# Today

- Please email requests for problems to discuss on Monday.
- Midterm Tuesday!
- Office hours M 11-12:30, M2-2:45, T10:30-12.
- Translating from word problem to IVP.
- Solving a linear ODE: $y' = a - by$.

A population grows proportional to the size of the population itself. Which of the following is a differential equation for the population size P(t)?

(A) $P'(t) = kt$

(B) $P(t) = e^{kt}$

(C) $P'(t) = kP(t)$

(D) $P'(t) = e^{kt}$

A population grows proportional to the size of the population itself. Which of the following is a differential equation for the population size P(t)?

(A) $P'(t) = kt$

(B) $P(t) = e^{kt}$ <-- This is a solution. So is $P(t)=P_0e^{kt}$ for any value $P_0$.

(C) $P'(t) = kP(t)$

(D) $P'(t) = e^{kt}$

# Doubling time

- Let $c(t) = c_0e^{kt}$.
- If $k>0$, $c(t)$ is increasing and doubles when $c_0e^{kt} = 2c_0$.
- That is when $t=\ln(2)/k$.
- This is called the doubling time.

# Half-life

- Let $c(t) = c_0e^{kt}$.
- If $k<0$, $c(t)$ is decreasing and halves when $c_0e^{kt} = c_0/2$.
- That is when $t=-\ln(2)/k$.
- This is called the half-life.

# Characteristic time / mean life

- Let $c(t) = c_0 e^{kt}$.
- If $k<0$, $c(t)$ is decreasing and reaches $1/e$ its original value when $c_0 e^{kt} = c_0/e$.
- That is when $t=-1/k$.
- This is called the characteristic time or mean life. Just like half-life but replace 2 with e (could be called 1/e-life).

The rate of change of an object's temperature is proportional to the difference between the objects temperature and the surrounding environment.

(A) $T'(t) = k\,(\,T(t) - E\,)$

(B) $T'(t) = k\,(\,E - T(t)\,)$

(C) $T'(t) = E - kT(t)$

(D) $T'(t) = kT(t) - E$

Assume $k>0$ and E is the temperature of the environment.

The rate of change of an object's temperature is proportional to the difference between the objects temperature and the surrounding environment.

(A) $T'(t) = k(T(t) - E)$

(B) $T'(t) = k(E - T(t))$

(C) $T'(t) = E - kT(t)$

(D) $T'(t) = kT(t) - E$

Make sure eq matches physical intuition.

If the coefficient on T(t) is +ive, the solution ---> ∞.

Units have to match!

Assume $k>0$ and E is the temperature of the environment.

An object dropped in water will accelerate under the influence of the constant downward force of gravity and an upward drag force proportional to the velocity.

(A) $v'(t) = \delta ( v(t) - g )$

(B) $v'(t) = \delta ( g - v(t) )$

(C) $v'(t) = \delta v(t) - g$

(D) $v'(t) = g - \delta v(t)$

Assume $\delta > 0$ and $g = 9.8\ m/s^2$.

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Assume $\delta > 0$ and $g = 9.8\ m/s^2$.

Newton's 2nd Law

$$ma = F_{grav} + F_{drag}$$
$$= mg - \gamma\,v(t)$$
$$a = g - \gamma/m\,v(t)$$

- Sign on $\delta v$ is independent of reference frame.
- Sign on g depends on ref. fr.

A drug delivered by IV accumulates at a constant rate $k_{IV}$. The body metabolizes the drug proportional to the amount of the drug.

(A) $d'(t) = k_{IV} - k_m d(t)$

(B) $d'(t) = (k_{IV} - k_m) d(t)$

(C) $d'(t) = k_{IV} d(t) - k_m$

(D) $d'(t) = -k_{IV} + k_m d(t)$

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$$d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0.$$

(A) $d(t) = k_{IV}/k_m\,(1-\exp(k_m t))$

(B) $d(t) = k_{IV}/k_m\,(1-\exp(-k_m t))$

(C) $d(t) = k_{IV}/k_m - \exp(k_m t)$

(D) $d(t) = k_{IV}/k_m - \exp(-k_m t)$

(E) Not sure how to do this one.

Note: $\exp(x)=e^x$.

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