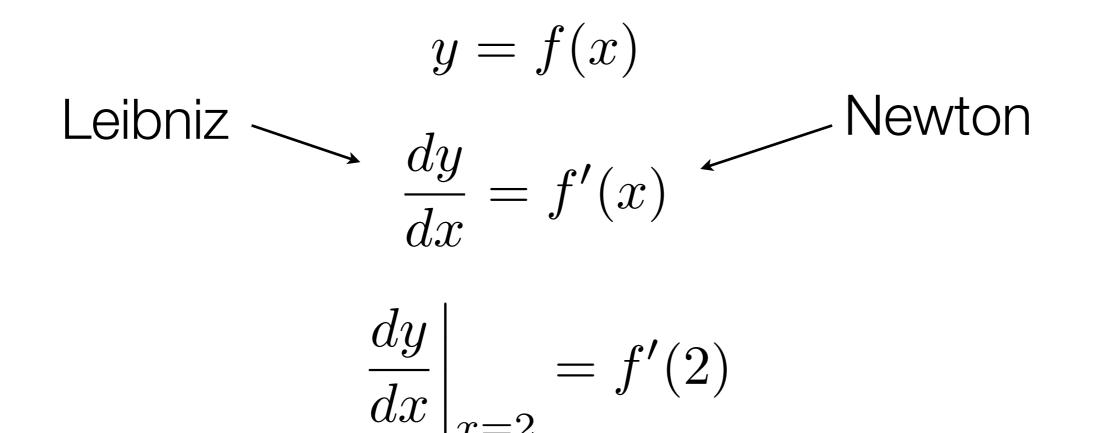
Today

- A comment on derivative notation.
- Power rule.
- Rules for differentiating sum, products and quotients of functions.
- Antiderivatives of power functions

A comment on derivative notation



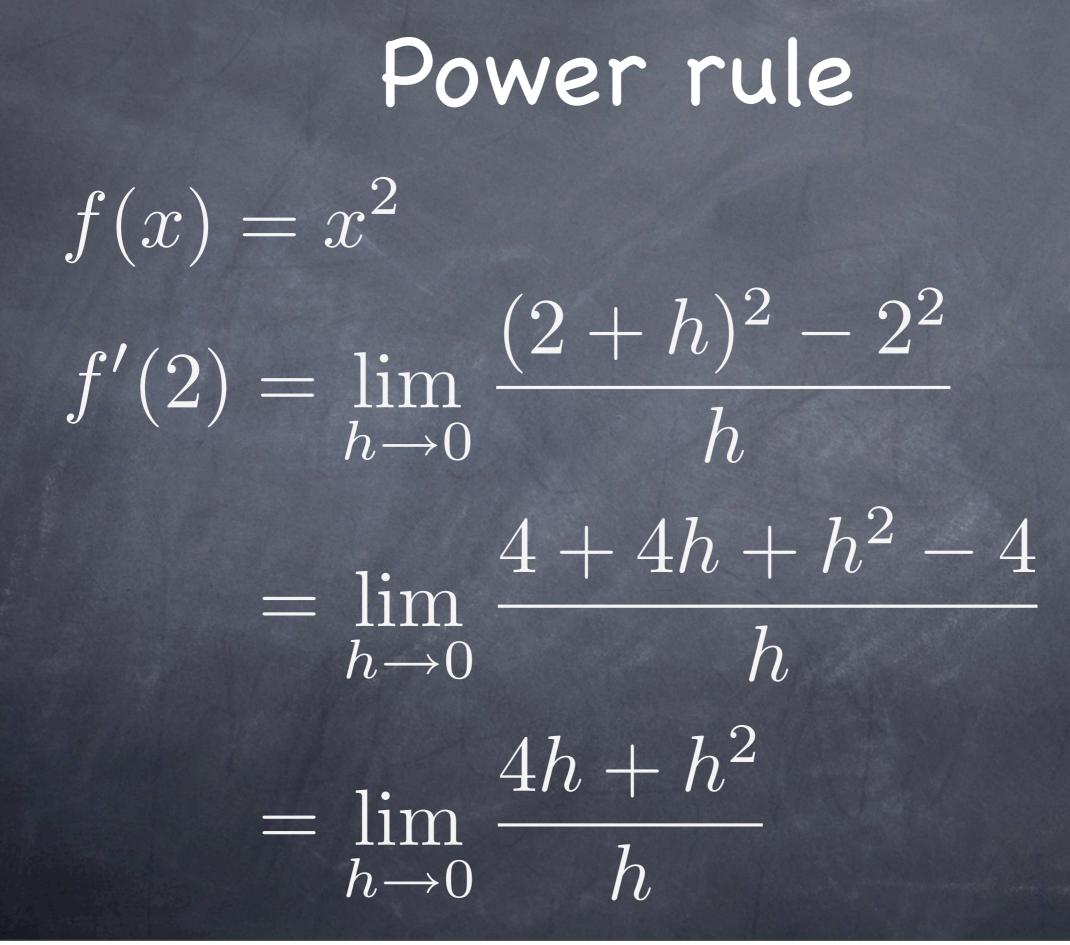
 $f(x) = x^2$

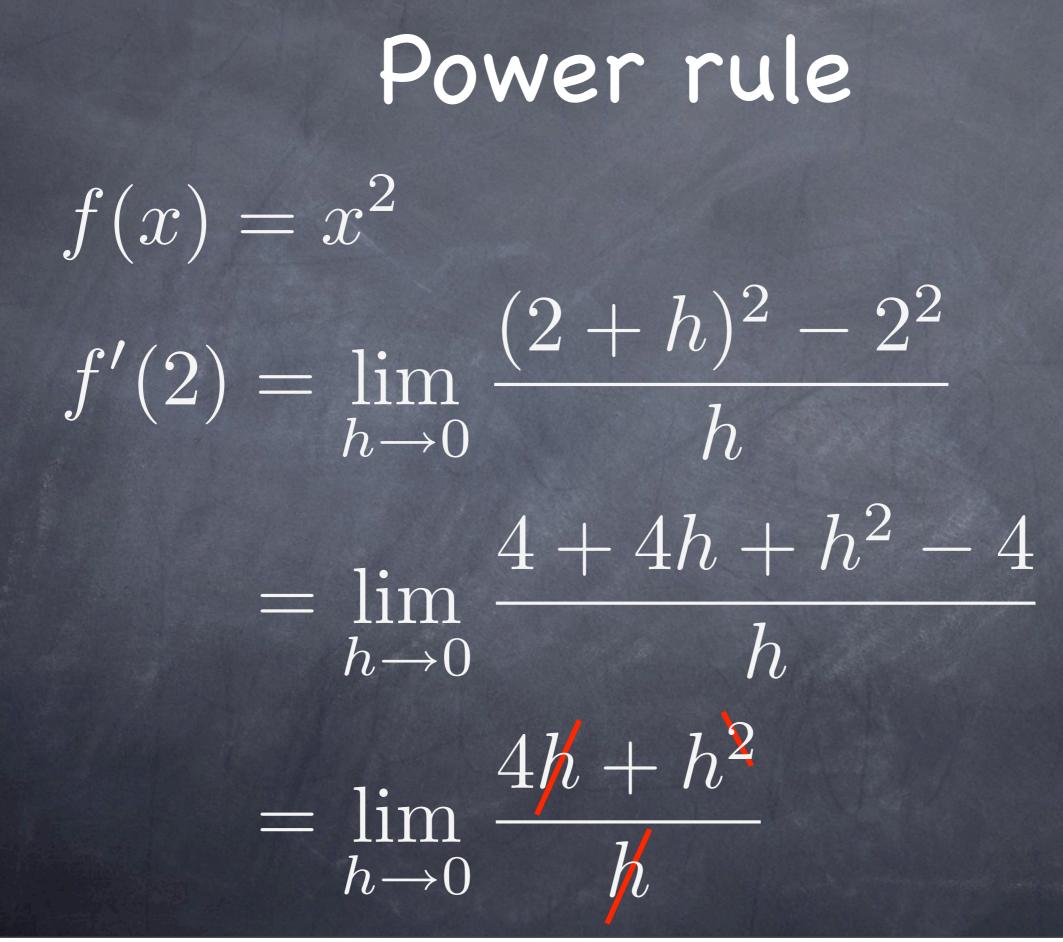
 $f(x) = x^2$

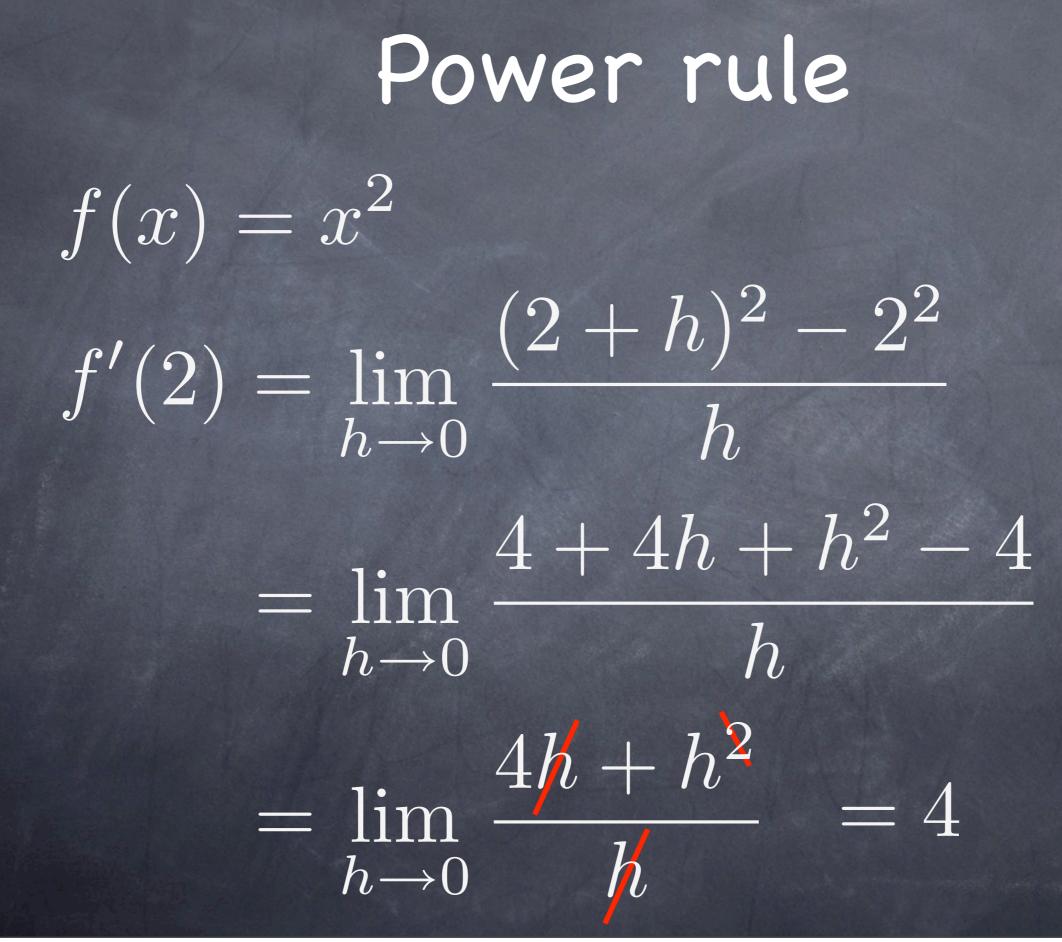
Find f' at x=2 (using the definition of the derivative).

Power rule $f(x) = x^{2}$ $f'(2) = \lim_{h \to 0} \frac{(2+h)^{2} - 2^{2}}{h}$

Power rule $f(x) = x^2$ $f'(2) = \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h}$ $= \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h}$







 $f(x) = x^2$

 $f(x) = x^2$

Find f' at all points x at the same time

$$f(x) = x^{2}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^{2} - x^{2}}{h}$$

 $f(x) = x^2$ $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$ $= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$

 $f(x) = x^2$ $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$ $= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$ $= \lim_{h \to 0} \frac{2hx + h^2}{h}$

 $f(x) = x^2$ $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$ $= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$ h $= \lim_{h \to 0} \frac{2hx + h^2}{h}$

 $f(x) = x^2$ $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$ $= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$ h $= \lim_{h \to 0} \frac{2hx + h^2}{h} = 2x$

 $f(x) = x^3$

Power rule $f(x) = x^{3}$ $f'(x) = \lim_{h \to 0} \frac{(x+h)^{3} - x^{3}}{h}$

Power rule

$$f(x) = x^{3}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^{3} - x^{3}}{h}$$

$$= \lim_{h \to 0} \frac{x^{3} + 3hx^{2} + 3h^{2}x + h^{3} - x^{3}}{h}$$

Power rule

$$f(x) = x^{3}$$

 $f'(x) = \lim_{h \to 0} \frac{(x+h)^{3} - x^{3}}{h}$
 $= \lim_{h \to 0} \frac{x^{3} + 3hx^{2} + 3h^{2}x + h^{3} - x^{3}}{h}$

Power rule

$$f(x) = x^{3}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^{3} - x^{3}}{h}$$

$$= \lim_{h \to 0} \frac{x^{3} + 3hx^{2} + 3h^{2}x + h^{3} - x^{3}}{h}$$

Power rule $f(x) = \overline{x^3}$ $f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$ $= \lim \frac{x^3 + 3hx^2 + 3h^2x + h^3 - x^3}{2}$ K $h \rightarrow 0$ $= 3x^{2}$

 $f(x) = x^n$

 $f'(x) = nx^{n-1}$

Suppose
$$f(x) = g(x) + k(x)$$
 and that $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

(A) 4

(B) 7

(C) 10

Suppose
$$f(x) = g(x) + k(x)$$
 and that $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

(A) 4

(B) 7

(C) 10

Suppose
$$f(x) = g(x)k(x)$$
 and that
 $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

(A) 3

(B) 10

(C) 11

Suppose
$$f(x) = g(x)k(x)$$
 and that
 $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

(A) 3

(B) 10

(C) 11

Suppose
$$f(x) = g(x)k(x)$$
 and that
 $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

(A) 3 Try g(x)=x and k(x)=x². (B) 10 (C) 11

Suppose
$$f(x) = g(x)k(x)$$
 and that
 $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

(A) 3	
	Try g(x)=x and $k(x)=x^2$.
(B) 10	If f'(x)=g'(x)k'(x) then

(C) 11

Suppose
$$f(x) = g(x)k(x)$$
 and that
 $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

(A) 3	
	Try g(x)=x and $k(x)=x^2$.
(B) 10	If f'(x)=g'(x)k'(x) then
(\bigcirc) 1 1	$f(x) = (x) (x^2)$ so $f'(x)=(1) (2x)=2x$.
(C) 11	

Suppose
$$f(x) = g(x)k(x)$$
 and that
 $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

(A) 3	
	Try g(x)=x and $k(x)=x^2$.
(B) 10	If f'(x)=g'(x)k'(x) then
(C) 11	$f(x) = (x) (x^2)$ so $f'(x)=(1) (2x)=2x$.
	But f(x)=x ³ and power rule says
(D) 17	

Suppose
$$f(x) = g(x)k(x)$$
 and that
 $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

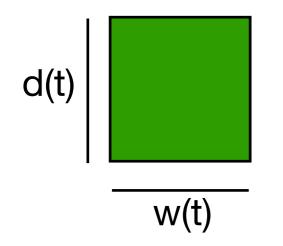
Try g(x)=x and $k(x)=x^2$.
If f'(x)=g'(x)k'(x) then
$f(x) = (x) (x^2)$ so $f'(x)=(1) (2x)=2x$.
But f(x)=x ³ and power rule says
$f'(x) = 3x^2$.

Suppose
$$f(x) = g(x)k(x)$$
 and that
 $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

(A) 3	
	Try g(x)=x and $k(x)=x^2$.
(B) 10	If f'(x)=g'(x)k'(x) then
	$f(x) = (x) (x^2)$ so $f'(x) = (1) (2x) = 2x$.
(C) 11	But f(x)=x ³ and power rule says
(D) 17	$f'(x) = 3x^2$.
	So g'(x)k'(x) can't be right.

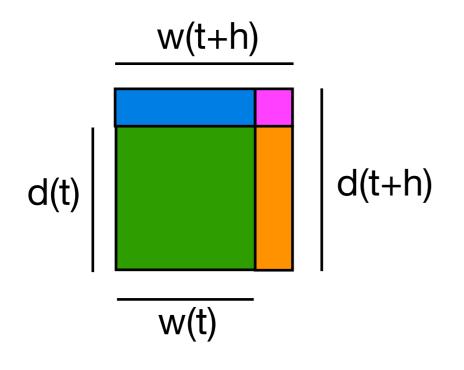
What is the correct derivative for f(x)=g(x)k(x)?

$$A(t) = d(t)w(t)$$



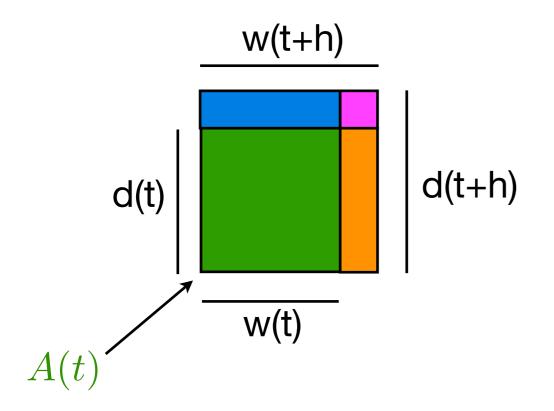
What is the correct derivative for f(x)=g(x)k(x)?

$$A(t) = d(t)w(t)$$

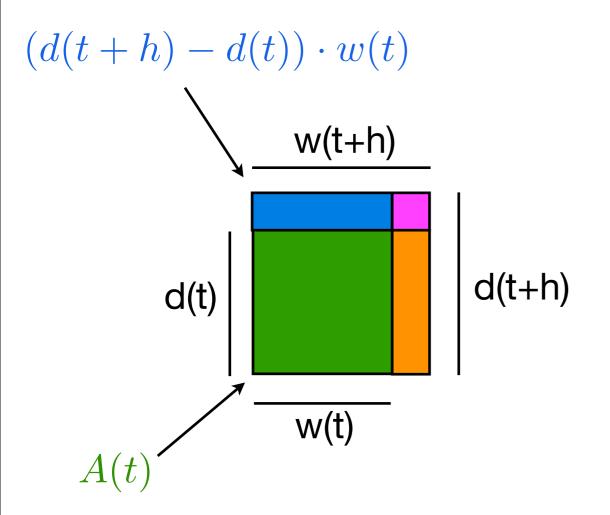


What is the correct derivative for f(x)=g(x)k(x)?

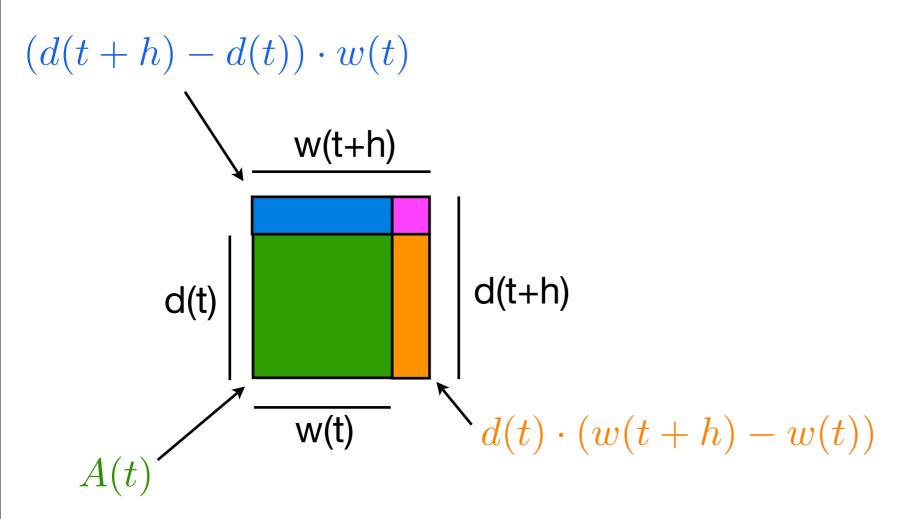
$$A(t) = d(t)w(t)$$



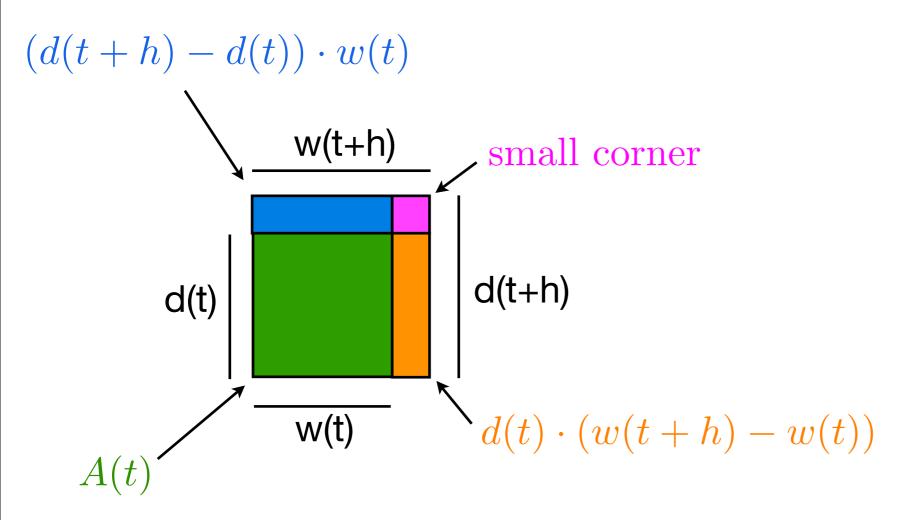
$$A(t) = d(t)w(t)$$



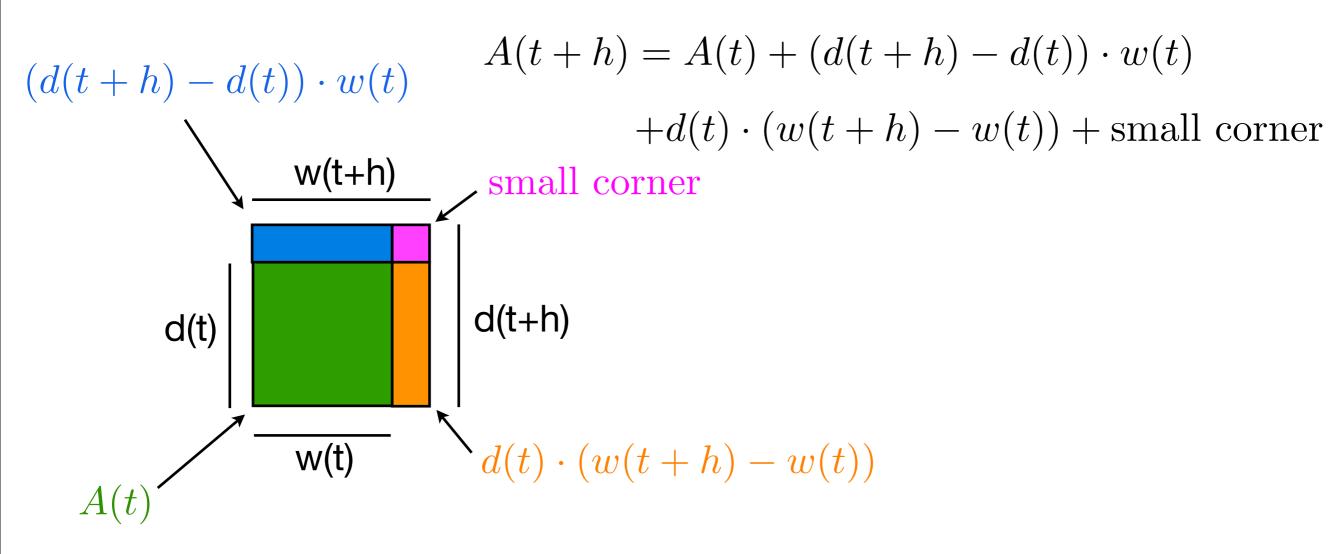
$$A(t) = d(t)w(t)$$



$$A(t) = d(t)w(t)$$

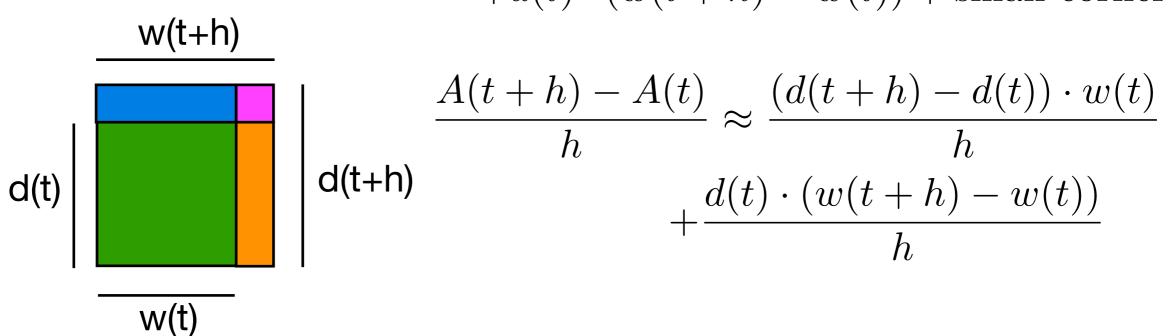


$$A(t) = d(t)w(t)$$



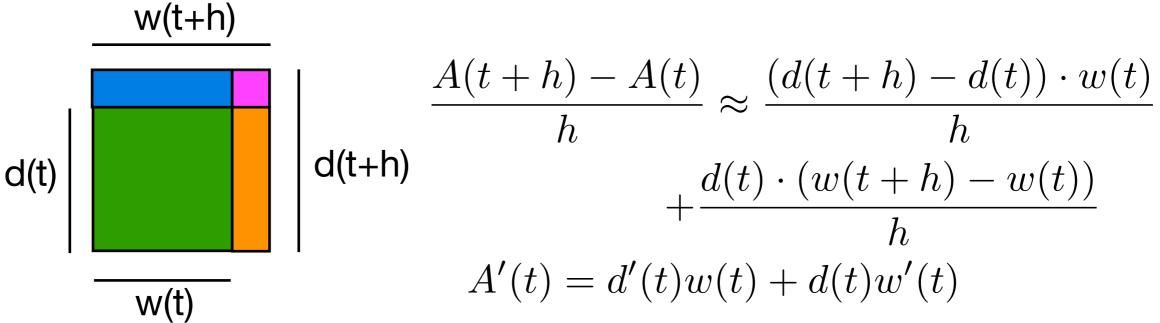
$$A(t) = d(t)w(t)$$

$$A(t+h) = A(t) + (d(t+h) - d(t)) \cdot w(t)$$
$$+d(t) \cdot (w(t+h) - w(t)) + \text{small corner}$$



$$A(t) = d(t)w(t)$$

$$A(t+h) = A(t) + (d(t+h) - d(t)) \cdot w(t)$$
$$+d(t) \cdot (w(t+h) - w(t)) + \text{small corner}$$



- Addition rule:
 - f(x) = g(x) + h(x)
 - f'(x) = g'(x) + h'(x)

- Addition rule:
 - f(x) = g(x) + h(x)
 - f'(x) = g'(x) + h'(x)
- Product rule:
 - f(x) = g(x)h(x)
 - f'(x) = g'(x)h(x) + g(x)h'(x)

- Addition rule:
 - f(x) = g(x) + h(x)
 - f'(x) = g'(x) + h'(x)
- Product rule:
 - f(x) = g(x)h(x)
 - f'(x) = g'(x)h(x) + g(x)h'(x)
- Quotient rule (can be justified once we cover chain rule):
 - $f(x) = g(x) / h(x) = g(x) (h(x))^{-1} < --- apply product and chain rules or$

•
$$f'(x) = [g'(x)h(x) - g(x)h'(x)] / g(x)^2$$

Suppose
$$f(x) = g(x)/k(x)$$
 and that
 $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

• What is f'(2)?

(A) -13

- (B) -13/25
- (C) 17

(D) 17/25

Suppose
$$f(x) = g(x)/k(x)$$
 and that $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5.$

• What is f'(2)?

(A) -13

(B) -13/25

(C) 17

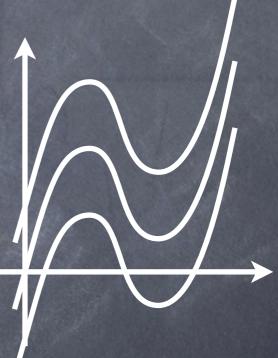
(D) 17/25

Antiderivatives - going backward If $f'(x) = 6x^2 + 4x - 1$, then (A) f(x) = 12x + 4(B) $f(x) = 2x^3 + 2x^2 - x$ (C) $f(x) = 2x^3 + 2x^2 - x + 2$ (D) $f(x) = 2x^3 + 2x^2 - x + C$

Antiderivatives - going backward If $f'(x) = 6x^2 + 4x - 1$, then (A) f(x) = 12x + 4(B) $f(x) = 2x^3 + 2x^2 - x$ (C) $f(x) = 2x^3 + 2x^2 - x + 2$ (D) $f(x) = 2x^3 + 2x^2 - x + C$

Antiderivatives – going backward If $f'(x) = 6x^2 + 4x - 1$, then

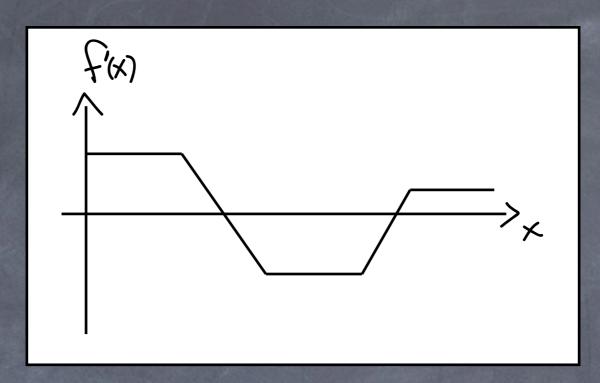
(A) f(x) = 12x + 4(B) $f(x) = 2x^3 + 2x^2 - x$ (C) $f(x) = 2x^3 + 2x^2 - x + 2$ (D) $f(x) = 2x^3 + 2x^2 - x + C$

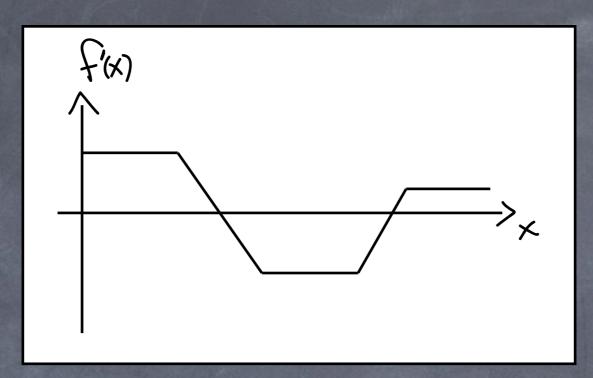


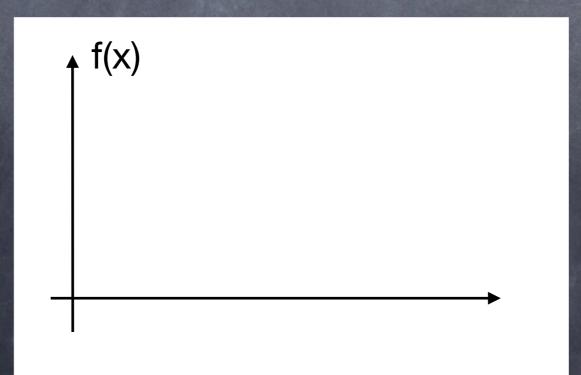
Slopes at each x don't change with vertical shift.

If
$$f'(x) = x^n$$
, which of the
following could be $f(x)$?
(A) $f(x) = \frac{1}{n+1}x^{n+1}$
(B) $f(x) = \frac{1}{n+1}x^{n+1} + C$
(C) $f(x) = nx^{n-1}$
(D) $f(x) = nx^{n-1} + C$
(E) $f(x) = x^n + C$

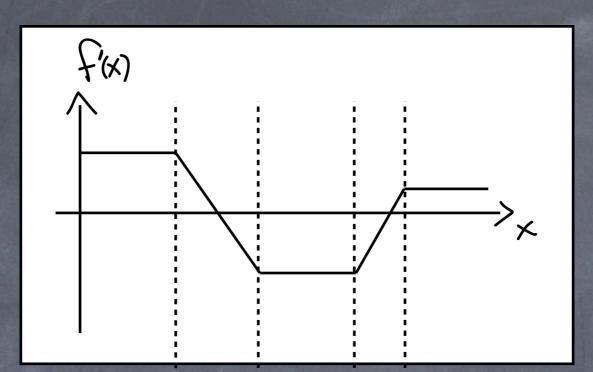
If $f'(x) = x^n$, which of the following could be f(x)? $f(x) = \frac{1}{n+1}x^{n+1}$ (A) $f(x) = \frac{1}{n+1}x^{n+1} + C$ **(B)** $f(x) = nx^{n-1}$ (C) $f(x) = nx^{n-1} + C$ (D) $f(x) = x^n + C$ (E)

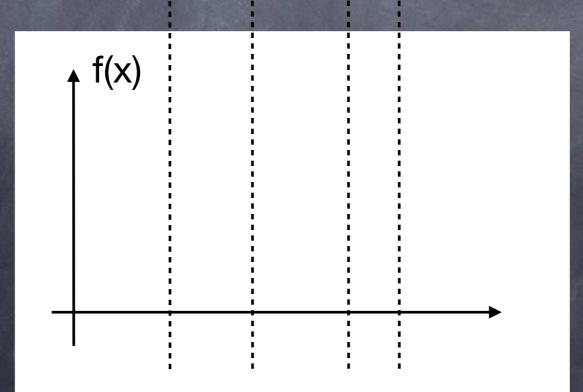


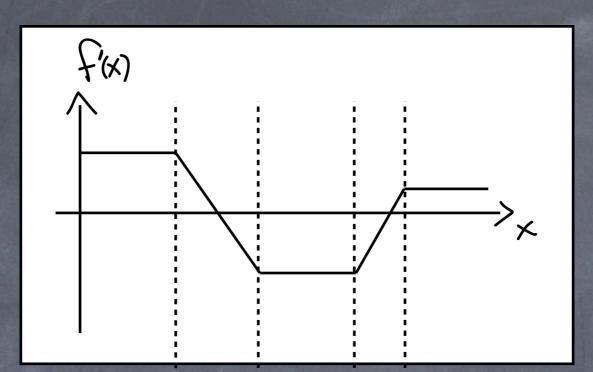


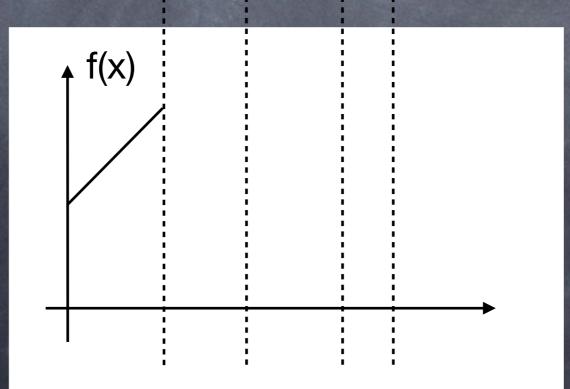


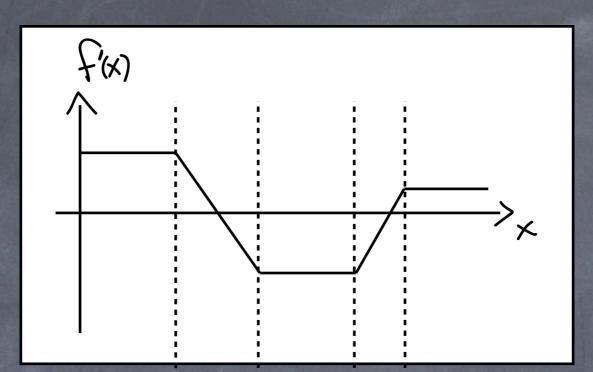
Thursday, September 18, 2014

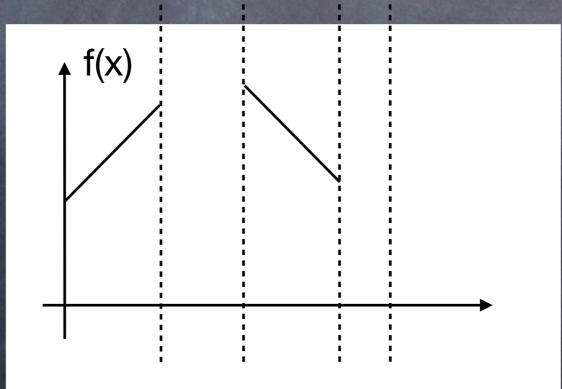




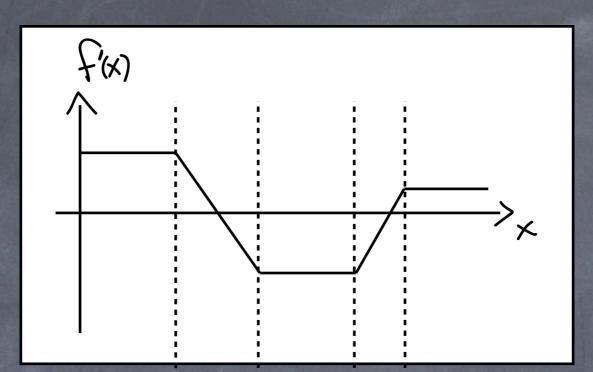


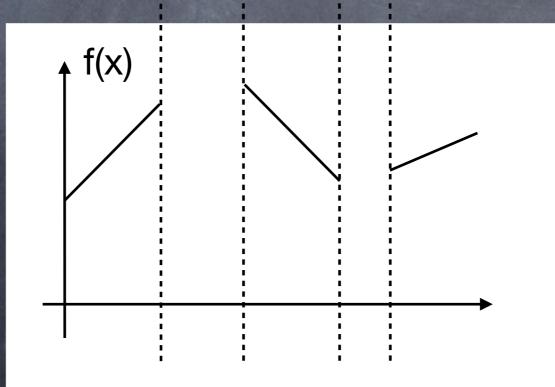


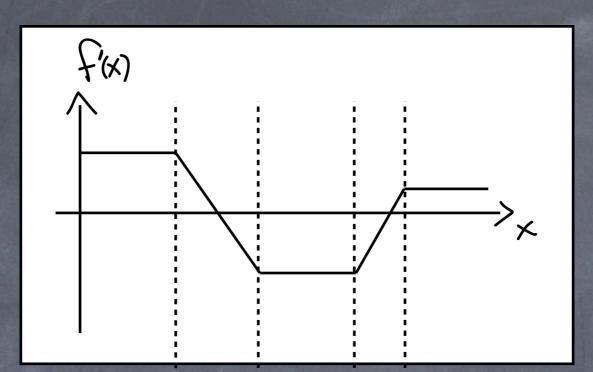


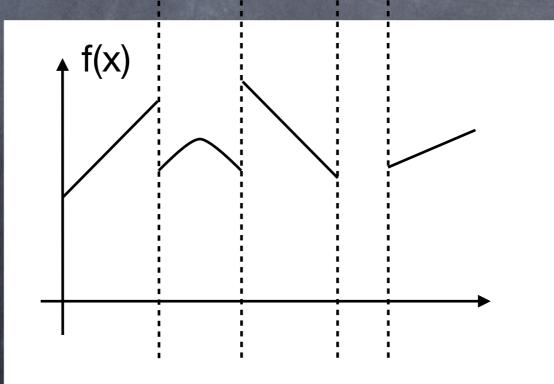


Thursday, September 18, 2014

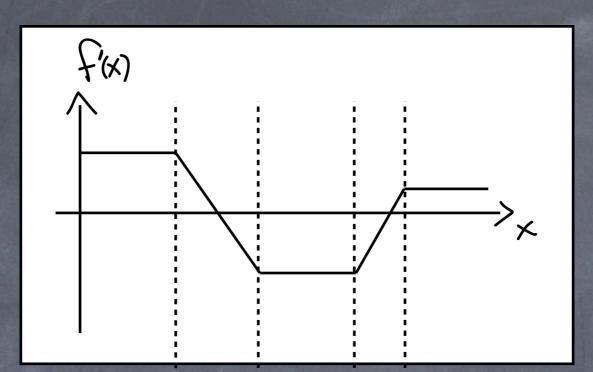


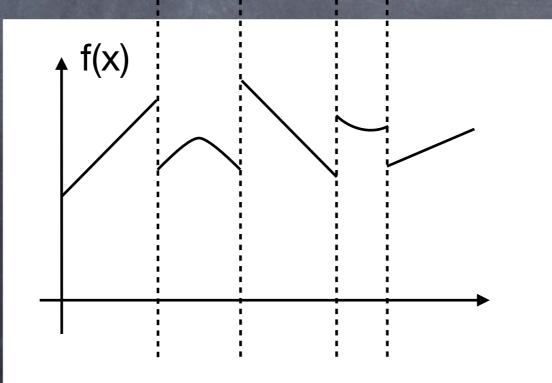


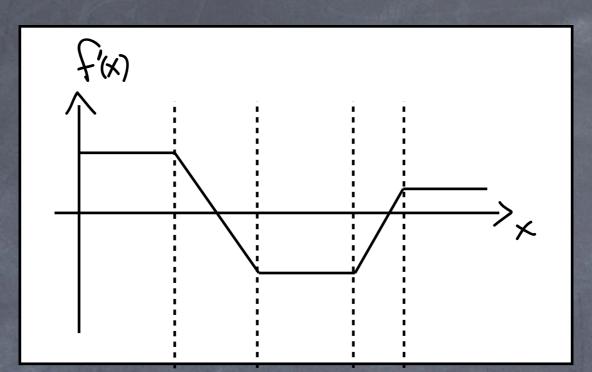


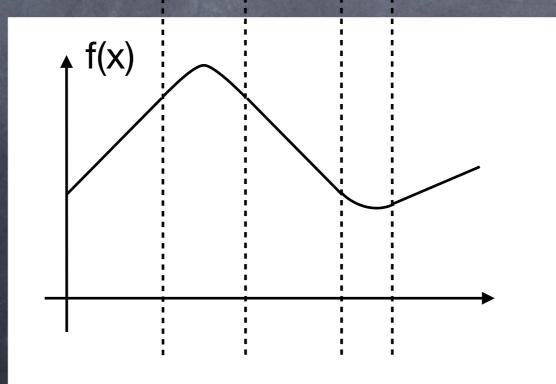


Thursday, September 18, 2014

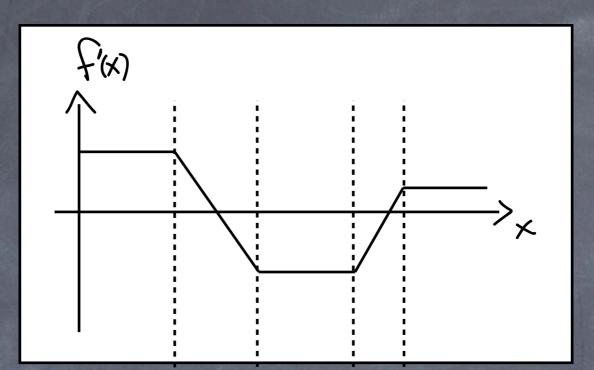


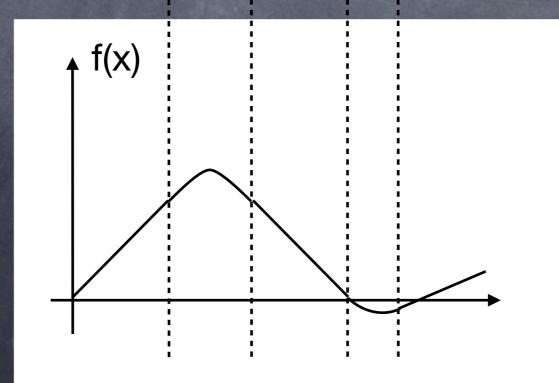






Thursday, September 18, 2014





Only determined up to a vertical shift.

 \odot If x(t) is position as a function of time,

If x(t) is position as a function of time,
 velocity v(t) = x'(t),

If x(t) is position as a function of time,
velocity v(t) = x'(t),
acceleration a(t) = v'(t) = x"(t).

If x(t) is position as a function of time,
velocity v(t) = x'(t),
acceleration a(t) = v'(t) = x"(t).
Constant acceleration a:

If x(t) is position as a function of time,
velocity v(t) = x'(t),
acceleration a(t) = v'(t) = x"(t).
Constant acceleration a:
v(t)

If x(t) is position as a function of time,
velocity v(t) = x'(t),
acceleration a(t) = v'(t) = x"(t).
Constant acceleration a:
v(t) = at + C

If x(t) is position as a function of time,
velocity v(t) = x'(t),
acceleration a(t) = v'(t) = x"(t).
Constant acceleration a:
v(t) = at + C = at + v₀

 \odot If x(t) is position as a function of time, \oslash velocity v(t) = x'(t), \oslash acceleration a(t) = v'(t) = x''(t). Constant acceleration a: $o v(t) = at + C = at + v_0$ ⊘ x(†)

 \odot If x(t) is position as a function of time, \oslash velocity v(t) = x'(t), \oslash acceleration a(t) = v'(t) = x''(t). Constant acceleration a: $o v(t) = at + C = at + v_0$ $x(t) = a/2 t^2 + v_0 t + D$

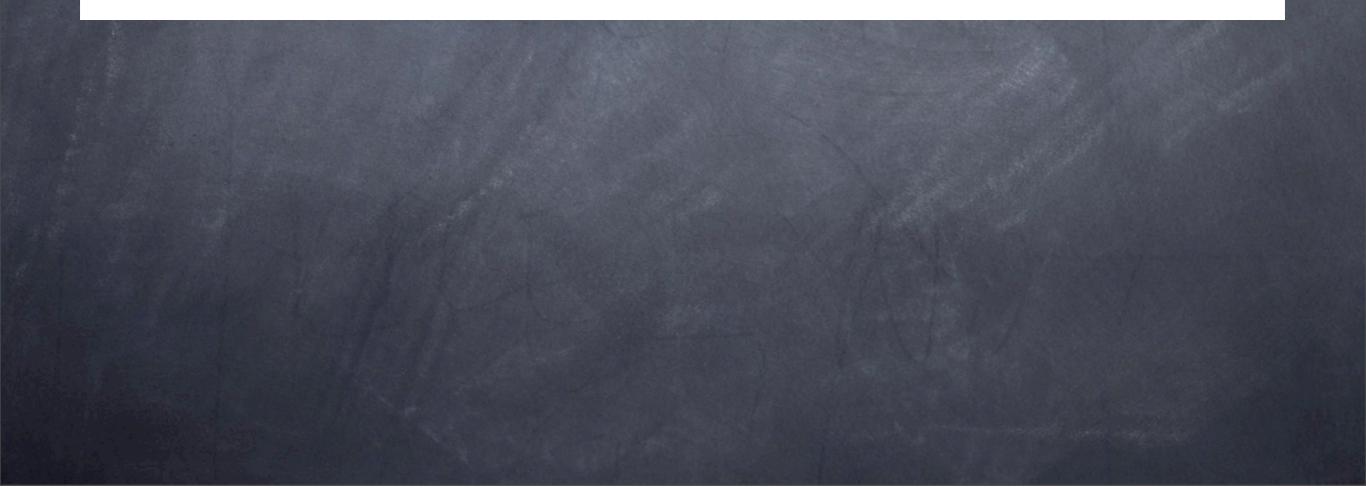
Position-Velocity-Acceleration

 \odot If x(t) is position as a function of time, \oslash velocity v(t) = x'(t), \oslash acceleration a(t) = v'(t) = x''(t). Constant acceleration a: o v(t) = at + C = at + v₀ $x(t) = a/2 t^2 + v_0 t + D = a/2 t^2 + v_0 t + x_0$

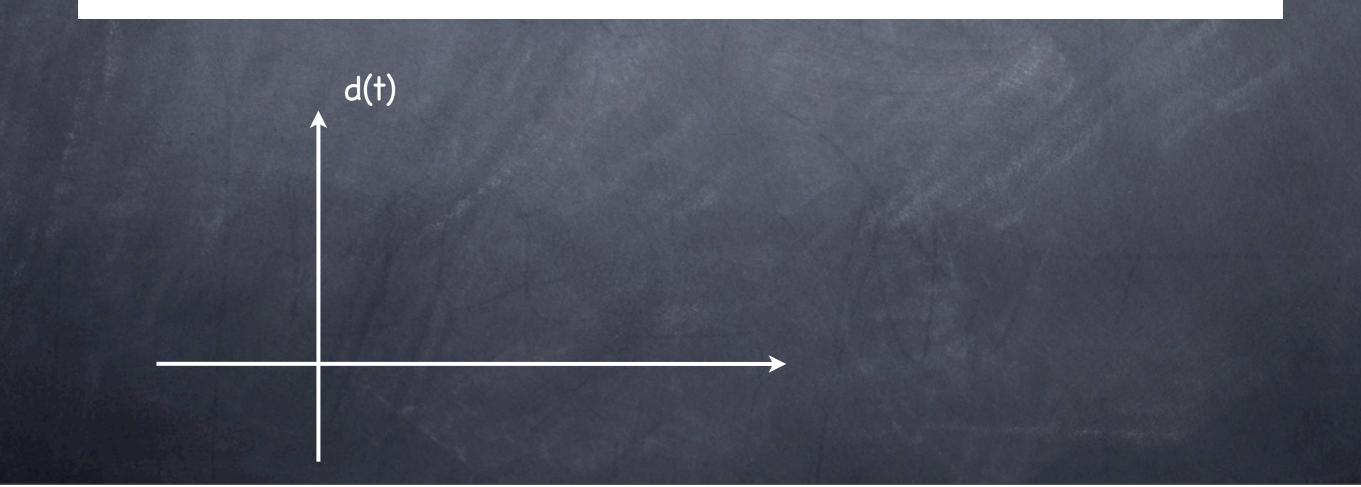
Examples of approximately constant acceleration

Ball dropping near surface of planet
Fireworks

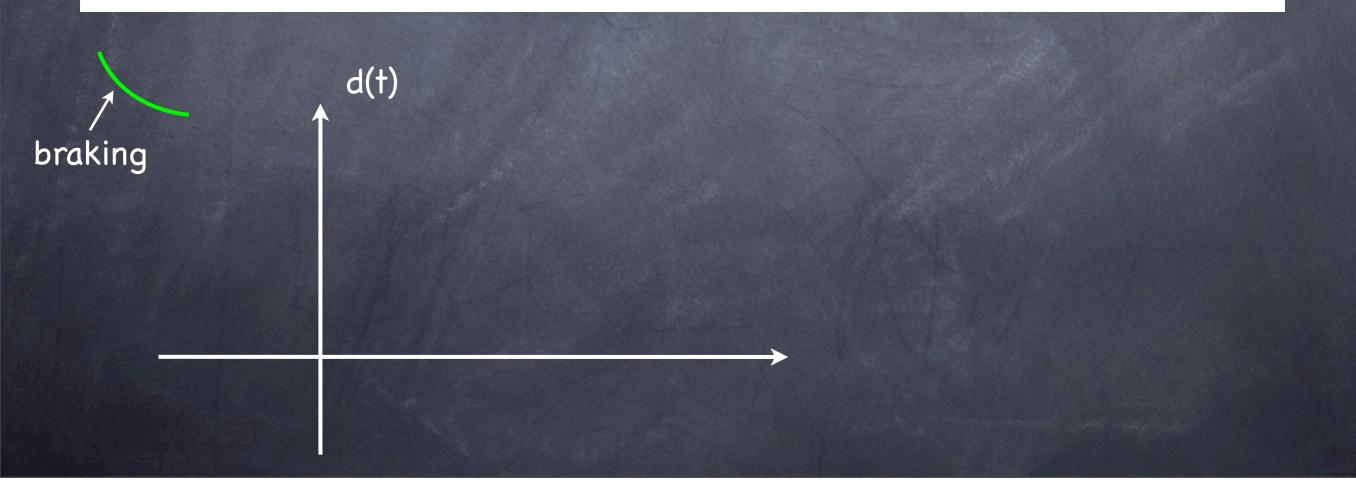
- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



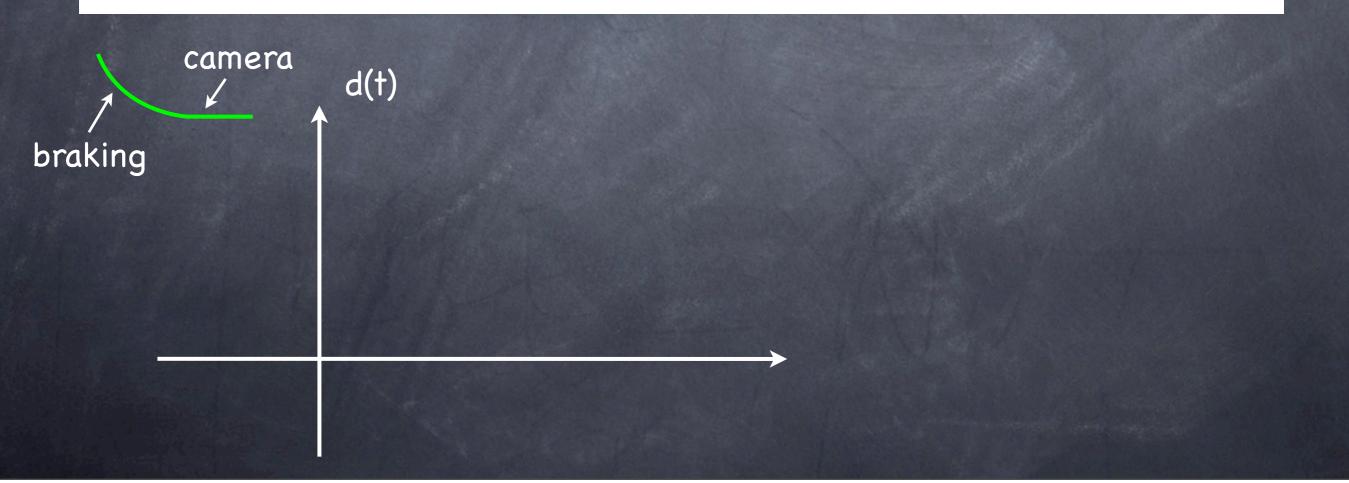
- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



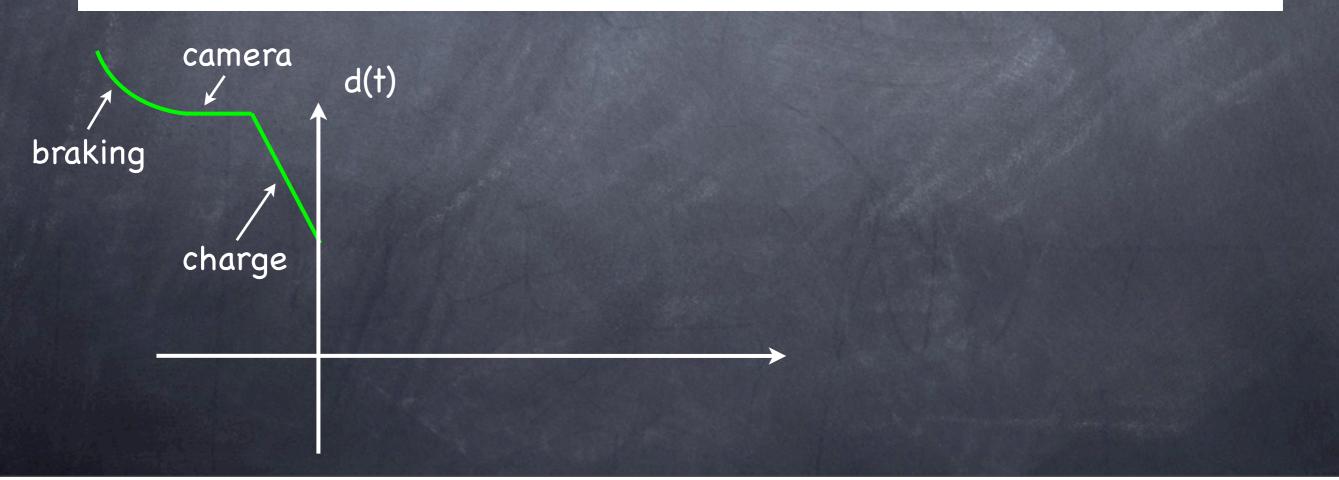
- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



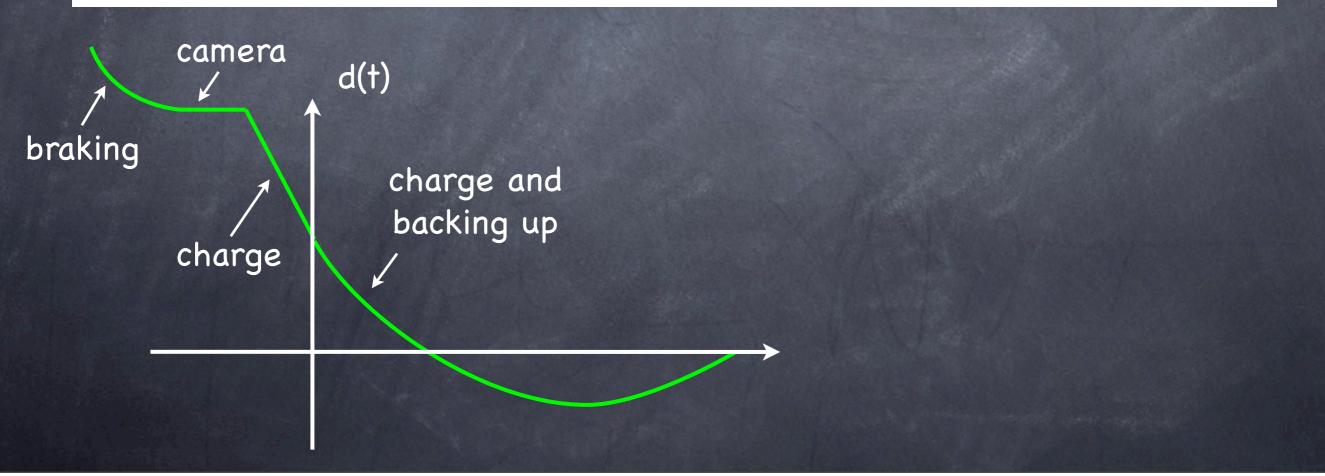
- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



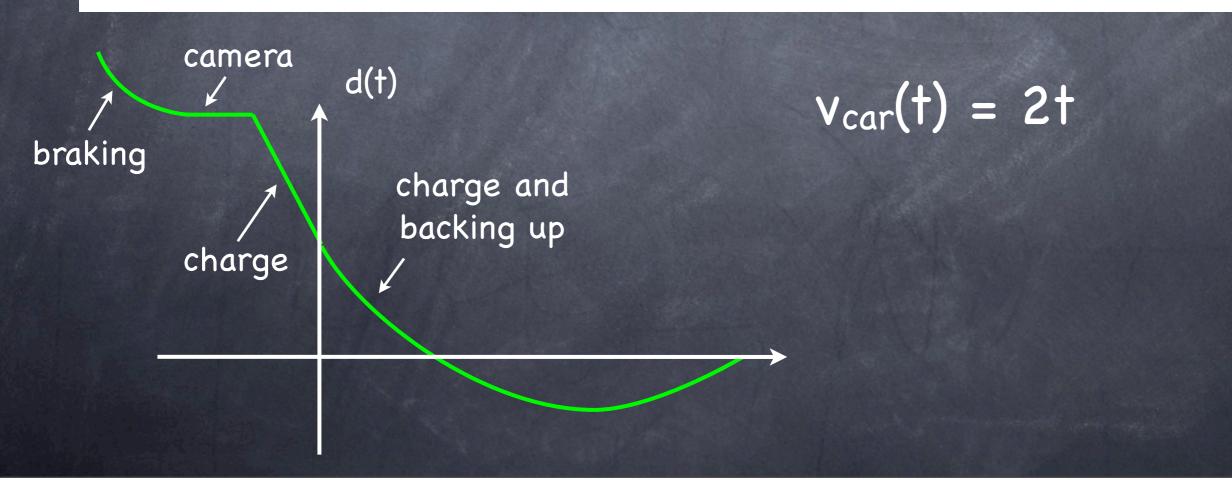
- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



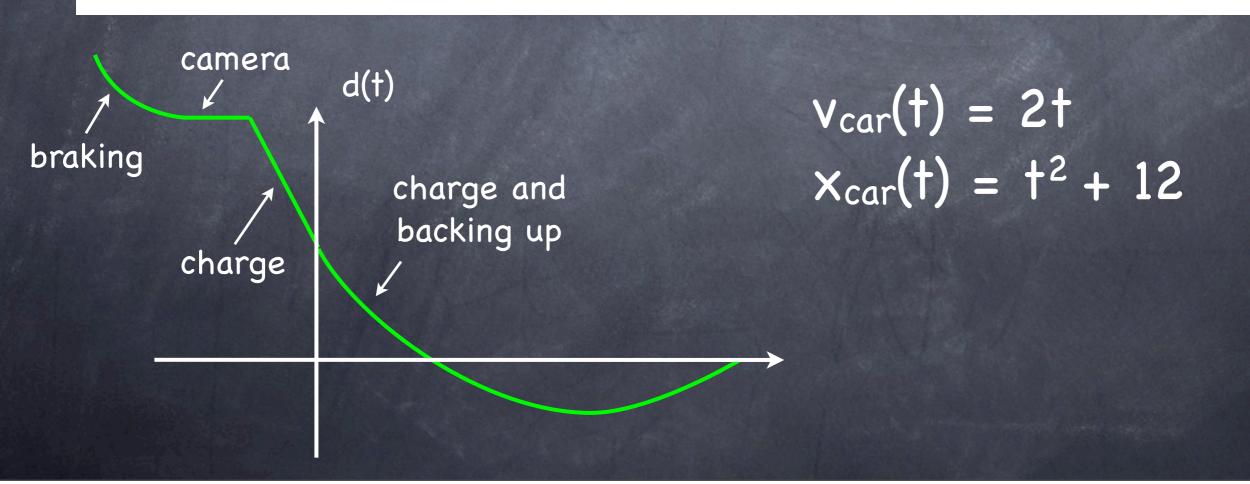
- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



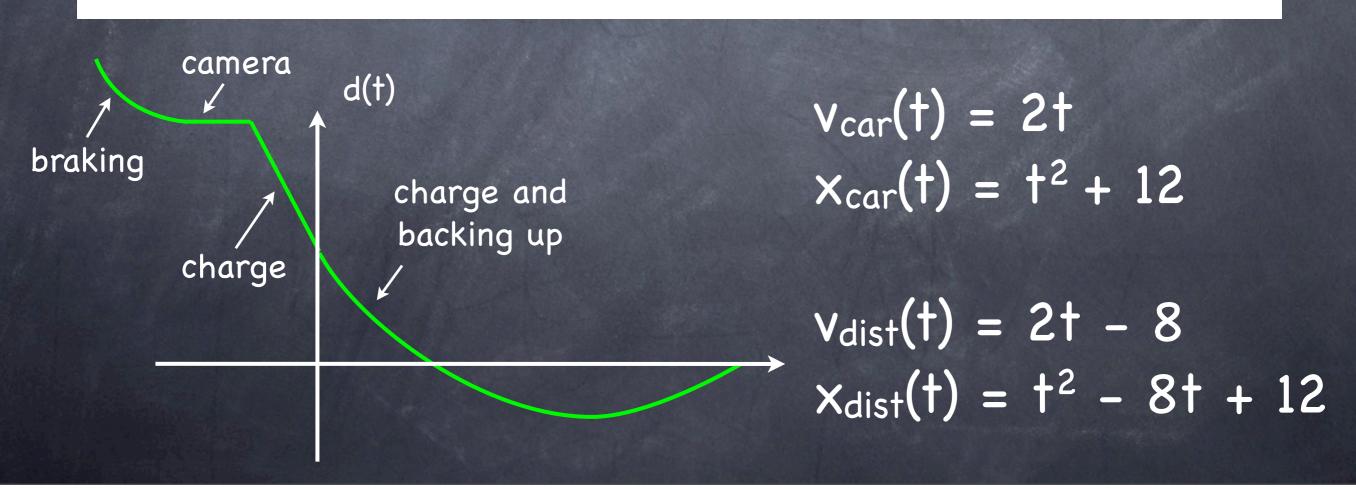
- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



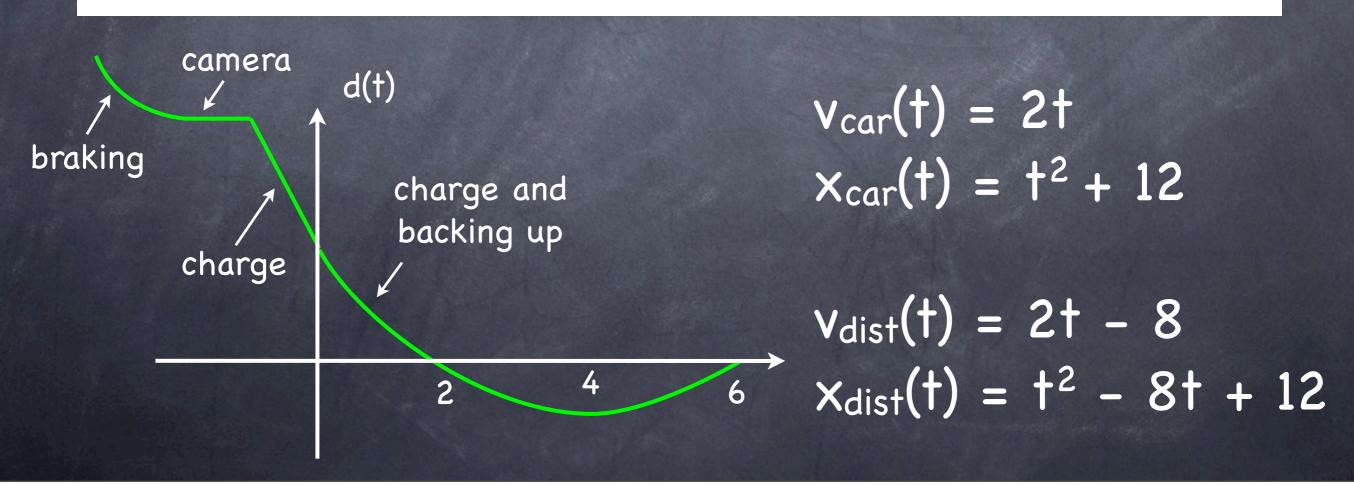
- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



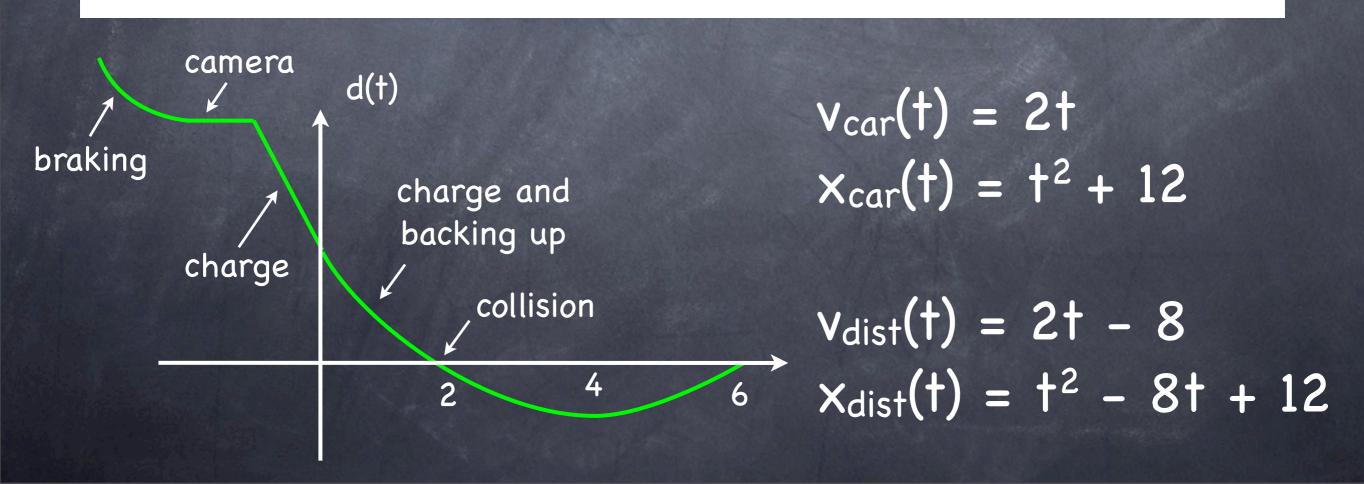
- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.

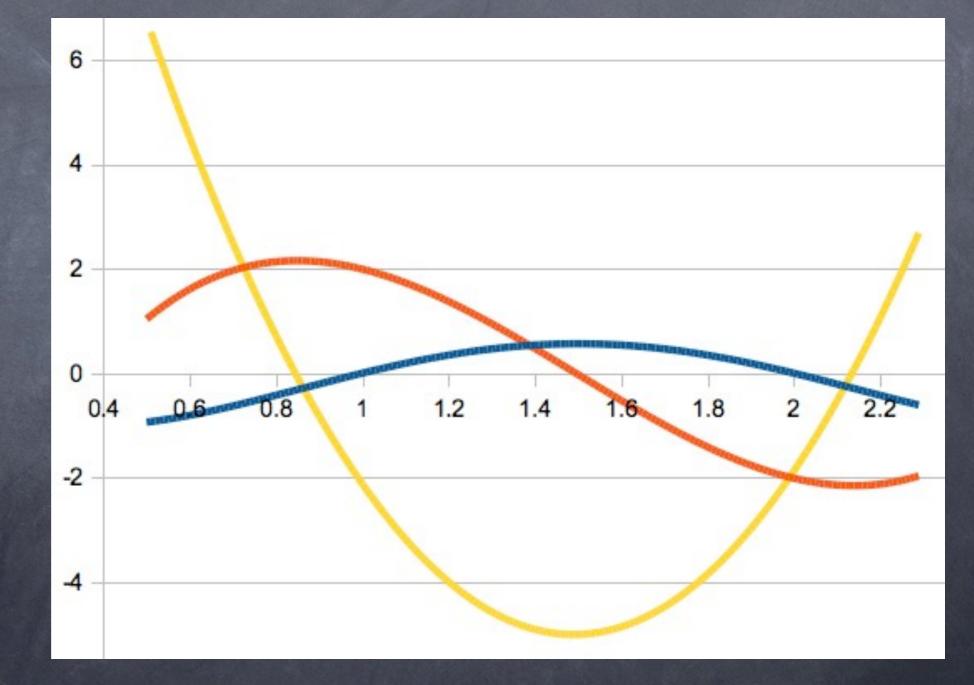


- 8. (10 points) You are driving down the highway when you see a sleeping moose. You apply the brakes and carefully stop your car 20m away from the animal. While you are looking for your camera the moose wakes up. It instantly charges toward your car at a constant speed of 8m/s. One second later, you start backing away from the moose at a constant acceleration of 2m/s².
 - i. (4 points) Write down a function d(t) that is the distance from your car to the moose where t = 0 indicates the moment when you start backing away.



Which is x, v, a?

 $(A) \times, \vee, \land$ $(B) \times, \vee, \land$ $(C) \times, \vee, \land$



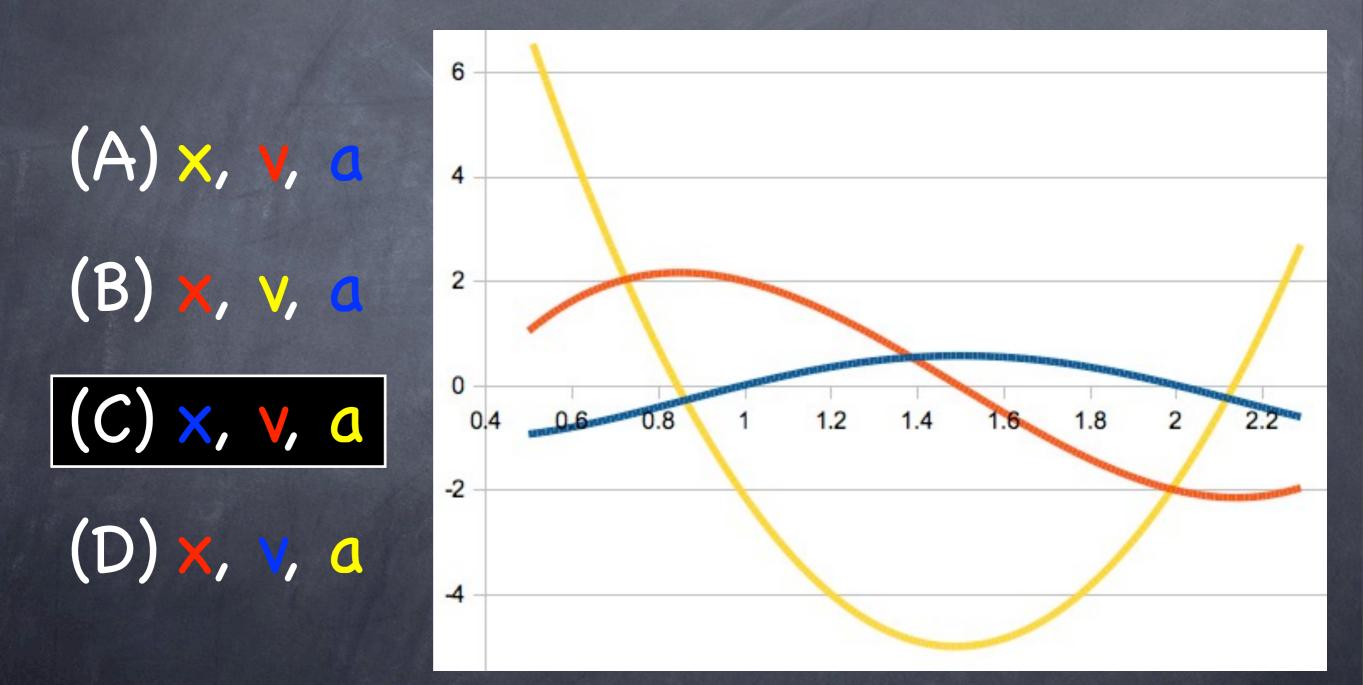
Which is x, v, a?

6 (A) ×, v, d (B) ×, ∨, ₫ 2 0 (C) ×, v, a 1.4 1.2 2 0.4 1.8 22 -2 (D) x, v, a -4

Check max/mins --> zeros, check inc/dec --> +/-.

Thursday, September 18, 2014

Which is x, v, a?



Check max/mins --> zeros, check inc/dec --> +/-.

Thursday, September 18, 2014

Product rule: If k(x)=f(x)g(x) then k'(x) = ?

(A) f'(x)g(x)
(B) f(x)g'(x)
(C) f'(x)g(x) + f(x)g'(x)
(D) f'(x)g'(x)

Example: $k(x)=(x^5-2x^3+x^2+3)(3x^3-x^2+1)$

Product rule: If k(x)=f(x)g(x) then k'(x) = ?

(A) f'(x)g(x)
(B) f(x)g'(x)
(C) f'(x)g(x) + f(x)g'(x)
(D) f'(x)g'(x)

Example: $k(x)=(x^5-2x^3+x^2+3)(3x^3-x^2+1)$

Quotient rule: If k(x)=f(x)/g(x) then k'(x) = ?

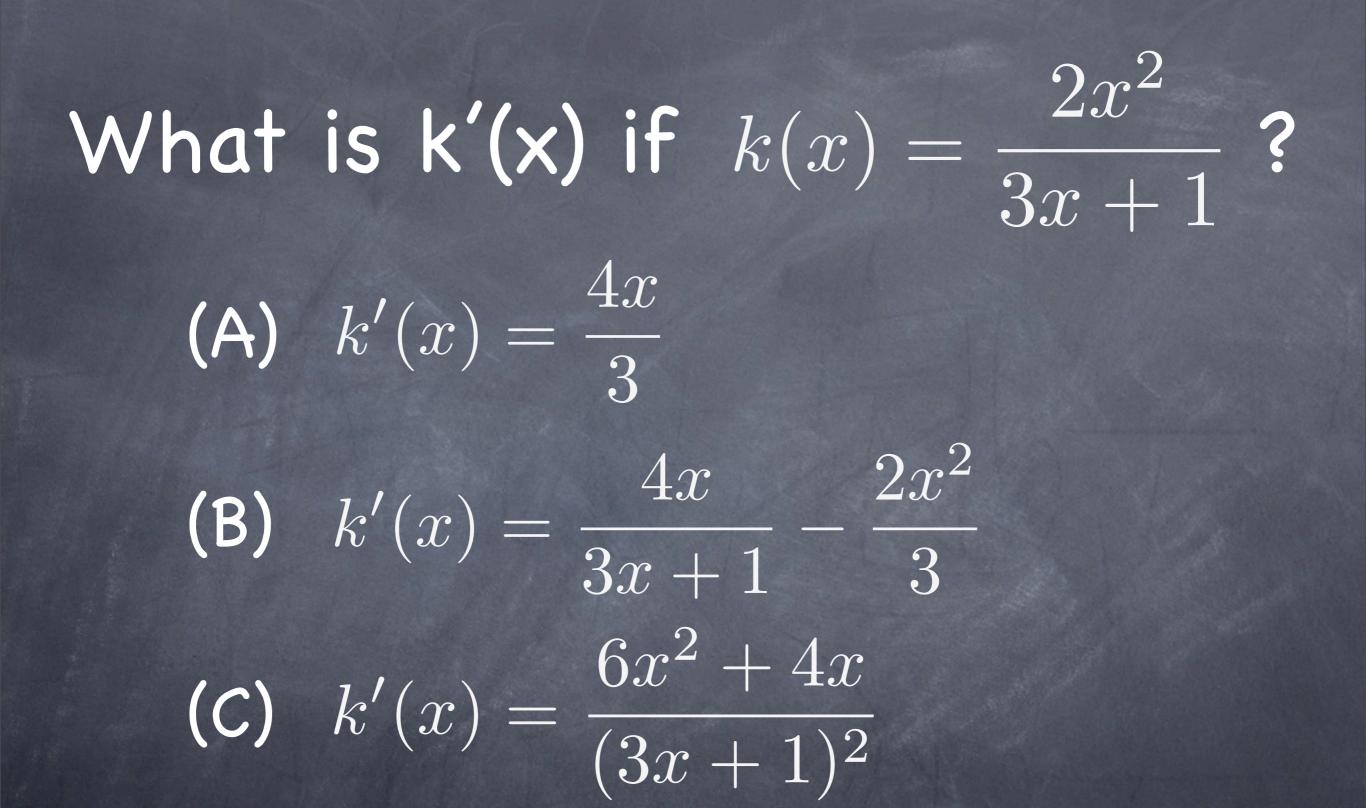
(A) f'(x)/g'(x)
(B) [f'(x)g(x) - f(x)g'(x)] / g(x)²
(C) f'(x)g(x) + f(x)g'(x)
(D) f'(x)/g(x)

Example: $k(x)=2x^{2}/(3x+1)$

Quotient rule: If k(x)=f(x)/g(x) then k'(x) = ?

(A) f'(x)/g'(x)
(B) [f'(x)g(x) - f(x)g'(x)] / g(x)²
(C) f'(x)g(x) + f(x)g'(x)
(D) f'(x)/g(x)

Example: $k(x)=2x^{2}/(3x+1)$



(D) $k'(x) = \frac{4x}{3x+1} - \frac{2x^2}{(3x+1)^2}$

