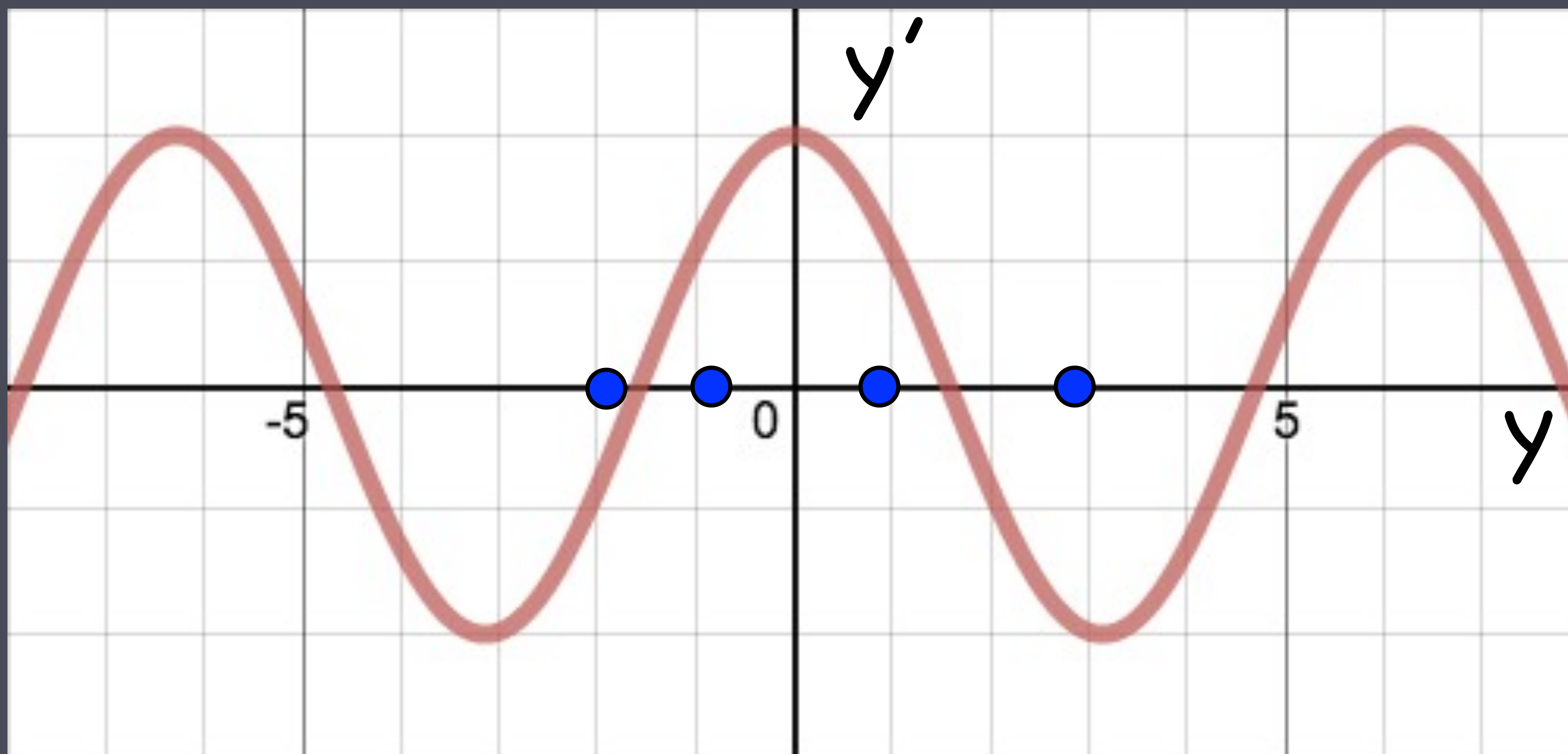
Today

- Phase-line to solution-sketching example (cont).
- Logistic equation in many contexts
    - Classic example of the power of mathematics - one unifying description for many apparently unrelated phenomena.

$y' = \cos(y)$

Sketch a few solutions y(t).



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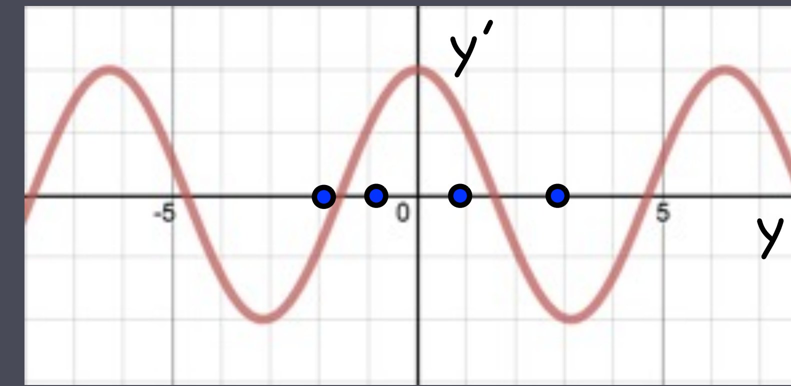

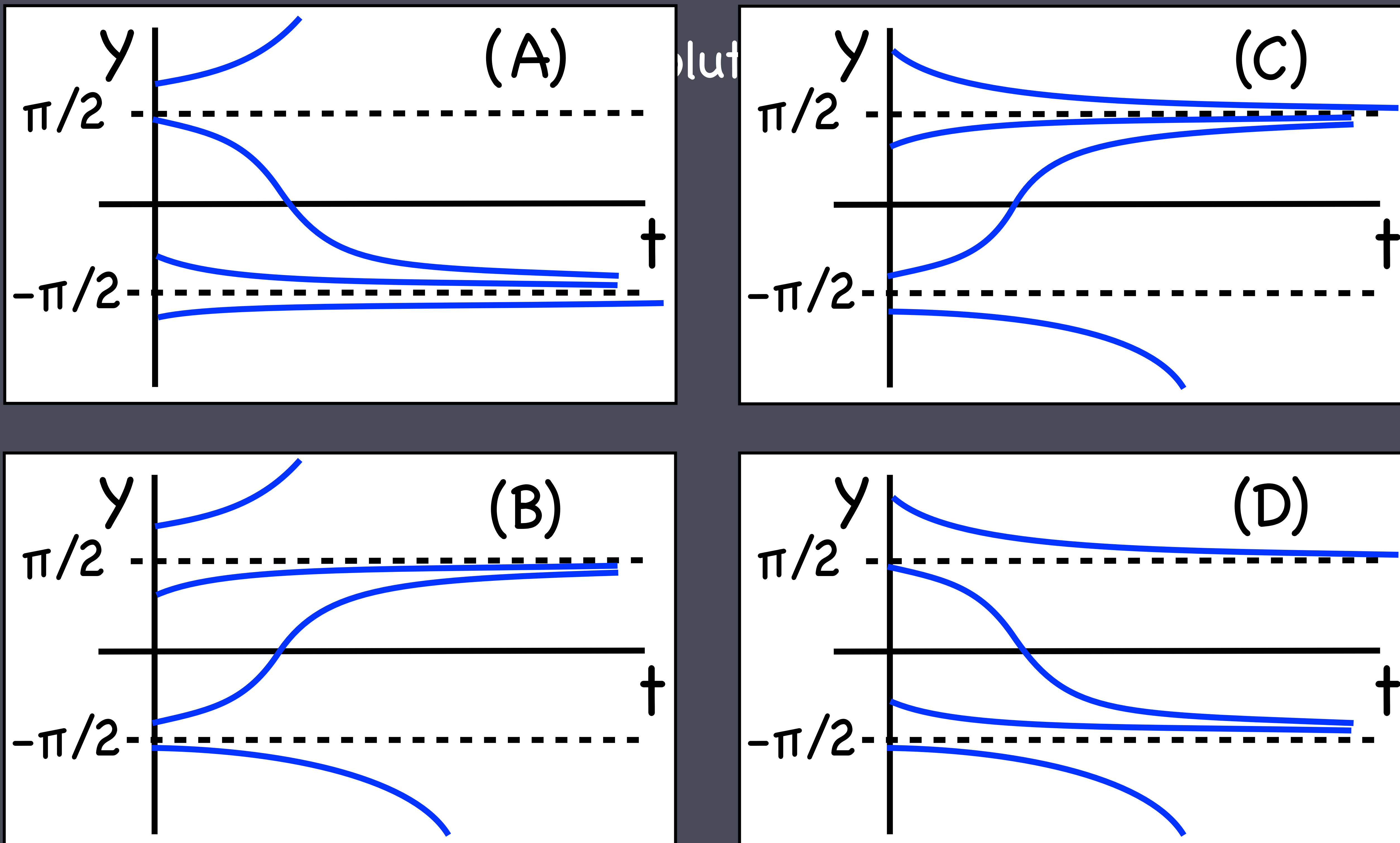

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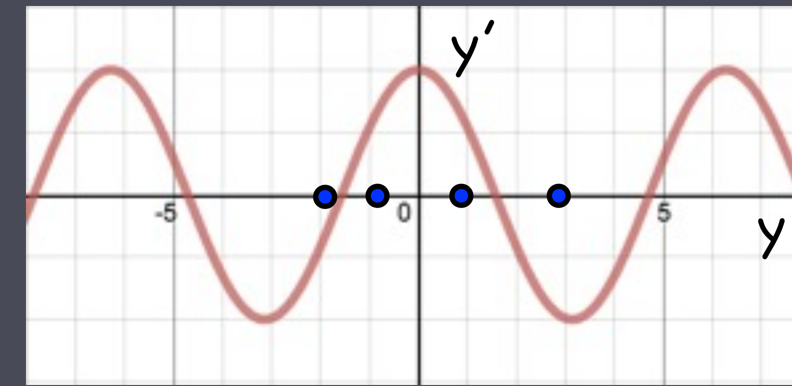
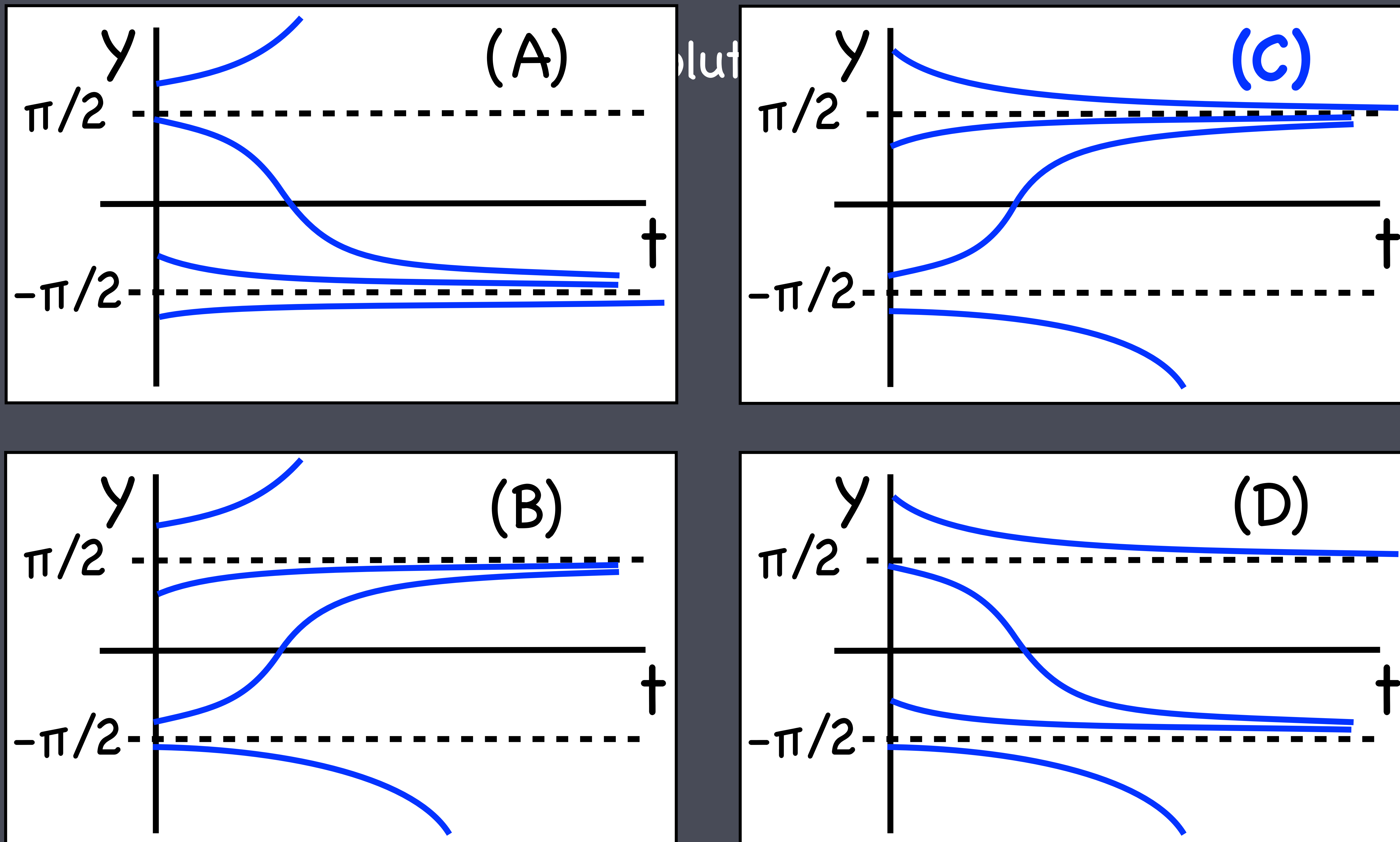


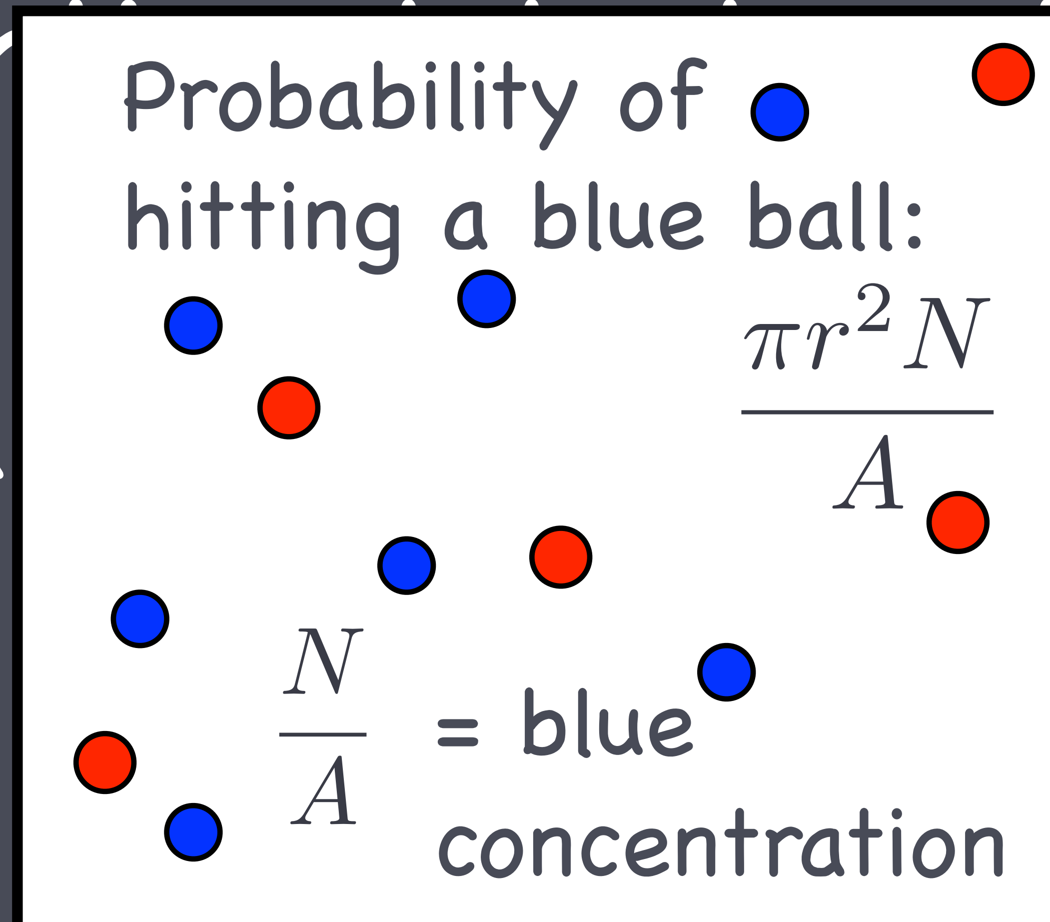


What you should be able to do:

- Identify steady states for a DE.
- Draw/interpret the phase line for a DE.
- Draw/interpret a slope field for a DE.
- Determine stability of steady states.
- Determine long-term behaviour of solutions.
- Sketch the graphs of solutions using phase line and/or slope fields (slopes, concavity, IPs, h-asymptotes).

Rates of change that are propor… things

- A chemical … reactant occurs at a … how much reactant is … decay):



- A chemical reaction with two reactants occurs at a rate proportional to the how much of both reactants are present:

$$\frac{dR_1}{dt} = -kR_1R_2$$

Logistic equation in different contexts...

Rates of change that are proportional to two things

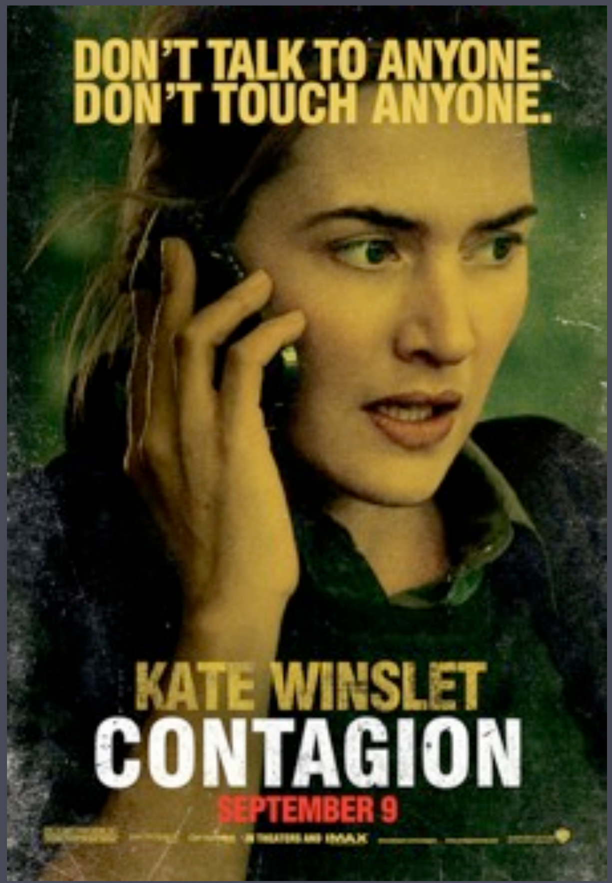
- Infectious disease: bSI (S=susceptible, I=infected)
- Spread of rumour: bNH (N = not heard rumour, H = heard rumour)
- Spread of new words: bNU (use word or not).
- Spread of new technologies: bNU (use tech or not).
- Active oil exploration sites: bUD (undiscovered and discovered).
- Waterlillies in a pond: bSW (waterlillies and space for waterwillies).

…two things that are just different forms of a single thing

- When X meets Y, there's a chance Y turns into X.
- Lose Y: $\frac{dY}{dt} = -bXY$ and gain X: $\frac{dX}{dt} = bXY$
- X+Y= constant = C so Y=C-X.
- $\frac{dX}{dt} = bX(C - X)$

Infectious disease

Dr. Erin Mears: Once we know the R_0, we'll be able to get a handle on the scale of the epidemic.
Minnesota Health #4: So, it's an epidemic now. An epidemic of what?
Dave: We sent samples to the CDC.
Dr. Erin Mears: In seventy two hours, we'll know what it is, if we're lucky.
Minnesota Health #4: Clearly, we're not lucky.



Infectious disease

- N individuals, I of them have a flu, S=N-I do not.
- If everyone interacts, new cases appear at a rate proportional to SI.
- The DE describing the spread of disease:

(A) $\frac{dI}{dt} = -bI(N-I)$

(B) $\frac{dI}{dt} = bI(N-I)$

(C) $\frac{dS}{dt} = -bSI$

(D) $\frac{dI}{dt} = bSI$

Infectious disease

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(D) $\frac{dI}{dt} = bSI$

- Compare this with $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$.

Infectious disease

What is the carrying capacity?

$$\frac{dI}{dt} = bI(N - I)$$

(A) b/N

(B) N/b

(C) I

(D) N

Infectious disease

What is the carrying capacity?

$$\frac{dI}{dt} = bI(N-I) = \underbrace{bN}_{\text{r}} I \left(1 - \frac{I}{\underbrace{N}_{\text{K}}}\right)$$

(A) b/N

(C) I

(B) N/b

(D) N

Everyone gets sick!

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

Infectious disease

- Suppose infected people recover at a rate proportional to how many there are.
- The DE describing the spread of disease with recovery:

(A) $\frac{dI}{dt} = bI(N-I) - \mu S$

(C) $\frac{dI}{dt} = -bI(N-I) + \mu I$

(B) $\frac{dI}{dt} = bI(N-I) - \mu I$

(D) $\frac{dI}{dt} = bI(N-I) + \mu I$

Infectious disease

$$\frac{dI}{dt} = bI(N - I) - \mu I$$

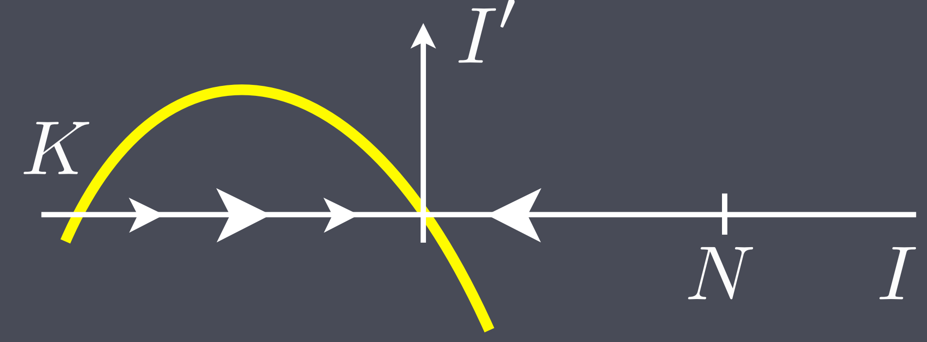
$$= bIN - bI^2 - \mu I$$

$$= bI\left(N - \frac{\mu}{b} - I\right)$$

K

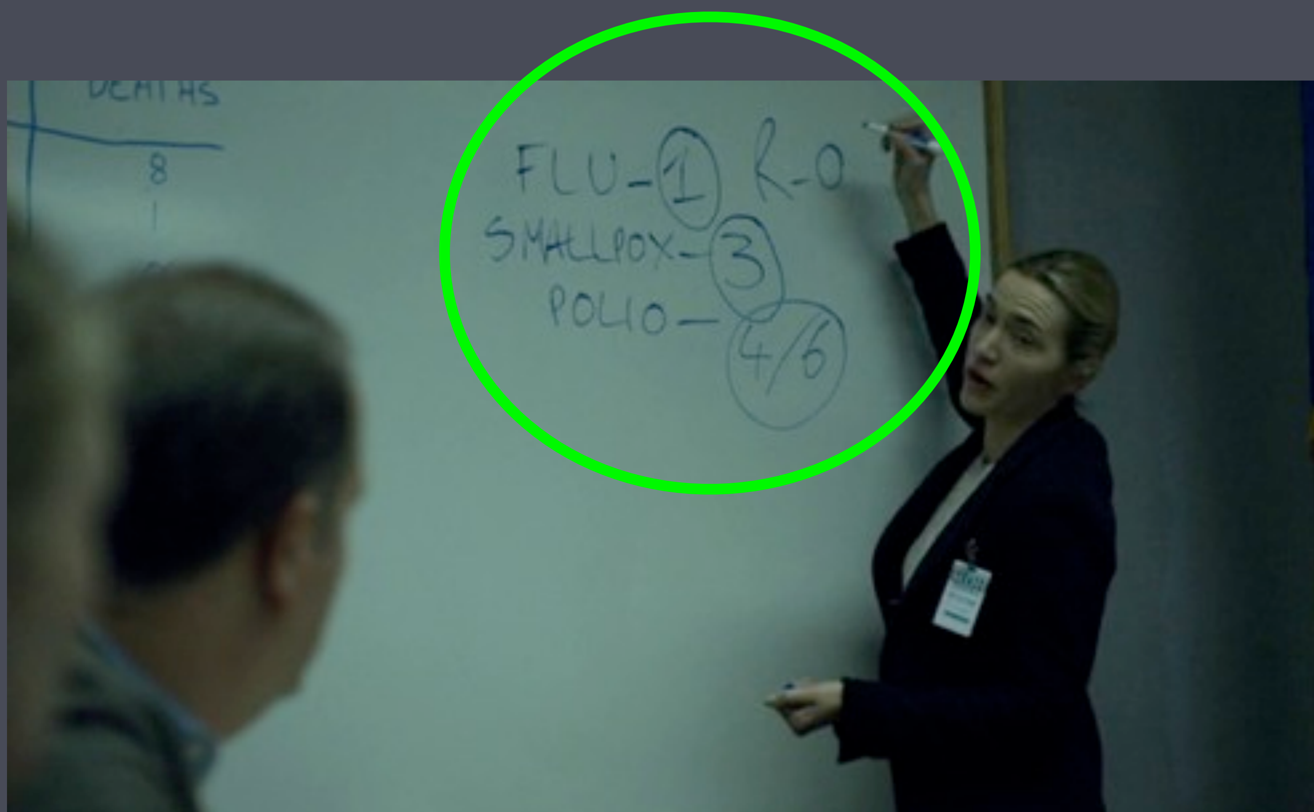
If $\frac{\mu}{b} > N$ then

$$K = N - \frac{\mu}{b} < 0$$



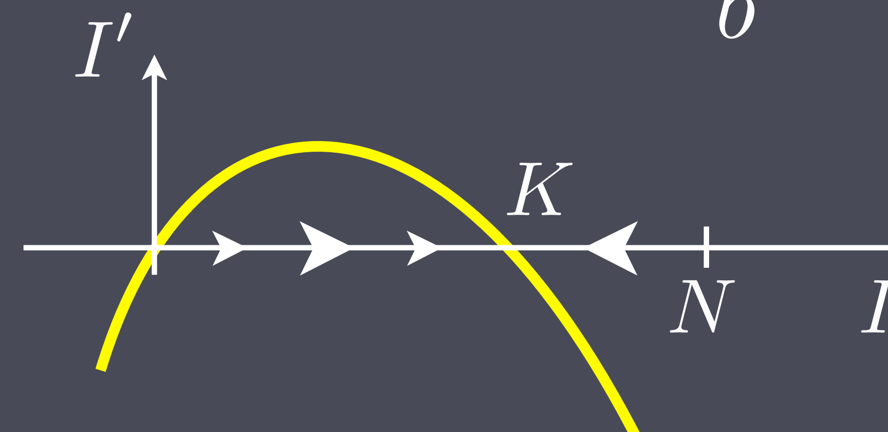
and the disease dies out.

Infectious disease



If $\dfrac{\mu}{b} < N$ then

$$K = N - \frac{\mu}{b} > 0$$

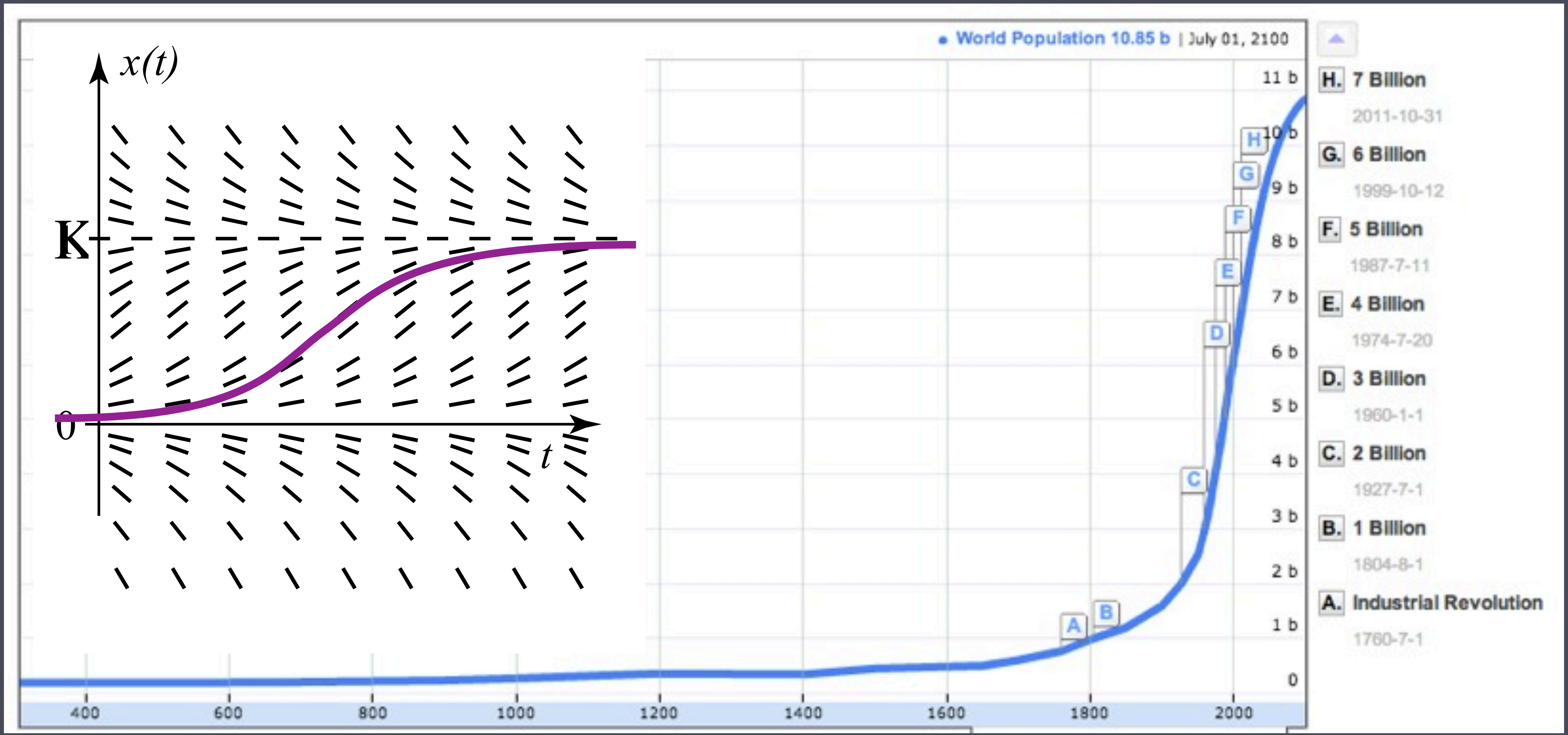


and the disease becomes an epidemic.

$$R_0 = \frac{Nb}{\mu} > 1$$

Some other logistic systems

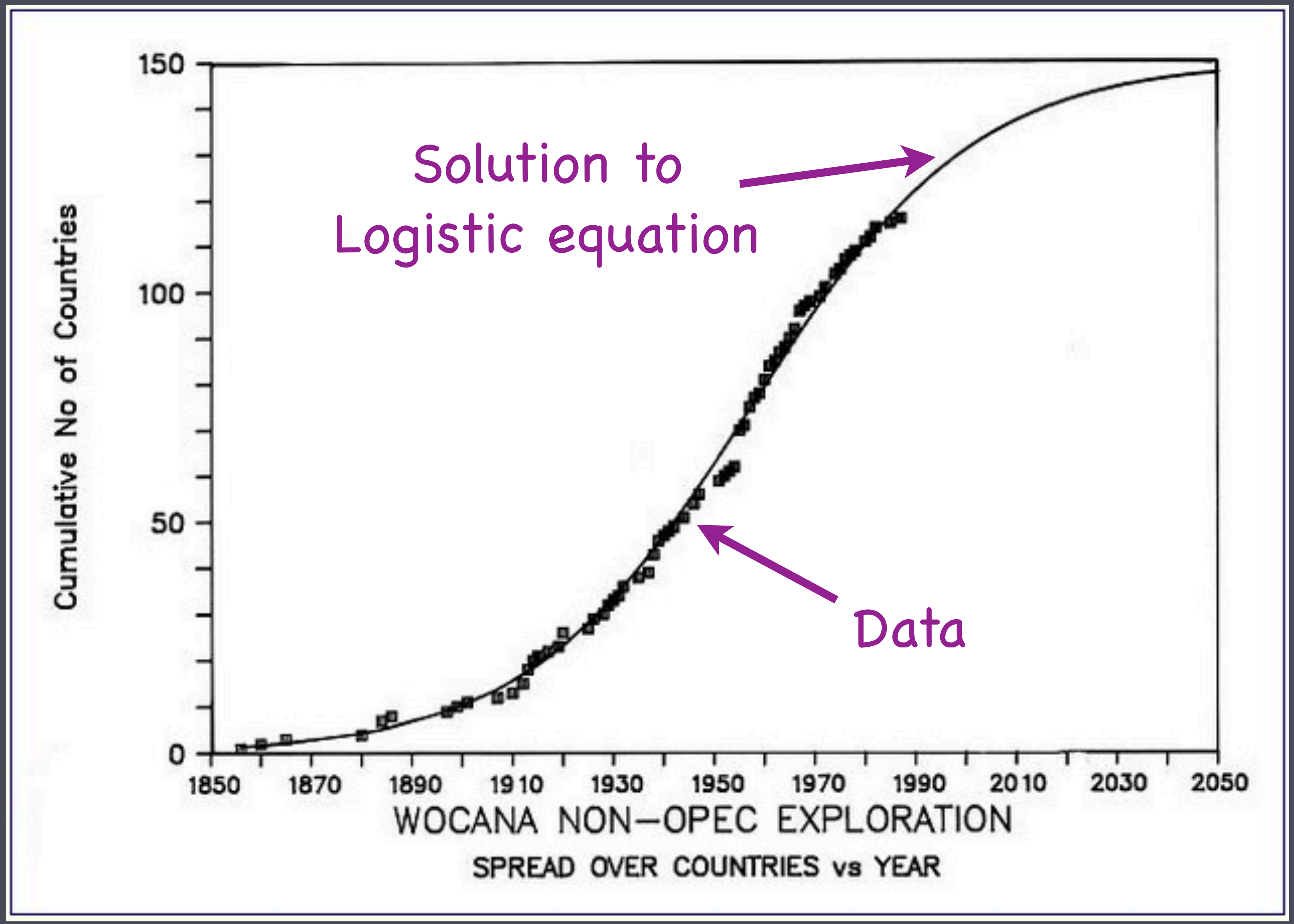
Human population



We're past the inflection point

- estimate K≈10 billion

Number of countries with active oil exploration



http://www.mhnederlof.nl/kinghubbert.html

American peak oil production

Oil econ
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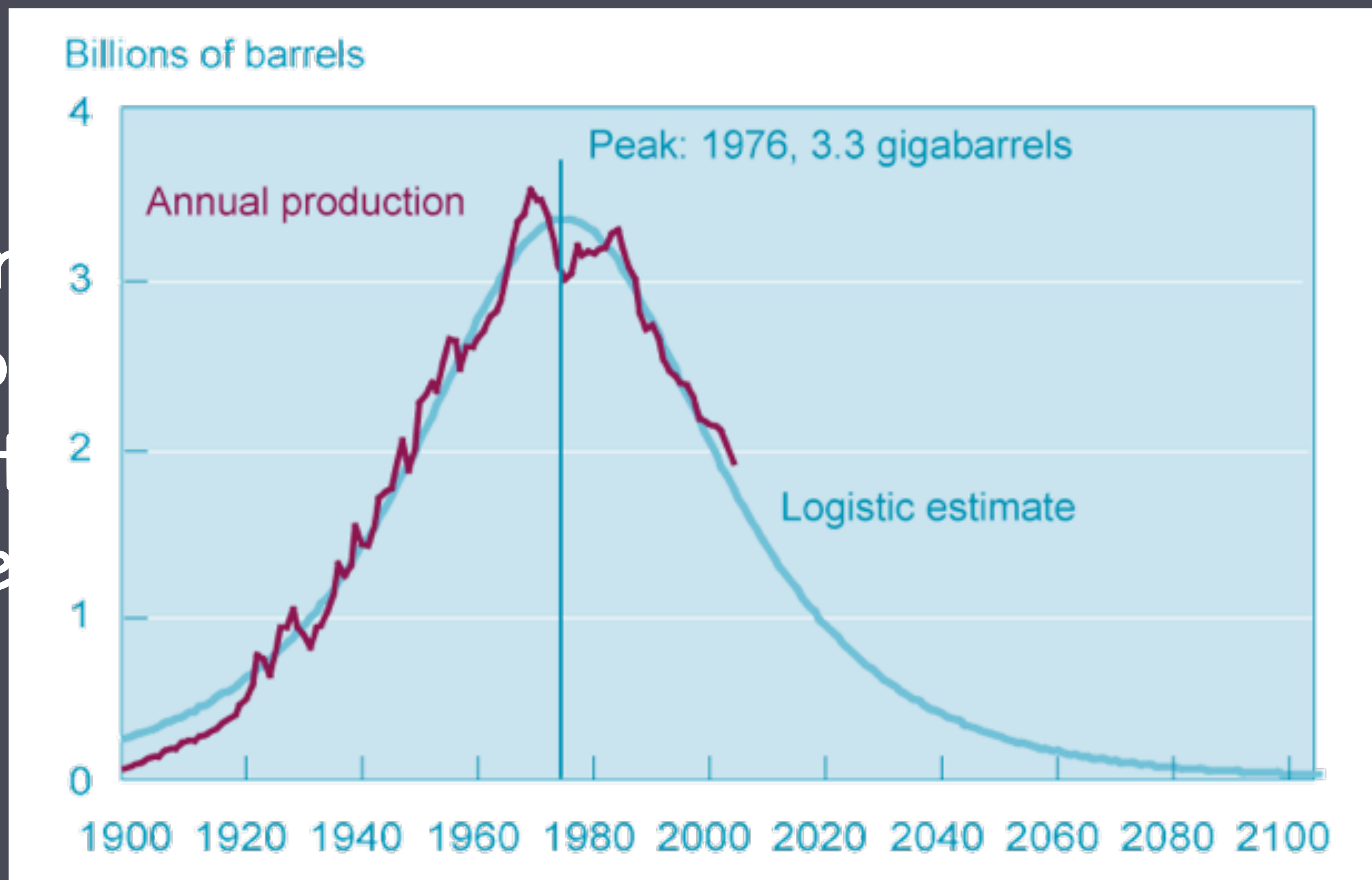

Source: Department of Energy, Energy Information Administration; and authors' calculations.

http://www.clevelandfed.org/research/commentary/2007/081507.cfm

World peak oil production