Today

- Reminders:
  - OSH 3 on Monday, Assign. 4a on Tues 7am,
  - Midterm 1 on Tues 6pm.
    - S.101 - HENN 200,
    - S.103 - Last name A-K: BUCH A203
    - S.103 - Last name L-Z: BUCH A103

Today

- How to choose x_0 for Newton's method.
- Inc/dec, critical points and extrema.
- First and Second derivative test.
- Concavity, potential IPs and actual IPs.

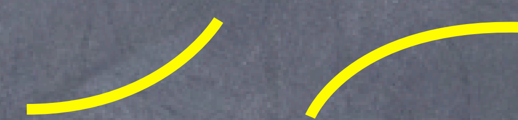
How to choose x_0

Desmos:

- https://www.desmos.com/calculator/hf5ll3di1l

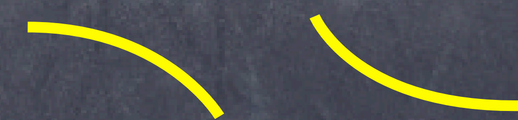
Increasing/decreasing

- We say a function is **increasing** on some interval if for any points a and b with **$a < b$** we have that **$f(a) < f(b)$**.



When f′ exists, same as $f'(x)>0$.

- We say a function is **decreasing** on some interval if for any points a and b with **$a < b$** we have that **$f(a) > f(b)$**.



When f′ exists, same as $f'(x)<0$.

- Notice - no reference to $f'(x)$!!

Local extrema (min/max)

- A point **a** is a **local minimum** of a function f(x) provided that **f(x) > f(a)** for all x on an interval around a (excluding a, of course).

Which of the following is a local minimum?

- A point **a** is a **local maximum** of a function f(x) provided that **f(x) < f(a)** for all x on an interval around a (excluding a, of course).

- If the function is differentiable at the minimum, then it must look like (A).

Critical points (CPs) of f(x)

- A CP of f(x) is a point **a** at which **f'(a)=0** or **f'(a) is not defined** even though f(a) is defined.
- Use of CPs of f(x):
  - If f'(x) changes sign at a CP, then the CP is an extremum (min/max) of f(x).
  - If f'(x) does not change sign at a CP, then the CP is not an extremum of f(x)!

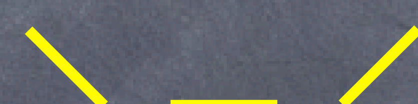
Critical points - examples

Desmos

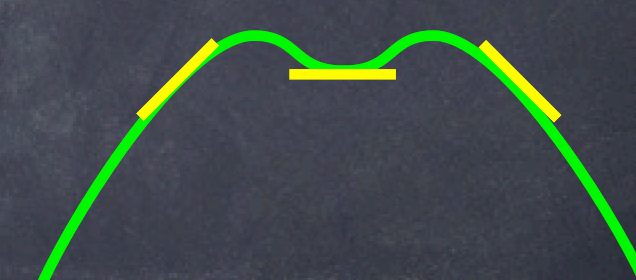
- https://www.desmos.com/calculator/nyquvpptn5

A critical point is an extremum when...

- First derivative test: A critical point x=a is an extremum when f′(x) changes sign at x=a.
    - If f′(x) goes from - to 0 to + then x=a is a min of f(x).



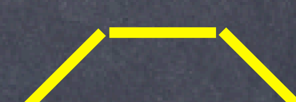
    - If f′(x) goes from + to 0 to - then x=a is a max of f(x).



A critical point is an extremum when...

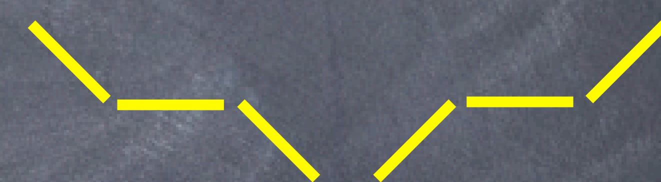
- Second derivative test: If $f'(x)$ is differentiable at $x=a$, then $x=a$ is an extremum when $f''(a) \neq 0$.
  - If $f''(a) > 0$, then $f'(x)$ goes from - to 0 to + so $x=a$ is a min of $f(x)$.

  - If $f''(a) < 0$, then $f'(x)$ goes from + to 0 to - so $x=a$ is a max of $f(x)$.



A critical point is NOT an extremum when...

- The First derivative test FAILS: A critical point $x=a$ is NOT an extremum when $f'(x)$ does not change sign at $x=a$. In this case, $f(x)$ keeps going in the same direction after the flat spot.



- Ex.: $f(x) = x^3$ --> $f'(x)=3x^2$ --> $f'(0) = 0$

  --> $f'(0^-) > 0$

  --> $f'(0^+) > 0$

A critical point is still mysterious when...

- The Second derivative test FAILS: If $f''(a) = 0$ then we can't be certain whether or not $x=a$ is an extremum.
    - Ex.1: $f(x)=x^4$ --> $f''(0)=0$ and $x=0$ is min.
    - Ex.2: $g(x)=x^5$ --> $g''(0)=0$ but $x=0$ is neither a min nor a max.

$g(x) = x^5 - x^3$ has...

(A) a maximum at x=0 and a minimum at x=$\sqrt{3/5}$.

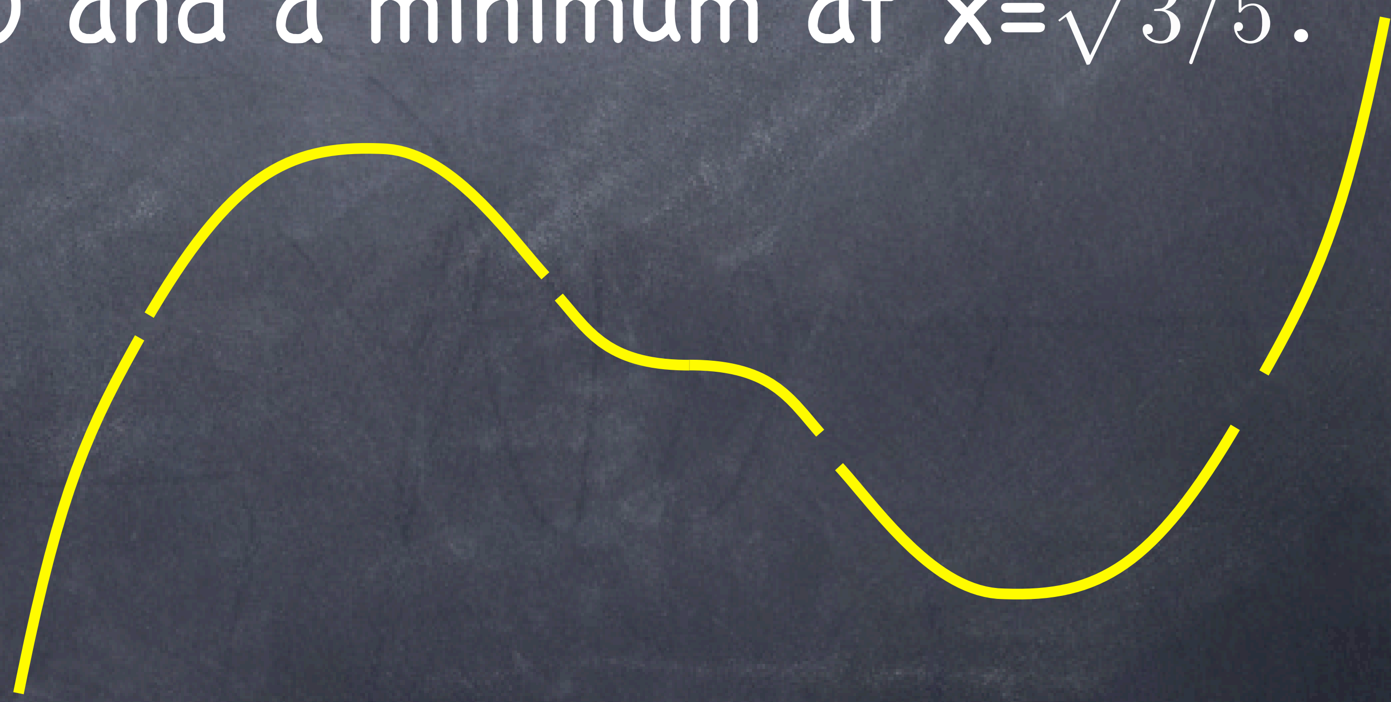
(B) a minimum at x=0 and a maximum at x=$\sqrt{3/5}$.

(C) no extremum at x=0 and a minimum at x= $\sqrt{3/5}$.

(D) a mystery pt at x=0 and a minimum at x=$\sqrt{3/5}$.

Note 1: asymptotics gives you a good start!

Note 2: g(x) is odd.



$g(x) = x^5 - x^3$ has...

(A) a maximum at x=0 and a minimum at x=$\sqrt{3/5}$.

(B) a minimum at x=0 and a maximum at x=$\sqrt{3/5}$.

(C) no extremum at x=0 and a minimum at x= $\sqrt{3/5}$.

(D) a mystery pt at x=0 and a minimum at x=$\sqrt{3/5}$.

$g'(x) = 5x^4 - 3x^2 = 0$ when x=0 or x=$\sqrt{3/5}$.

(i) FDT: $g'(x) \approx -3x^2$ near x=0 so $g'(x)$ doesn't change sign!

(ii) SDT: $g''(x) = 20x^3 - 6x = 2x(10x^2 - 3)$ so $g''(0)=0$ --> mystery!

Concave up/down

- We say a function is **concave up** on some interval if $f'(x)$ is increasing on that interval.

When $f''(x)$ exists, same as $f''(x)>0$.

- We say a function is **concave down** on some interval if $f'(x)$ is decreasing on that interval.

When $f''(x)$ exists, same as $f''(x)<0$.

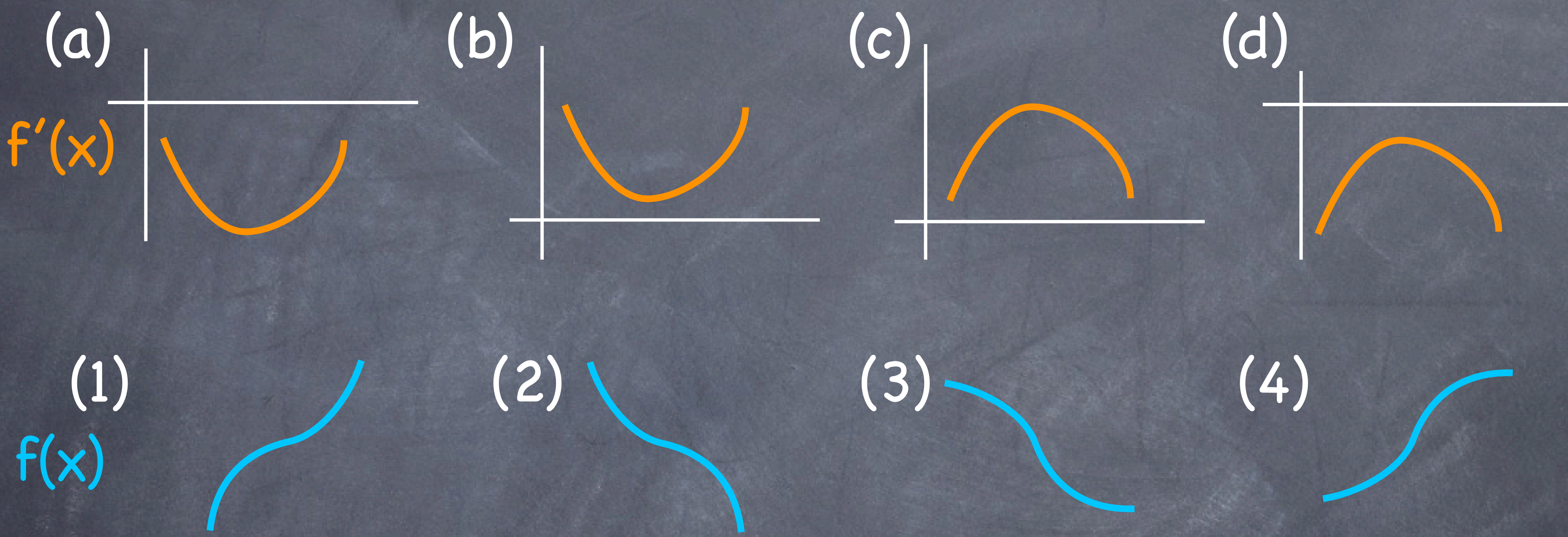
Inflection points

- An inflection point of f(x) is a point at which the concavity changes from up to down or down to up.

better!!

- A point **a** is an **inflection point** of a function f(x) provided that **a** is a **local minimum or a local maximum of f'(x).**

Match f′(x) to f(x)



(A) 1d, 2b, 3a, 4c

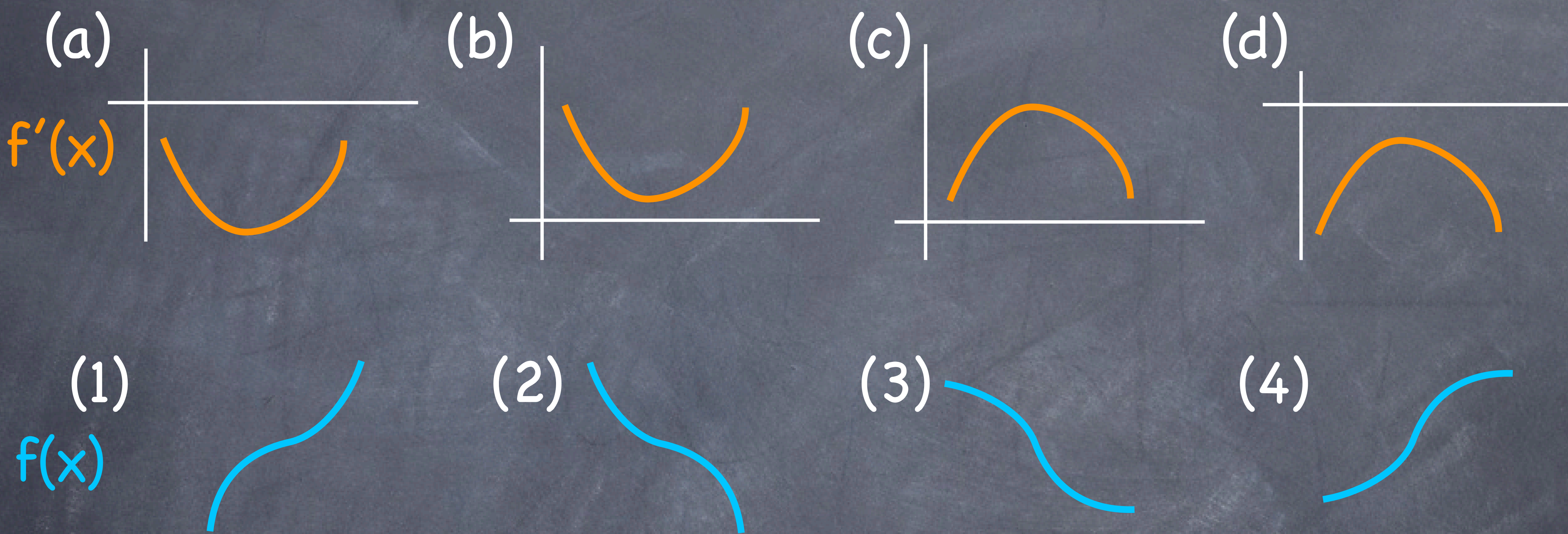
(B) 1b, 2d, 3a, 4c

(C) 1b, 2d, 3c, 4a

(D) 1c, 2a, 3d, 4b

(E) Don′t know.

Match f′(x) to f(x)



(A) 1d, 2b, 3a, 4c

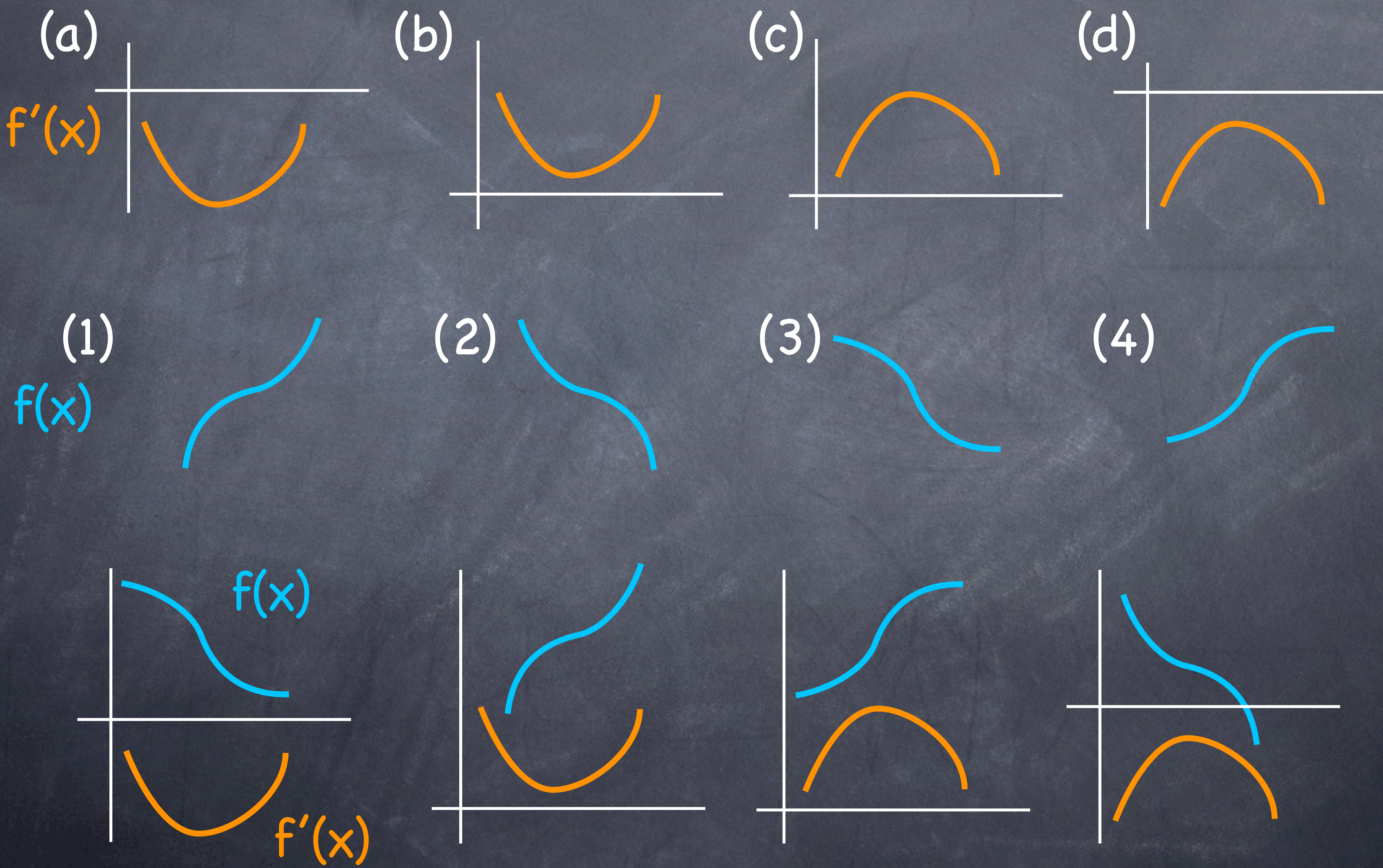
(B) 1b, 2d, 3a, 4c

(C) 1b, 2d, 3c, 4a

(D) 1c, 2a, 3d, 4b

(E) Don't know.

Match f′(x) to f(x)



If you want to find a min/max of $f'(x)$, look for points at which. . .

(A) $f'(x) = 0$. --> potential extremum of $f(x)$

(B) $f'(x) = 0$ and $f''(x) \neq 0$. --> extremum of $f(x)$

(C) $f''(x) = 0$. --> potential extremum of $f'(x)$

(D) $f''(x) = 0$ and $f'''(x) \neq 0$. --> extremum of $f'(x)$

(E) Don't know.

If you want to find a min/max of $f'(x)$, look for points at which. . .

(A) $f'(x) = 0$. --> potential extremum of $f(x)$

(B) $f'(x) = 0$ and $f''(x) \neq 0$. --> extremum of $f(x)$

(C) $f''(x) = 0$. --> potential extremum of $f'(x)$

(D) $f''(x) = 0$ and $f'''(x) \neq 0$. --> extremum of $f'(x)$

(E) Don't know.

This is "SDT" where the function considered is f' instead of f! Would usually use "FDT".