Today...

- Experiment revisited.
- Calculating the derivative from the definition.
- Limits and continuity examples.
- Reminders
    - Today: OSH 1
    - Sun: DT (ww)
    - Mon: PL3.1 (ww)

    Wed: PL3.2 (ww)

    Thurs: A2 (ww)

    Fri: Quiz 1

Studying experiment

- Test your partner
  - name (first and last),
  - date of birth,
  - location of birth,
  - intended major,
  - career ambitions (dream big!),
  - a list of places lived,
  - 3 favourite subjects from high school,
  - first pet's name or an instrument play(ed),
  - phone number (again, lie if necessary).

Studying experiment

(A) All 9 pieces of information correct.

(B) 8 pieces of information correct.

(C) 7 pieces of information correct.

(D) 5-6 pieces of information correct.

(E) 0-4 pieces of information correct.

Calculate derivative from definition

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Calculate $f'(2)$ where $f(x) = 1/x$ on the board.

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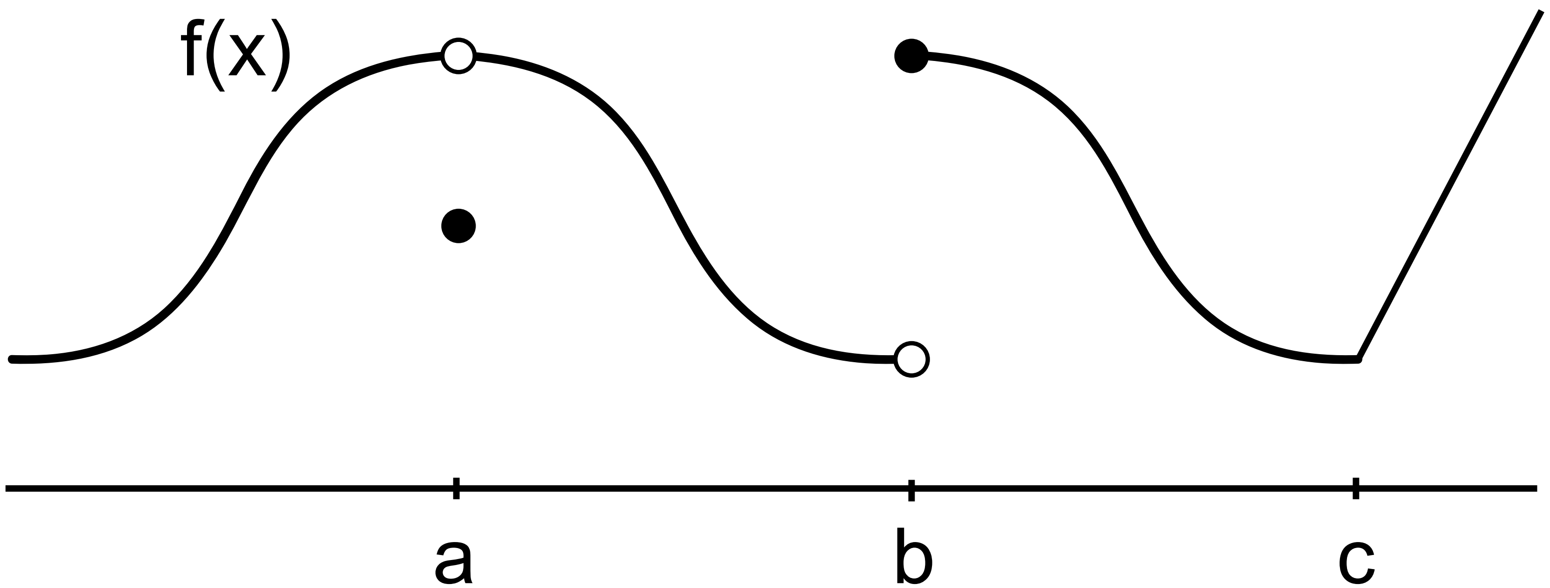
Common notation mistake:

Do not drop the “lim” along the way!

First eliminate the 0/0 problem, evaluate, then drop “lim”.

Limits



Which of the following are true?

1. $\lim_{x \to a} f(x) = f(a)$
2. $\lim_{x \to b} f(x) = f(b)$
3. $\lim_{x \to c} f(x)$ does not exist.
4. $\lim_{x \to a} f(x)$ exists.
5. $\lim_{x \to b} f(x)$ exists.

(A) 1, 4

(B) 2, 5

(C) 3

(D) 4

(E) 5

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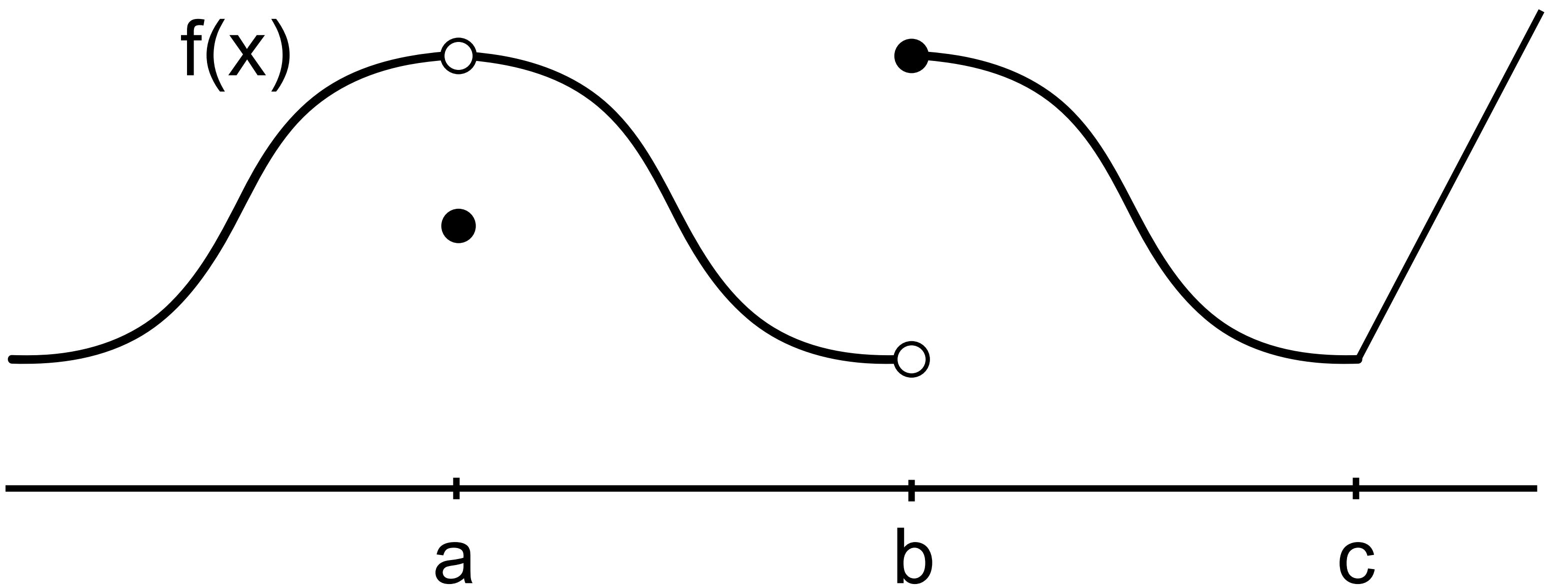

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(C) 3

(D) 4

(E) 5

Left and right limits

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- The right limit at **a** - plug in x values approaching **a** from above (x>**a**):

$$\lim_{x \to a^+} f(x)$$

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$$\lim_{x \to a^-} f(x)$$

Left and right limits

- The right limit at **a** - plug in x values approaching **a** from above (x>**a**):

$$\lim_{x \longrightarrow a^+} f(x)$$

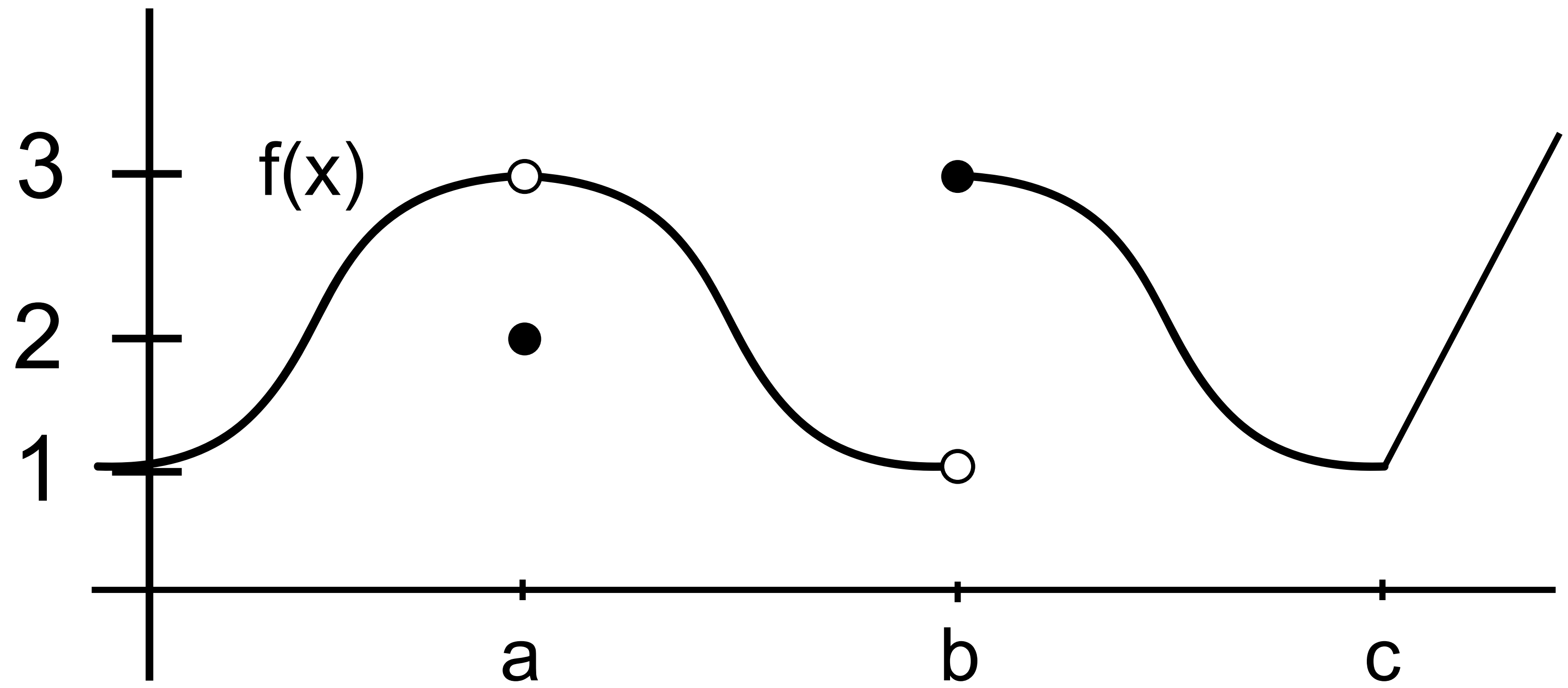
- The left limit at **a** - plug in x values approaching **a** from below (x<**a**):

$$\lim_{x \longrightarrow a^-} f(x)$$

- When these exist and are equal, $\lim_{x \longrightarrow a} f(x)$ exists

$$\lim_{x \longrightarrow a} f(x) = \lim_{x \longrightarrow a^+} f(x) = \lim_{x \longrightarrow a^-} f(x).$$

Limits



(A) $\lim_{x \to a} f(x)$ = 2

(B) $\lim_{x \to b^-} f(x)$ = 3

(C) $\lim_{x \to a} f(x)$ = 3

(D) $\lim_{x \to b} f(x)$ = 3

(E) $\lim_{x \to b^+} f(x)$ does not exist

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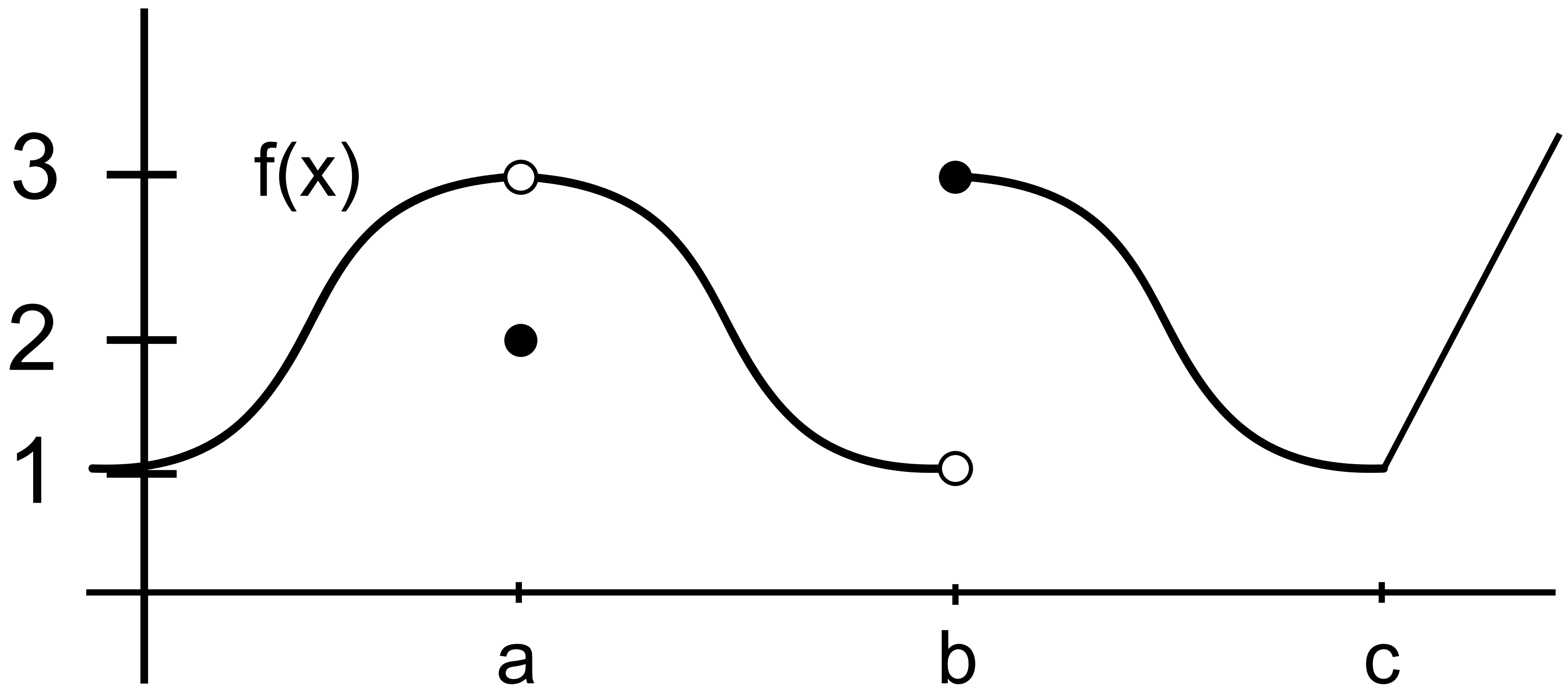

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When $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$

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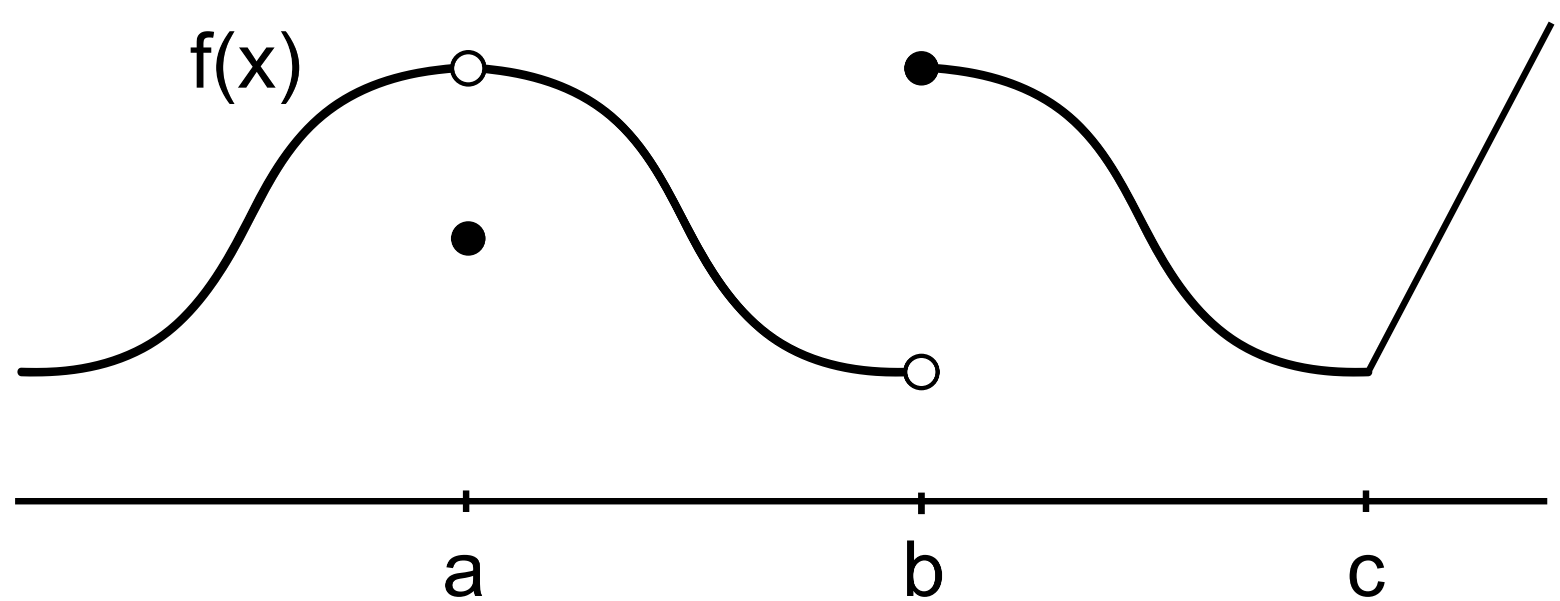

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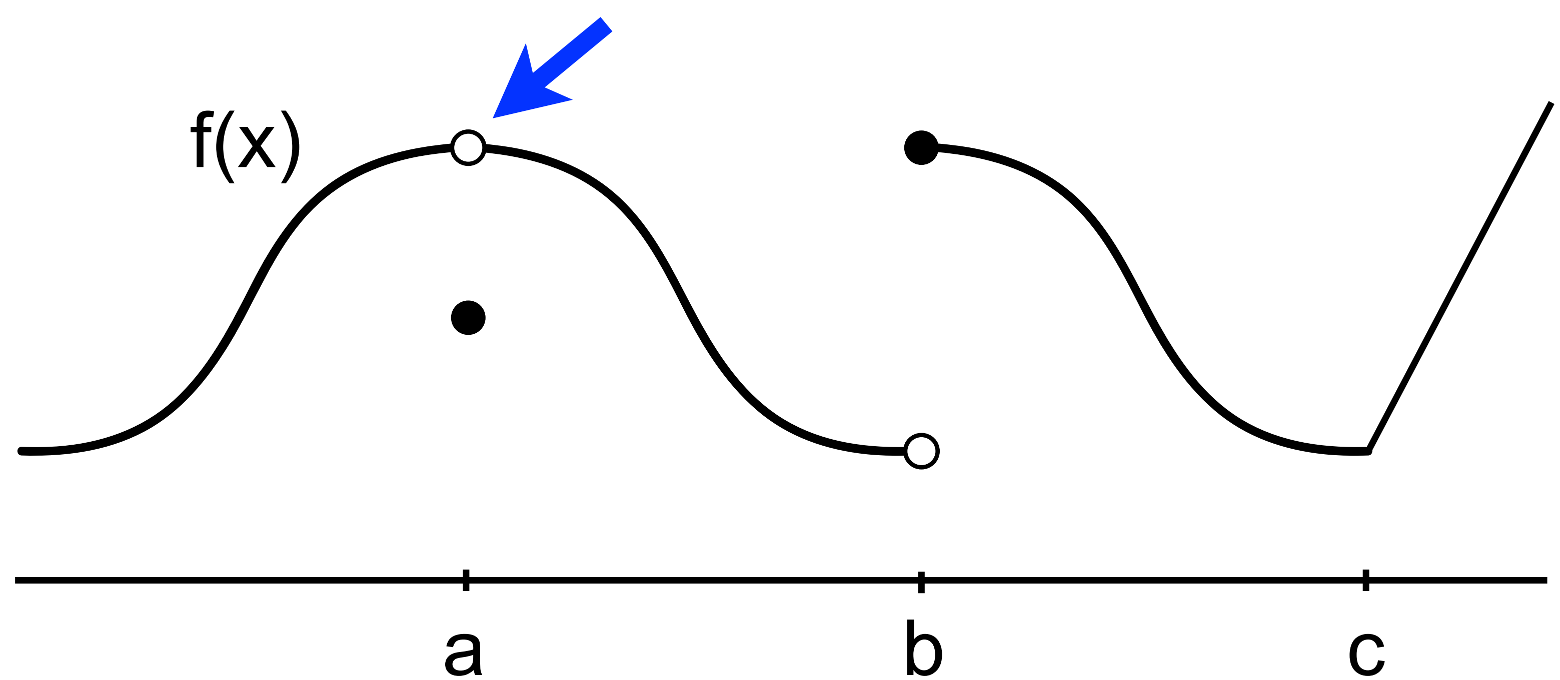

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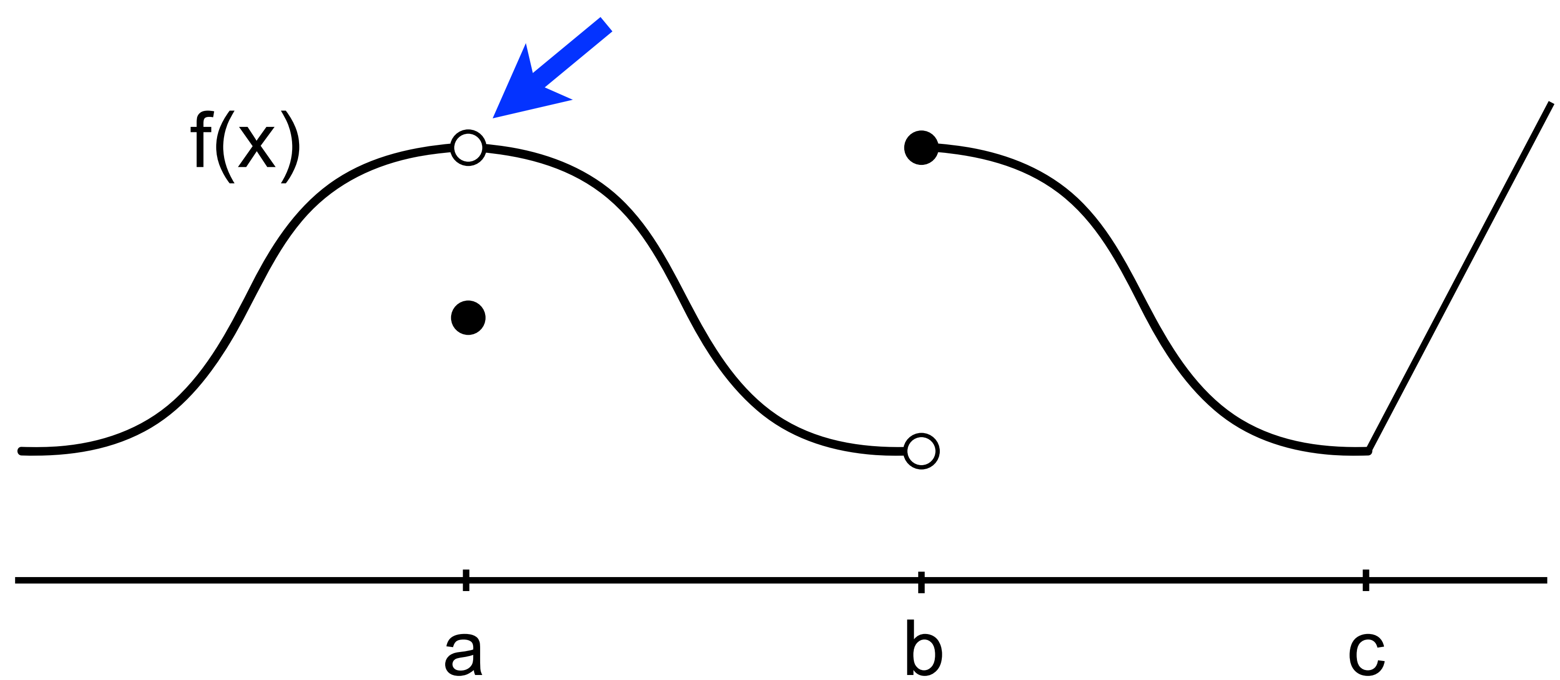

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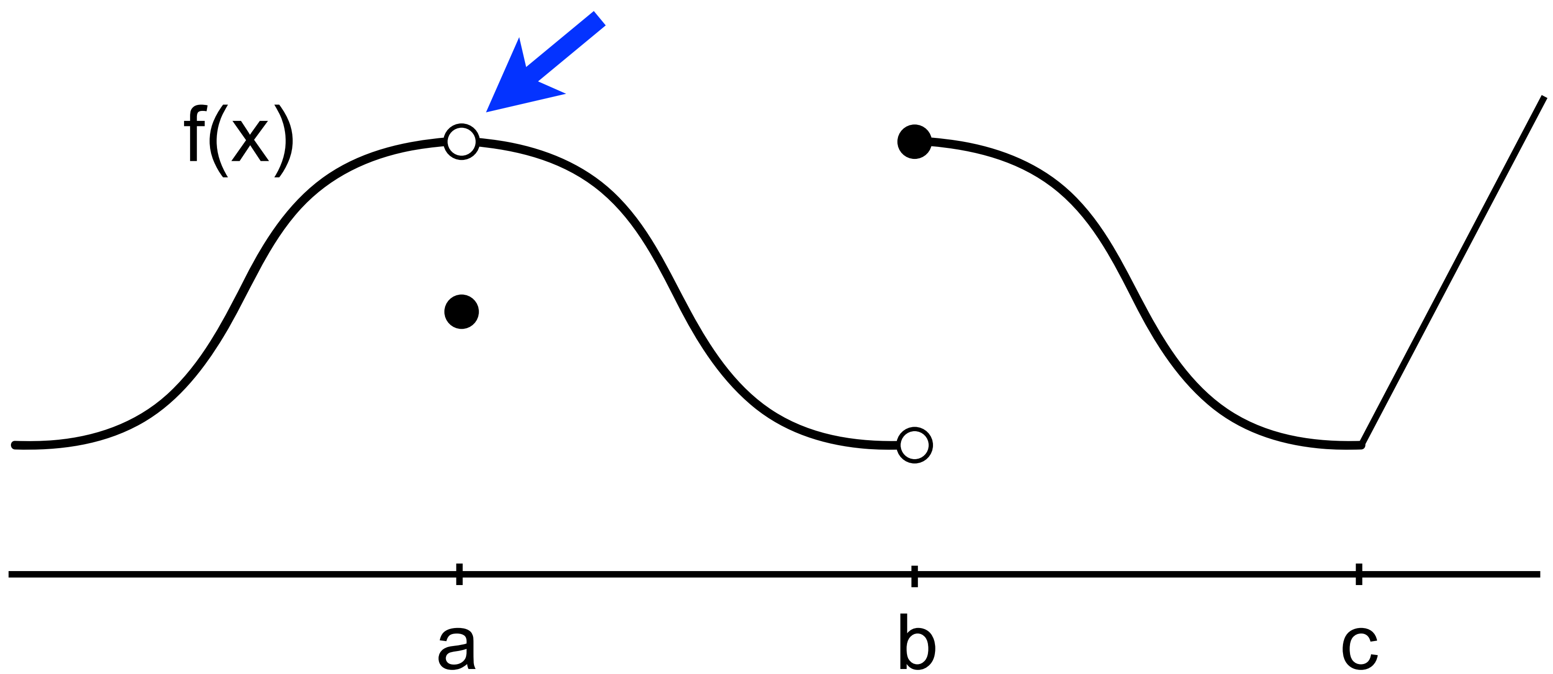

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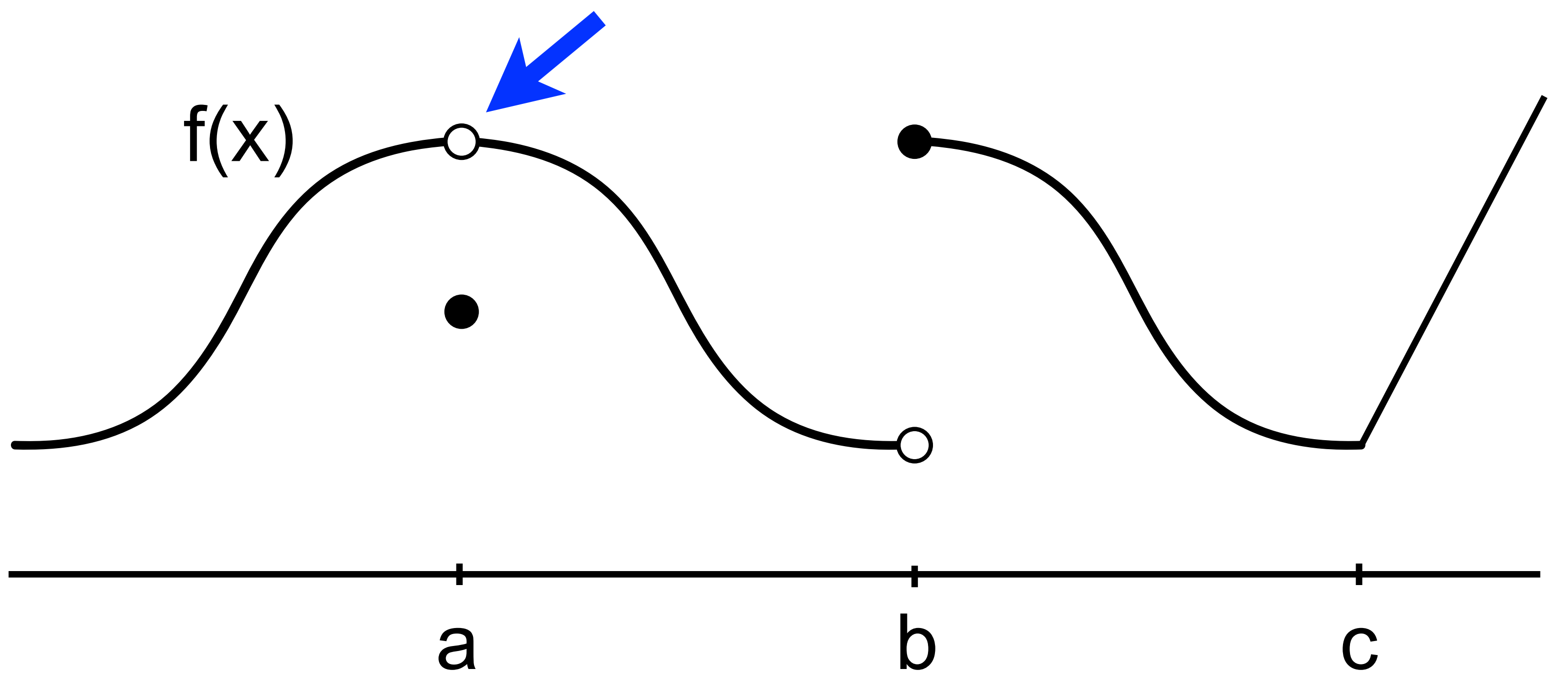

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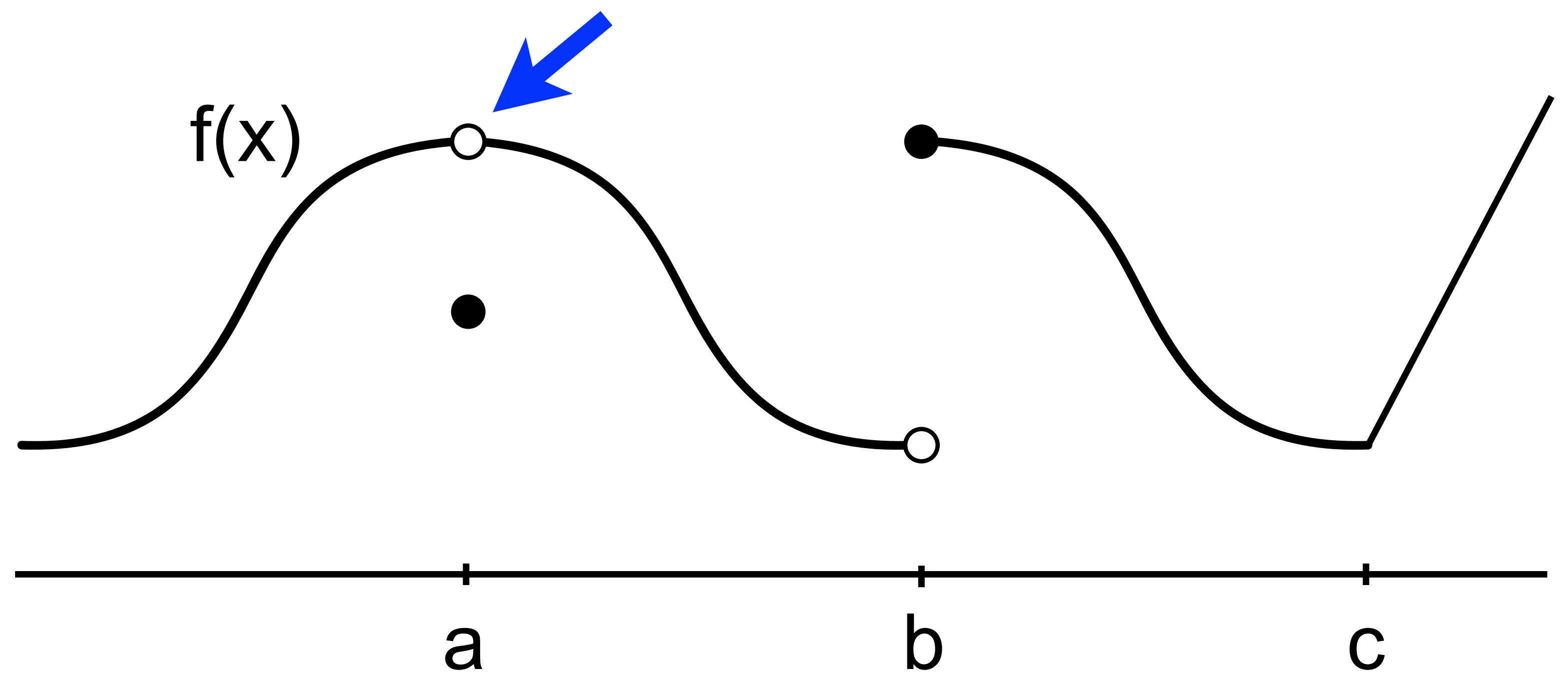

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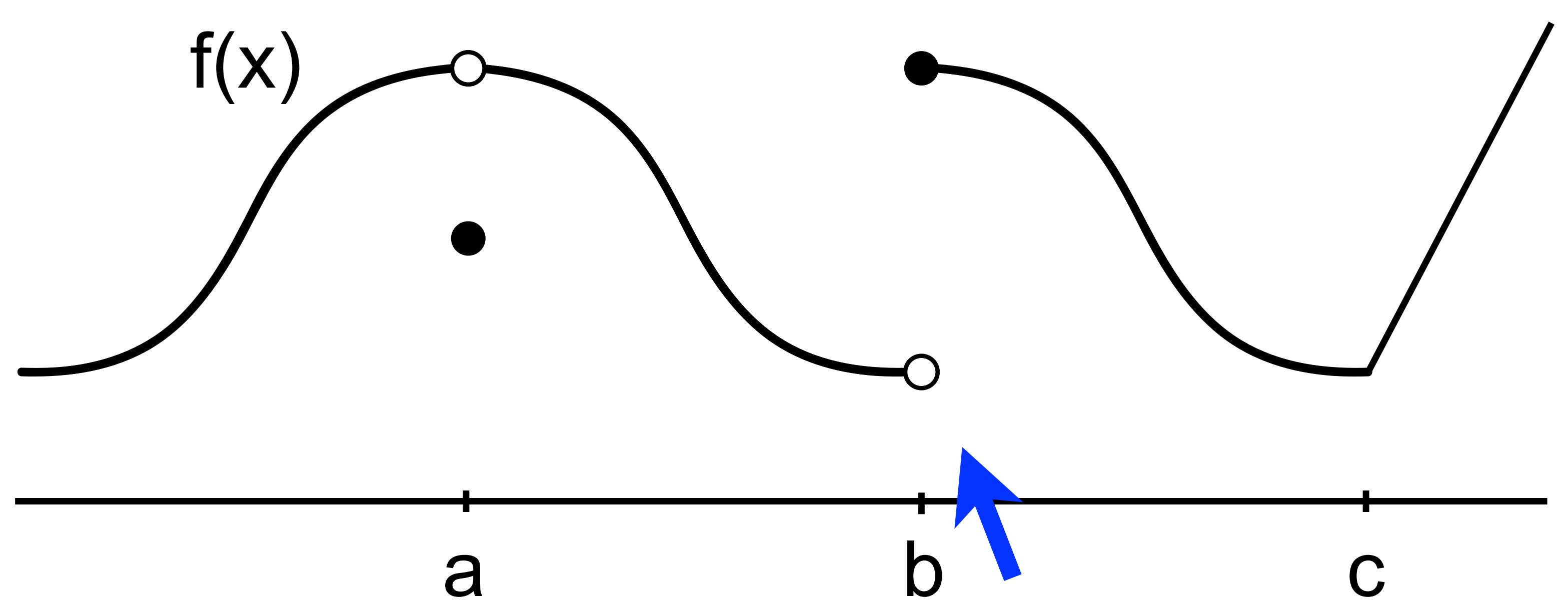

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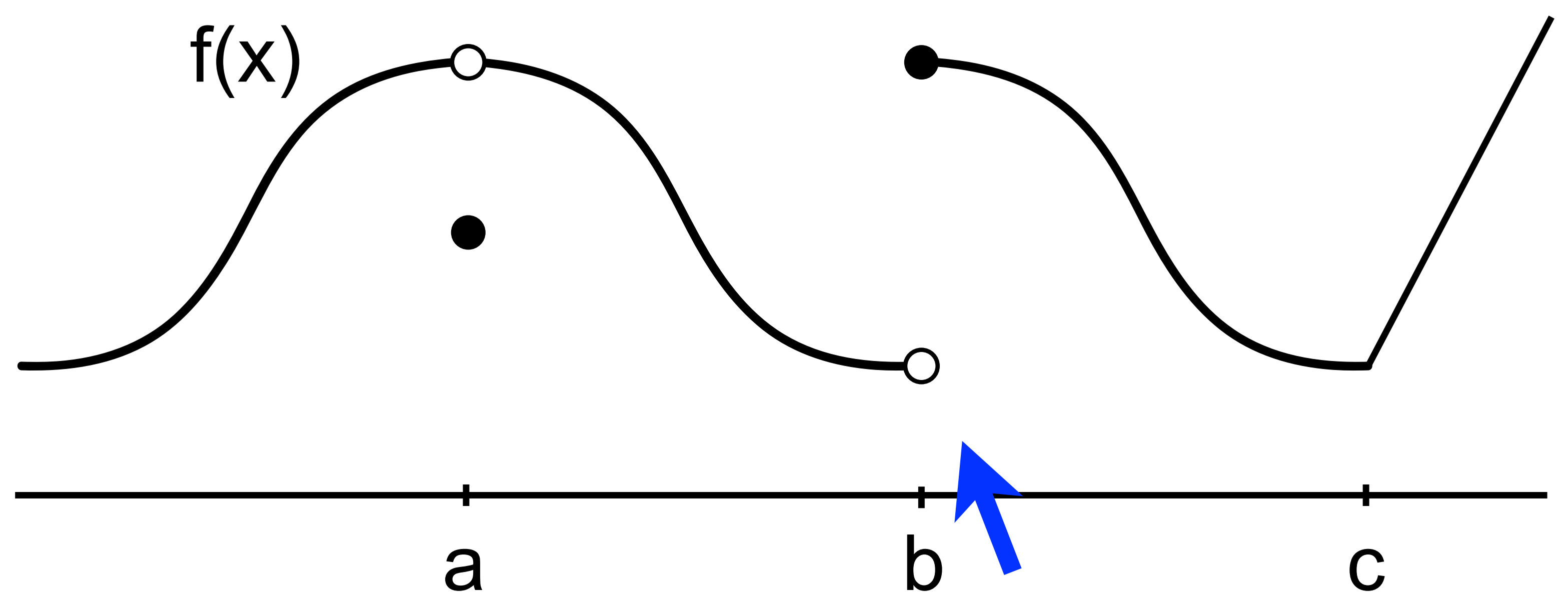

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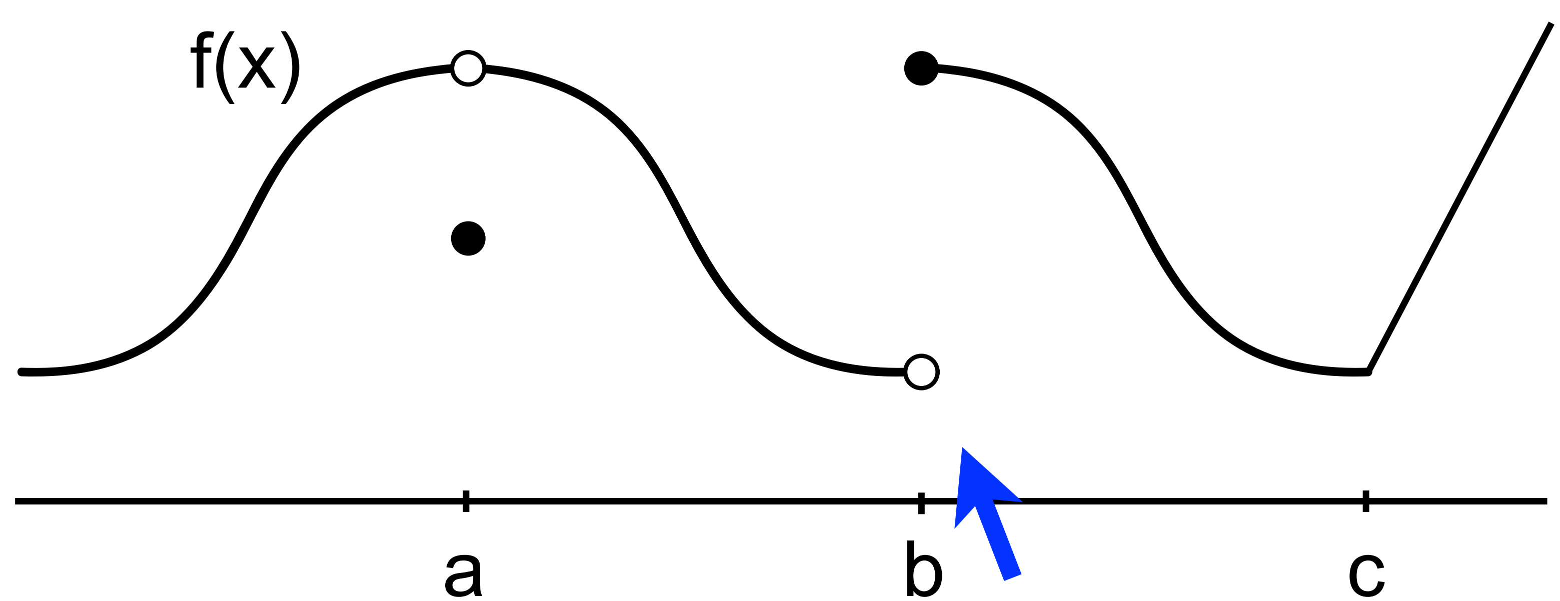

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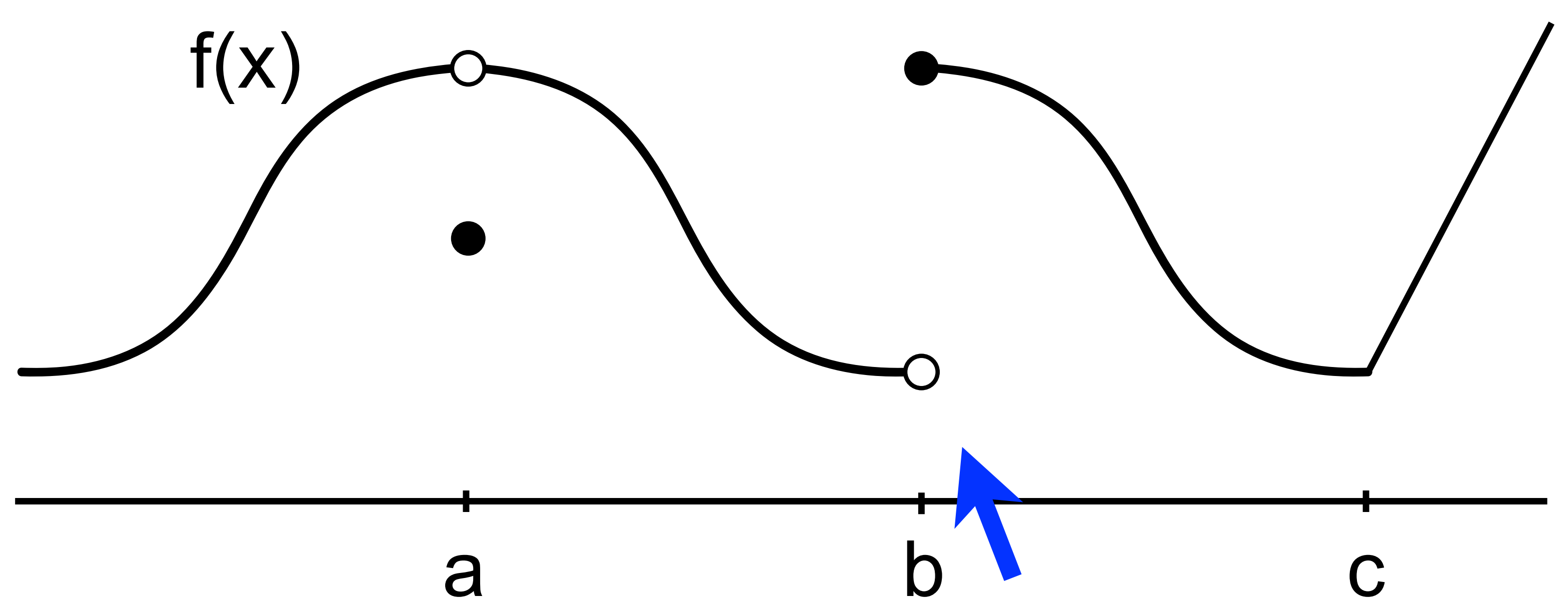

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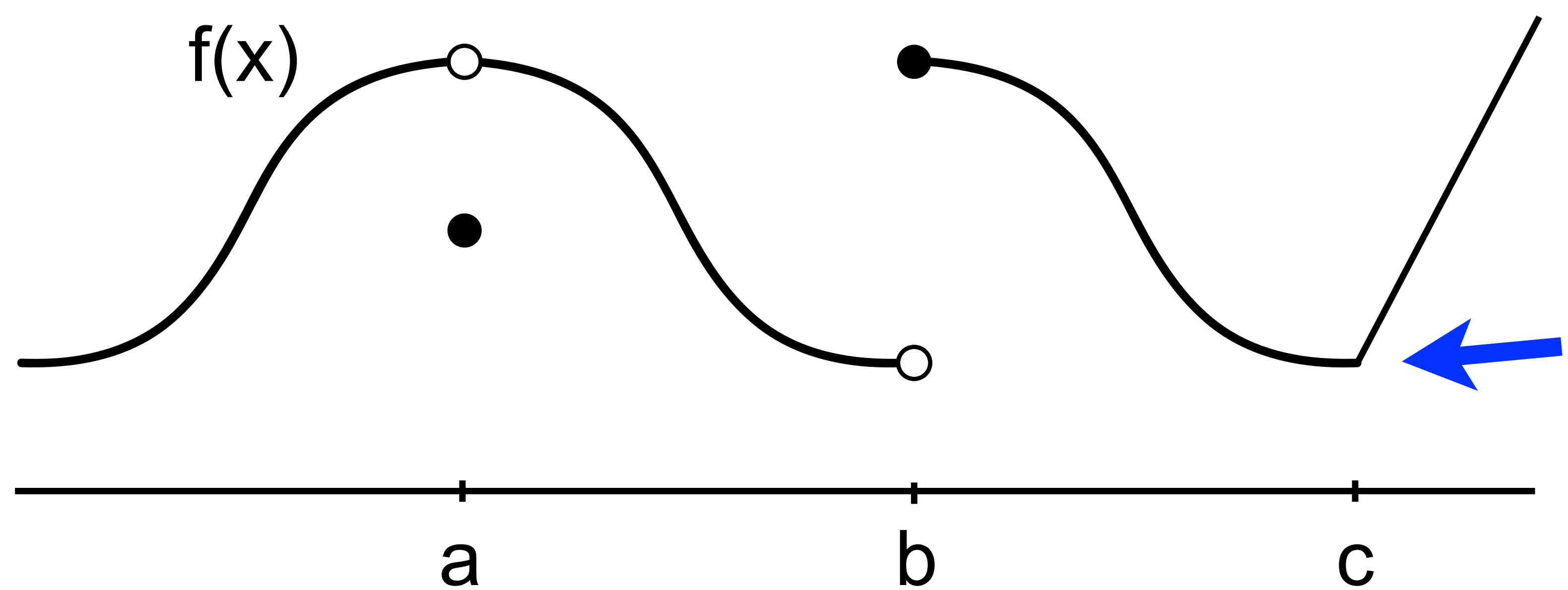

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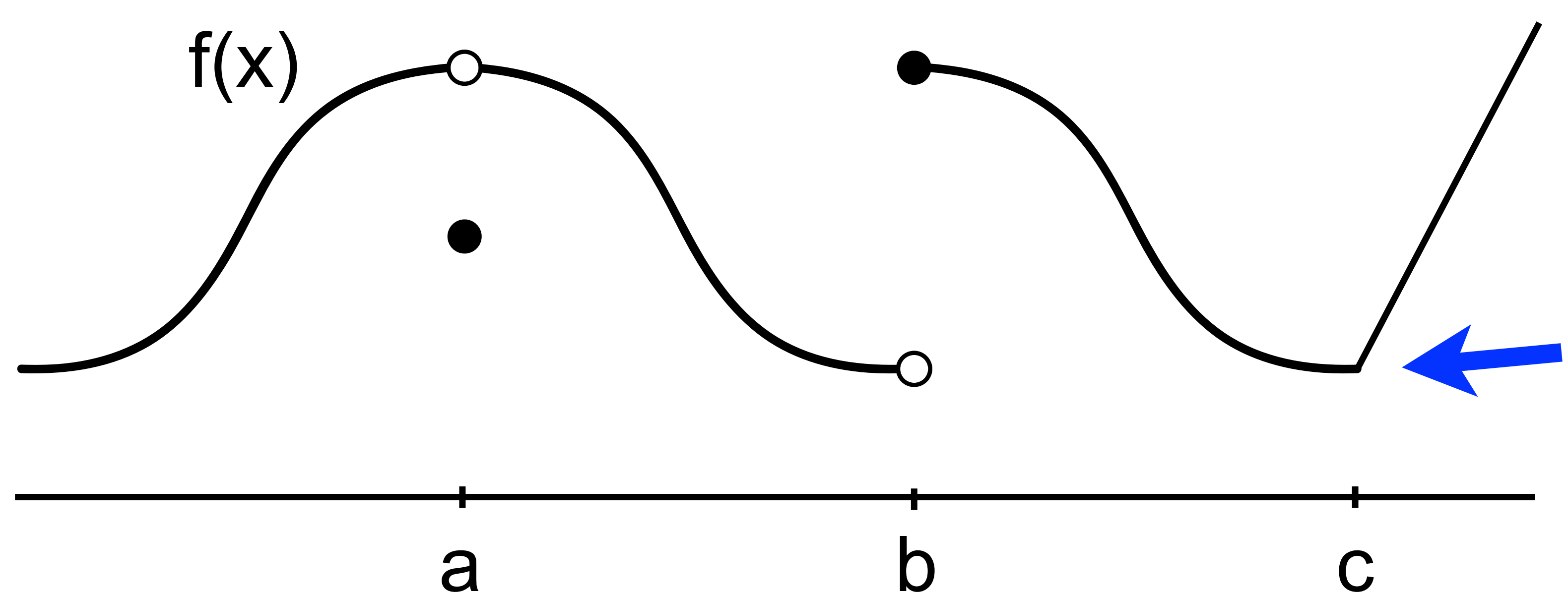

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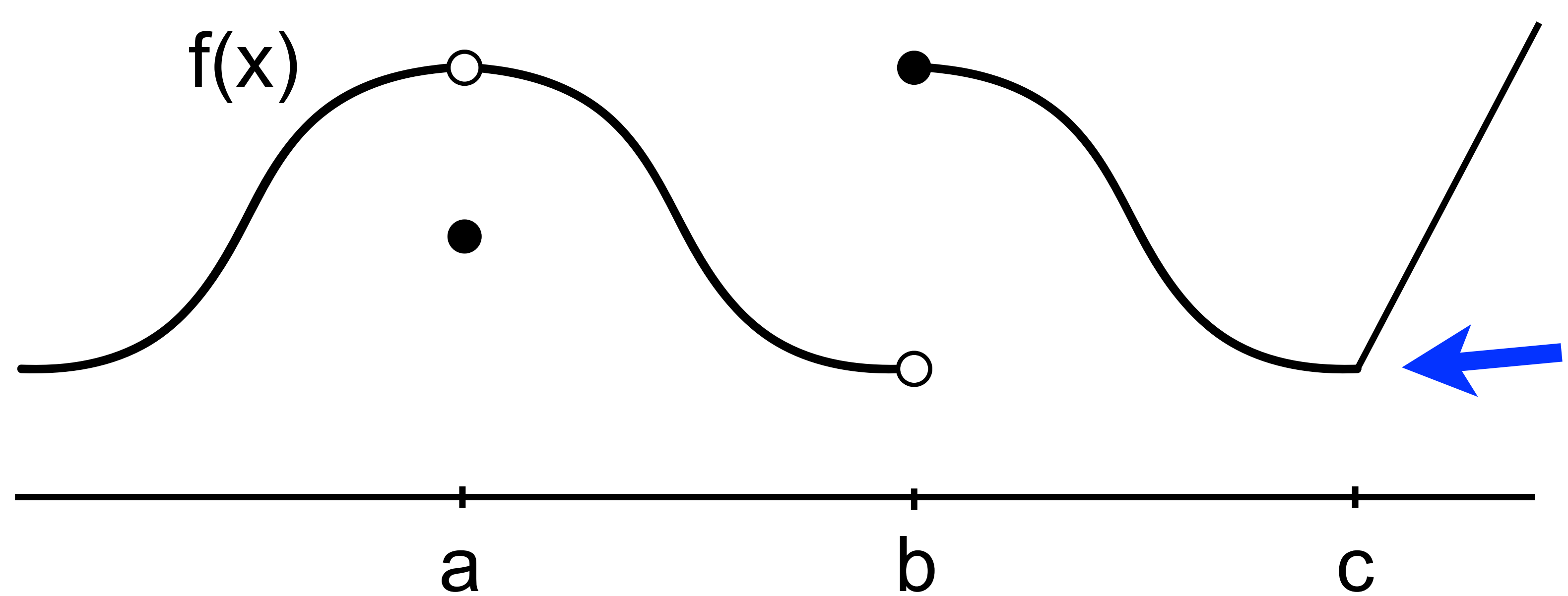

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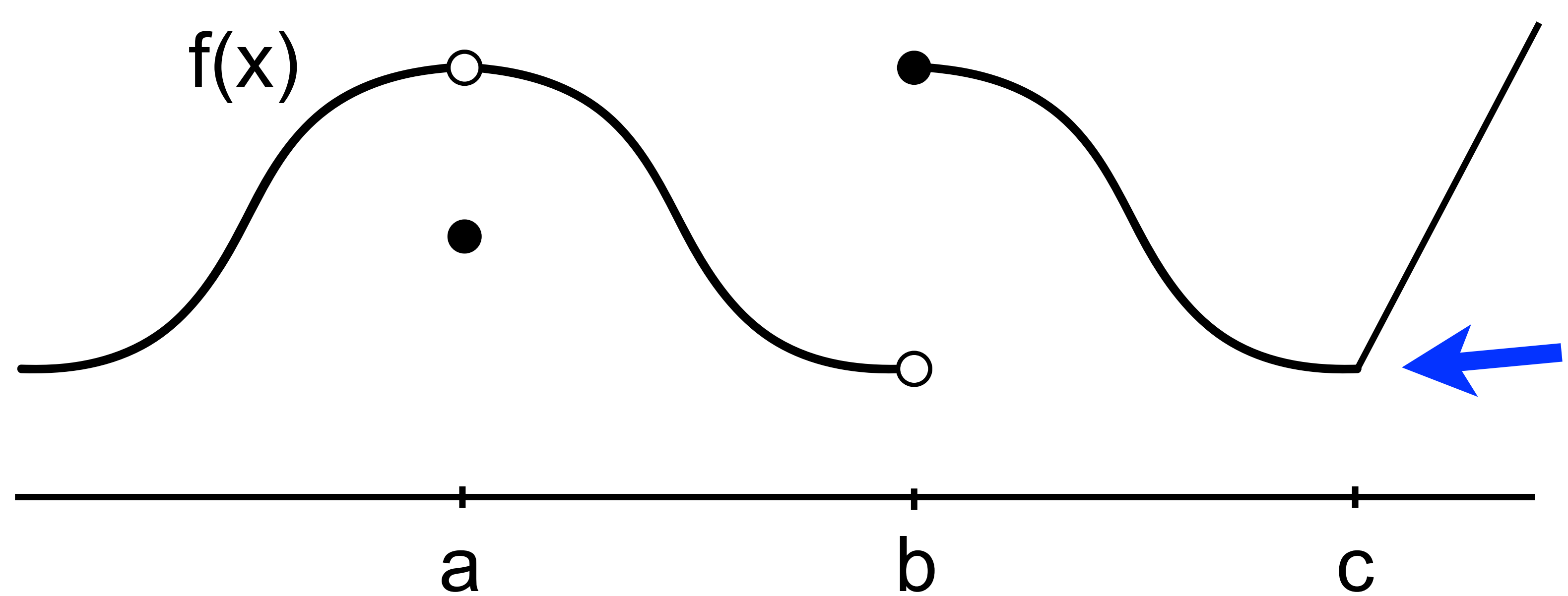

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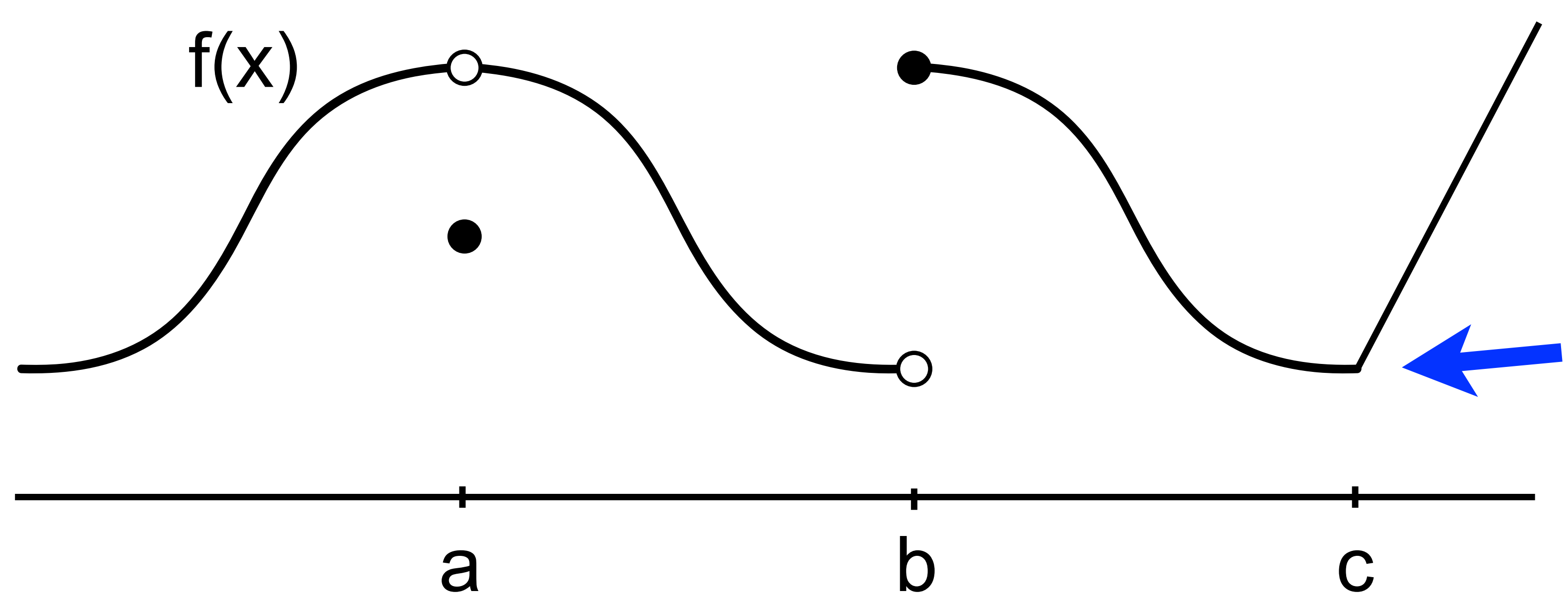

f(x) is continuous at all x except at x=a and x=b.

Continuous functions

When $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$

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- Examples of categories of continuous functions:
  - Polynomials
  - Exponentials
  - sin, cos
- These are all continuous for all real x.

Ensuring continuity

For what value of a is the following function continuous?

$$f(x) = \begin{cases} 4 - a^2 + 3x & x < 1 \\ x^2 + ax & x \geq 1 \end{cases}$$

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(E) Don’t know.

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https://www.desmos.com/calculator/obtqmika1u

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