Today

- Logistic equation appears in many places
    - Classic example of the power of mathematics - one unifying description for many apparently unrelated phenomena.
- A couple review questions.

Population models

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  - When $P \approx 0$, growth rate constant is $\approx rK>0$.
  - When $P=K$, growth rate constant is 0.

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- Steady state? P=0, P=K.
- Stability?

(A) P=0 is stable, P=K is unstable.

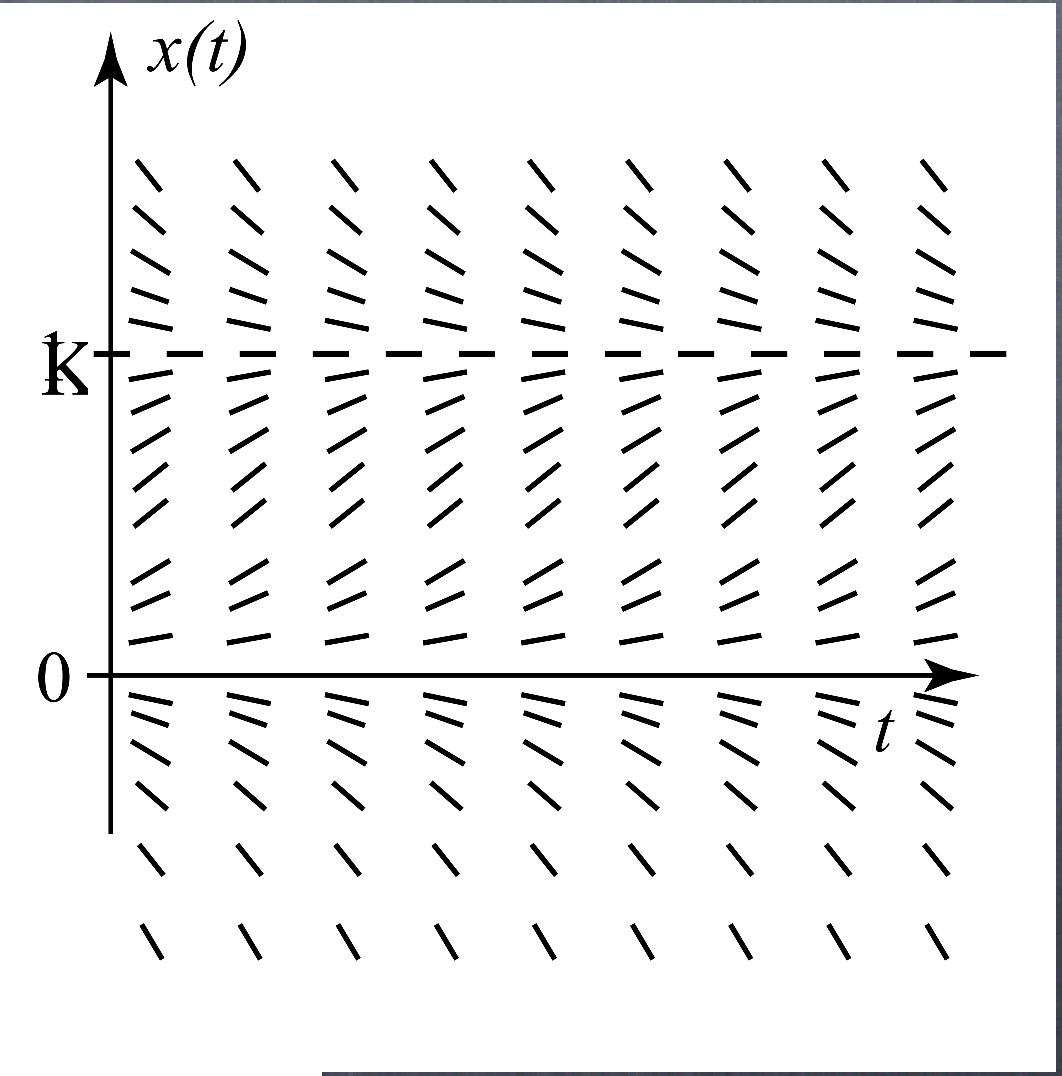
(B) P=0 is unstable, P=K is unstable.

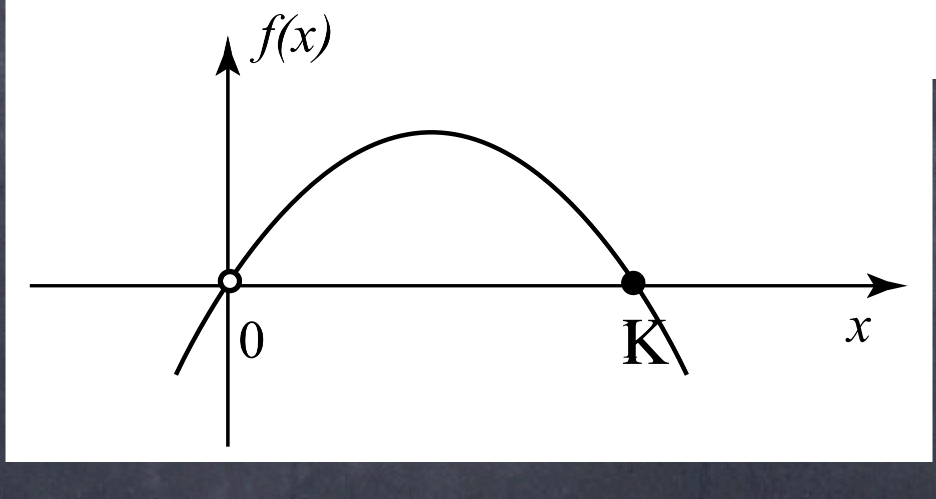
(C) P=0 is unstable, P=K is stable.

(D) P=0 is stable, P=K is stable.

Population models

- dP/dt = rP(1-P/K)
  - Called the Logistic Equation or sometimes Verhulst Equation.
  - Arises in many other contexts.





Logistic equation in different contexts...

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- $dx/dt = bX(C-X)$

The many faces of the Logistic equation

- Infectious disease: bSI (S=susceptible, I=infected)
- Spread of rumour: bNH (N = not heard rumour, H = heard rumour)
- Spread of new words in a language.
- Spread of new technologies.
- Oil exploration sites.
- Waterlillies in a pond.

Some data…

First an important feature

Where does inflection point occur?

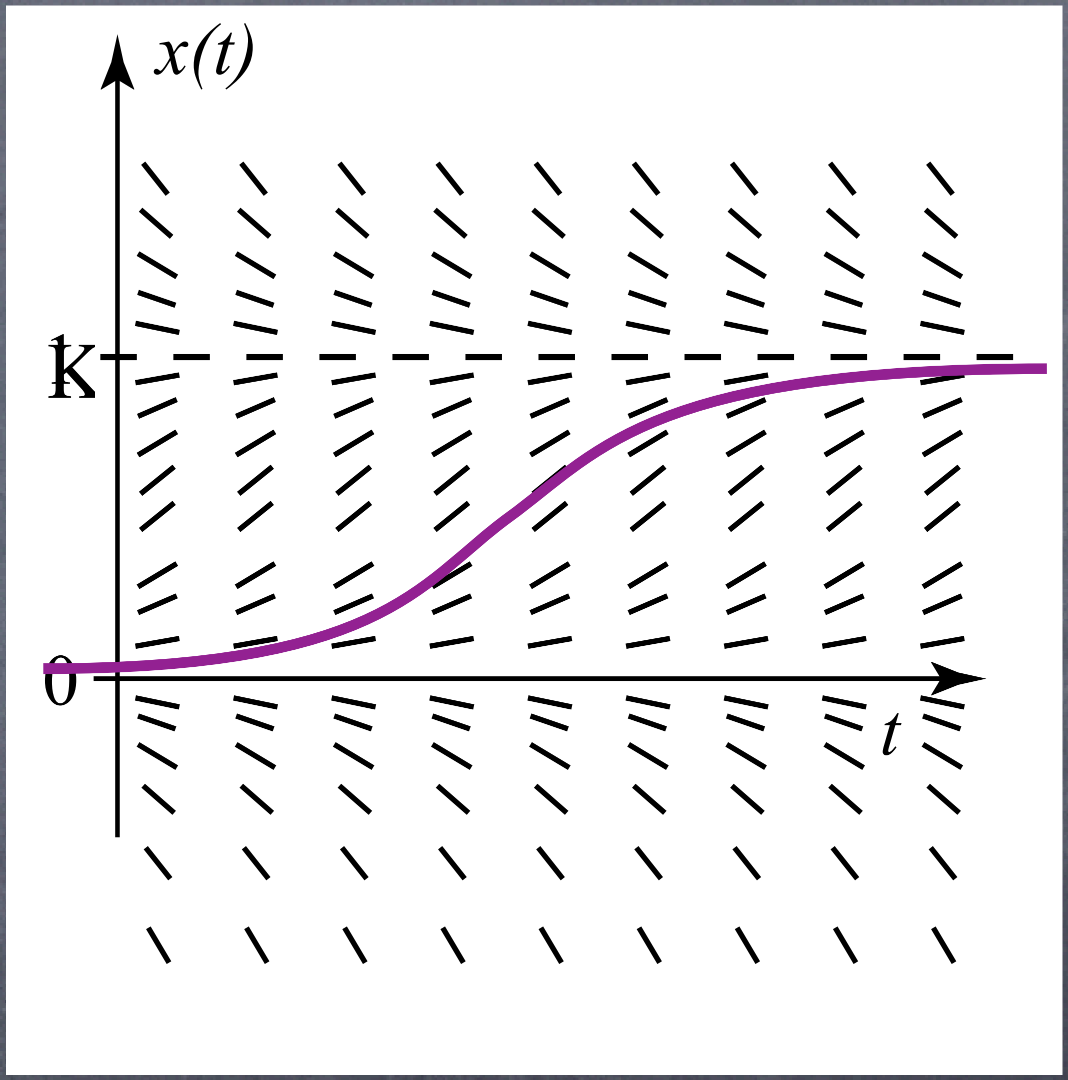
(A) x=0

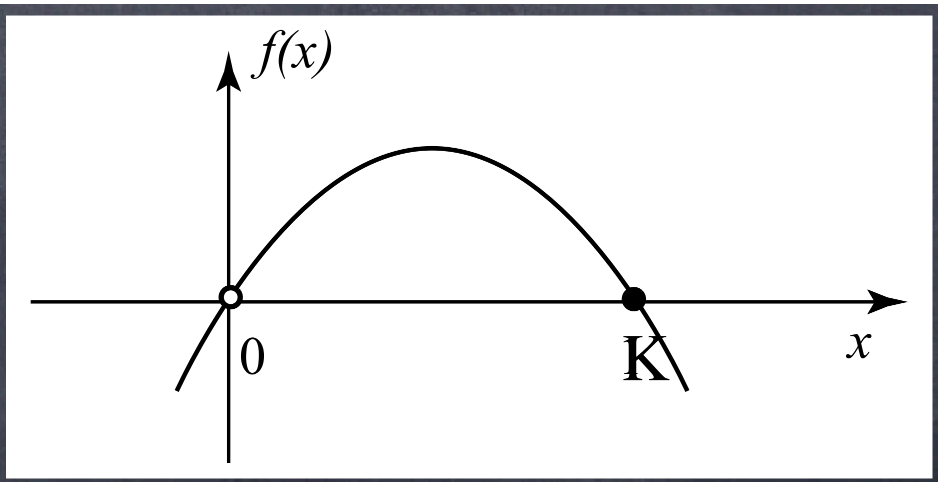
(B) x=1/2

(C) x=K/2

(D) x=1/4

(E) x=K





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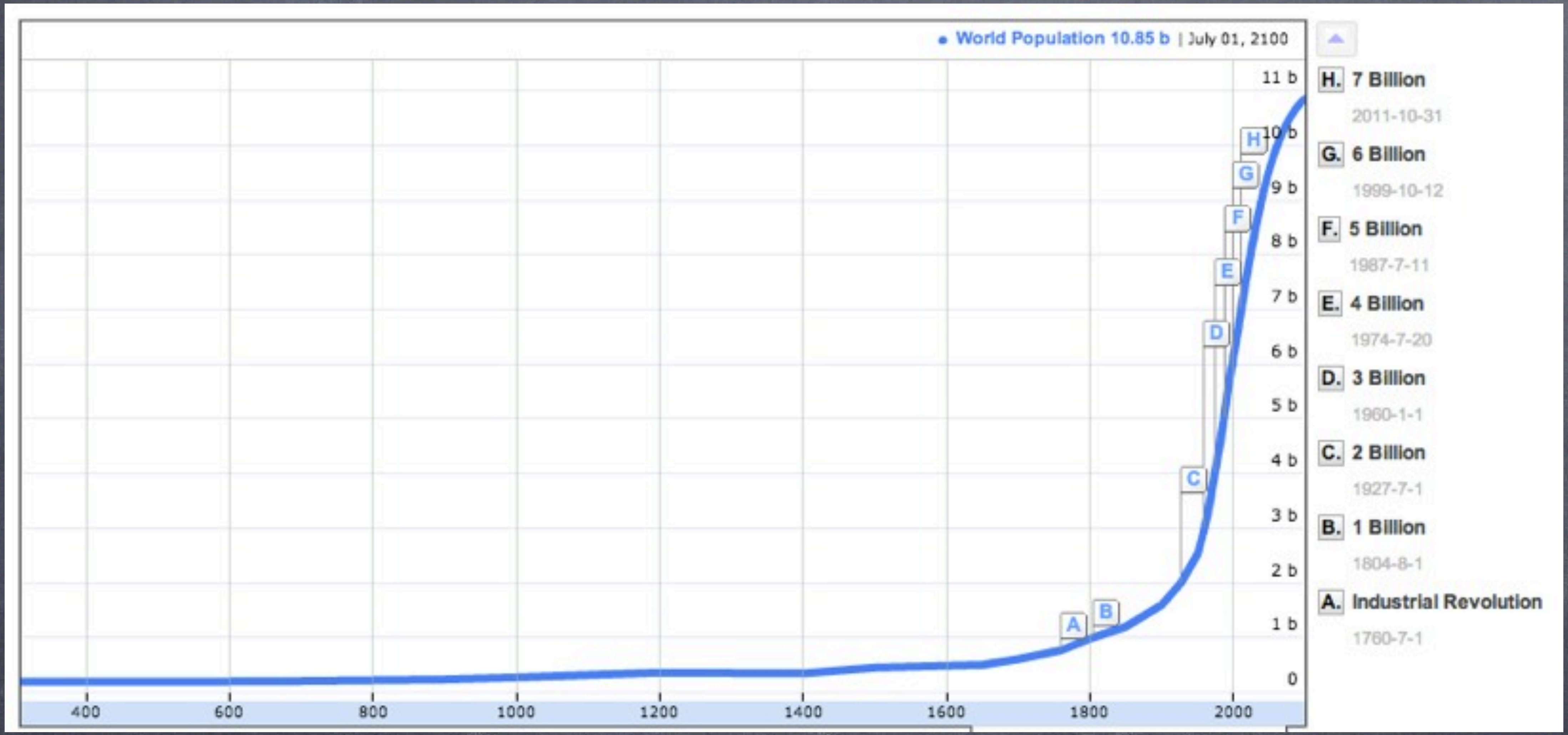
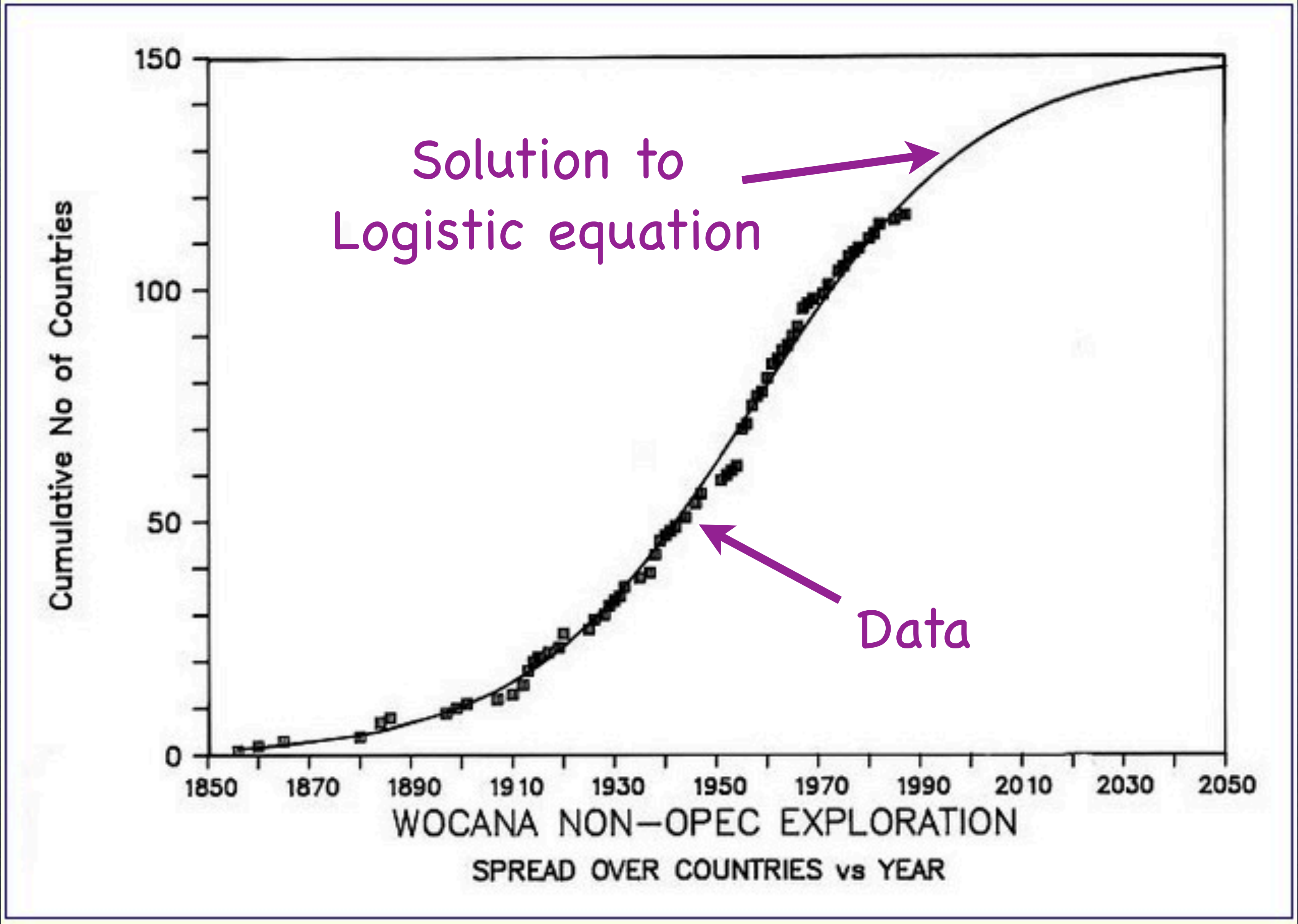


We're past the inflection point

- estimate K≈10 billion

Number of countries with active oil exploration



http://www.mhnederlof.nl/kinghubbert.html

American peak oil production

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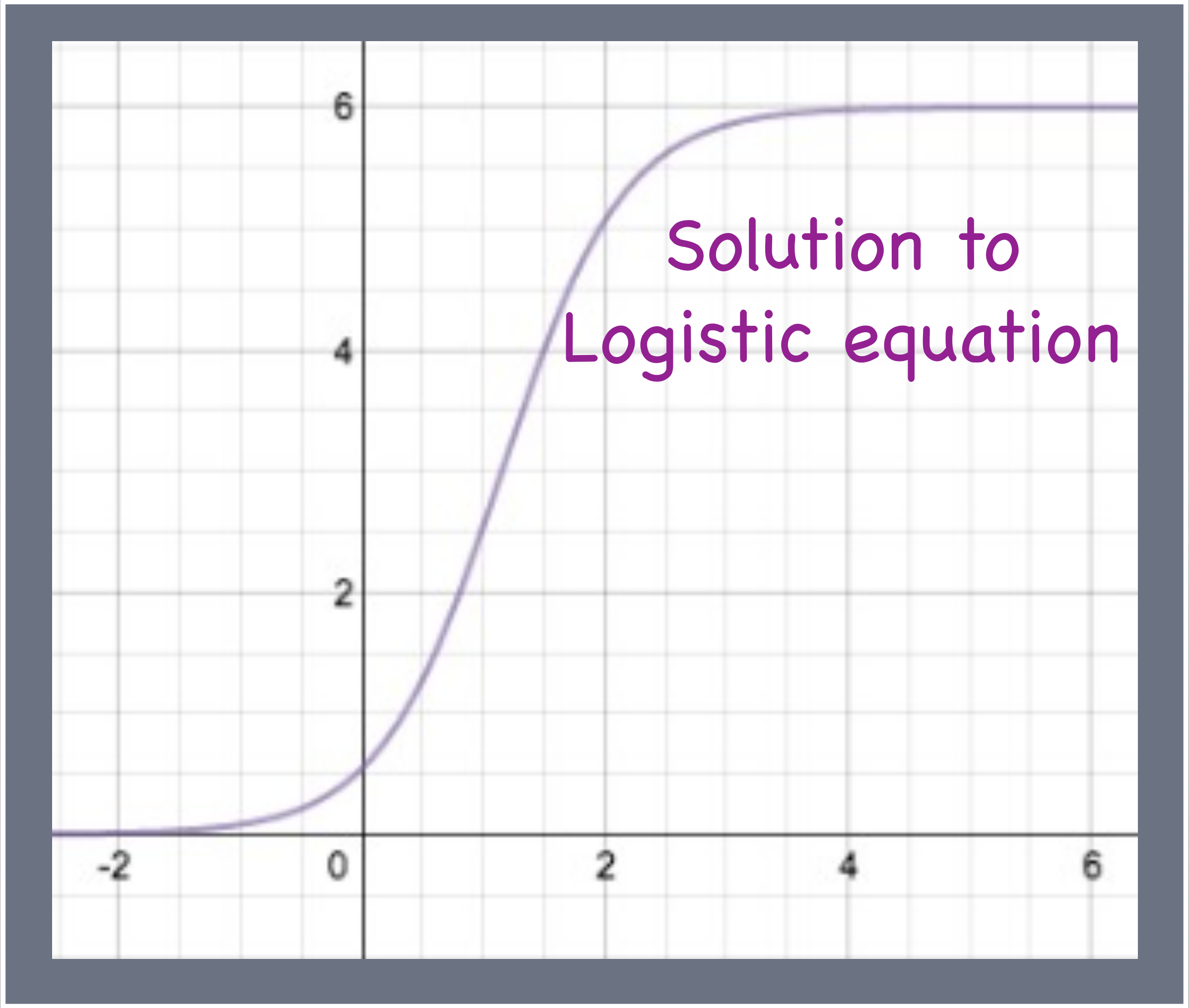

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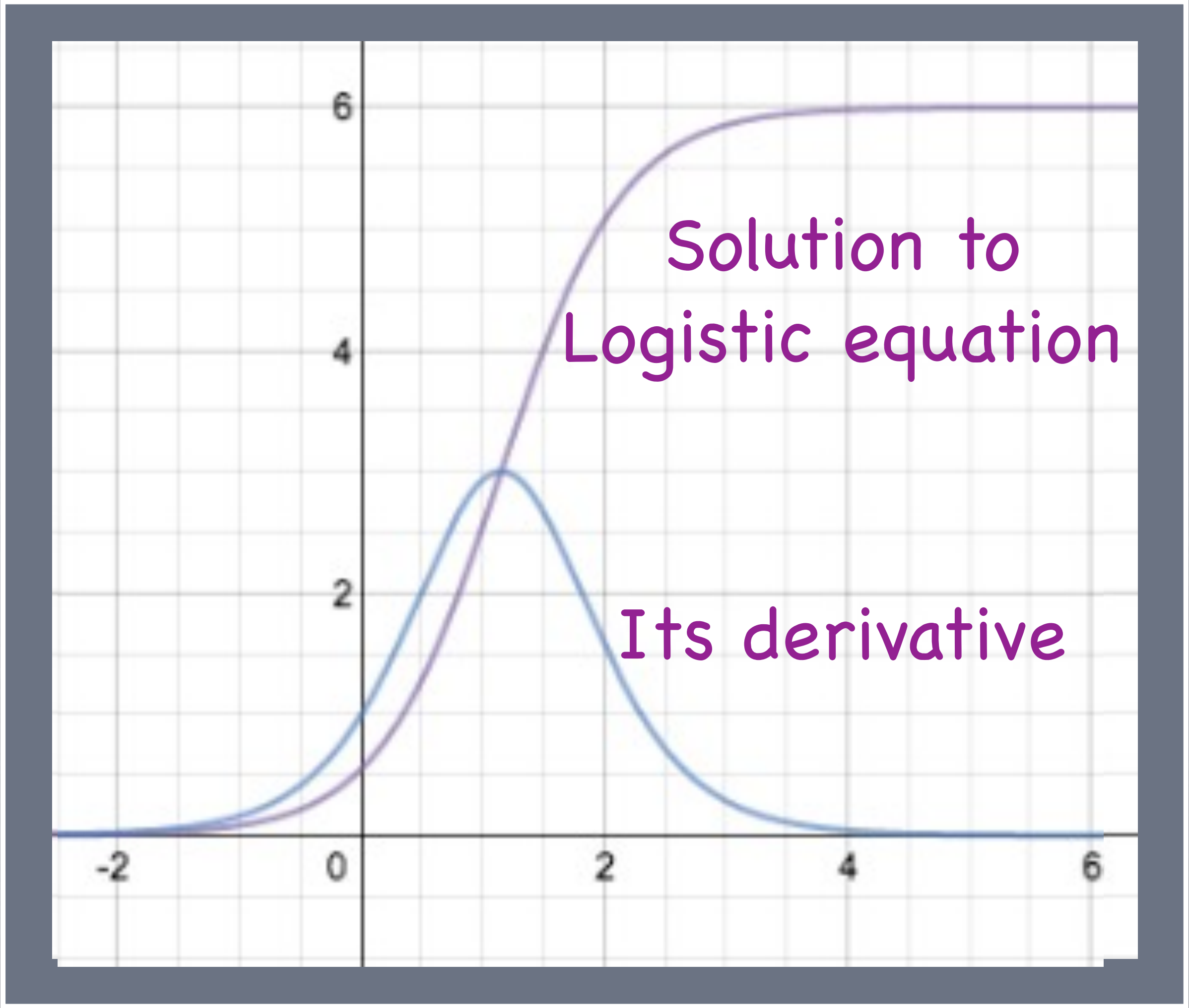

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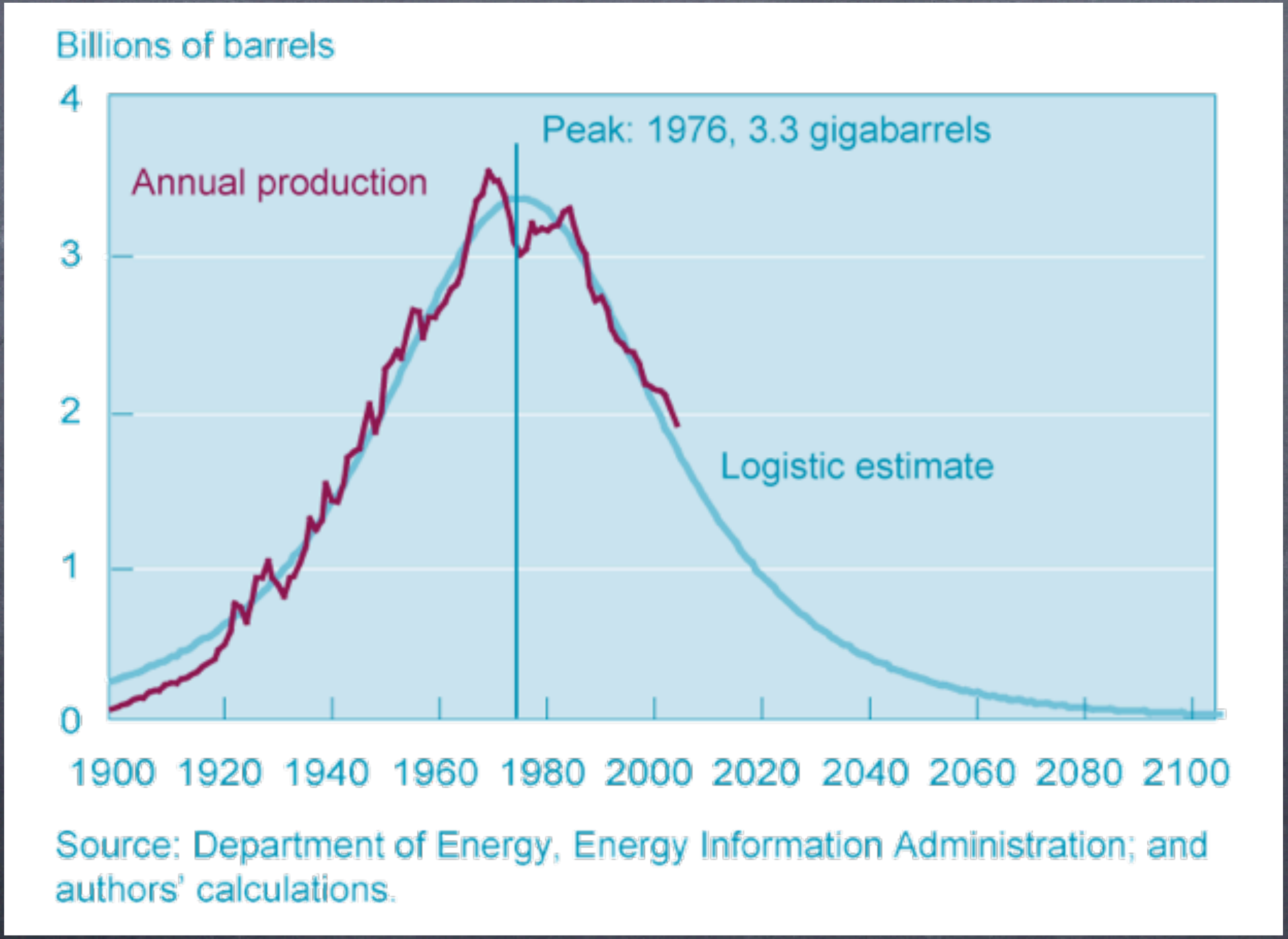
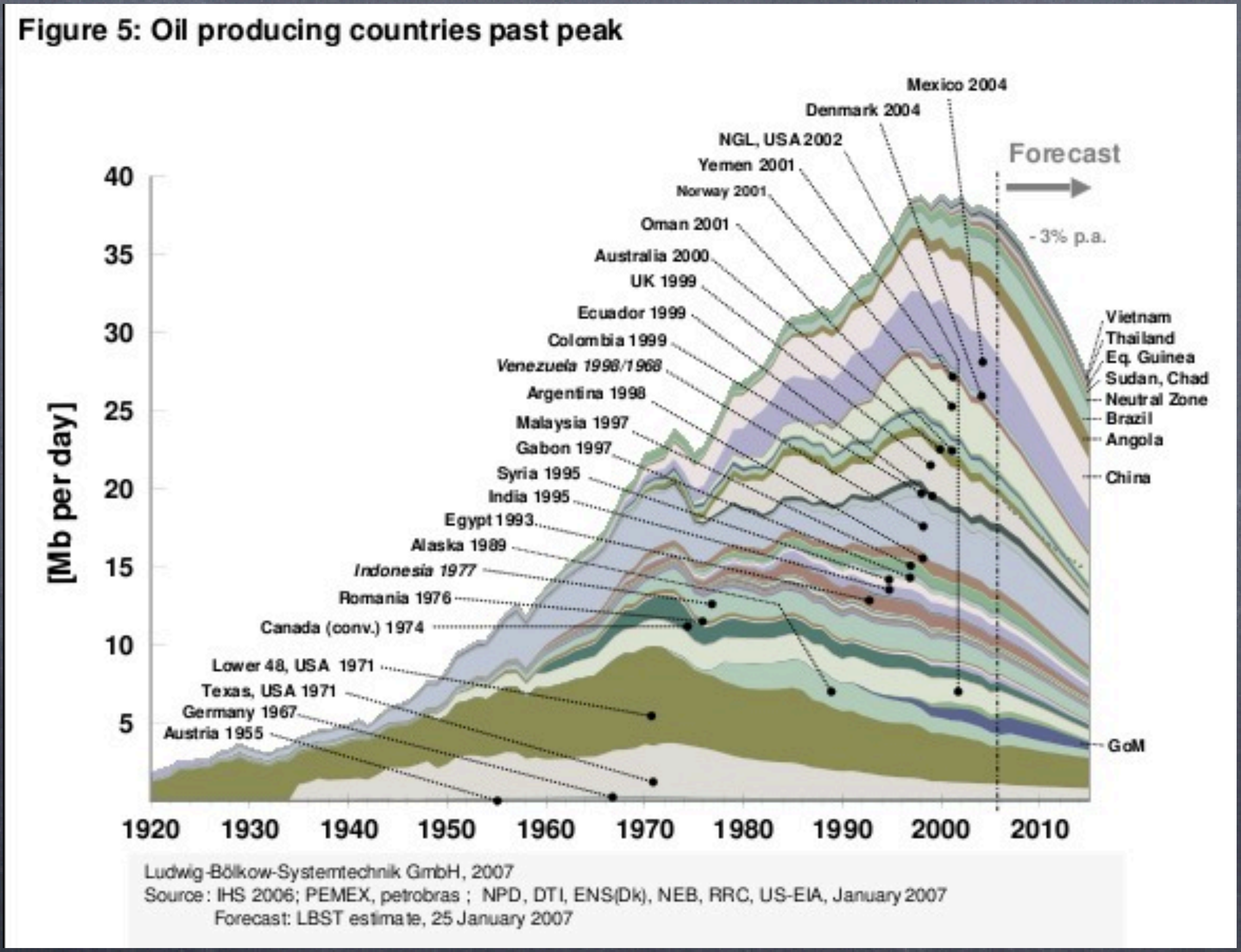


http://www.clevelandfed.org/research/commentary/2007/081507.cfm

World peak oil production



A couple review questions...

f(x)=ln(x) and g(x)=(x-1)/(e-1) both go through (1,0) and (e,1). At what point in between are they farthest apart?

(A) x = (e+1)/2

(B) x = sqrt(e)

(C) x = e-1

(D) x = e^2-1

(E) x= 2

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Define h(x)=f(x)-g(x) and find max of h(x).

Which equation gives the point on the graph of $f(x)=x^2+b$ whose tangent line goes through the origin?

(A) $a^2 = b$

(B) $y = 2a\ (x-a) + a^2+b$

(C) $x^2+b = 0$

(D) There is no such point.

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A population grows according to the equation $y' = y(100-y)(y-20)$. If the initial population size is 18, what size does the population eventually approach?

(A) 0

(B) 20

(C) 100

(D) 40 + 20 sqrt(7/3)

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The girls say good luck with your exams!!