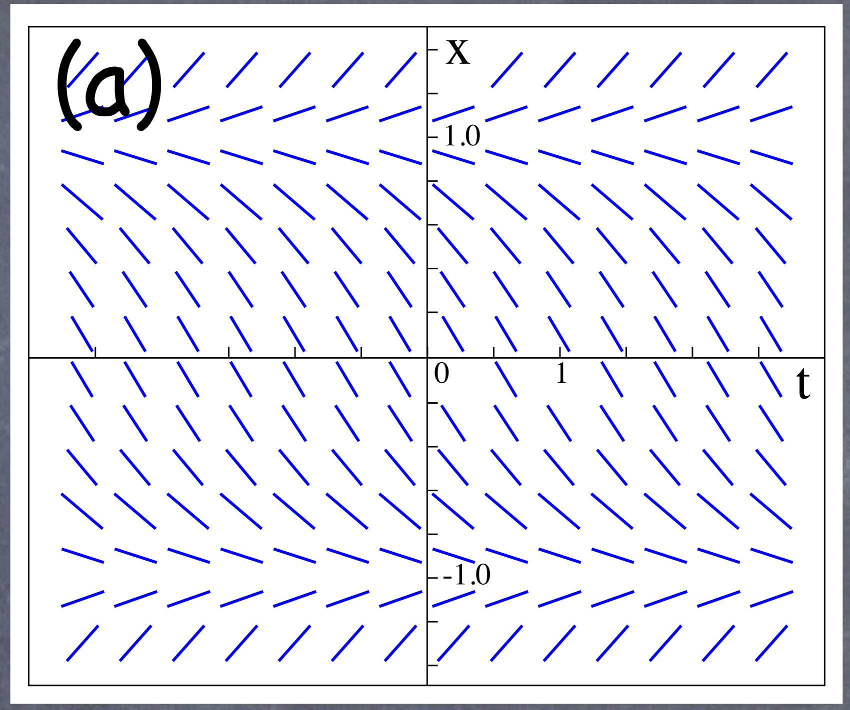
At (1,1), $y' = 0$.

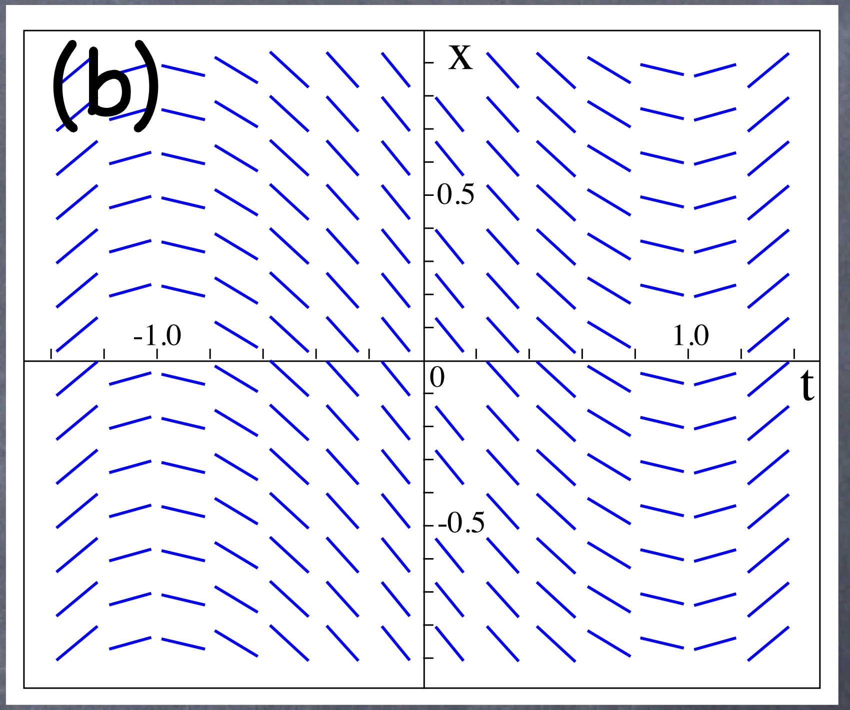
At (0,a), $y' = -a$.

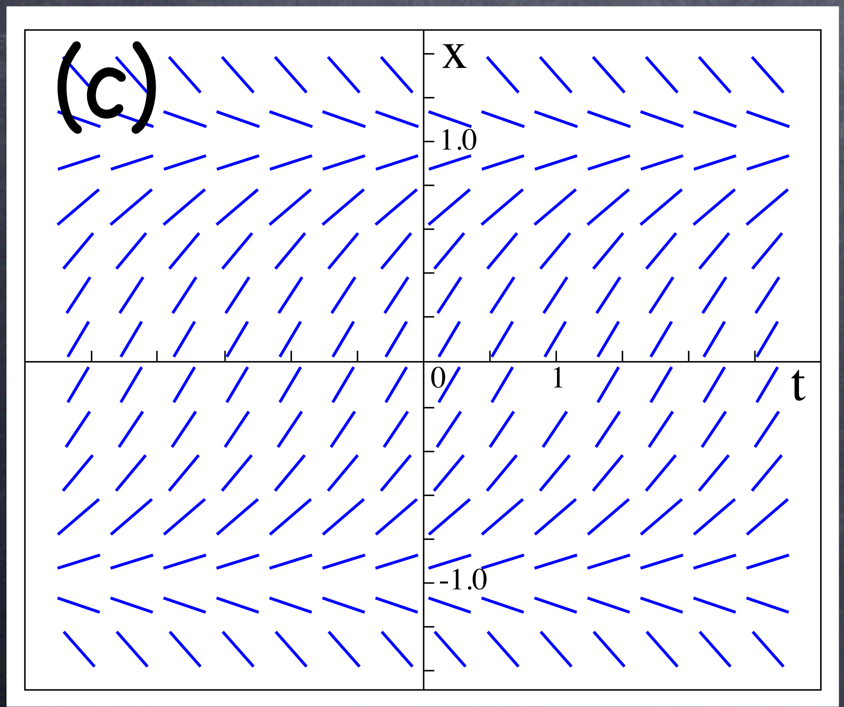
Along the line $y=t-2$,
$y' = t-(t-2) = 2$.

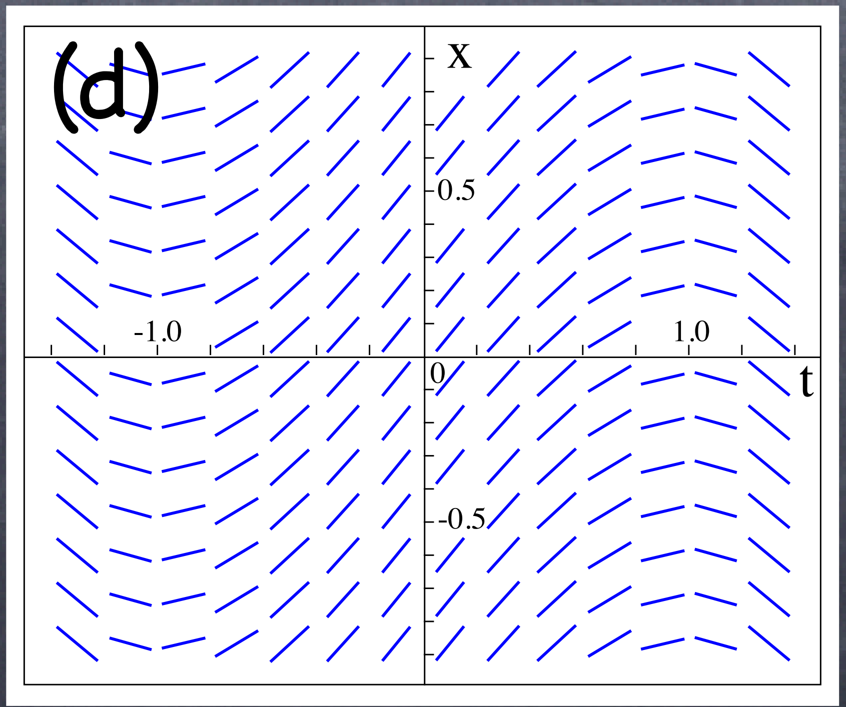
Match the equation to the slope field.

1. $x'=1-x^2$
2. $x'=x^2-1$
3. $x'=t^2-1$
4. $x'=1-t^2$









(A) 1c, 2a, 3d, 4b

(B) 1d, 2b, 3a, 4c

(C) 1a, 2c, 3b, 4d

(D) 1c, 2a, 3b, 4d

Match the equation to the slope field.

1. $x' = 1 - x^2$
2. $x' = x^2 - 1$
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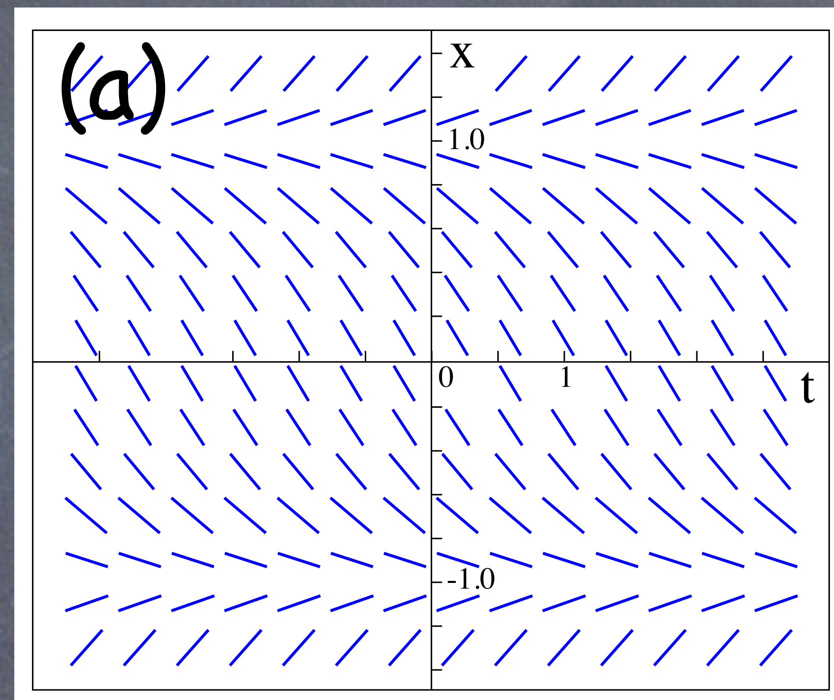

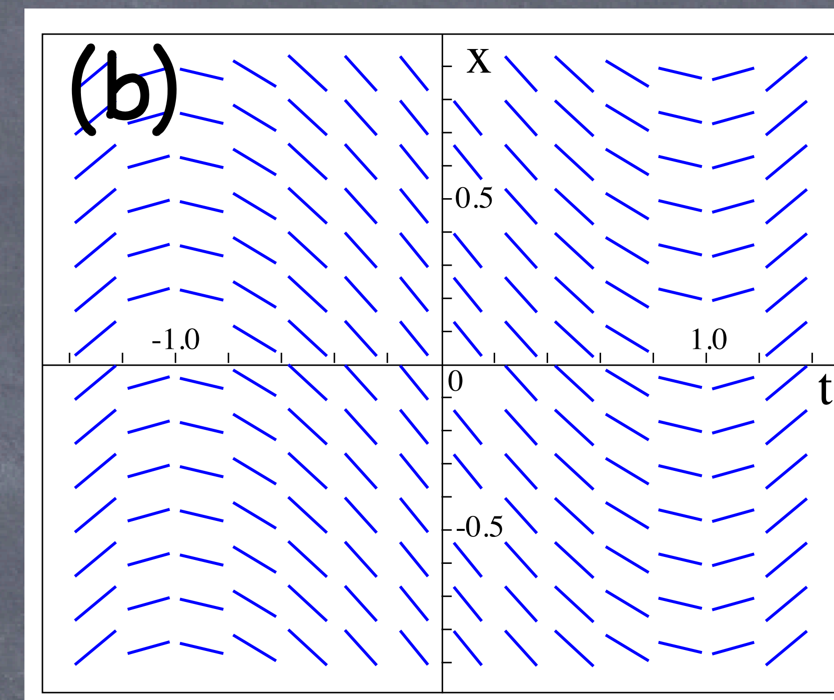

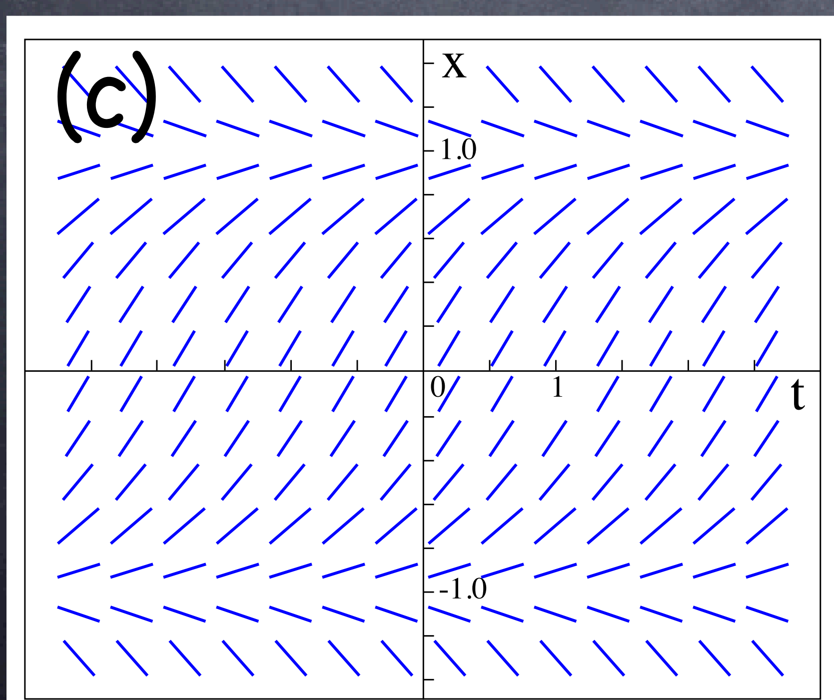

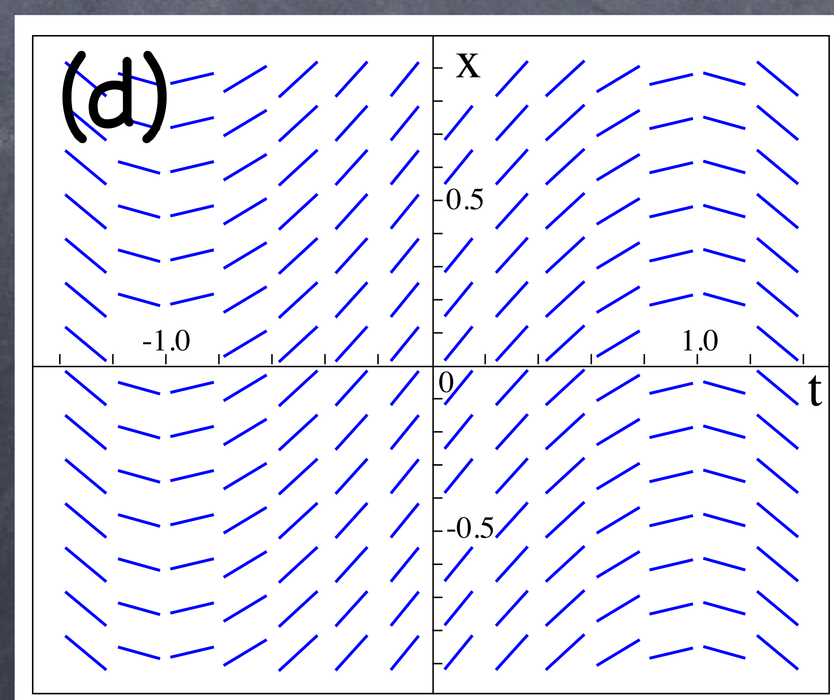

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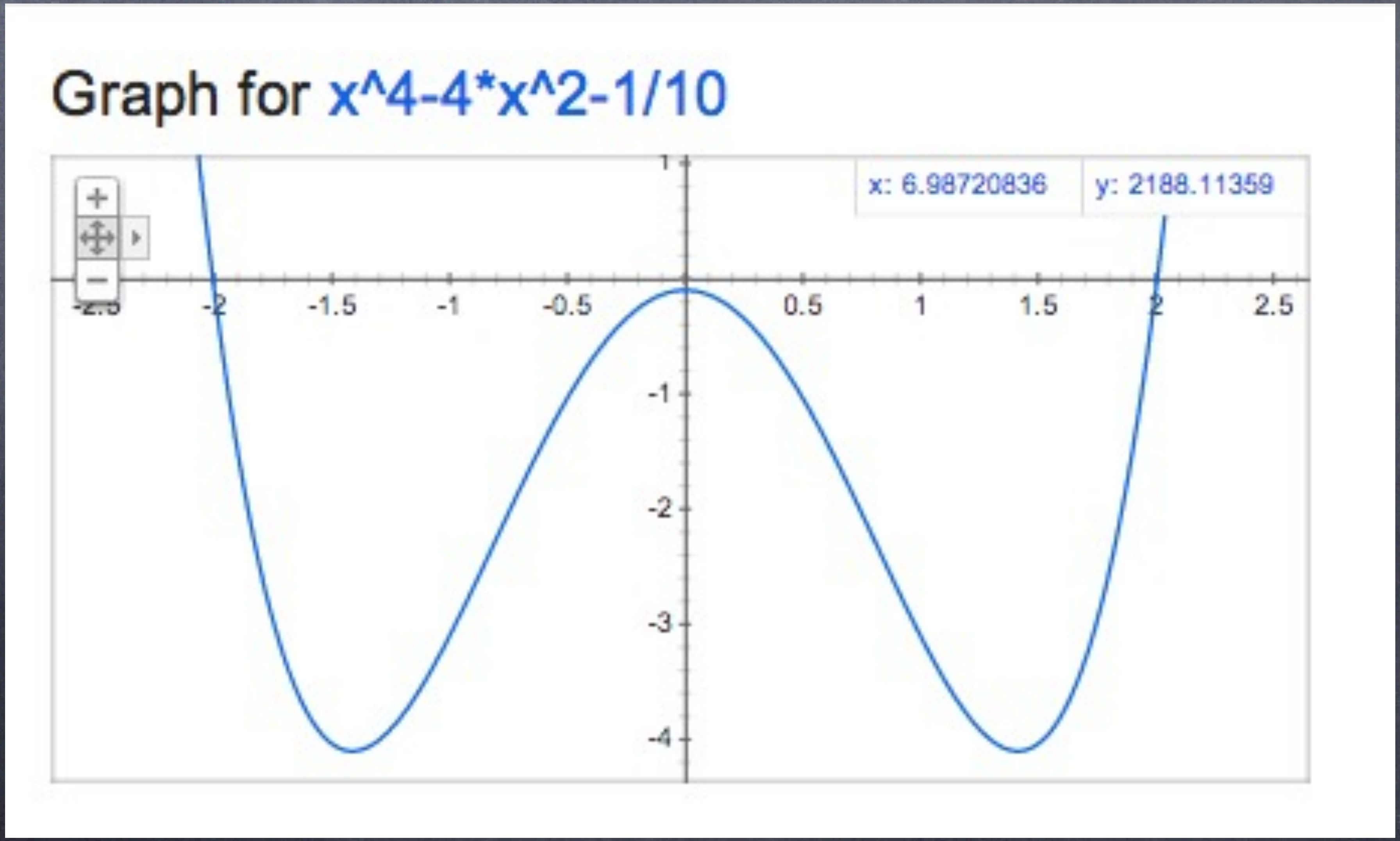
Find the largest zero of $f(x) = x^4 - 4x^2 - 1/10$.

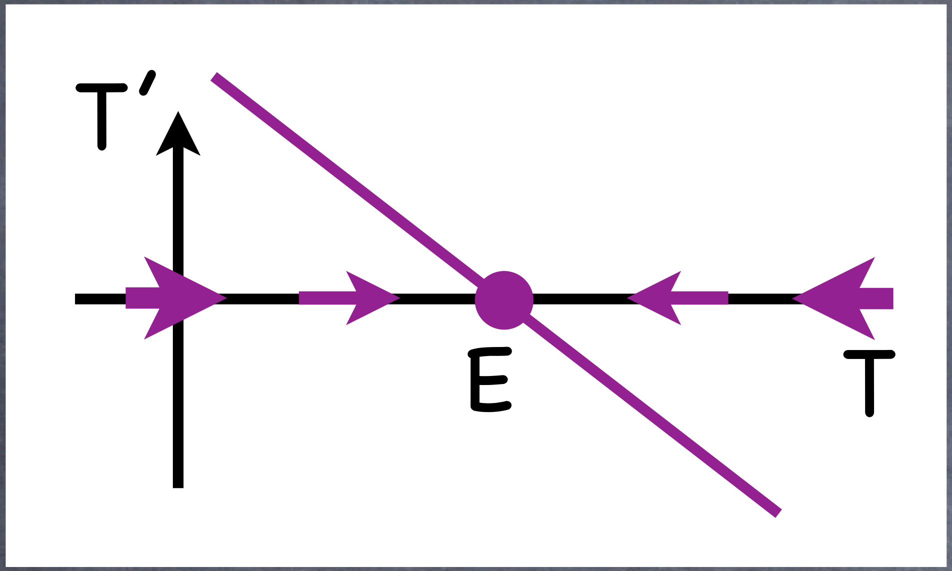
- Let's do this in a spreadsheet...



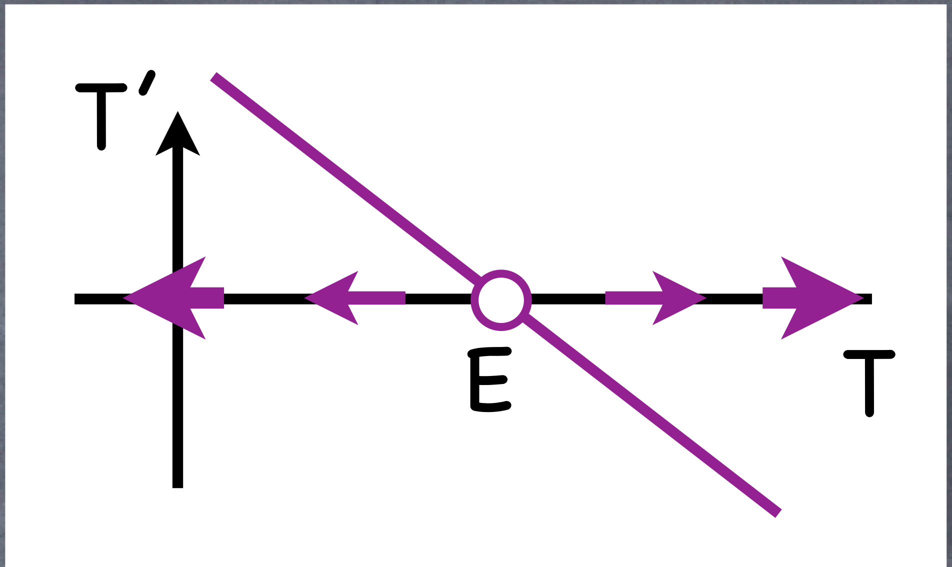
Phase line for NLC:

$$\frac{dT}{dt} = k(E - T)$$

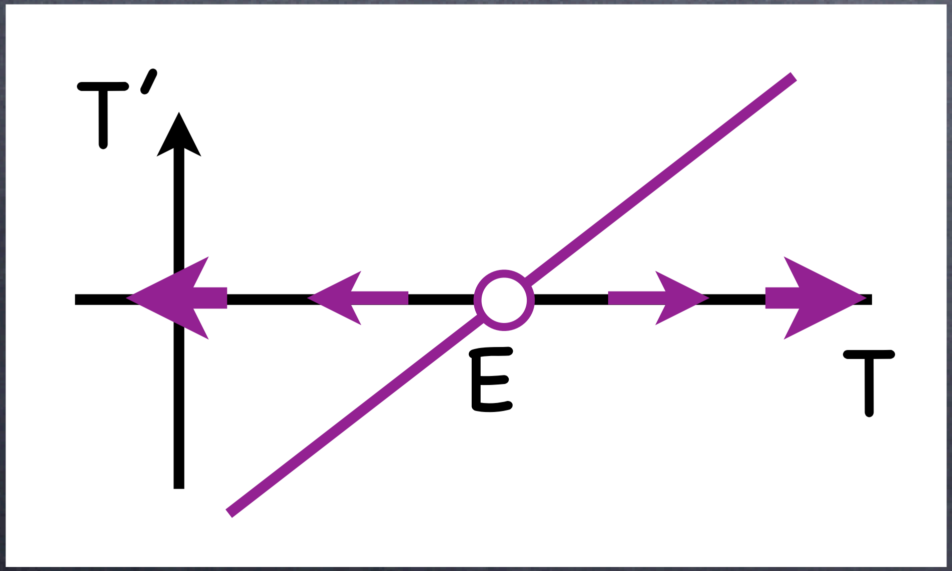
(A)



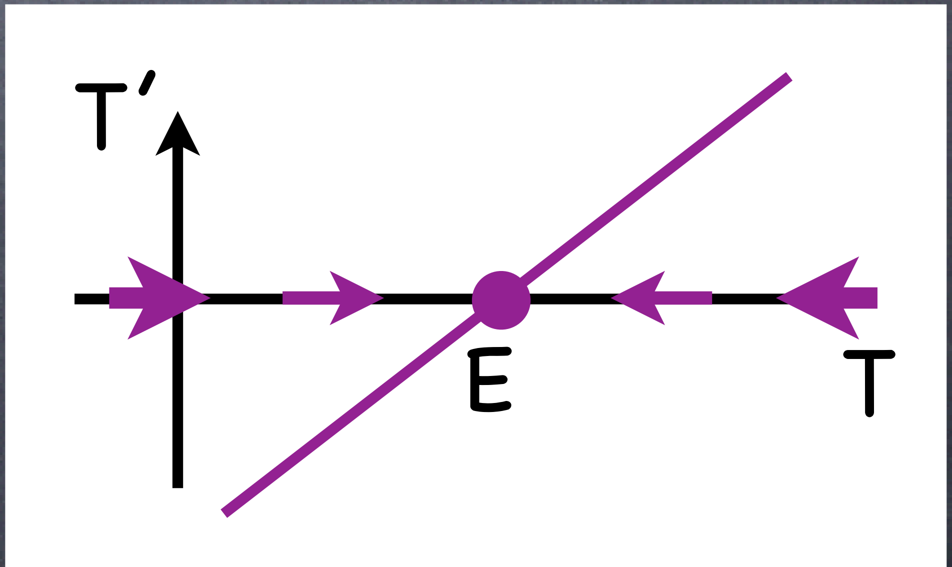
(B)



(C)



(D)



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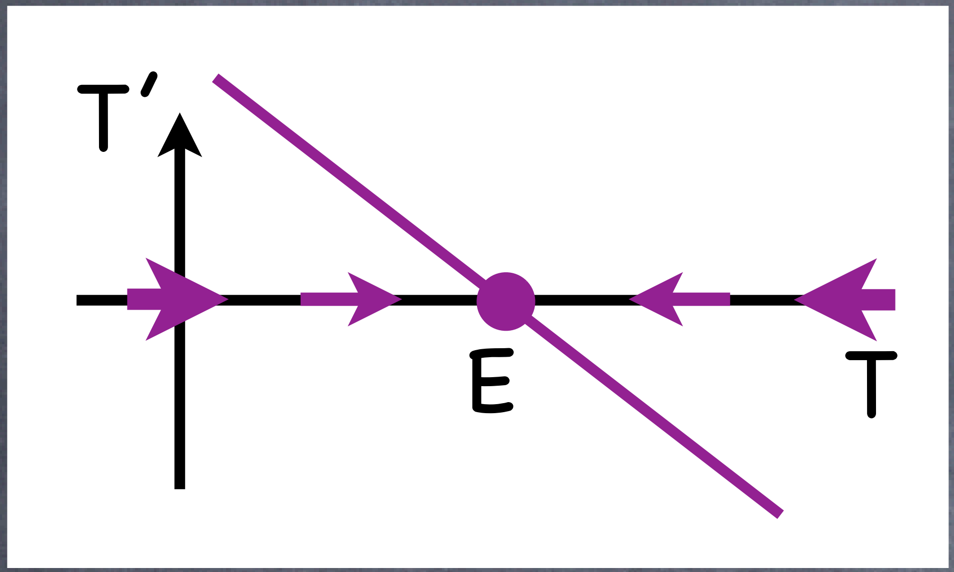

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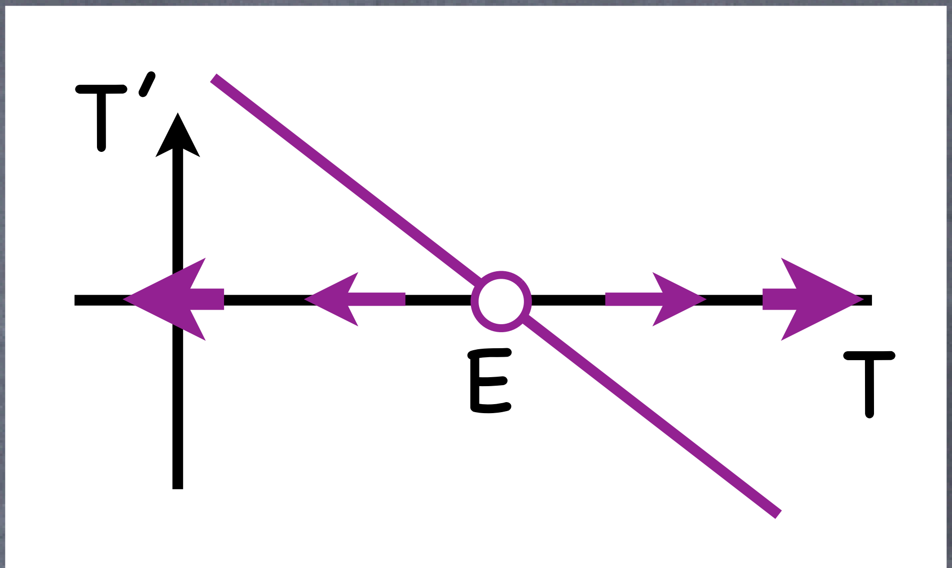

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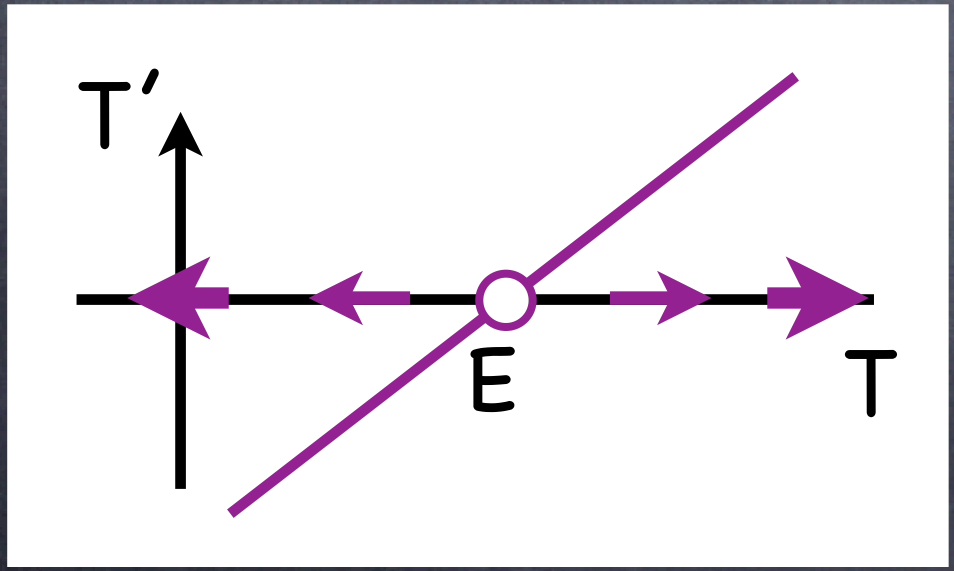

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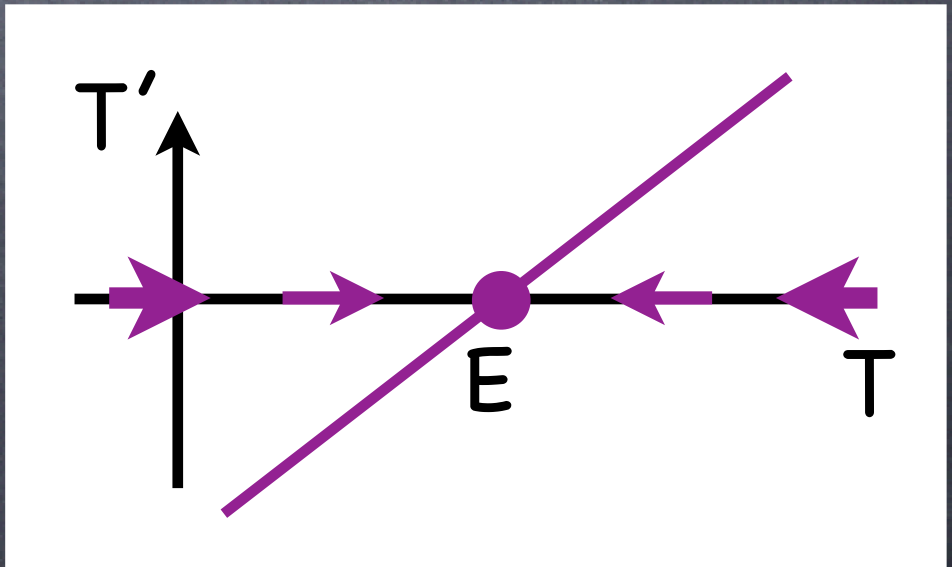

What influence does k have on this diagram?

$y' = \cos(y)$

- A solution satisfying the initial condition $y(0)=y_0$ will approach y^* as $t \to \infty$. Which y_0 and y^* pair is correct?

(A) $y_0 = 0$, $y^* = \pi$.

(B) $y_0 = -\pi$, $y^* = -\pi/2$.

(C) $y_0 = 2\pi$, $y^* = 3\pi/2$.

(D) $y_0 = \pi/4$, $y^* = 0$.

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