Today...

- A Hill function clicker question.
- From secant line to tangent line.
- The Definition of the Derivative.

Comparing Hill functions with different n values

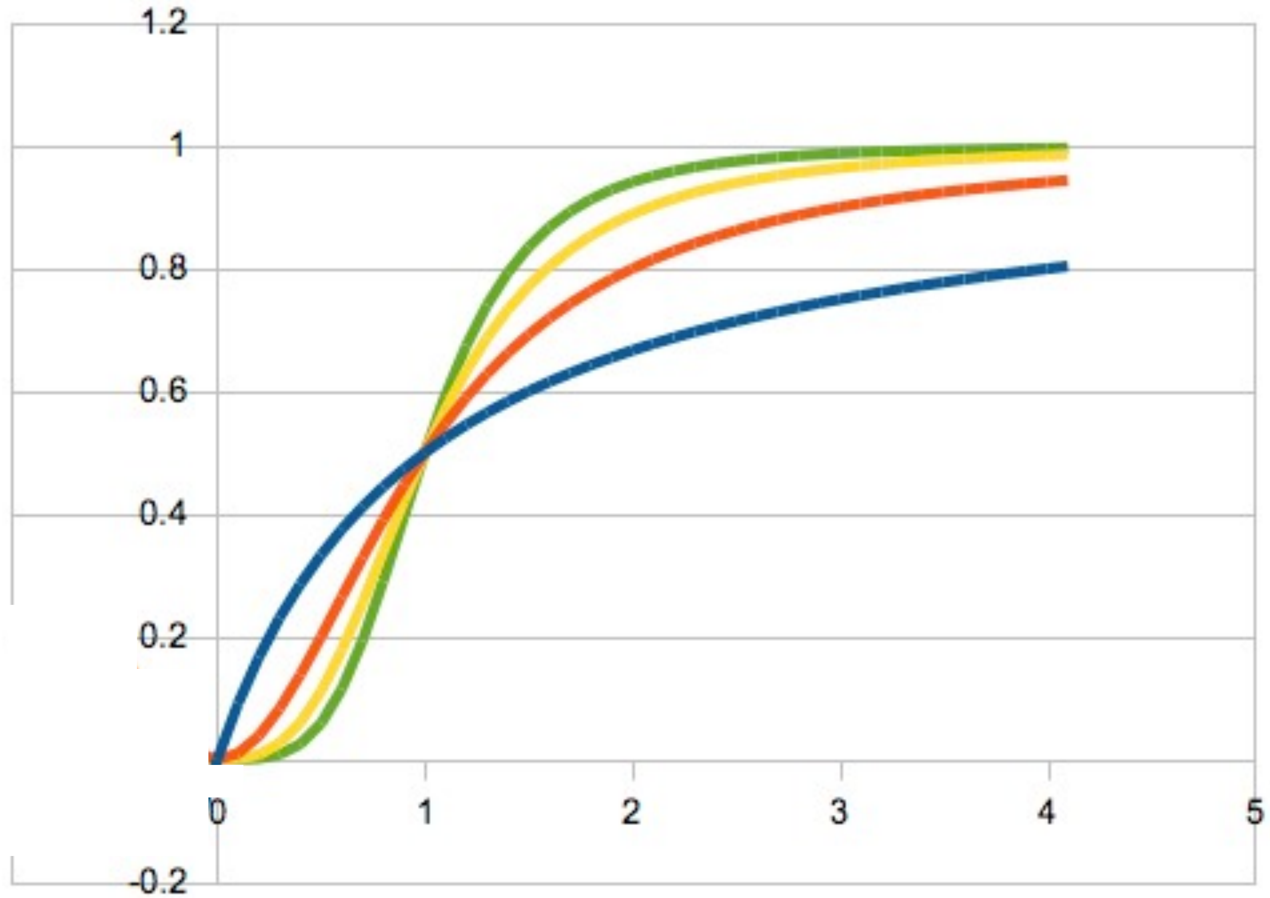
$$f(x) = \frac{ax^n}{b^n + x^n}$$

(A) Green: n=2, yellow: n=3, red: n=4, blue: n=5.

(B) Green: n=4, yellow: n=3, red: n=2, blue: n=1.

(C) Green: n=5, yellow: n=4, red: n=3, blue: n=2.

(D) Either (B) or (C) (not enough info).



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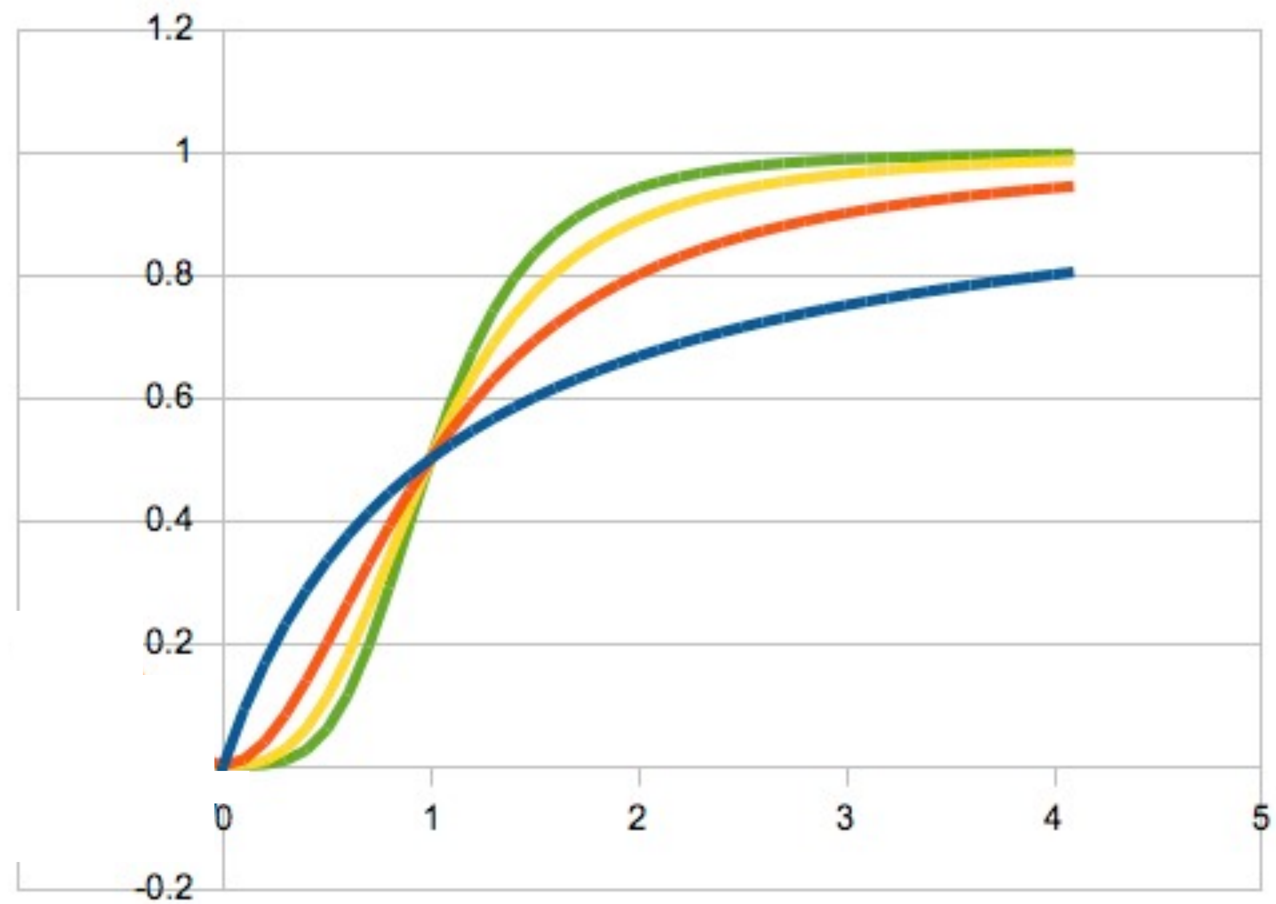

What is the slope of the line connecting the points?

(A) $m=(x_1-x_2)/(y_1-y_2)$

(B) $m=(x_2-x_1)/(y_1-y_2)$

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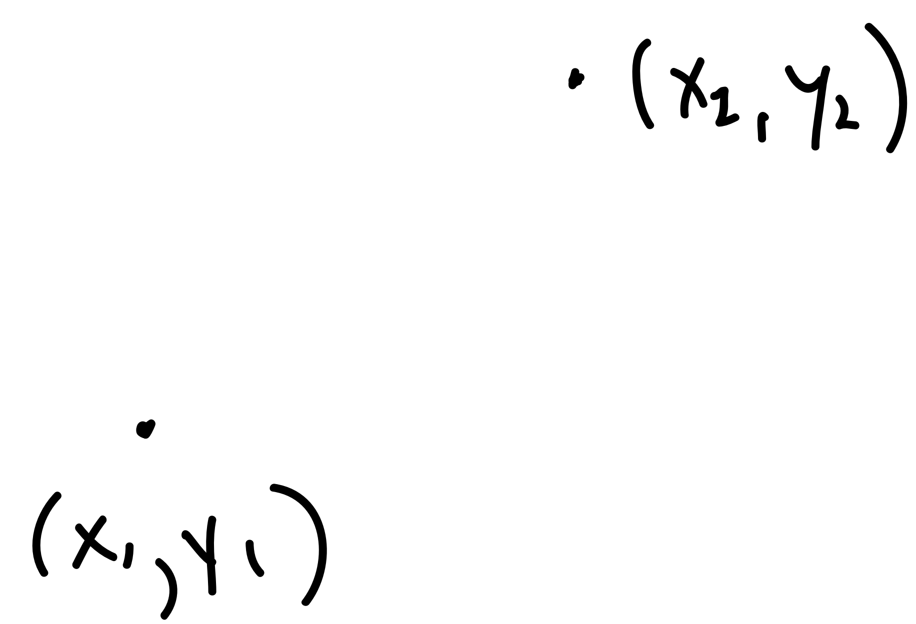

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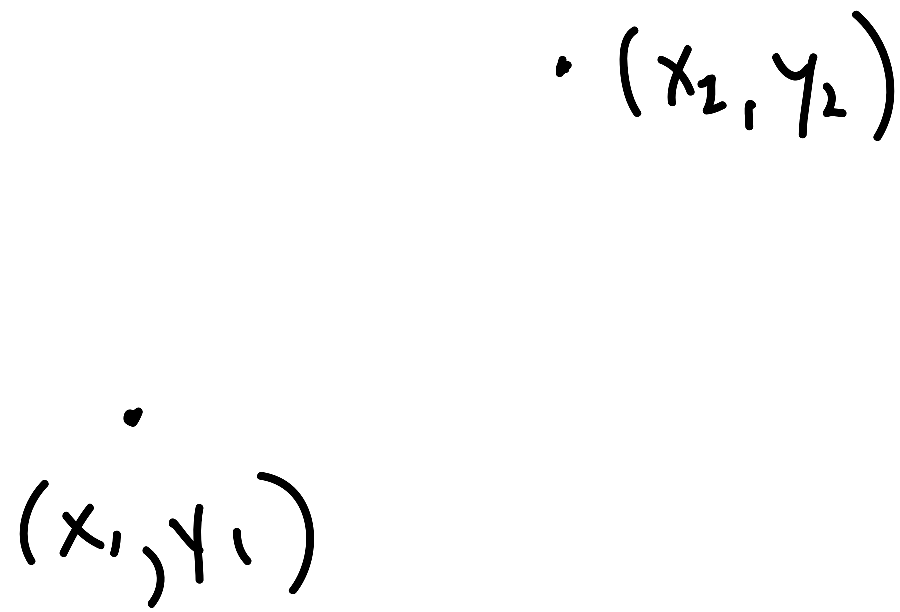

What is the slope of the secant line to the graph of f(x)?

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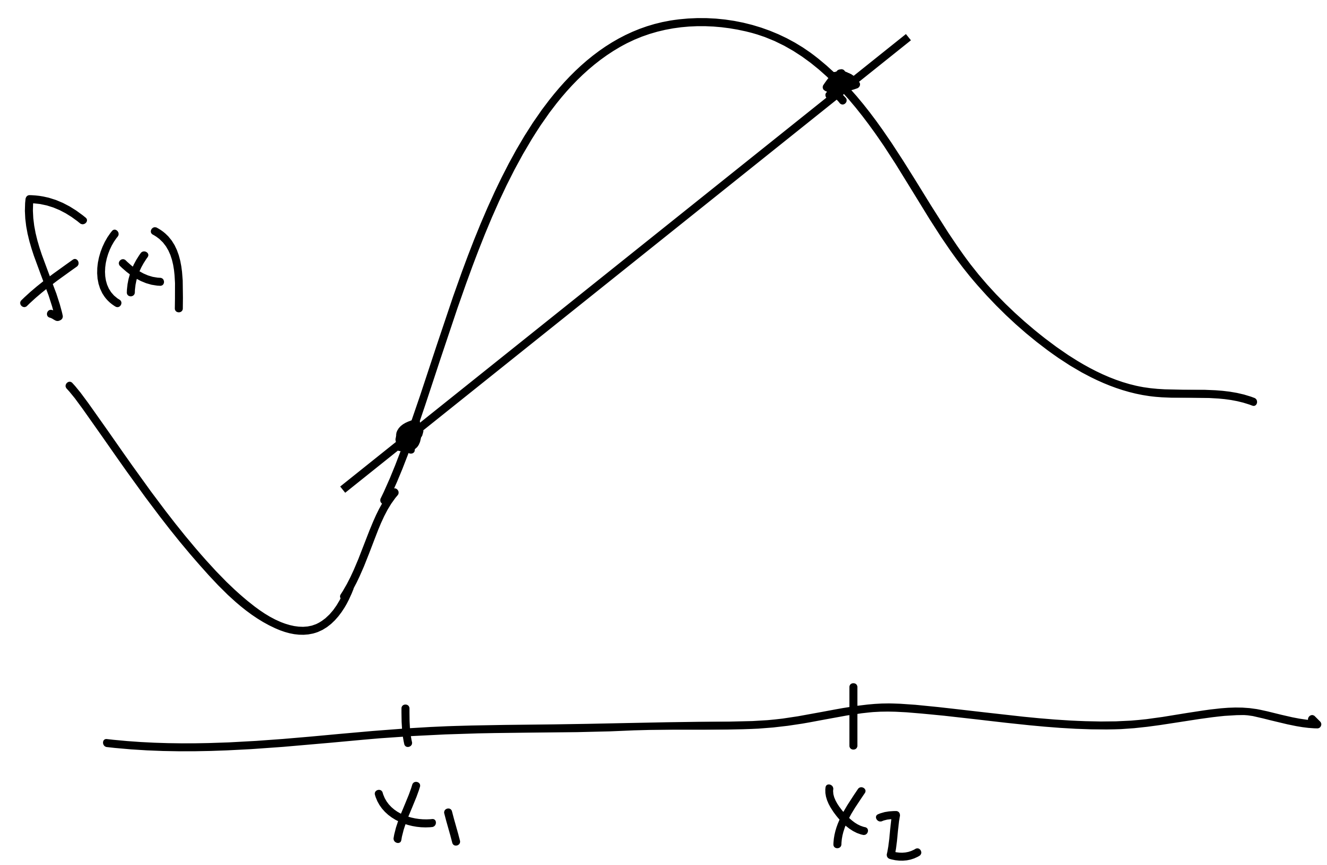

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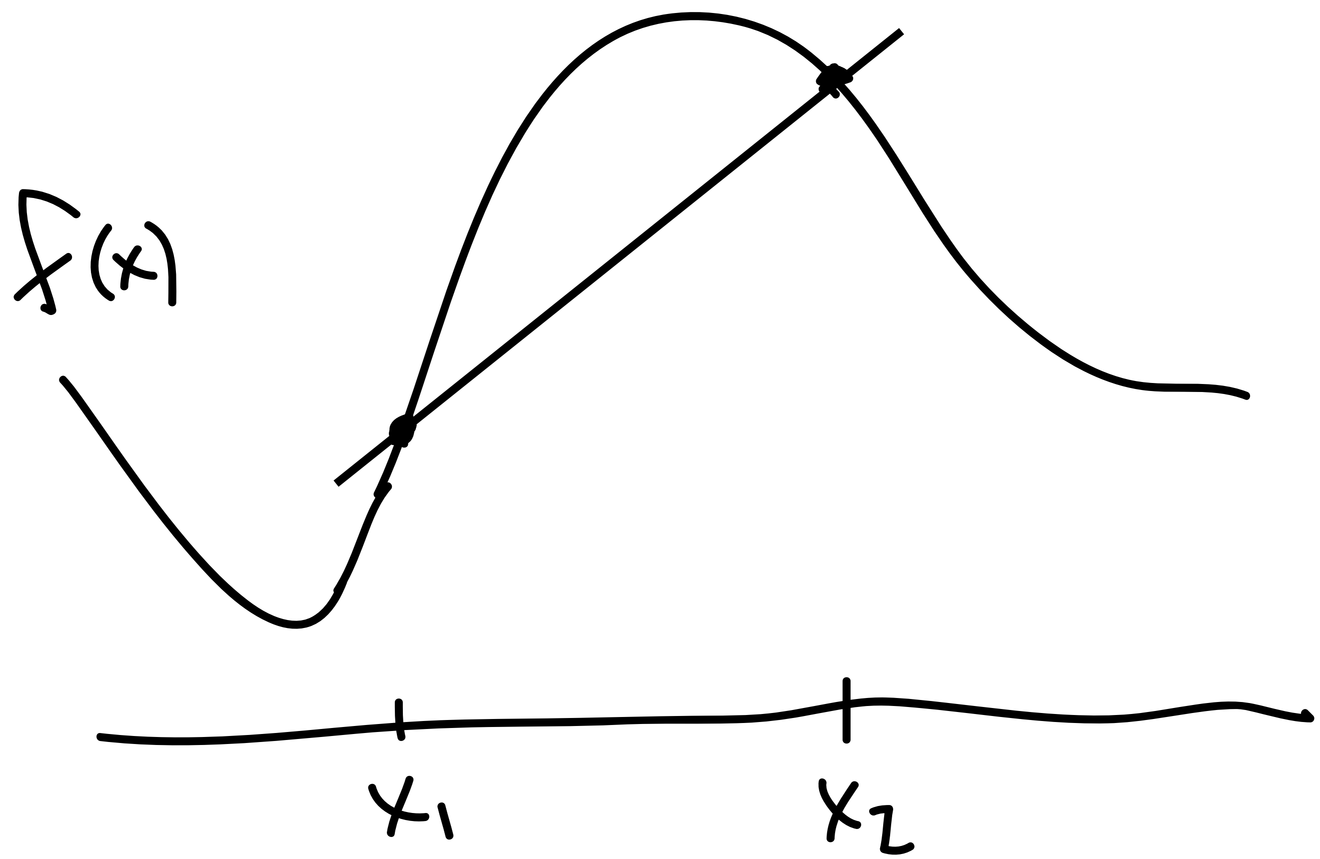
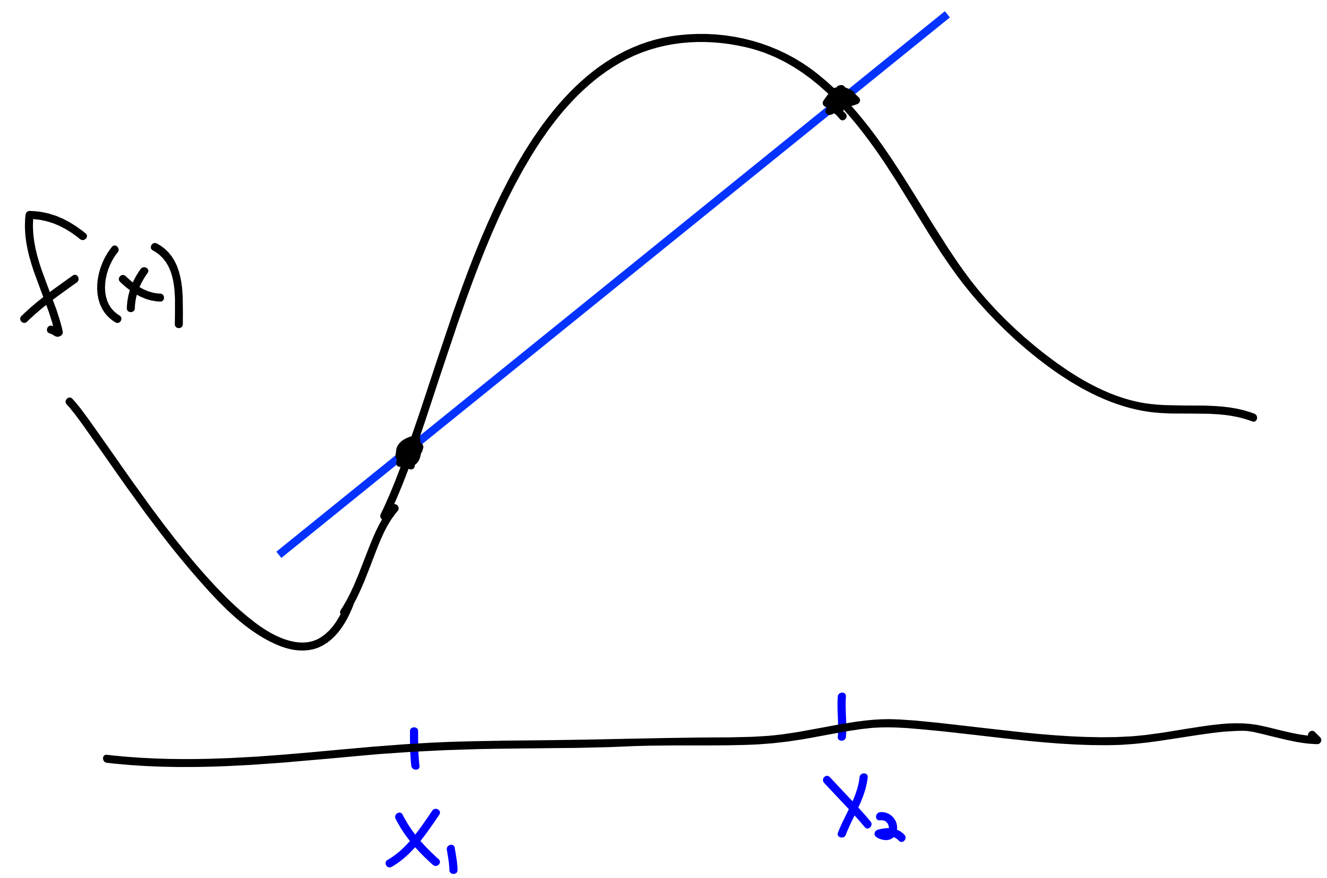


Slope of secant line = average rate of change from x_1 to x_2.

What if you want the rate of change AT x_1?

(instantaneous instead of average)



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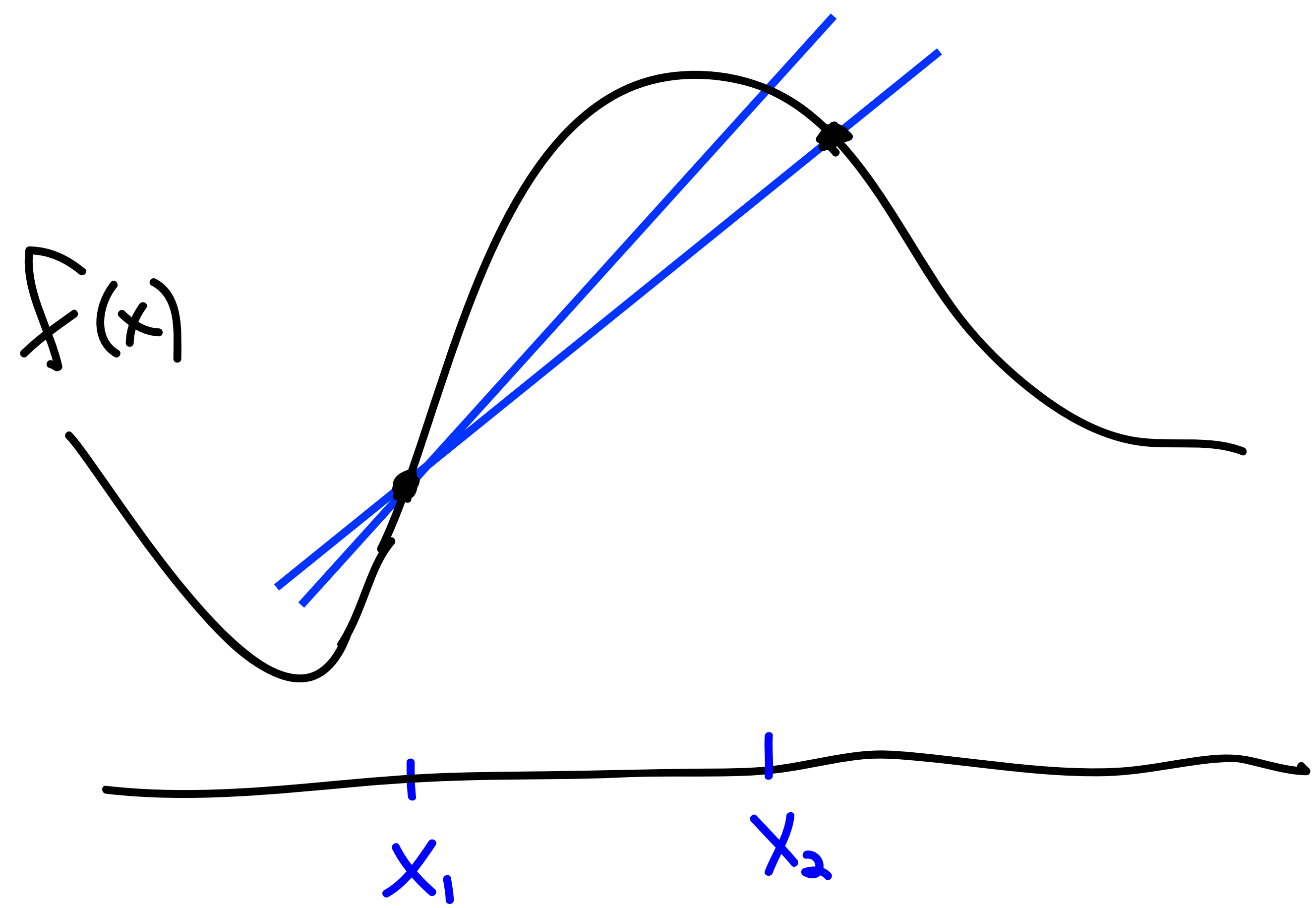
Take a point x_2 so that the secant line is closer to the "secant line" AT x_1.



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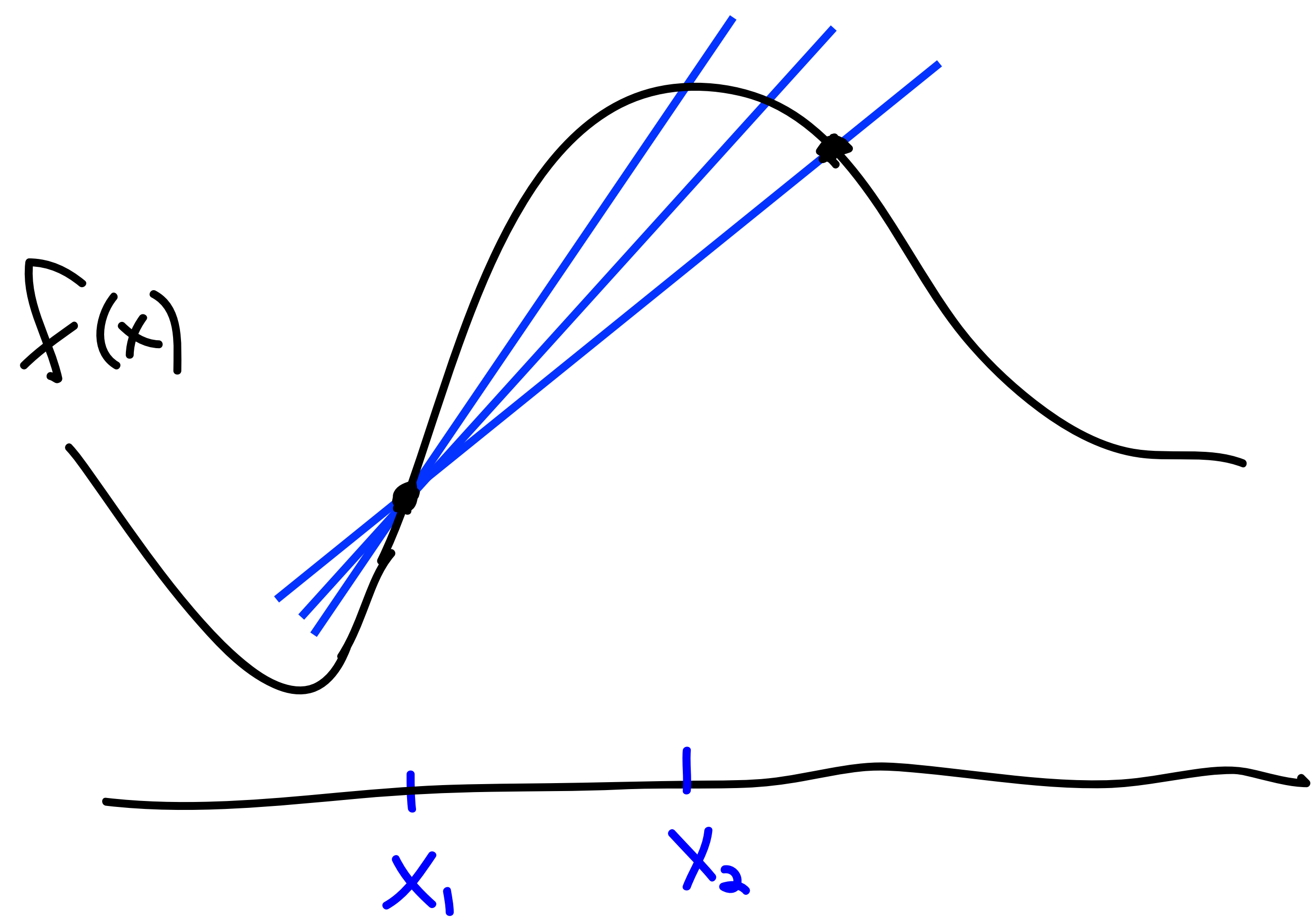

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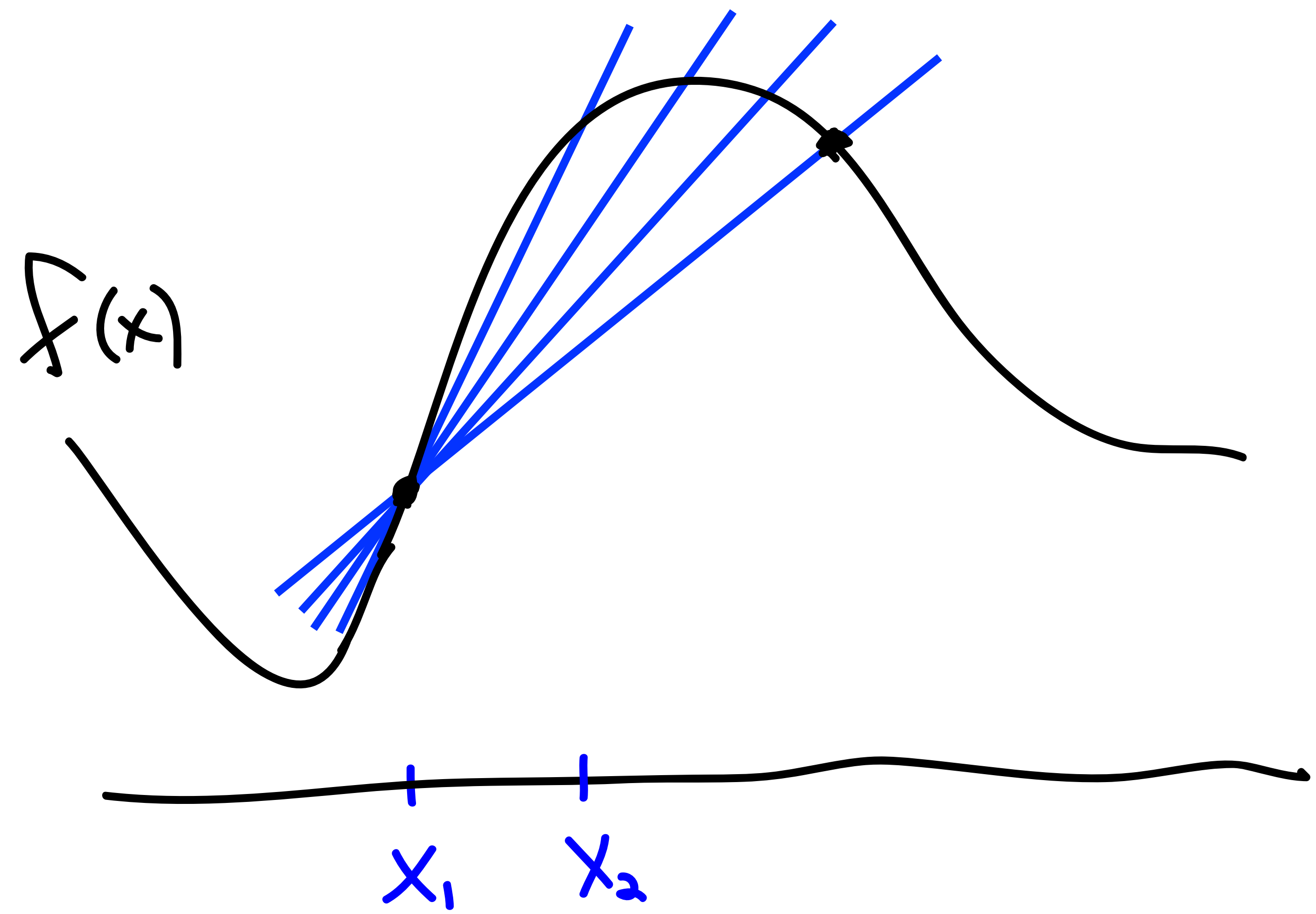

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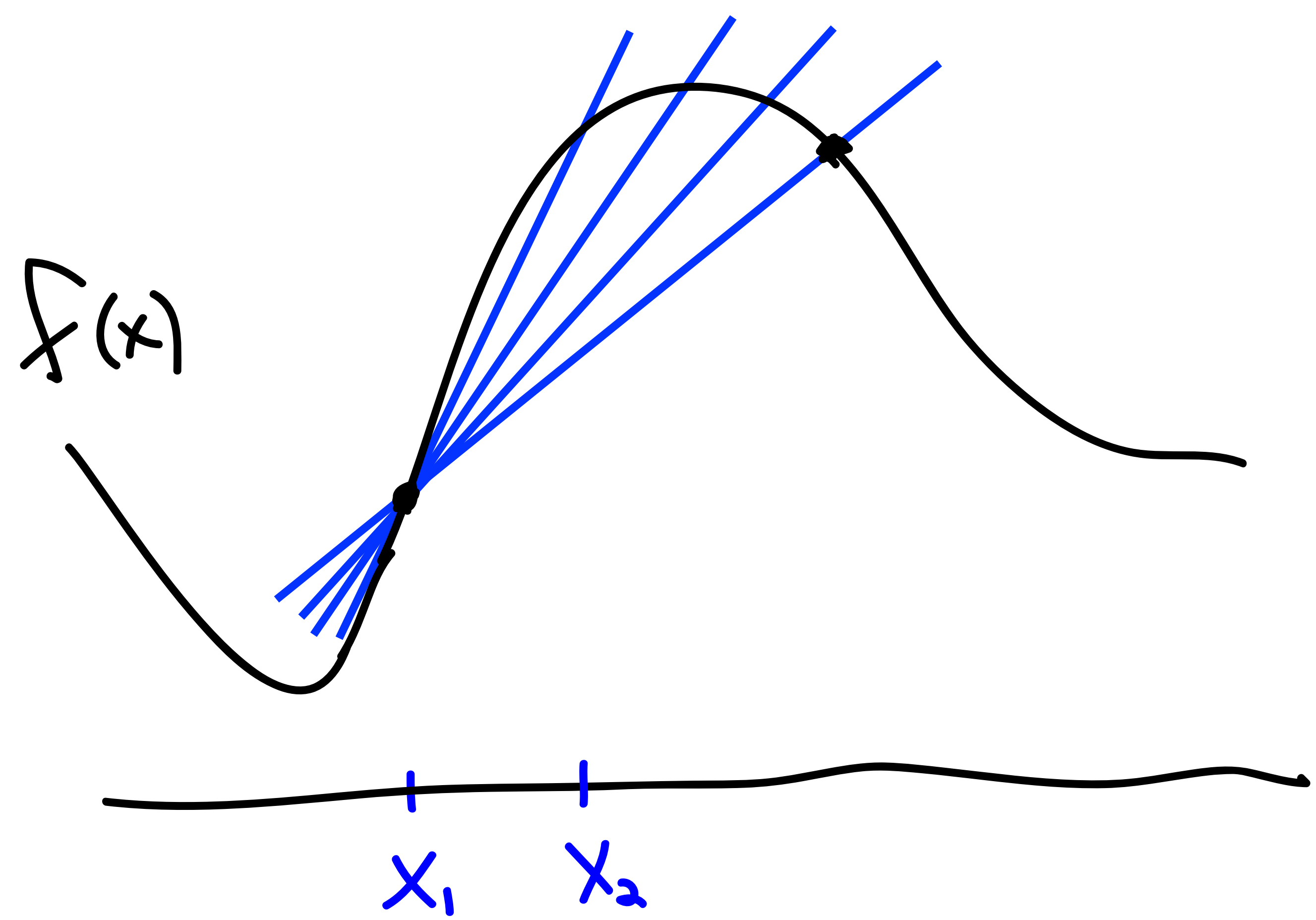

Alternate notation: let $x_2 = x_1+h$ so that

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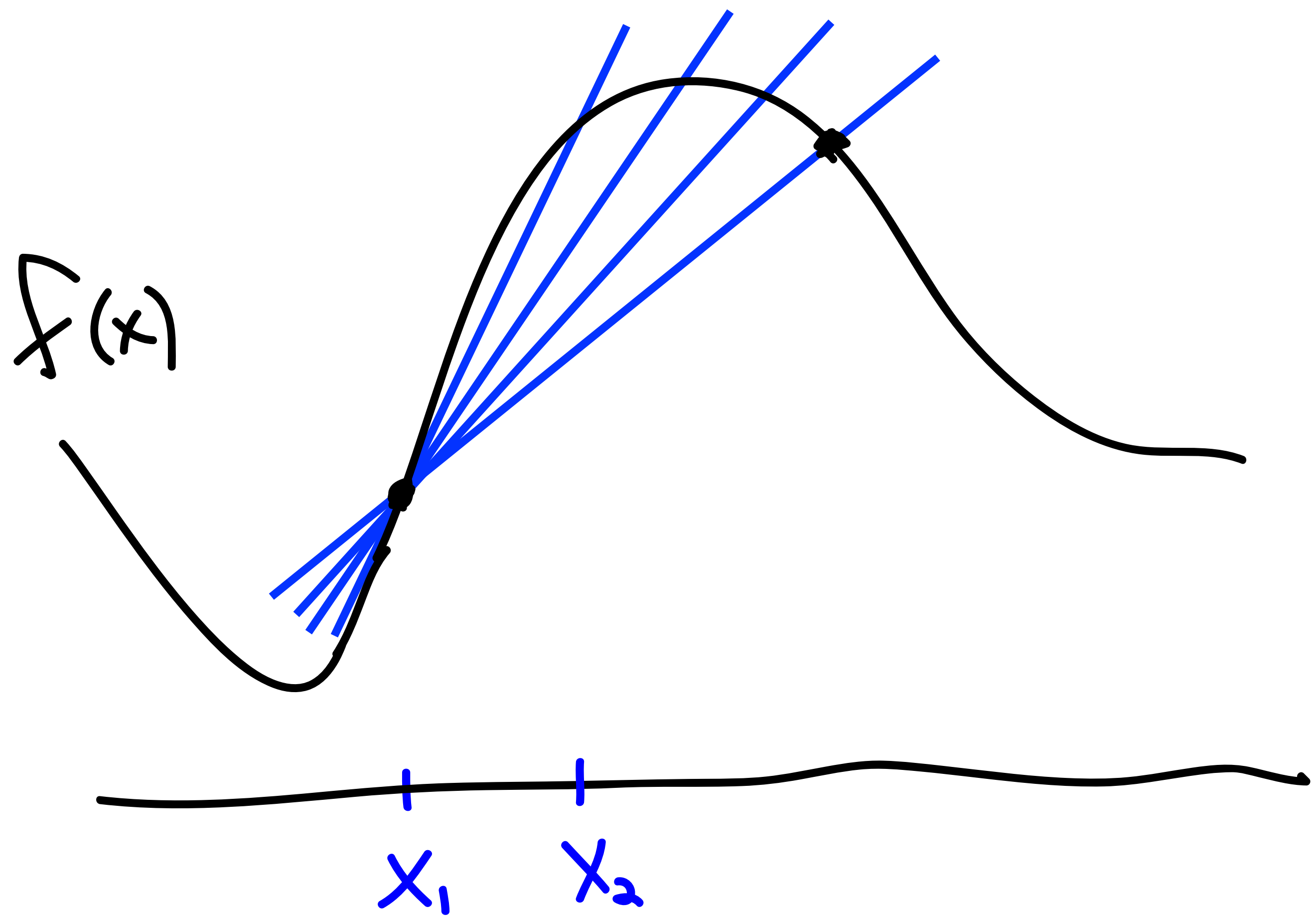

Alternate notation: let $x_2 = x_1+h$ so that

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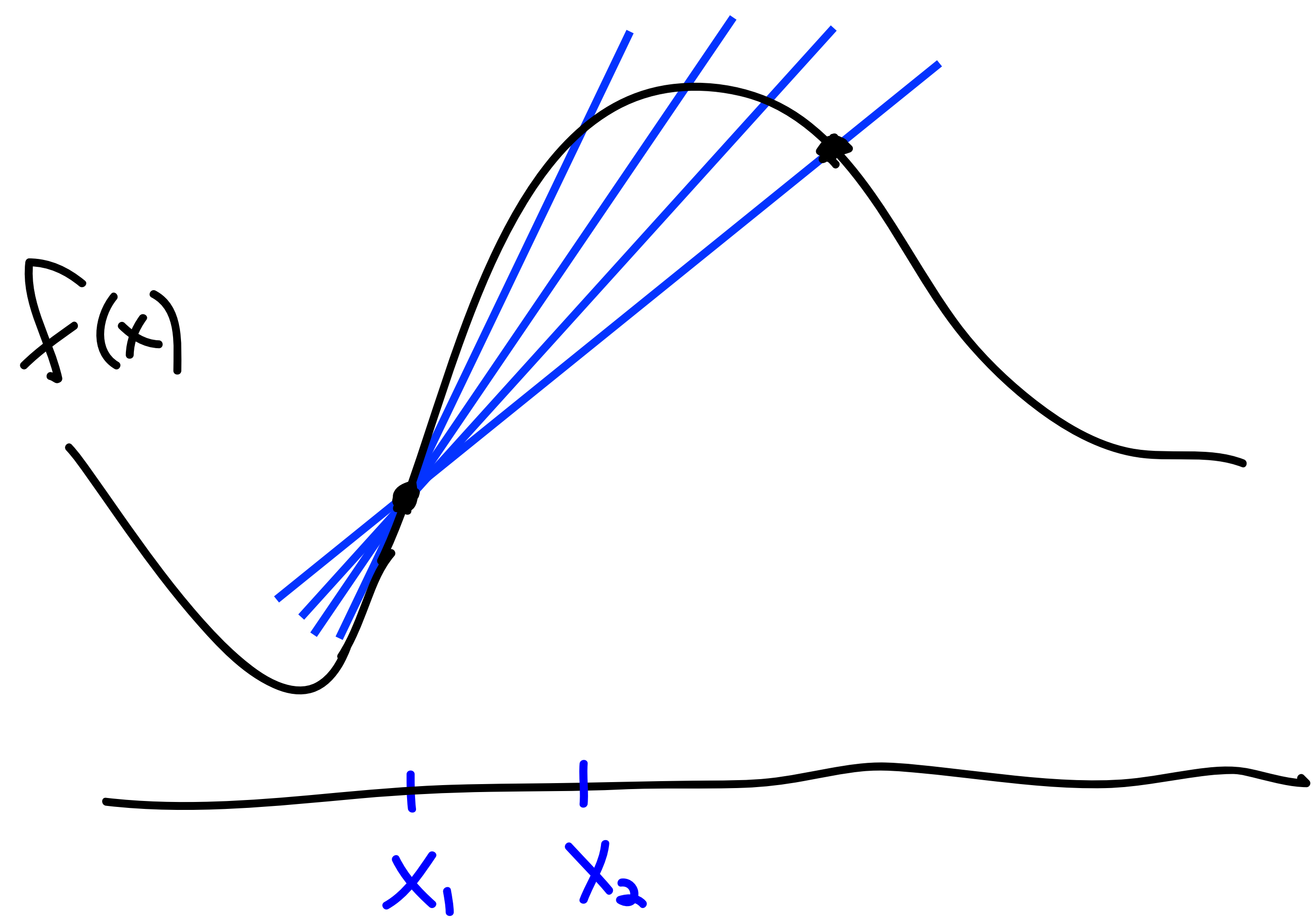

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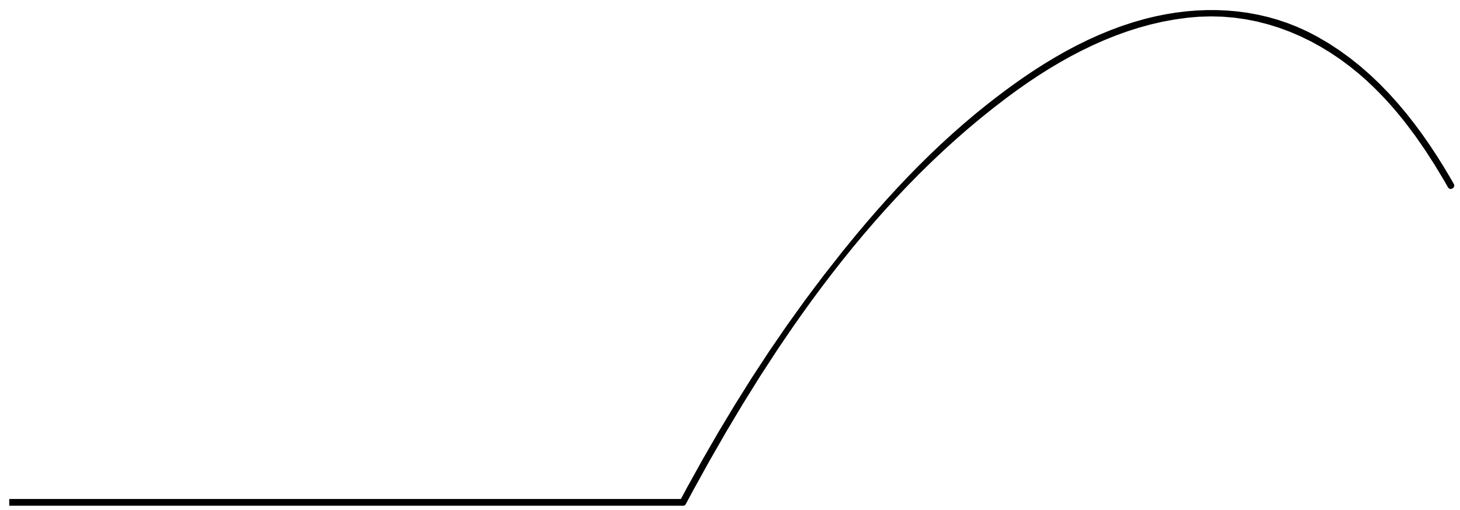
$$\text{slope at } x_1 = f'(x_1) = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$

Another example



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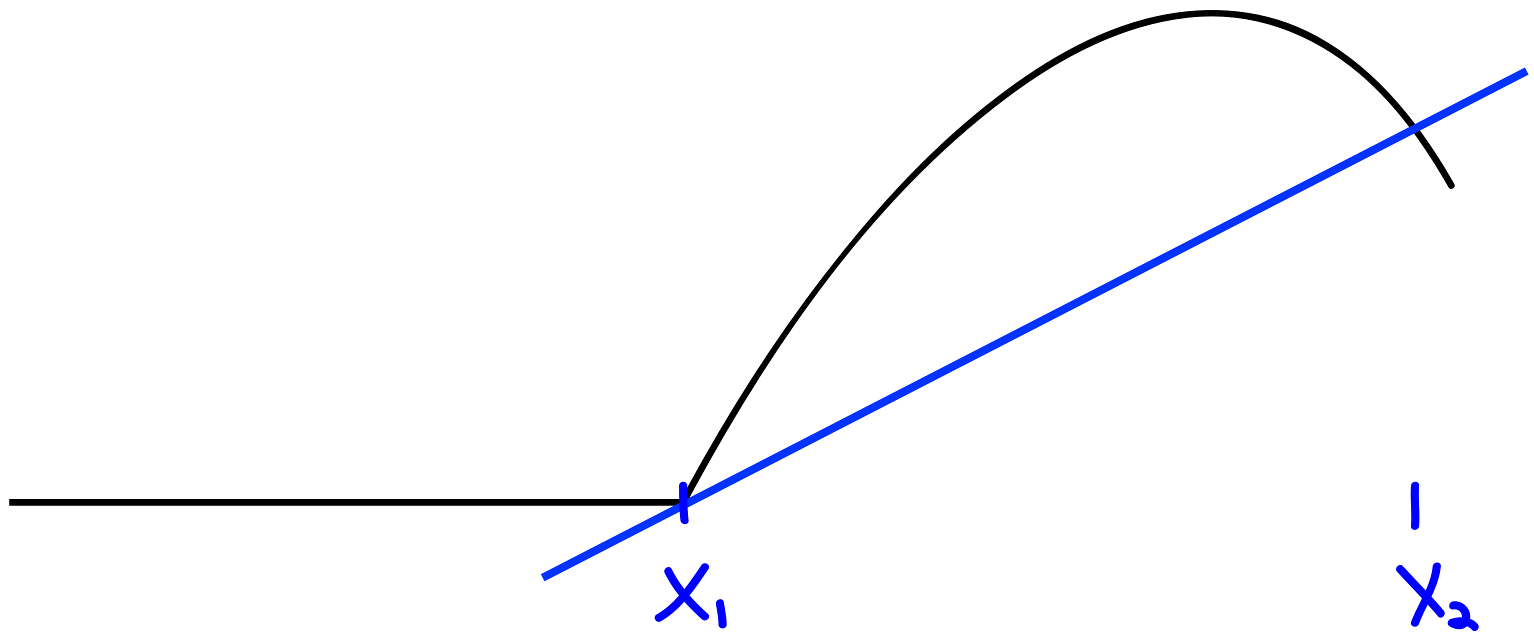

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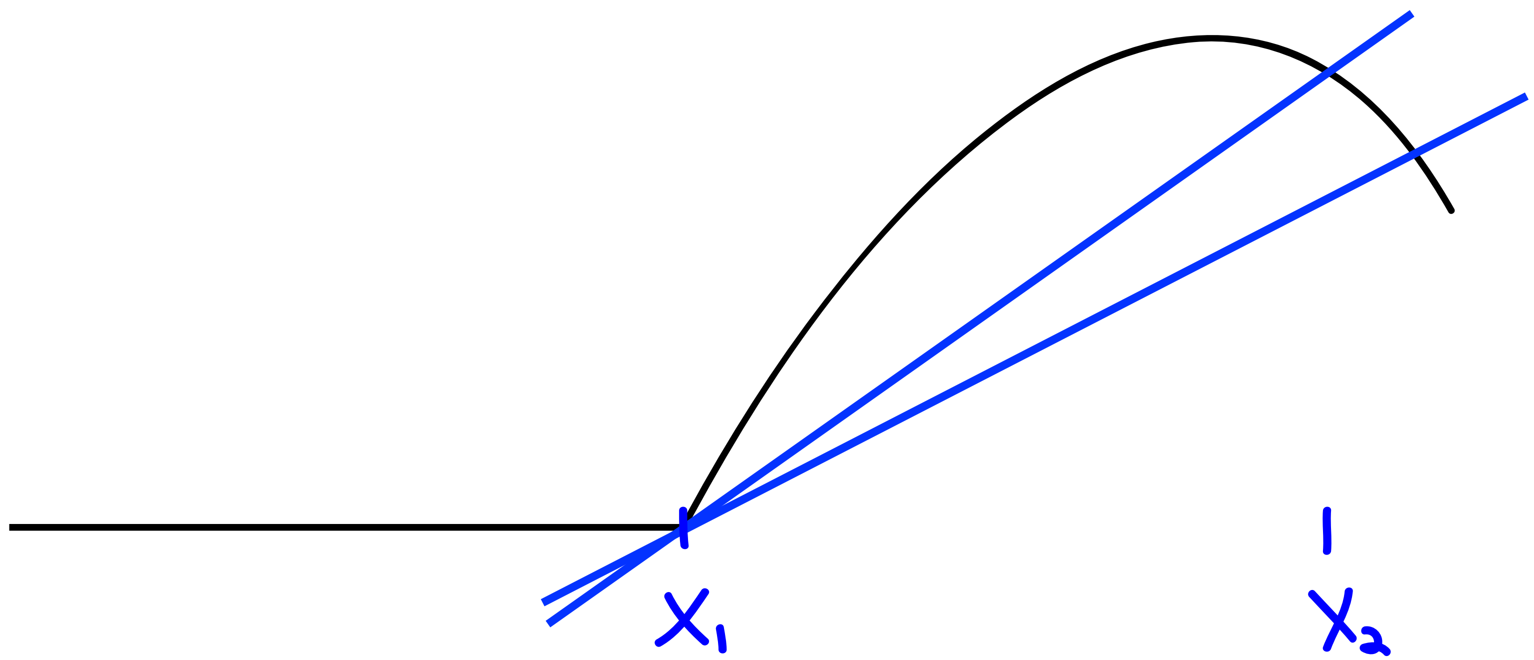

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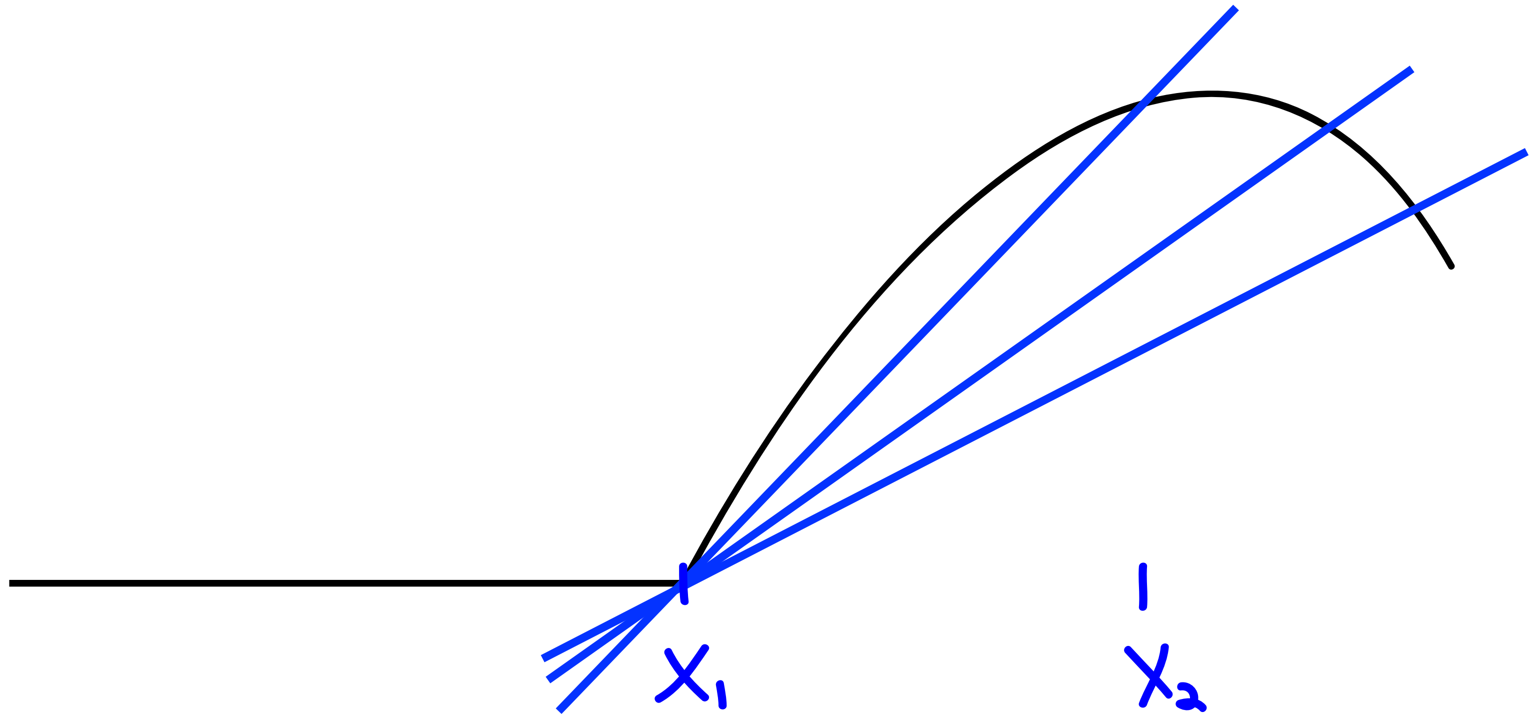

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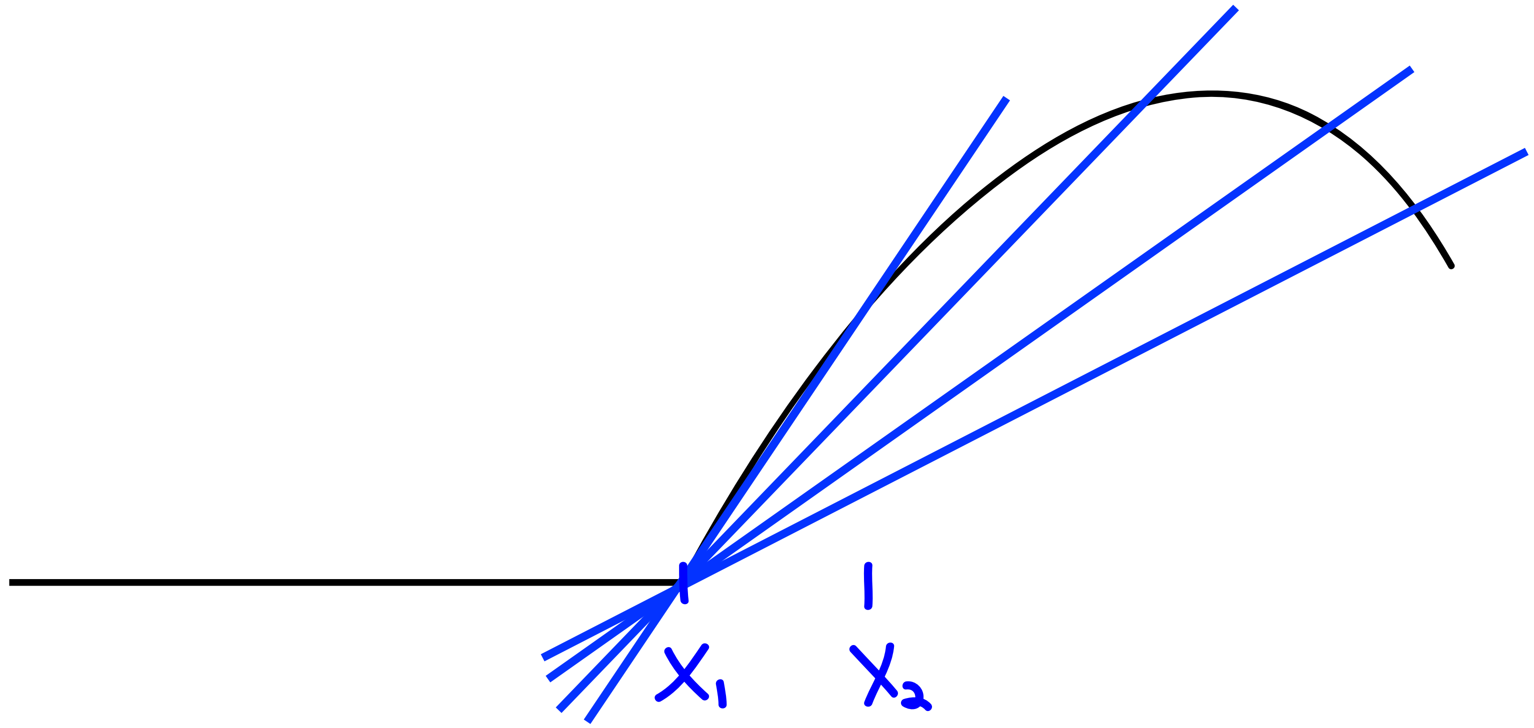

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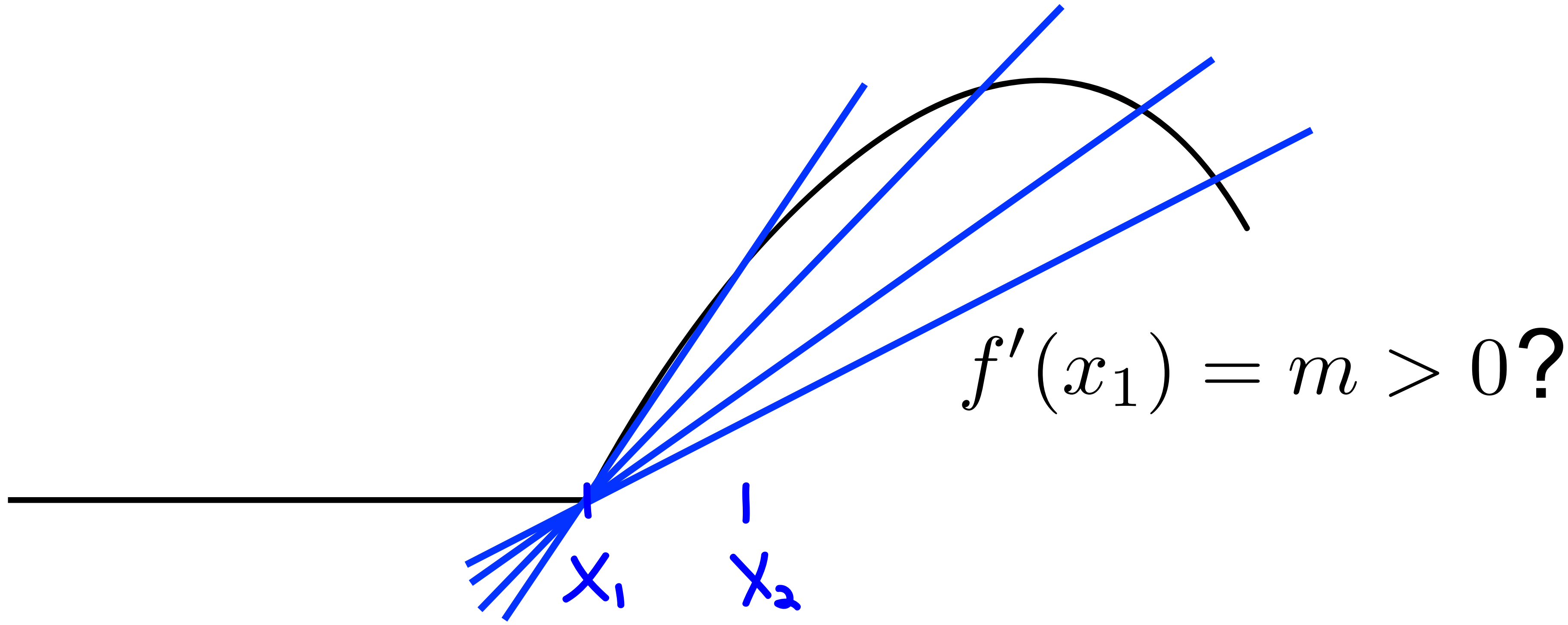

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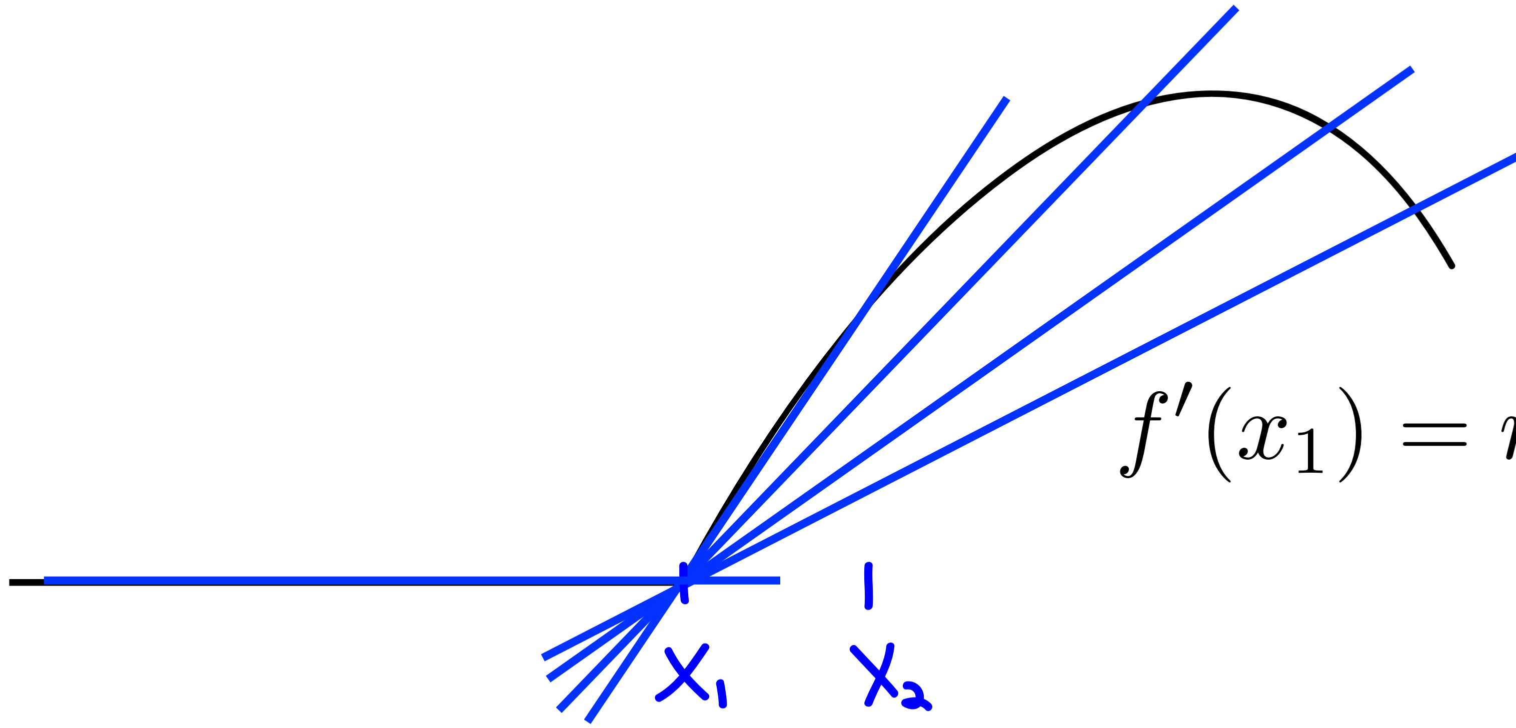

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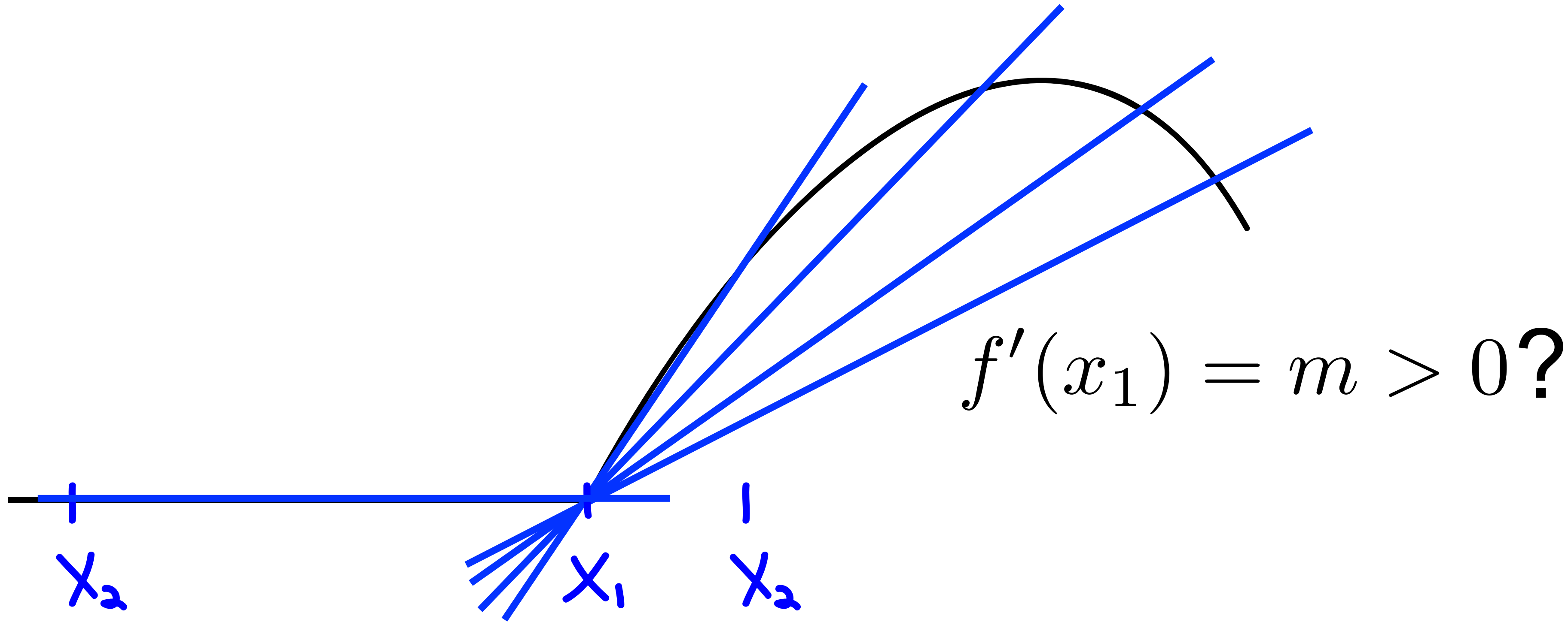

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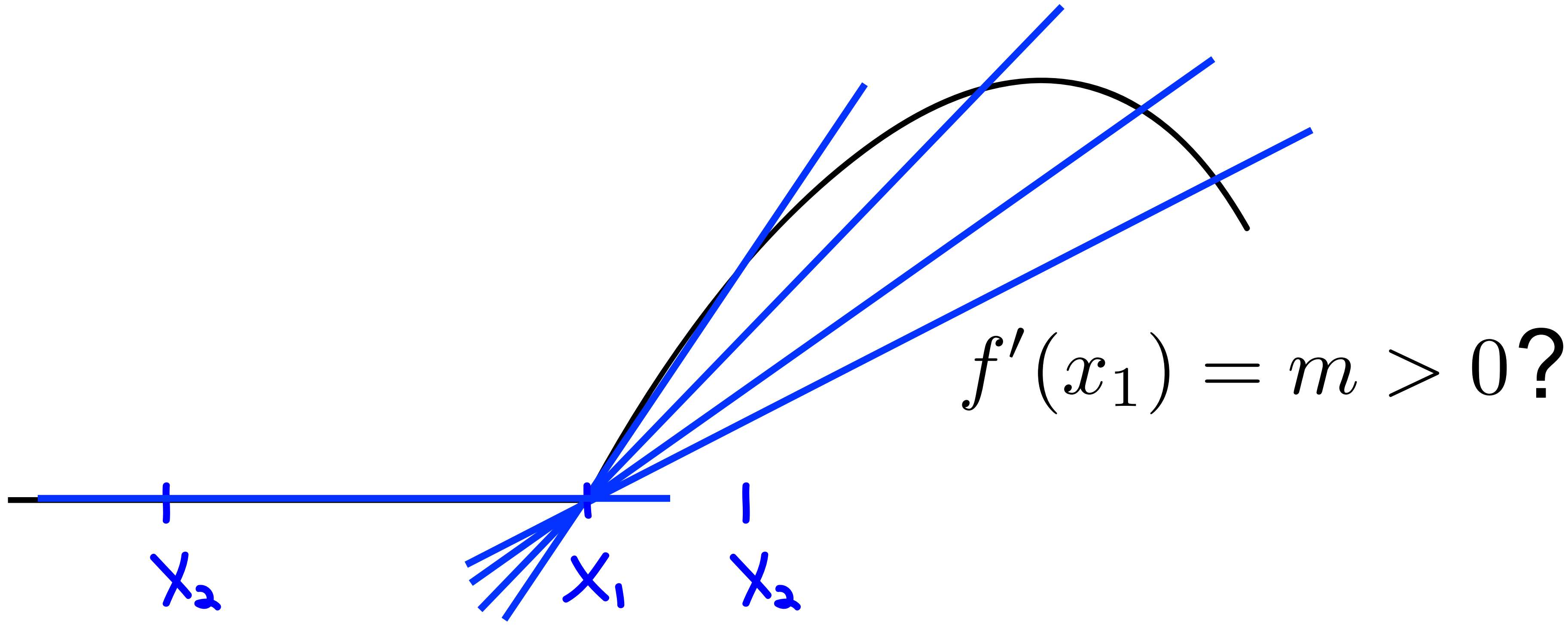

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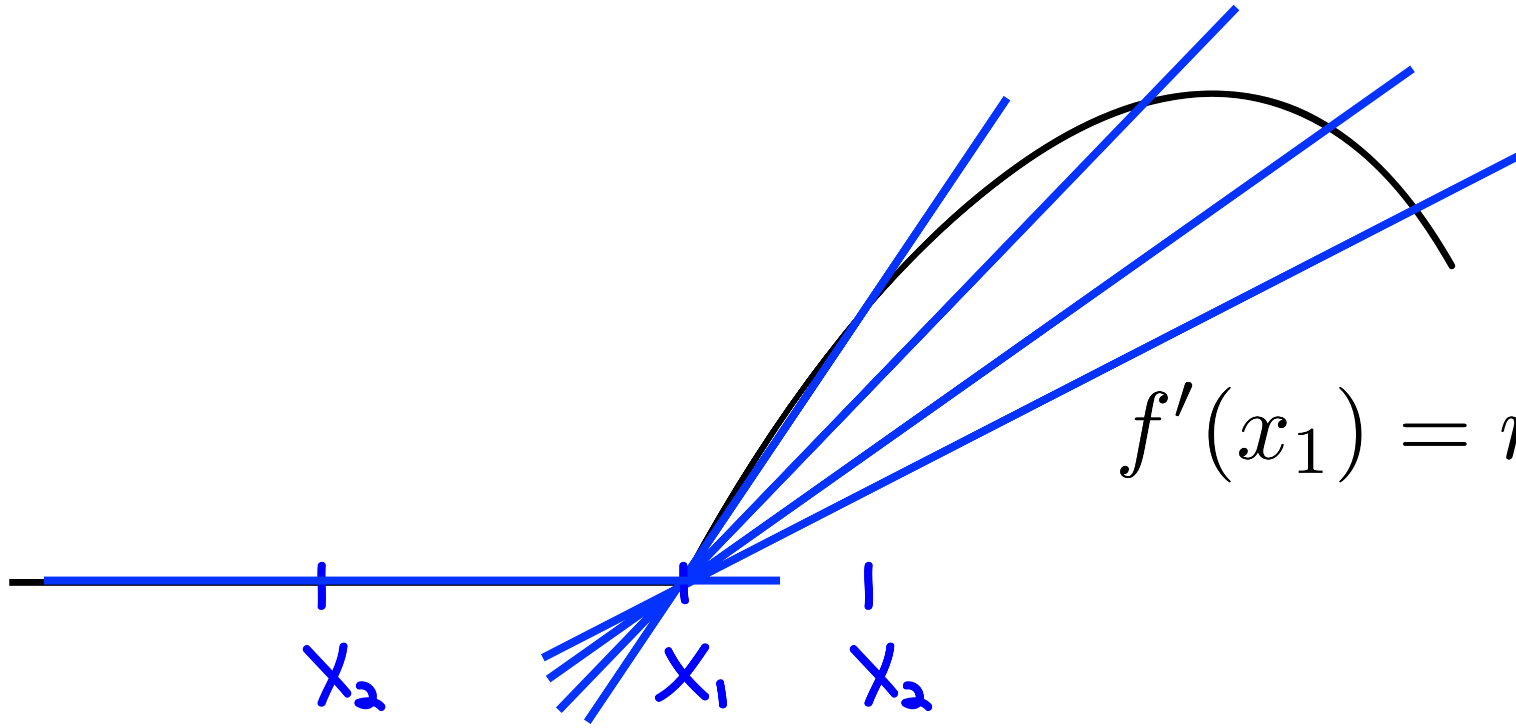

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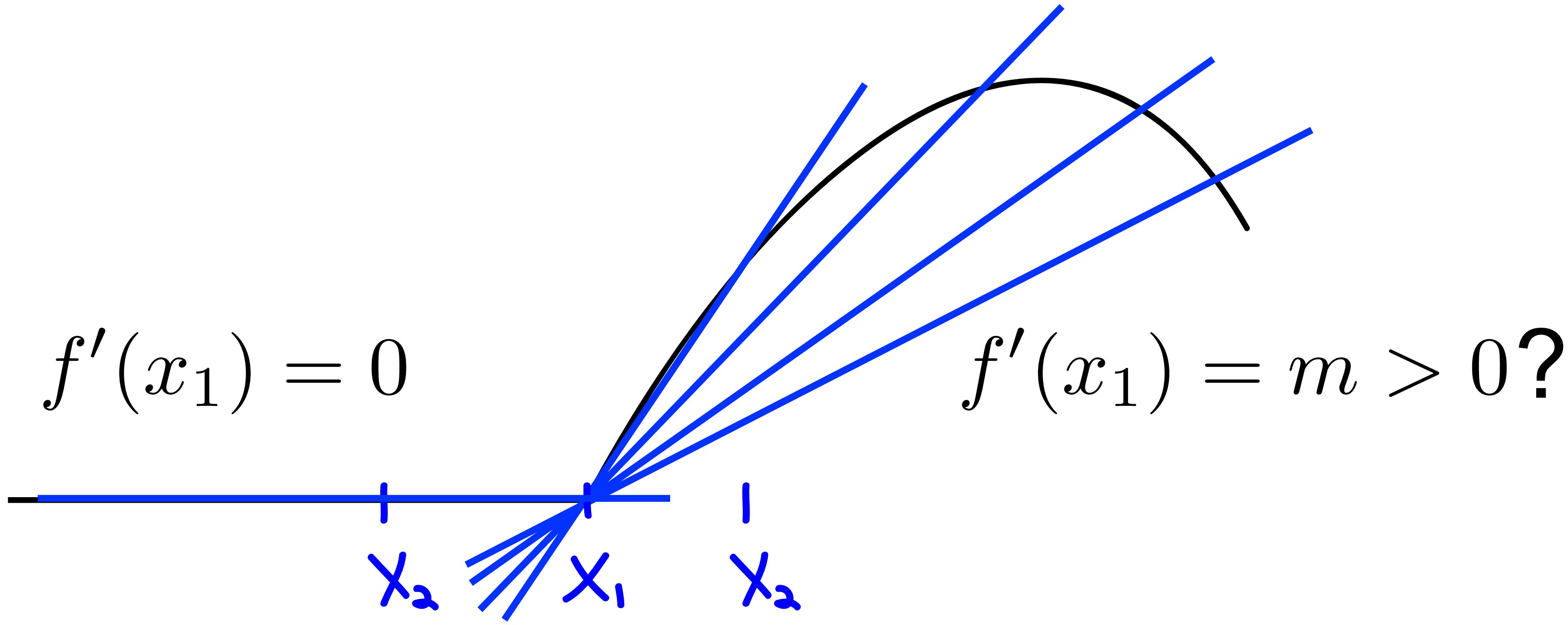

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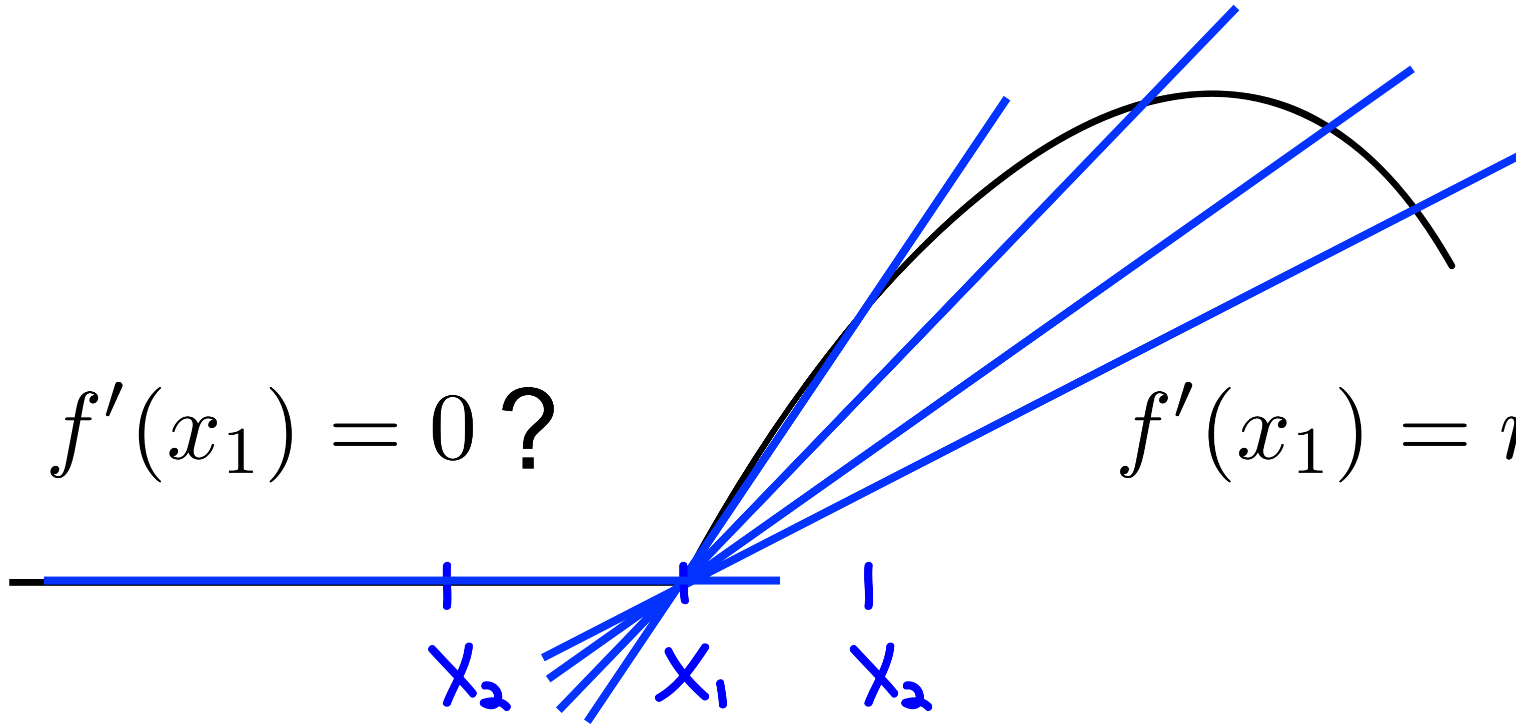

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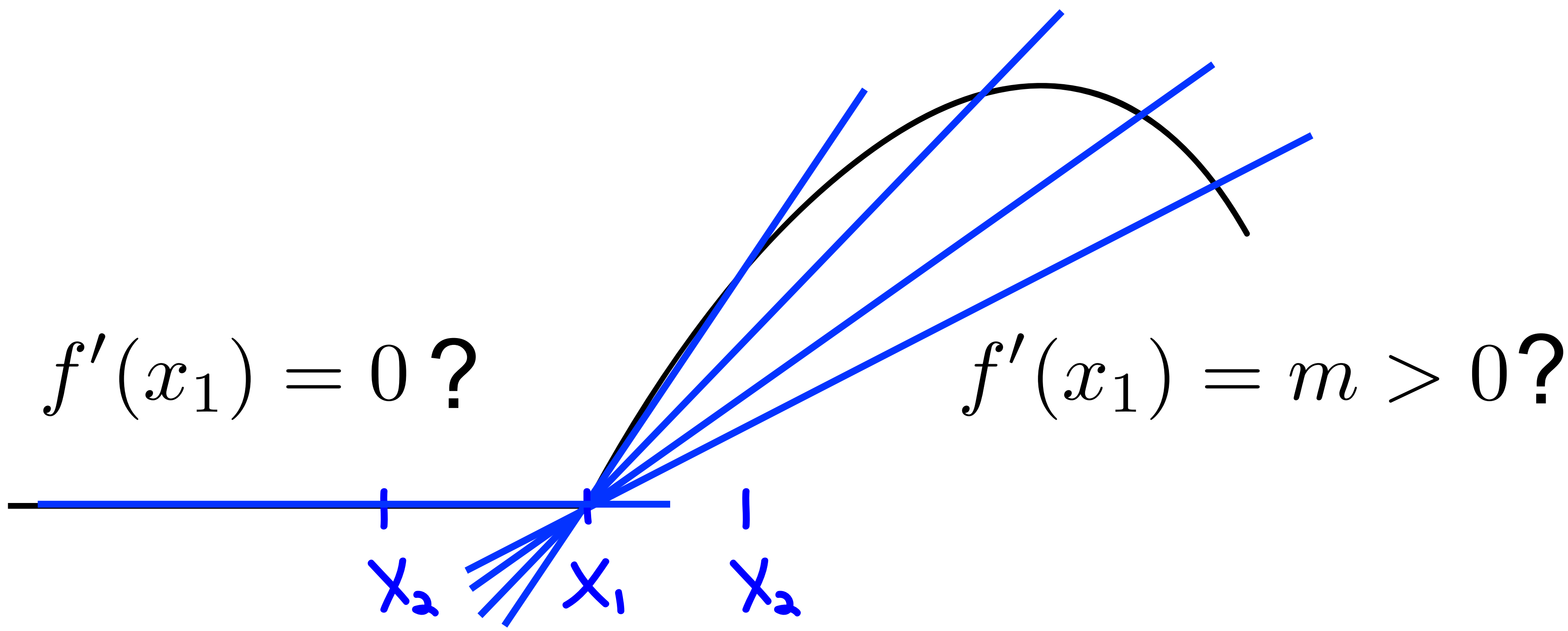

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(C) Both (A) and (B)

(D) The limit does not exist.

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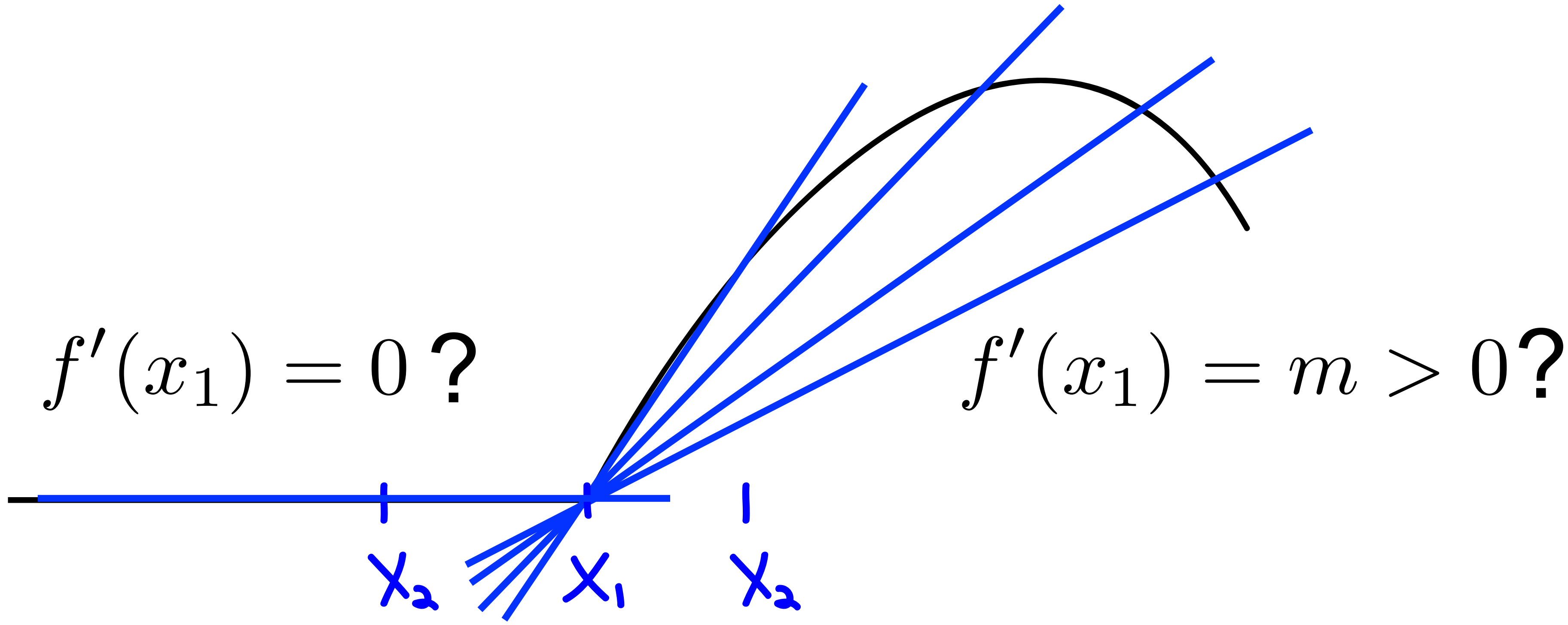

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The derivative of f(x) at x=a...

(A) ...touches the function at x=**a** but does not cross it.

(B) ...looks more and more like the function as you zoom in around x=**a**.

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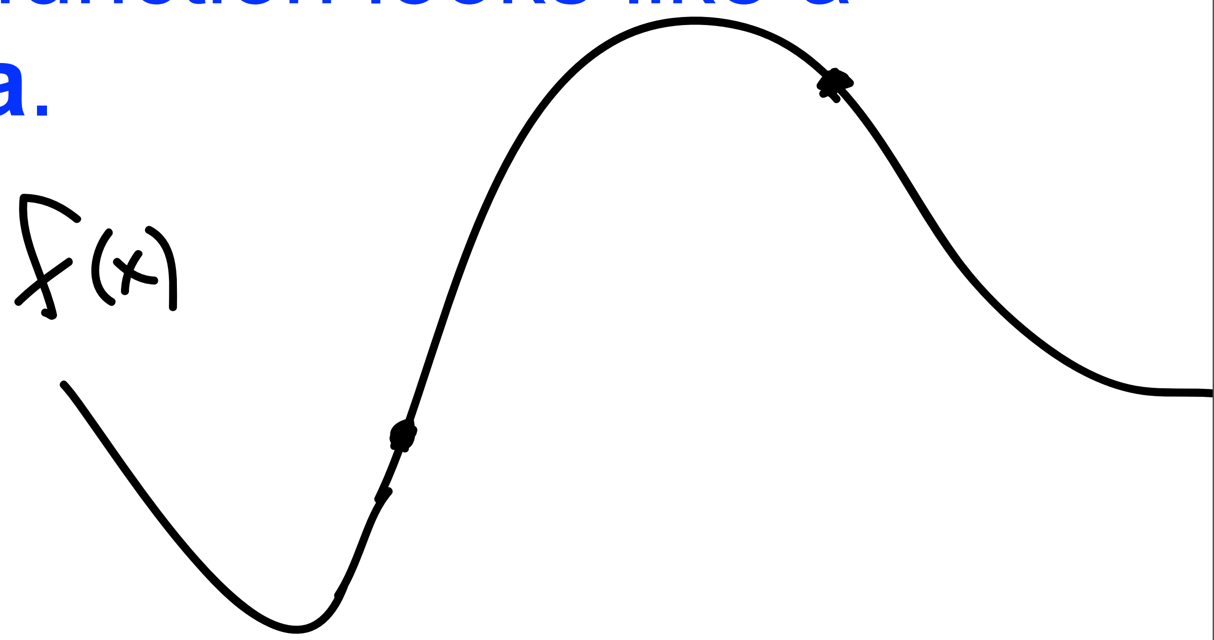

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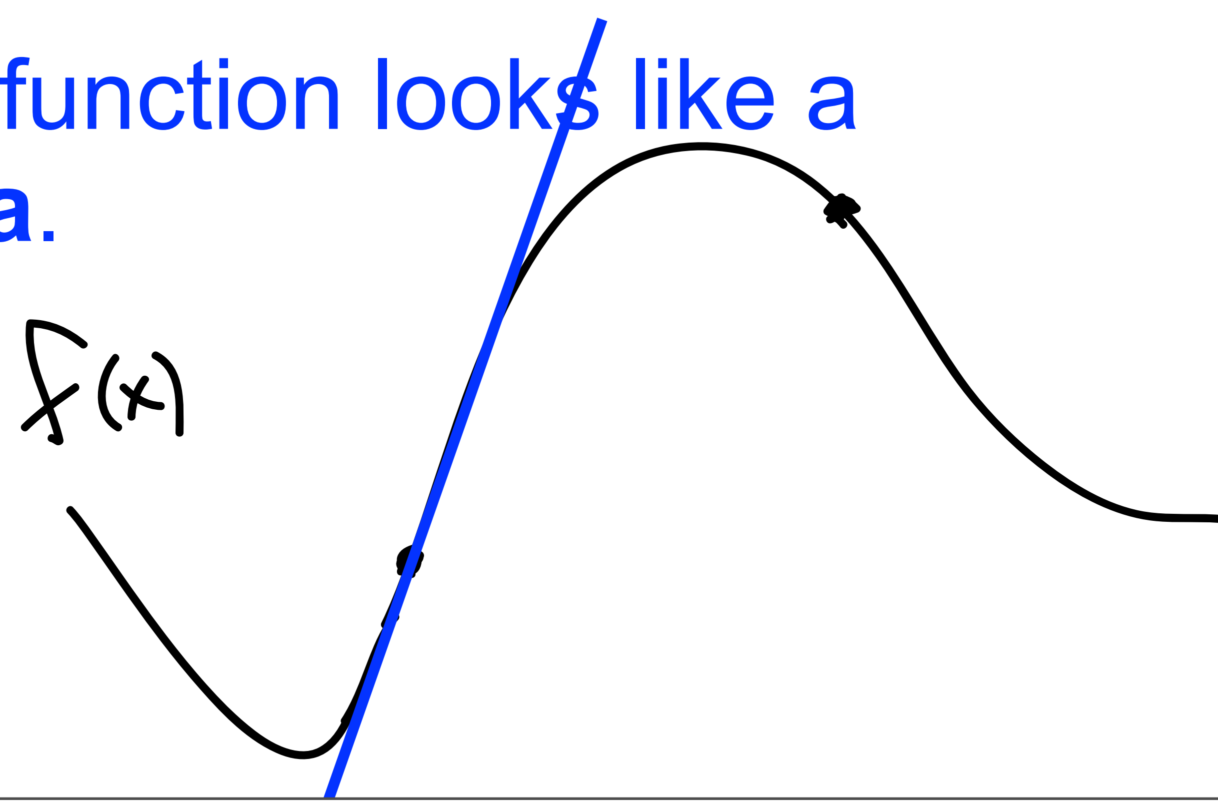

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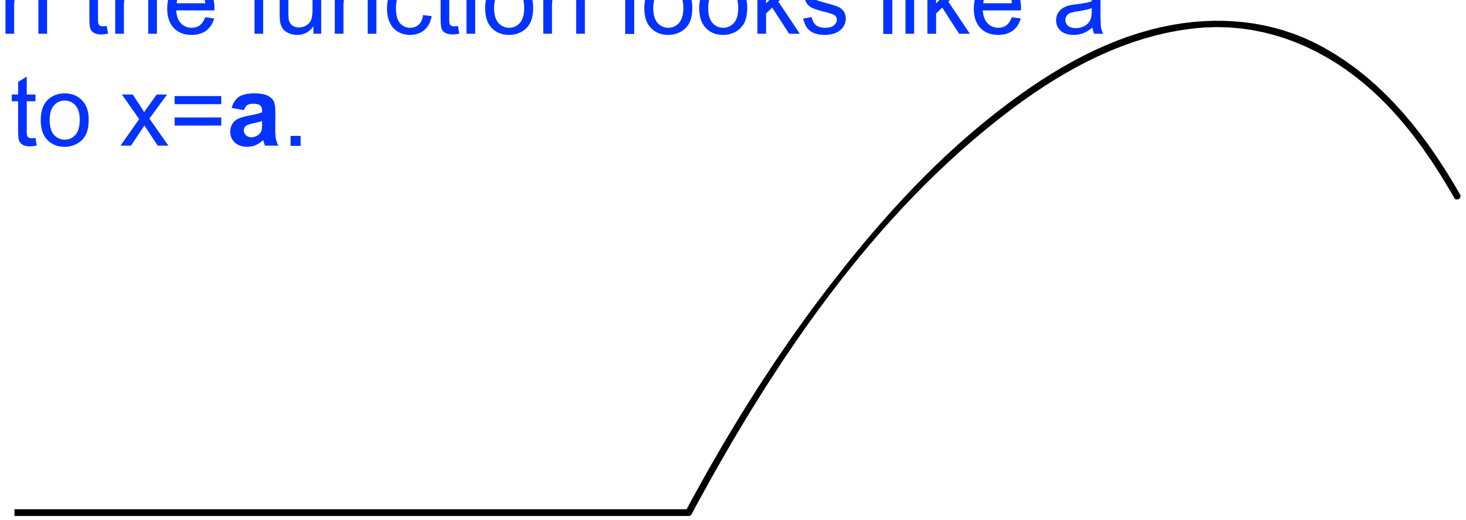

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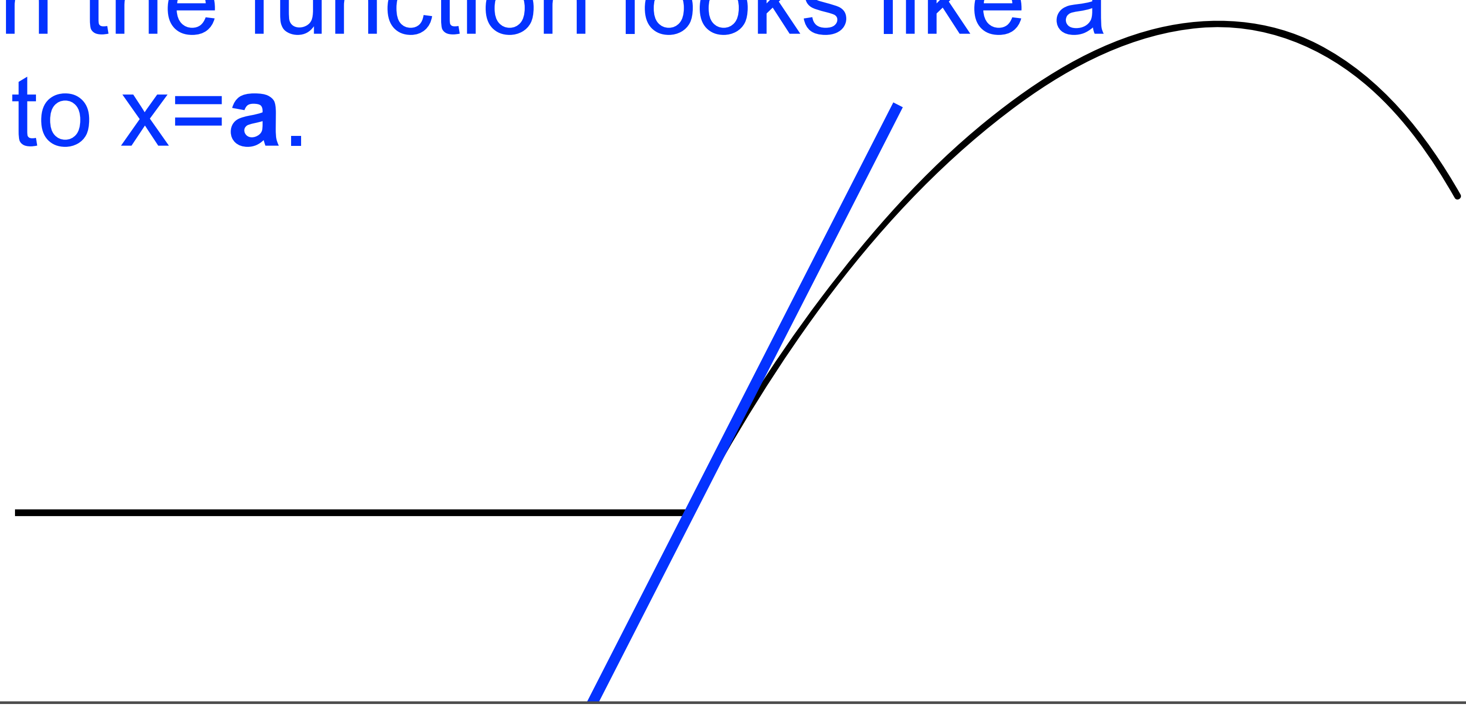

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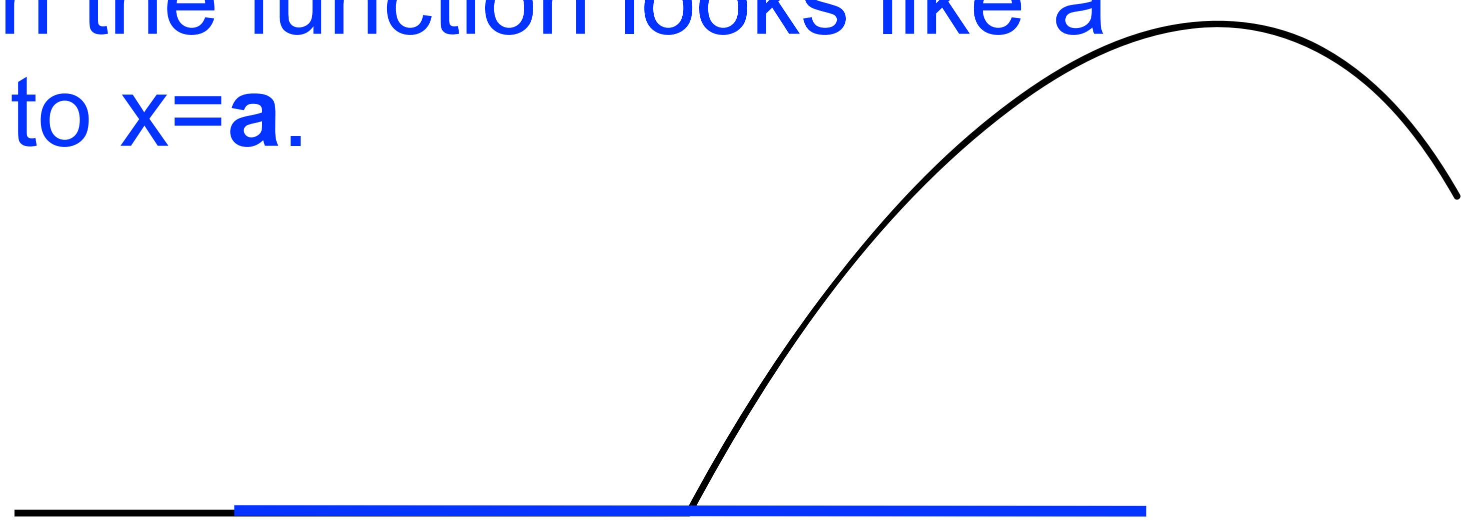

To evaluate a limit

To evaluate $\lim_{x \to a} f(x)$, you plug in values closer and closer to ***a*** but you never get to ***a***. In fact, ***f(a)*** may not even be defined. If you always get the same number no matter how you approach **a**, then the limit exists.

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Note: the limit involved in the derivative is only one special case. The limit above is concerned with the **value** of the function. When a limit has the form

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

we're talking about the slope of f (in this case, at x=2).

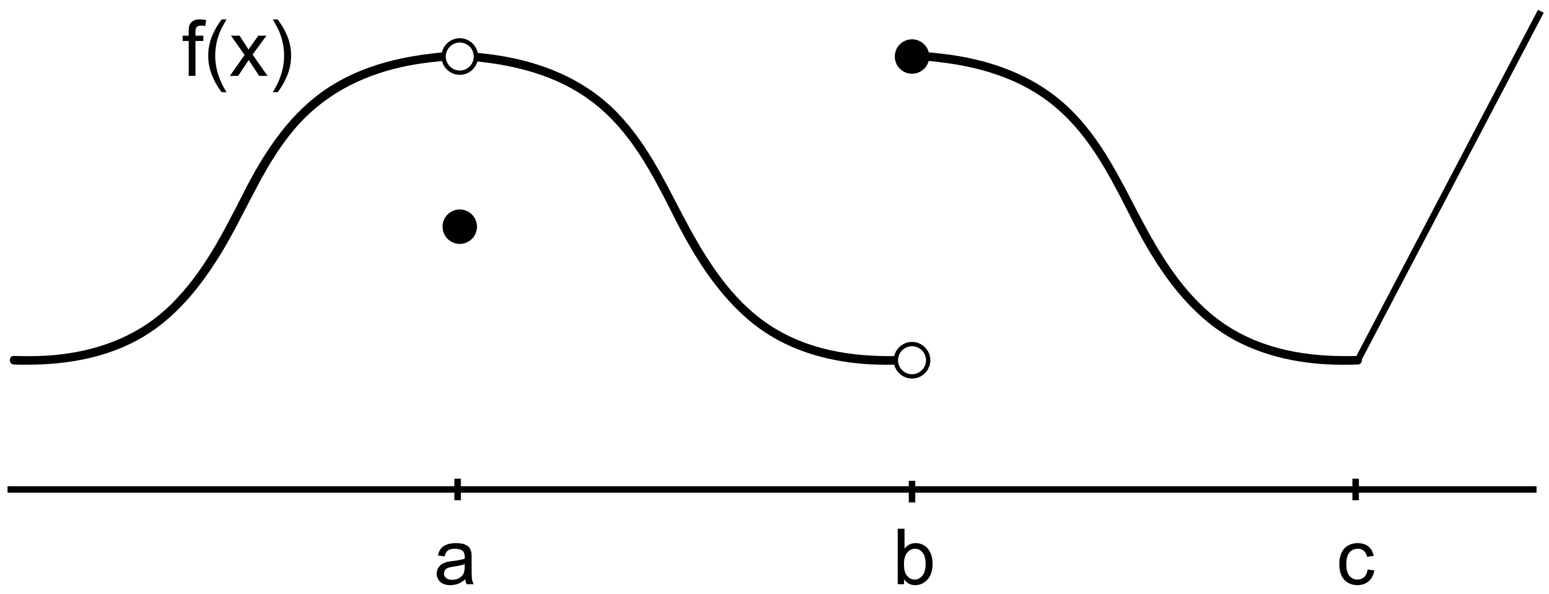
A WeBWorK limit example

Guess the value of the limit (if it exists) by evaluating the function at values close to where the limit is to be done. If it does not exist, enter DNE below.

$$\lim_{h \to 0} \frac{\sin(\frac{\pi}{4} + h) - \sin(\frac{\pi}{4})}{h}$$

Limit: [ ]

Limits



Which of the following are true?

1. $\lim_{x \to a} f(x) = f(a)$
2. $\lim_{x \to b} f(x) = f(b)$
3. $\lim_{x \to c} f(x)$ does not exist.
4. $\lim_{x \to a} f(x)$ exists.
5. $\lim_{x \to b} f(x)$ exists.

(A) 1, 4

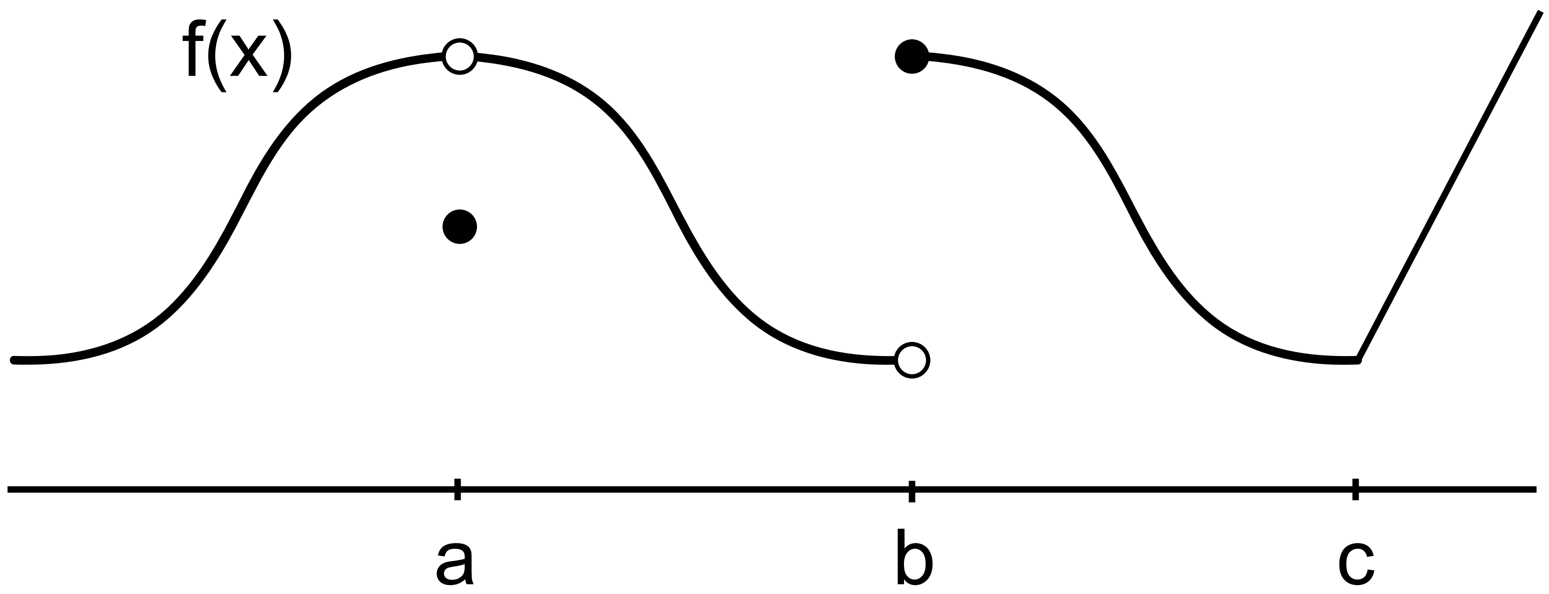
(B) 2, 5

(C) 3

(D) 4

(E) 5

Limits



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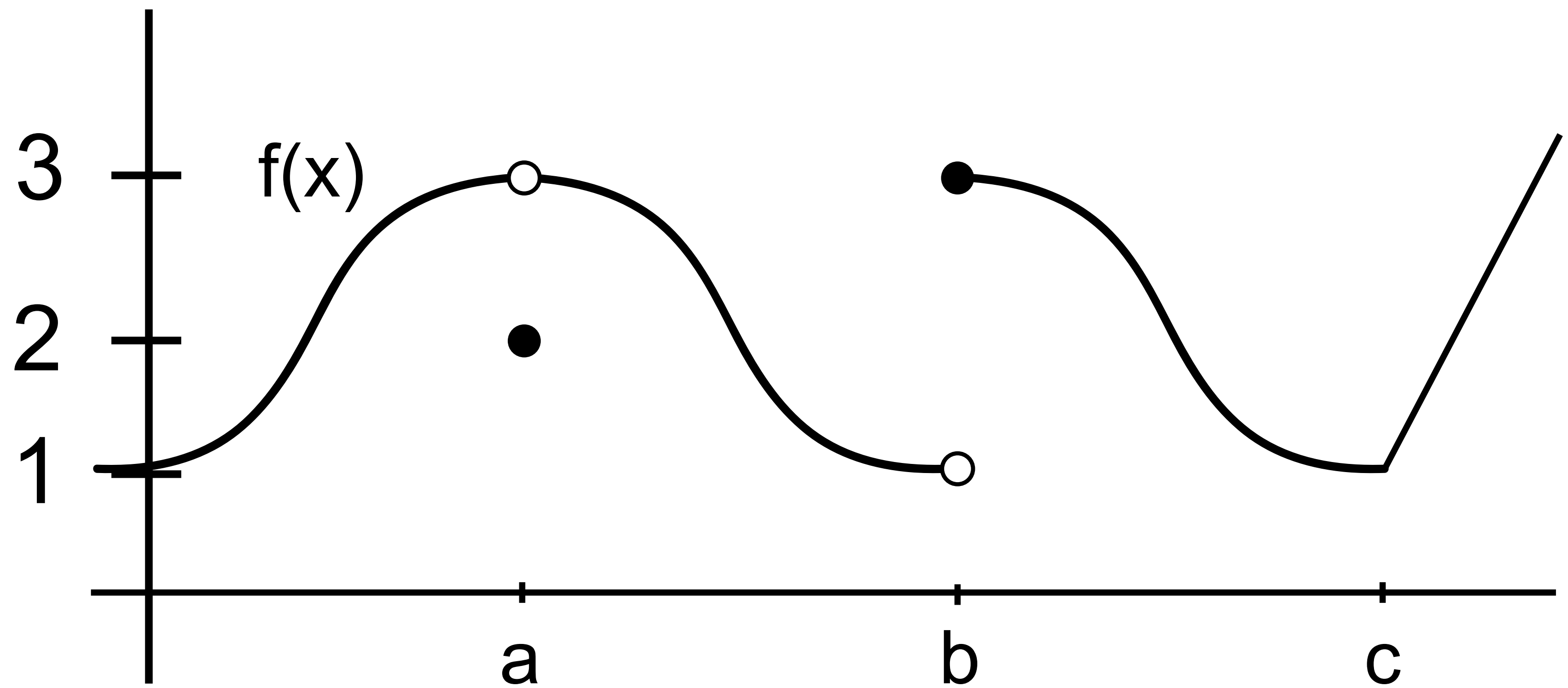

(A) $\lim_{x \to a} f(x) = 2$

(B) $\lim_{x \to b^-} f(x) = 3$

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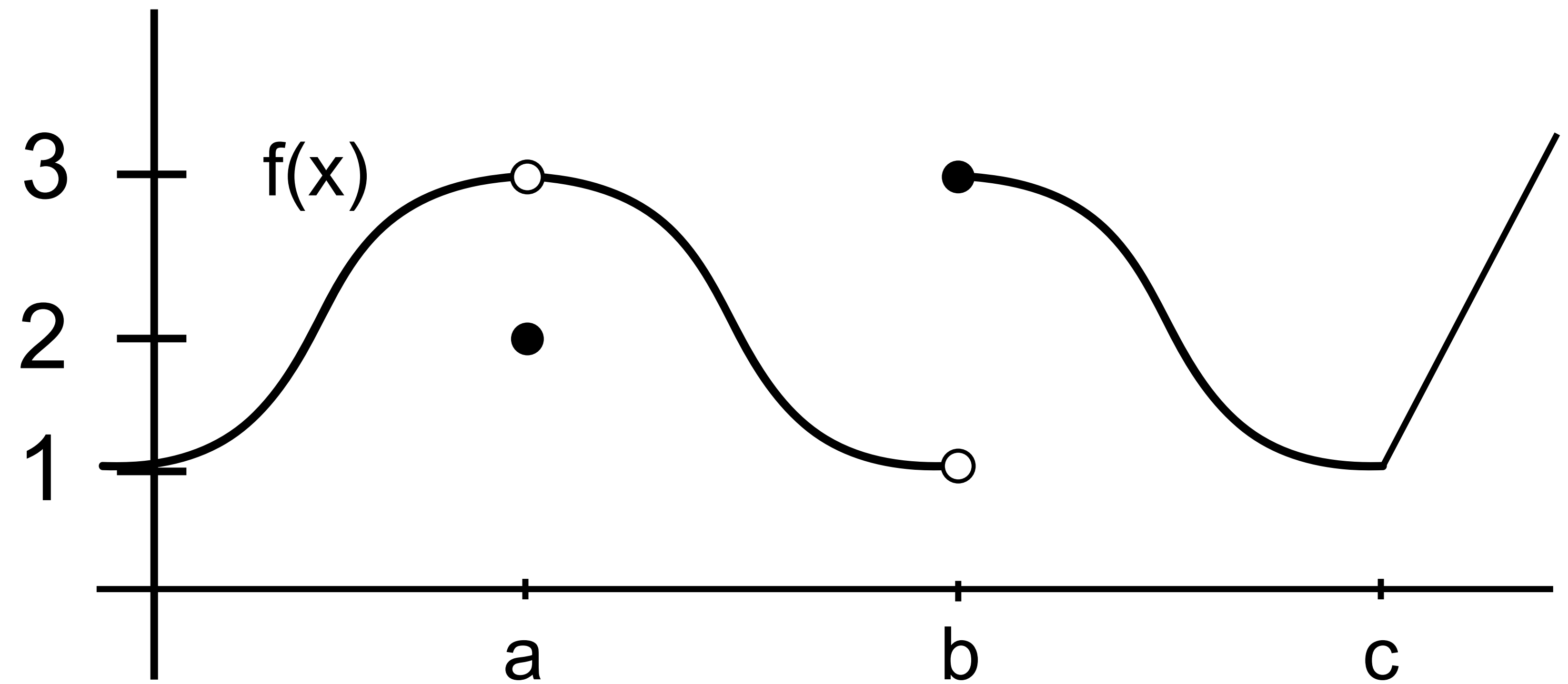

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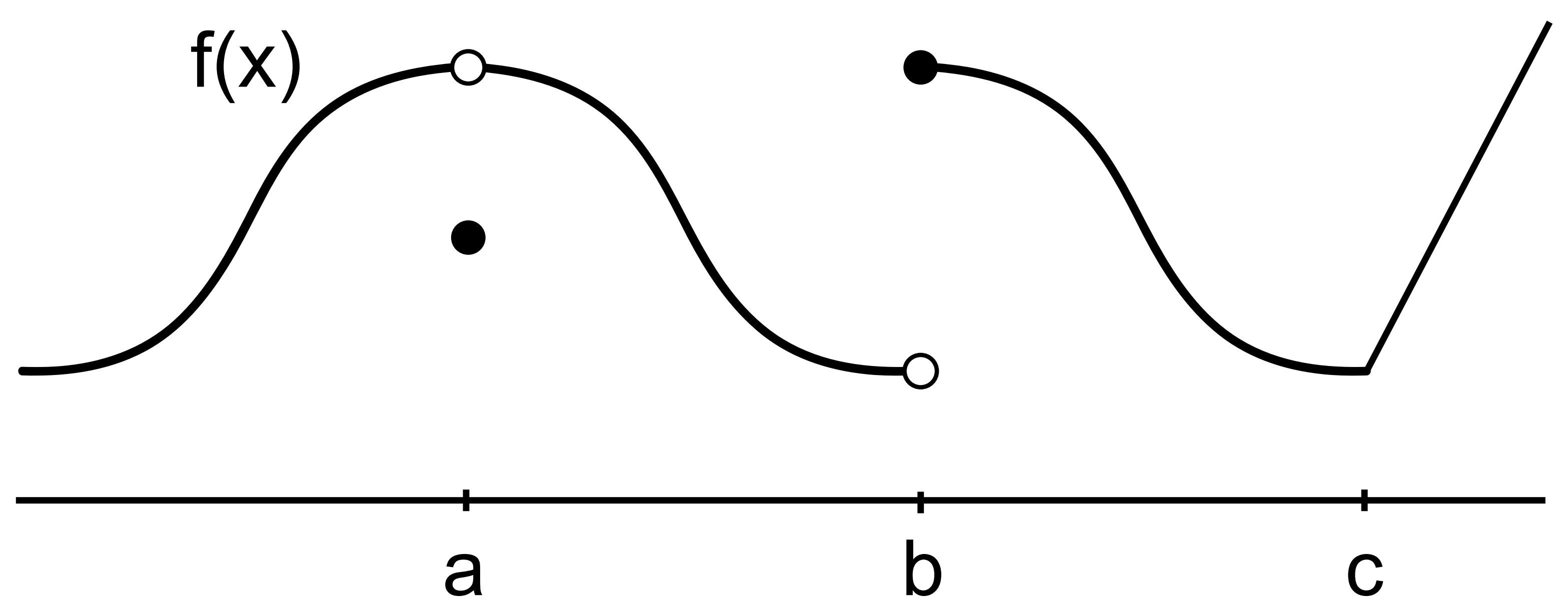

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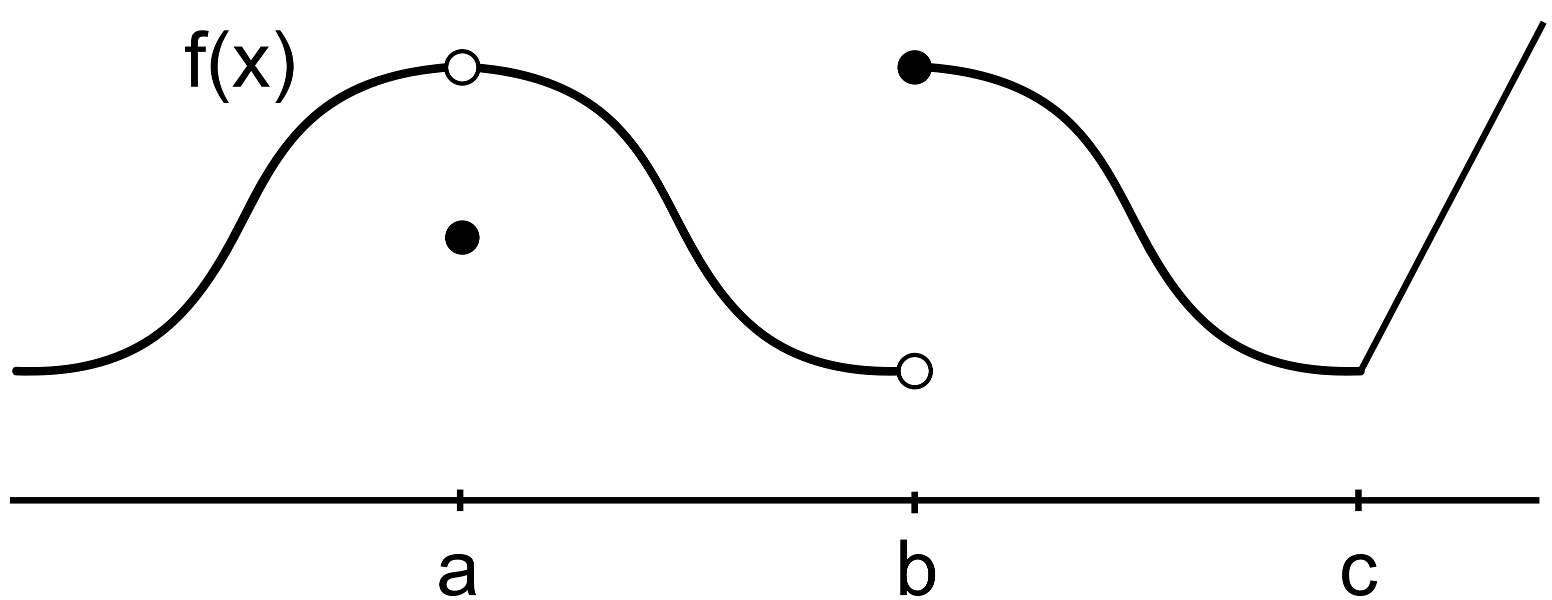

f(x) is continuous at all x except at x=a and x=b.