Today

- A lot of problems (hopefully).
- Safety reminder - don't walk on campus alone in the evening.
- AMS Safewalk (604 822 5355) or
- Campus Security (604-822-2222)

Requested problems

- Chain rule: sqrt(x+sqrt(x+sqrt(x)))
- Implicit diff: $e^{x/y} = 2x - 2y$
- Tangent line: function involving exp and/or log ($e^{xy} = 2x - 2y$ at (0,-1/2)
- Exponential function: twin survival (2012 midterm) - on the board
- Log rules: ln(x+1)-ln(x-1) = ln(x) (WW8 #11) - on the board
- Log derivatives: f(x) = x ln(x) - x, f'(x) = ??? - on the board
- Optimization: A wire 10 m long is cut into two pieces. - on the board
- Optimization: Cone inside a cone - partially on board
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- Optimization: Find pts on $16x^2+y^2=16$ farthest from (1,0) - on the board
- Optimization: Linear regression
- Related rate: height of water in cone
- Related rates: Baseball diamond (WW7 #16)
- Related rates: Sailing ships (WW7 # 13)
- Solve a DE/IVP: dA/dT=-9A , A(0)=6
- Check solution to DE: Bucket emptying of water (WW9 #9)
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- $y'\,(2 - e^{x/y}x/y^2) = 2 - e^{x/y}/y$ <-- factor y'
- $y' = (2 - e^{x/y}/y) / (2 - e^{x/y}x/y^2)$ <-- solve for y'

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$e^{xy} = 2x - 2y$ at $(0,-1/2)$

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- $y' = (2+1/2) / 2 = 5/4$
- Tangent line: $y = 5/4\,(x-0) - 1/2$.

Long Answer Problems

5. During pregnancy, a fraction of twins are at risk of death due to entanglement of their umbilical cords. To prevent such deaths, the twins are removed early by surgery (Caesarean section). However, removing them earlier puts them at risk of death due to premature birth. In this problem, you will determine when to schedule delivery so as to maximize their chance of survival in the face of these opposing risk factors.

   (a) [**5 pt**] Out of 25 such pregnancies that were identified at 24 weeks into the pregnancy, 16 were still alive at 32 weeks. Write down an exponential function that describes the number of surviving twins, $T(t)$, as a function of the time (measured in weeks) into the pregnancy.

On the board.

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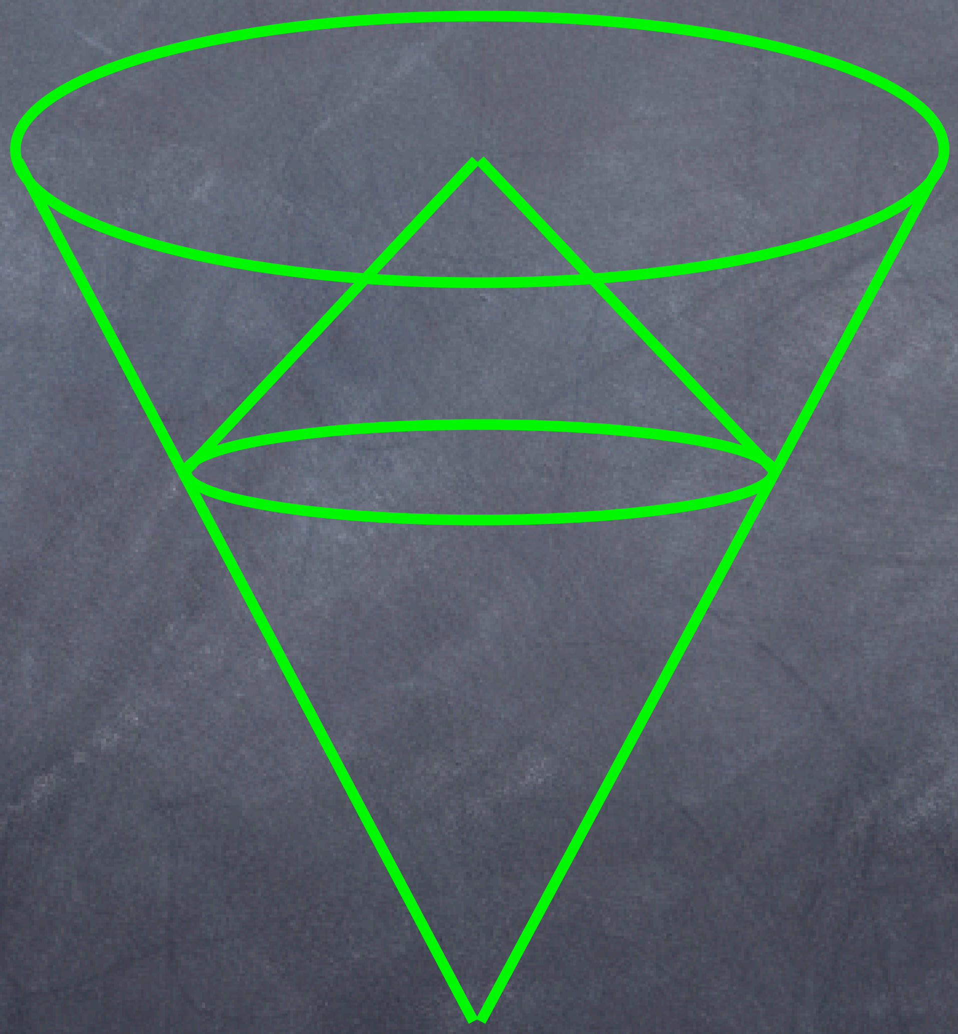
A cone with height h is inscribed in a larger cone with height H so that its vertex is at the center of the base of the larger cone. Find the height of the inner cone which maximizes the volume of the inner cone.



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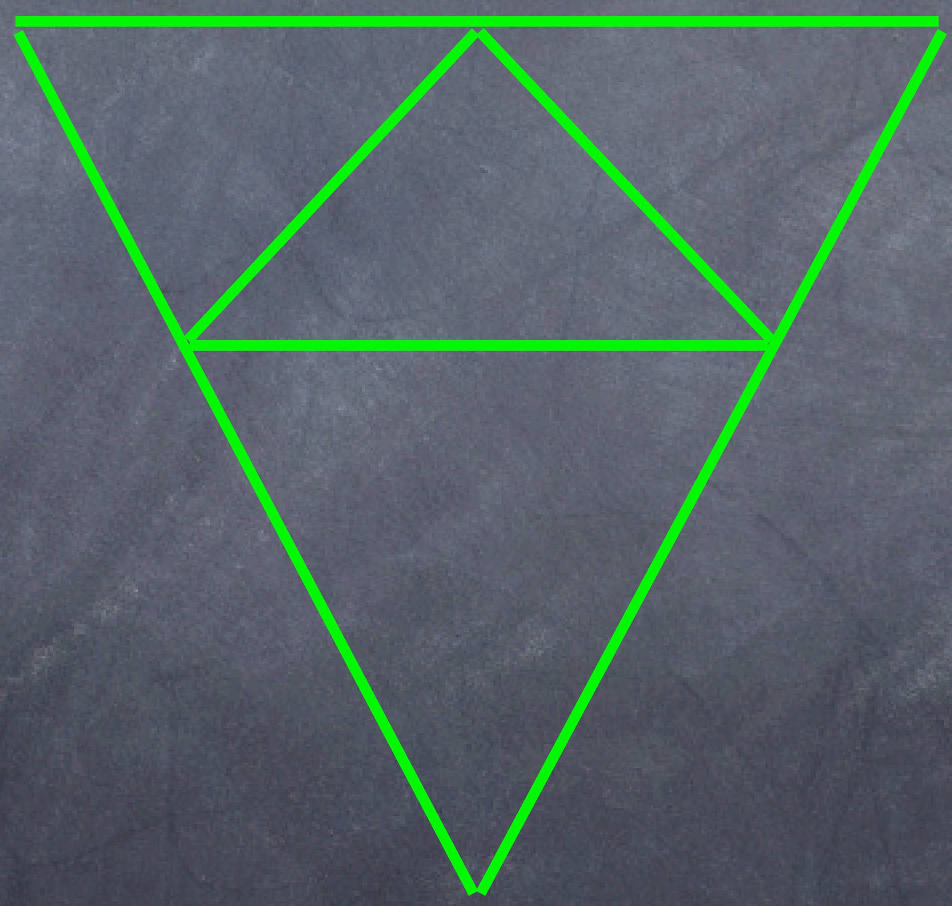

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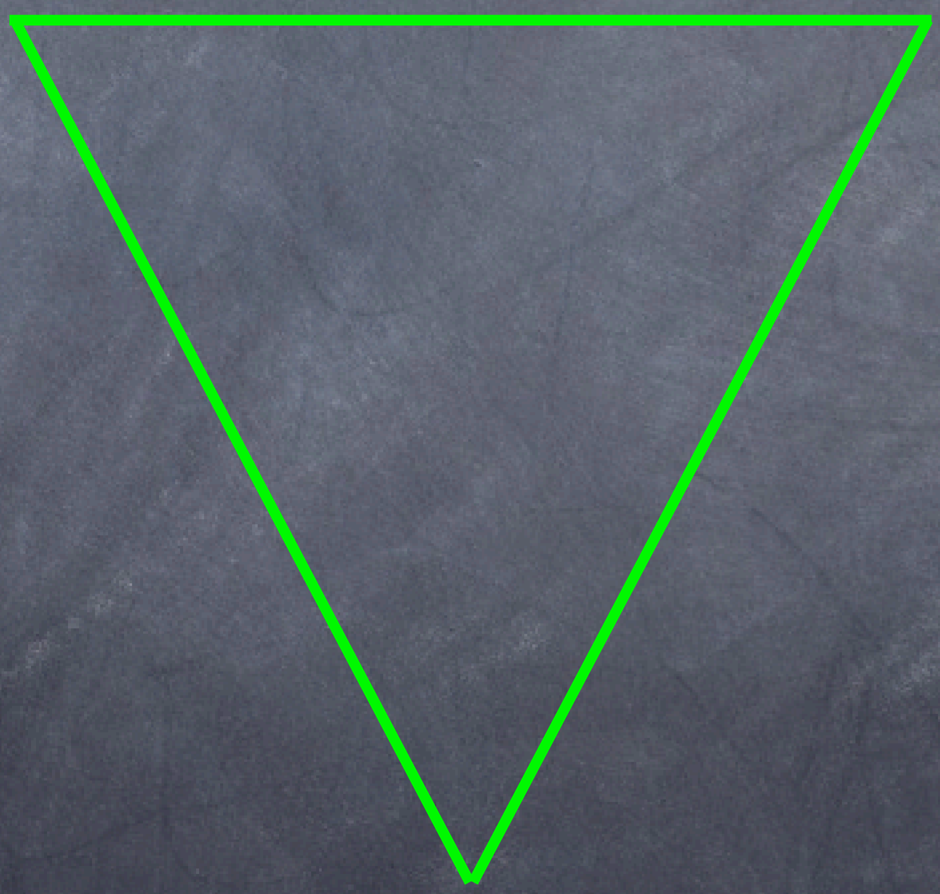

Step 2 - draw several versions.

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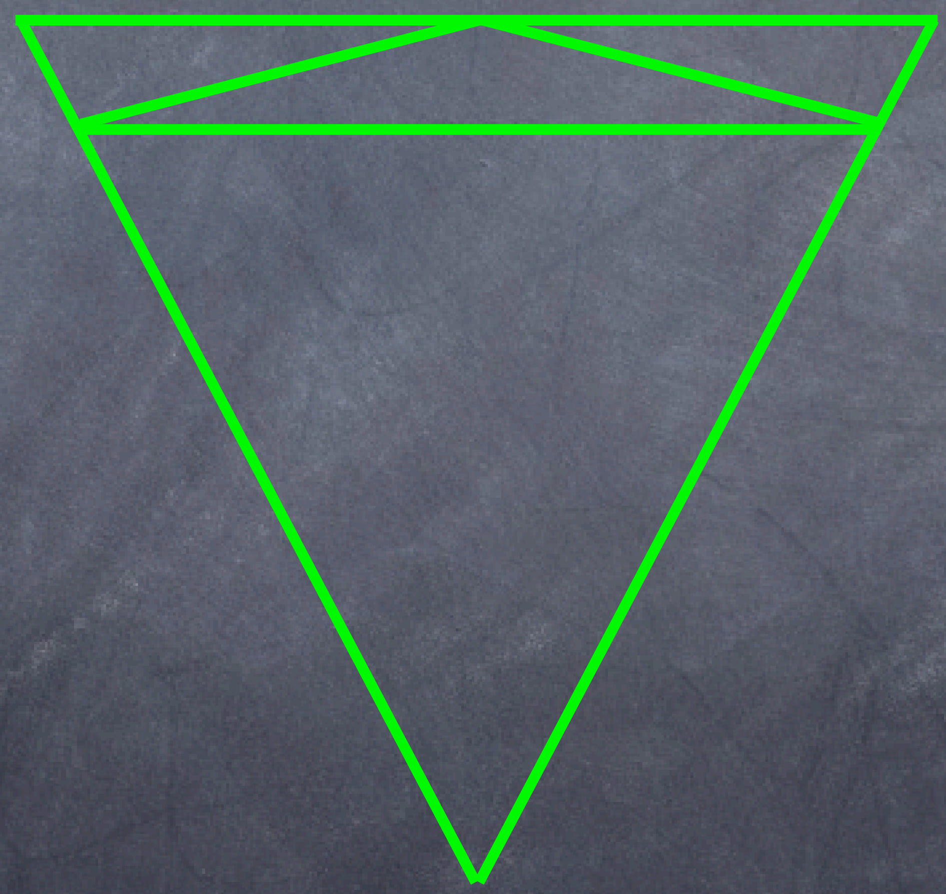

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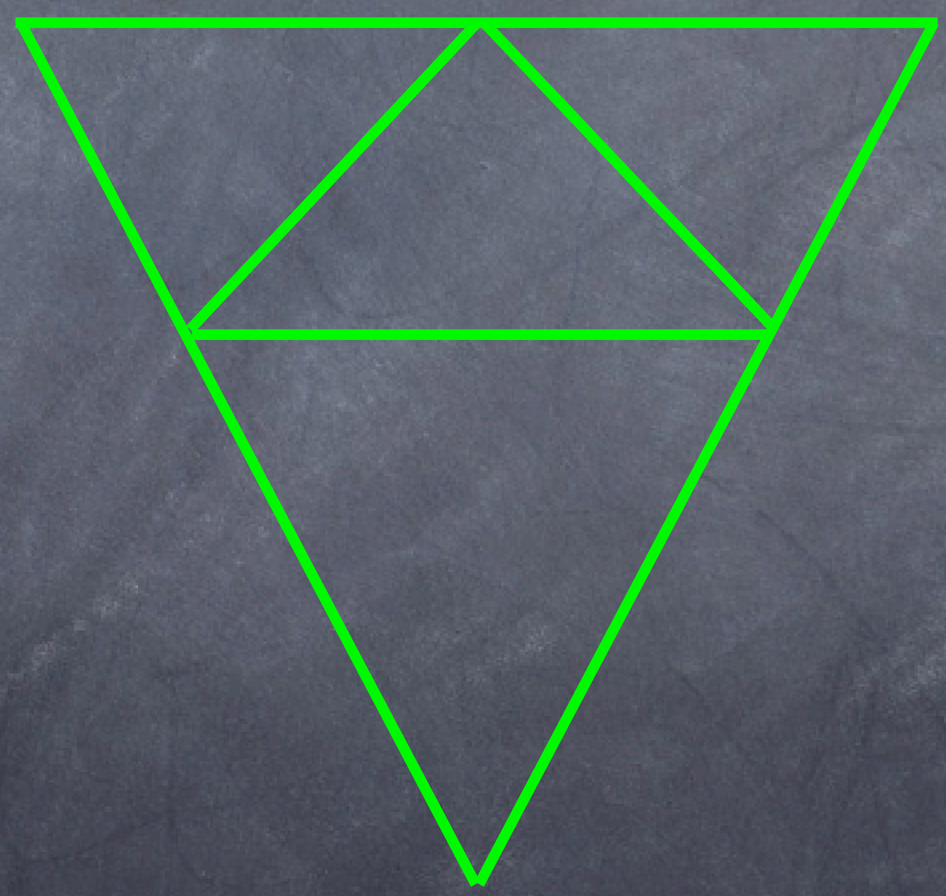

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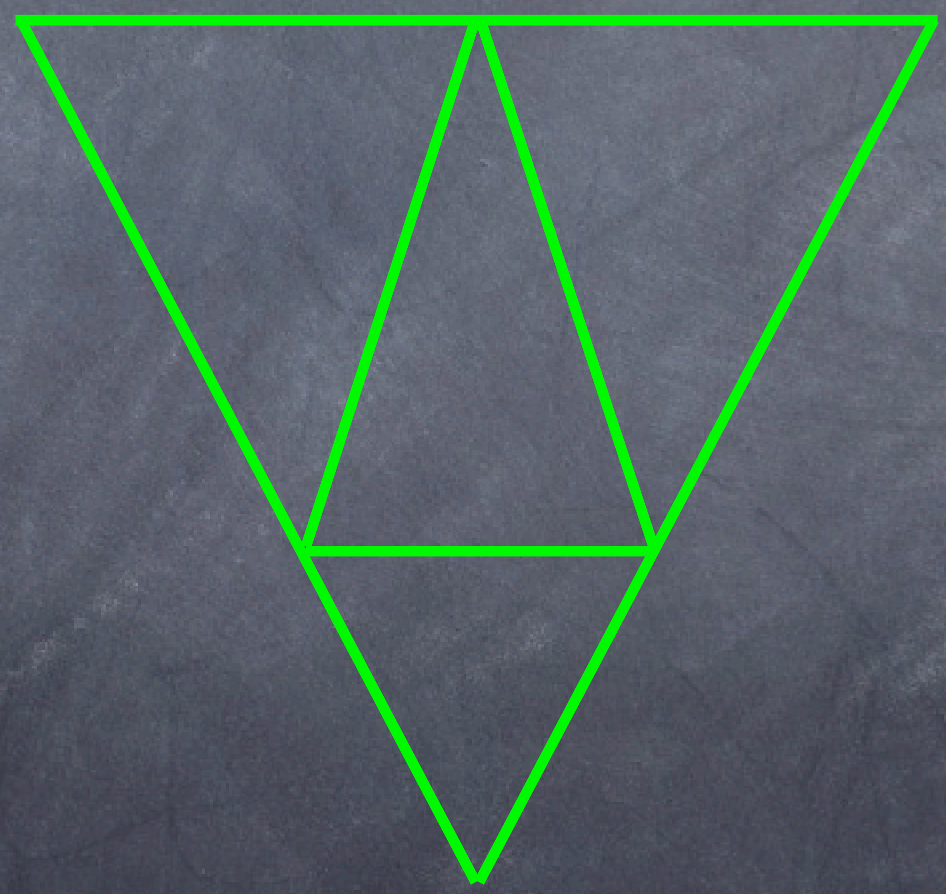

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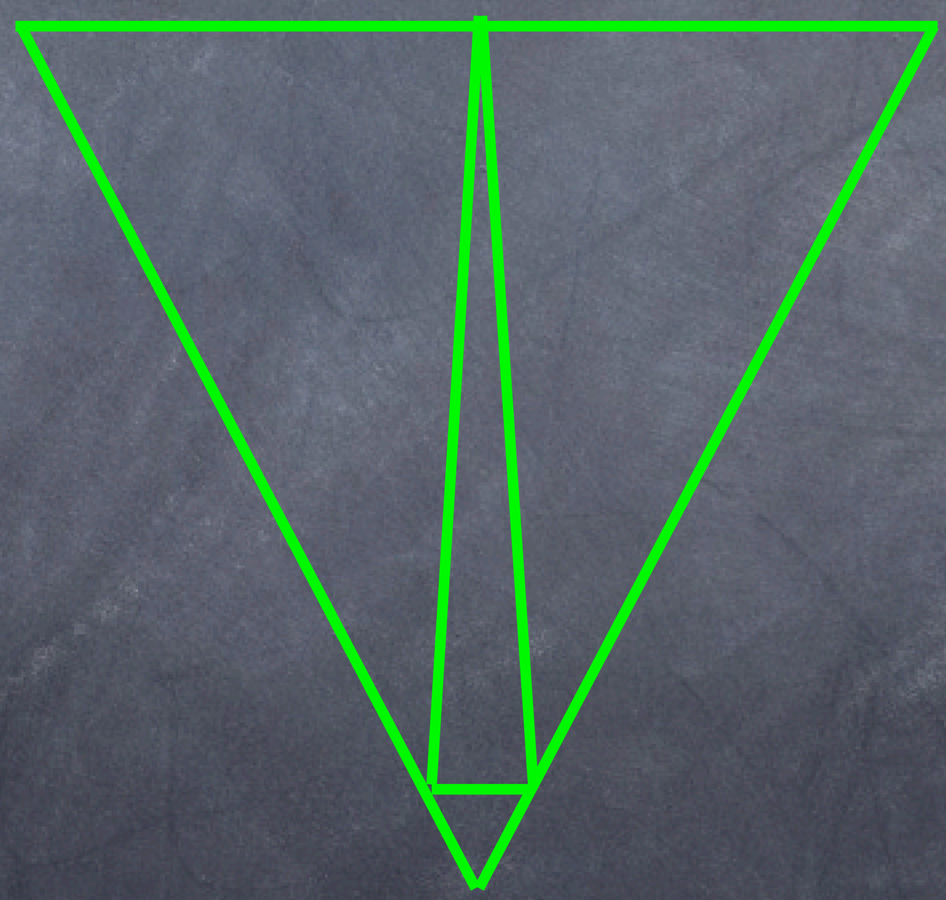

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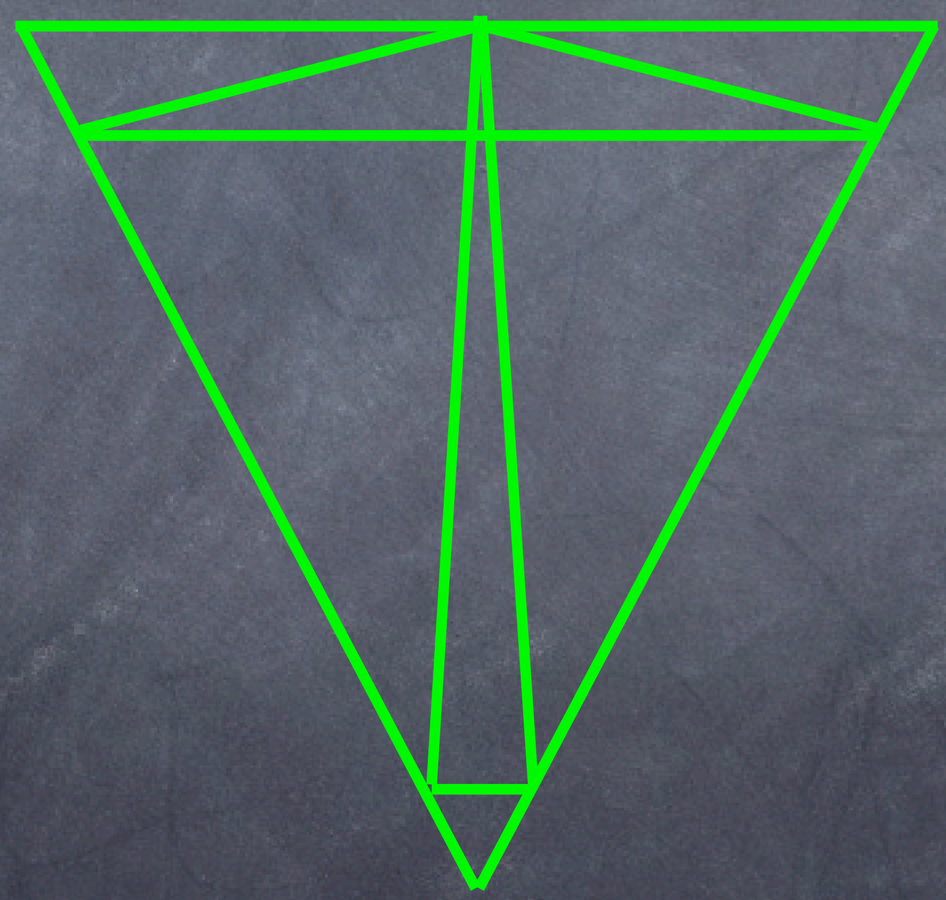

Step 3 - establish expectation - not a boundary max.

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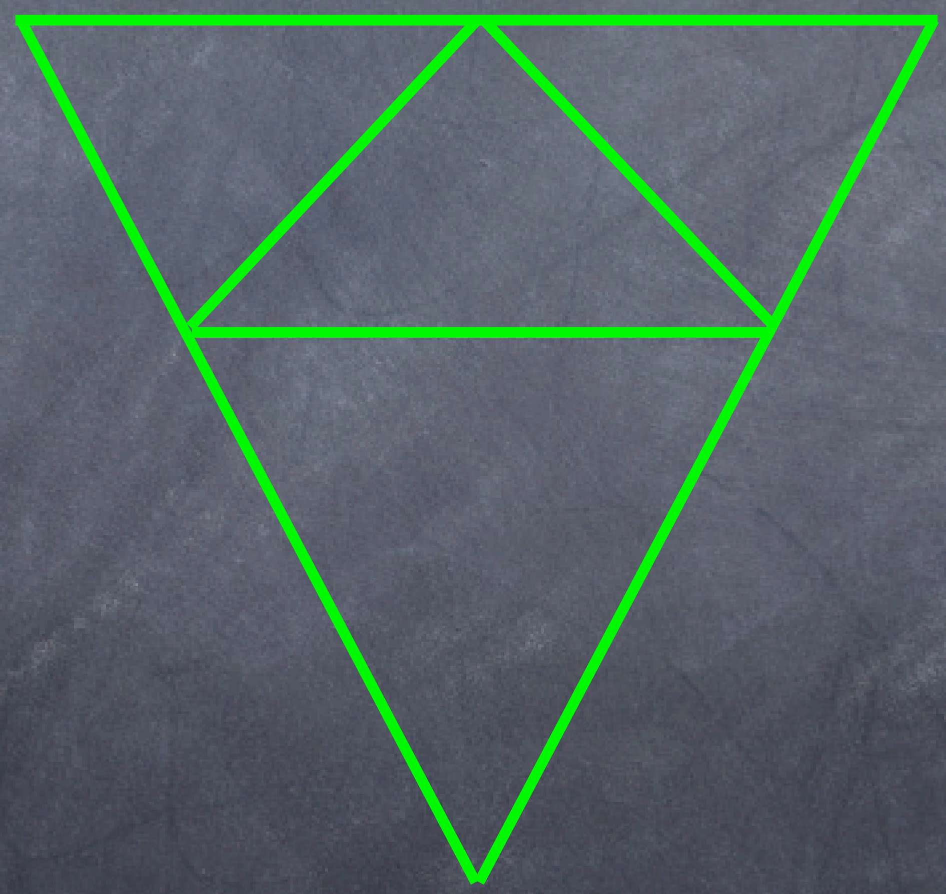

Step 4 - find objective function, constraint (on board)

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   (b) [**3 pt**] The overall probability of survival for a set of such twins, accounting for both survival to delivery and survival once delivered, is

$$S(t) = A(t-7)\exp(-t/20)$$

   where A is a constant. At which time t should the twins be removed to maximize their overall probability of survival?

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A cylindrical bucket has a hole in the bottom. If $h(t)$ is the height of the water at any time t in hours, then the differential equation describing this leaky bucket is given by the equation:

$$\frac{dh(t)}{dt} = -6\sqrt{h(t)}.$$

If initially, there are 4 inches of water in the bucket $(h(0) = 4)$, what is the solution to this differential equation?

A. $h(t) = (2 - 3t)^2$
B. $h(t) = \sqrt{16 - 2t}$
C. $h(t) = (3 - 3t)^2$
D. $h(t) = 4 - 6t^2$