Today

- A second order differential equation.
- Inverse trig functions review.
- Derivatives of inverse trig functions.
- Midterm - pick up today 11-12:30, 2:30-4,

  - Monday 11-12, Tuesday 10-4(??).

Find a solution to the equation $y'' = -a^2 y$.

(A) $y = a \sin(x)$

(B) $y = a \cos(x)$

(C) $y = \sin(ax)$

(D) $y = \sin(a^2 x)$

(E) $y = e^{ax}$

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(C) $y = \sin(ax)$ $y=\cos(ax)$ also solves it.

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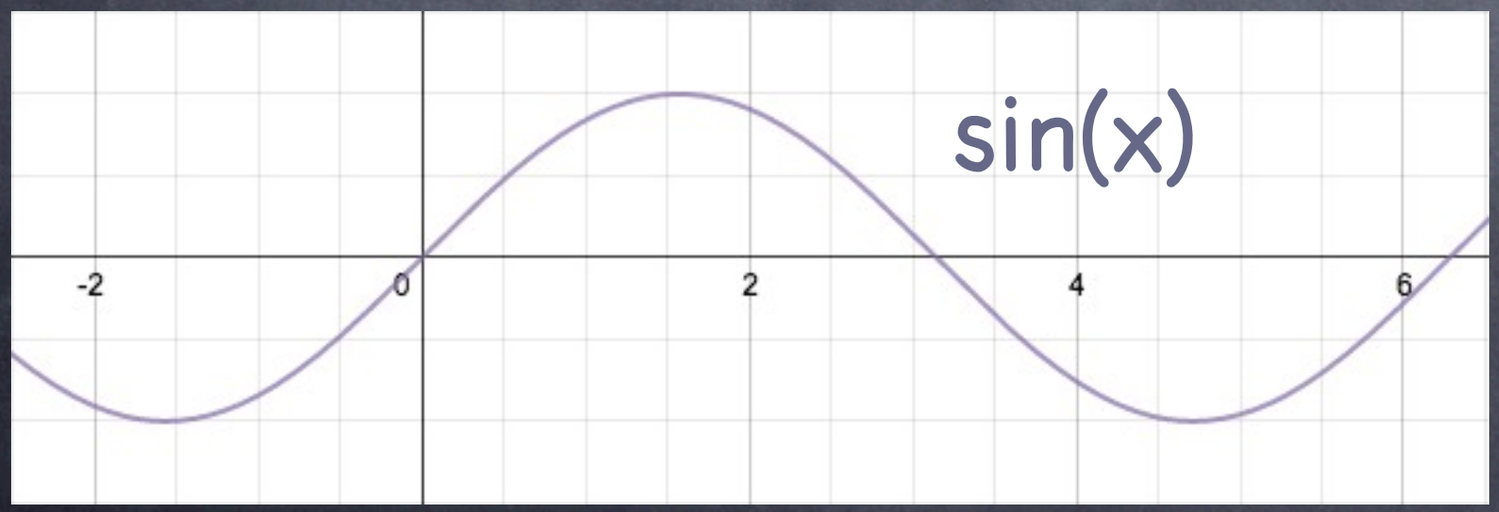
This is a "second order" equation because it includes a second derivative of y(t).

Inverse trig

- $f(x) = \sin(x) \dashrightarrow f^{-1}(x) = \arcsin(x)$



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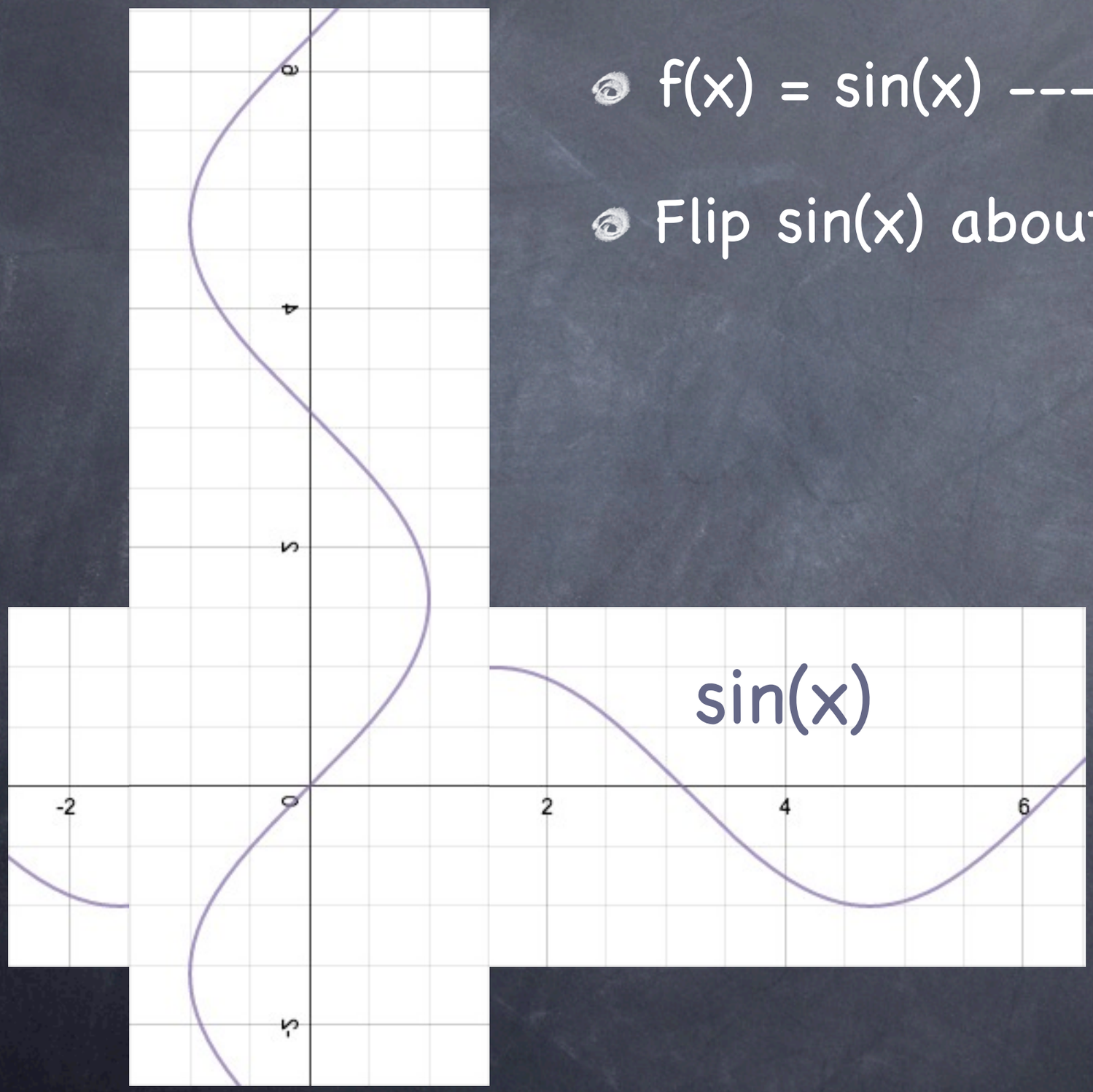

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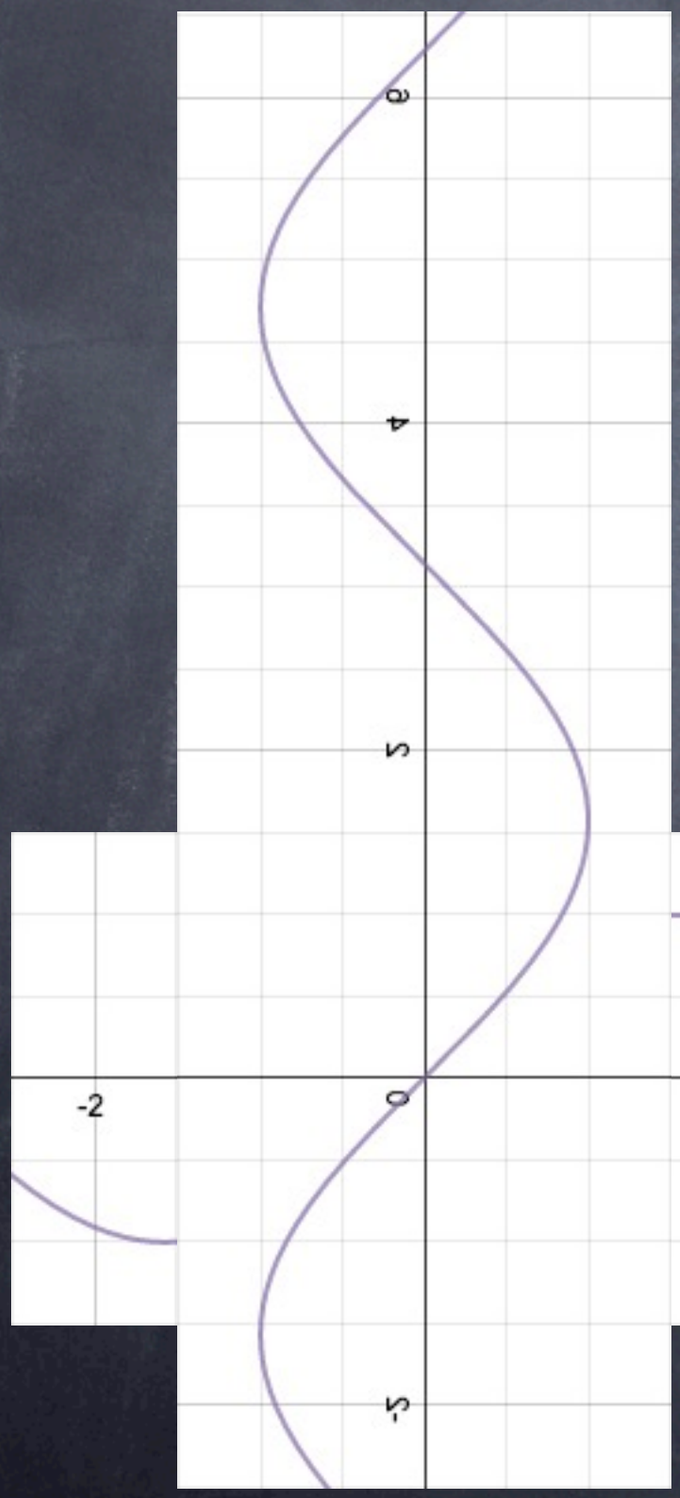

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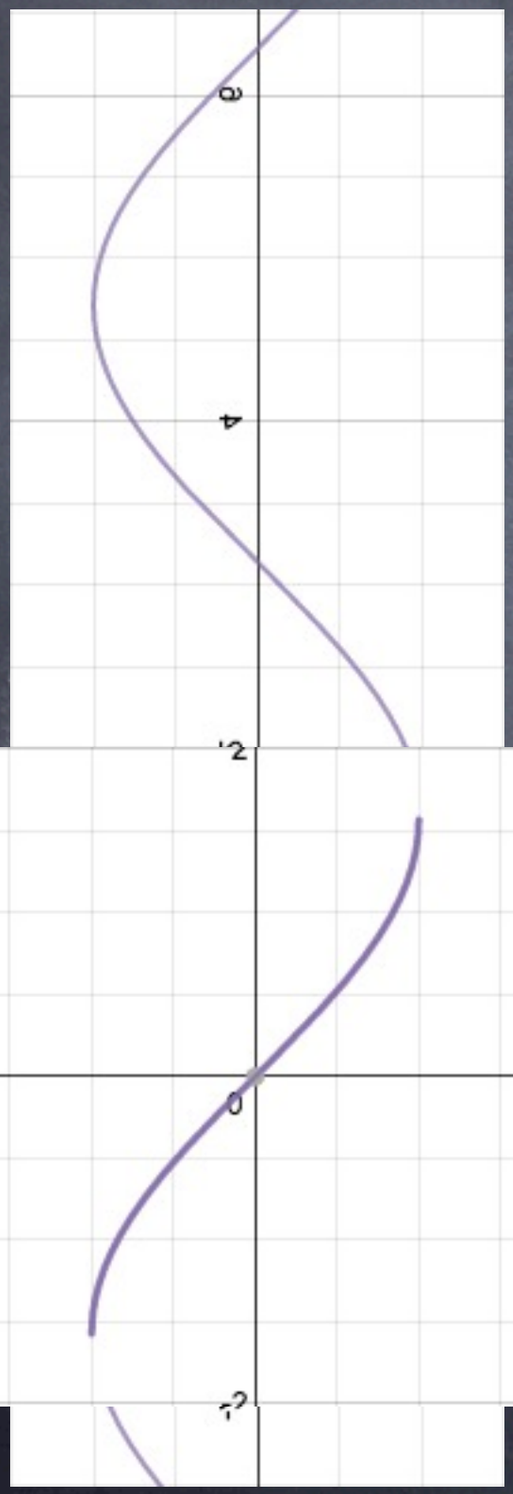

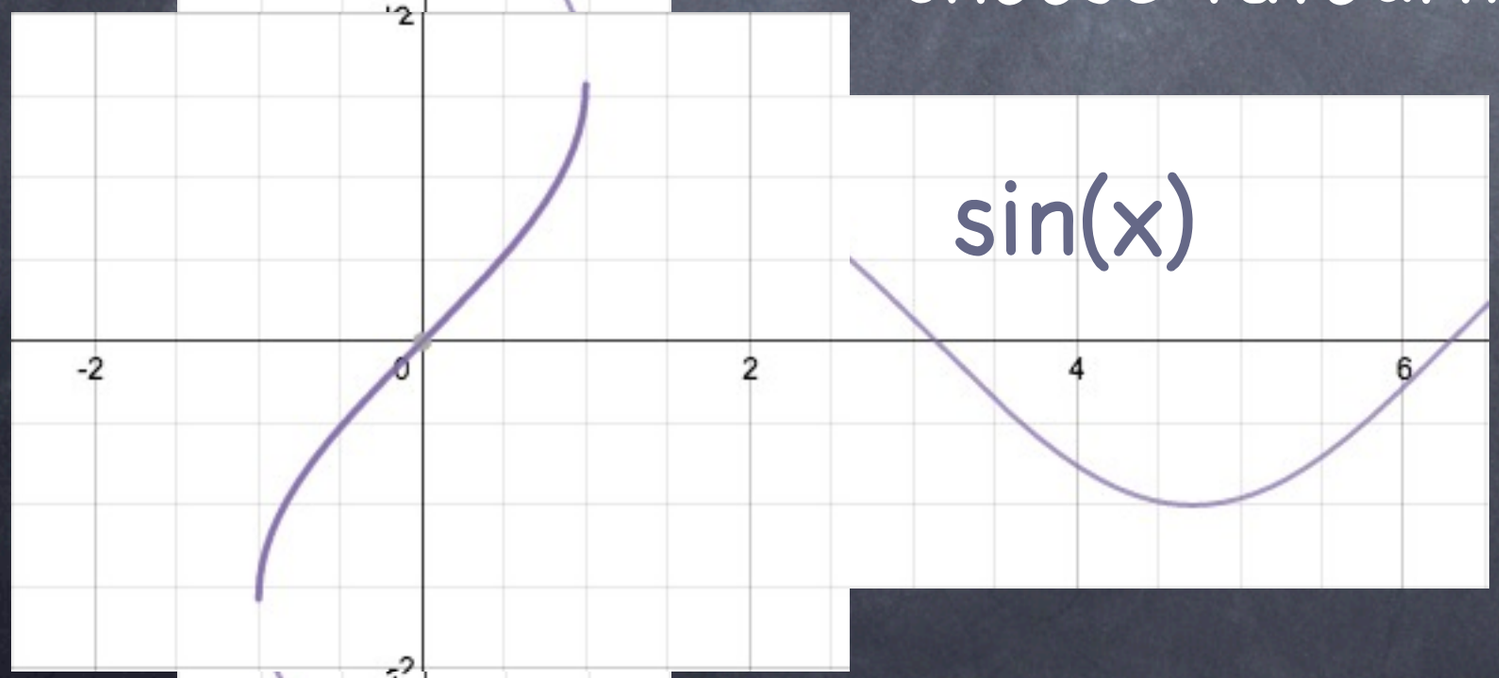

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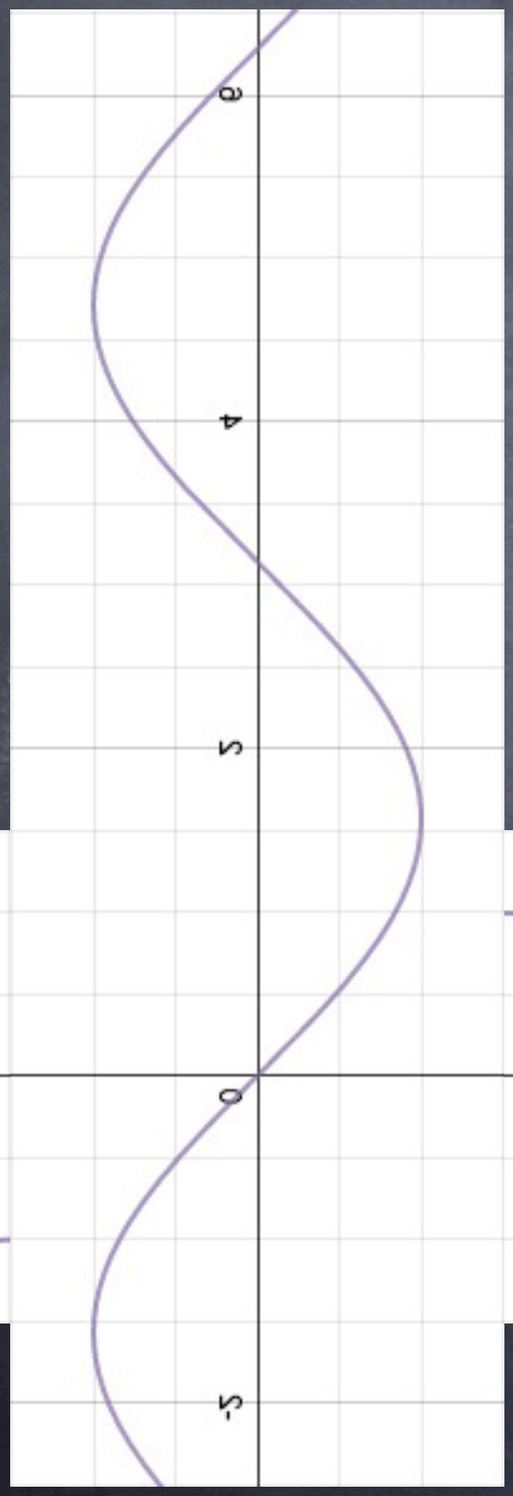

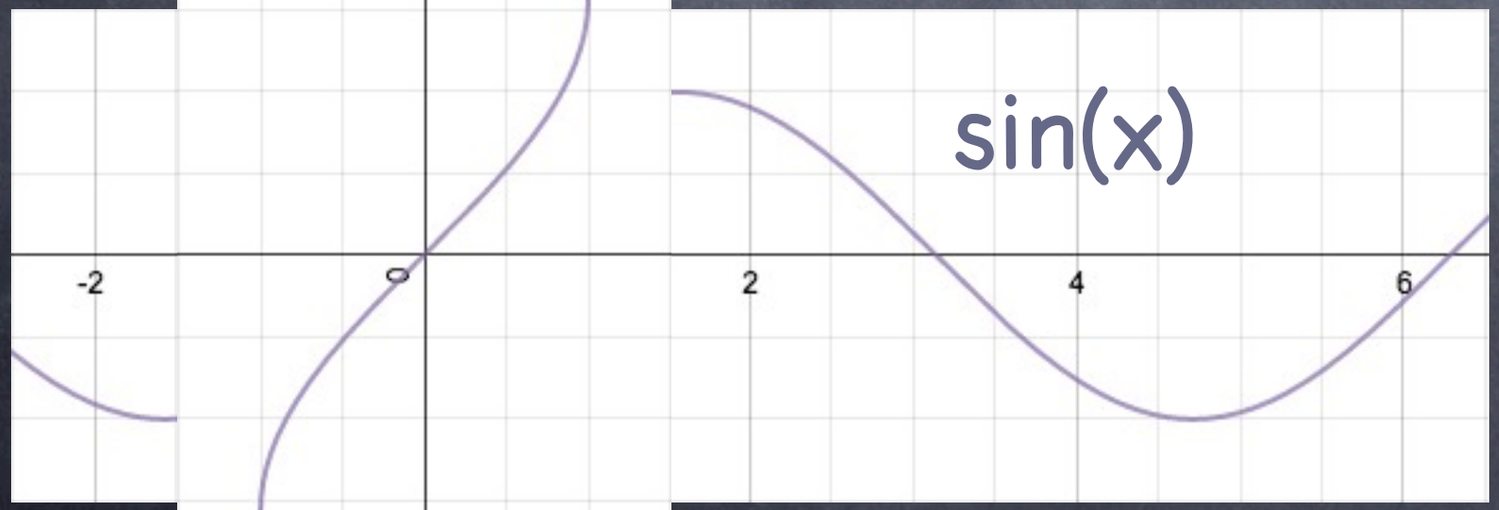

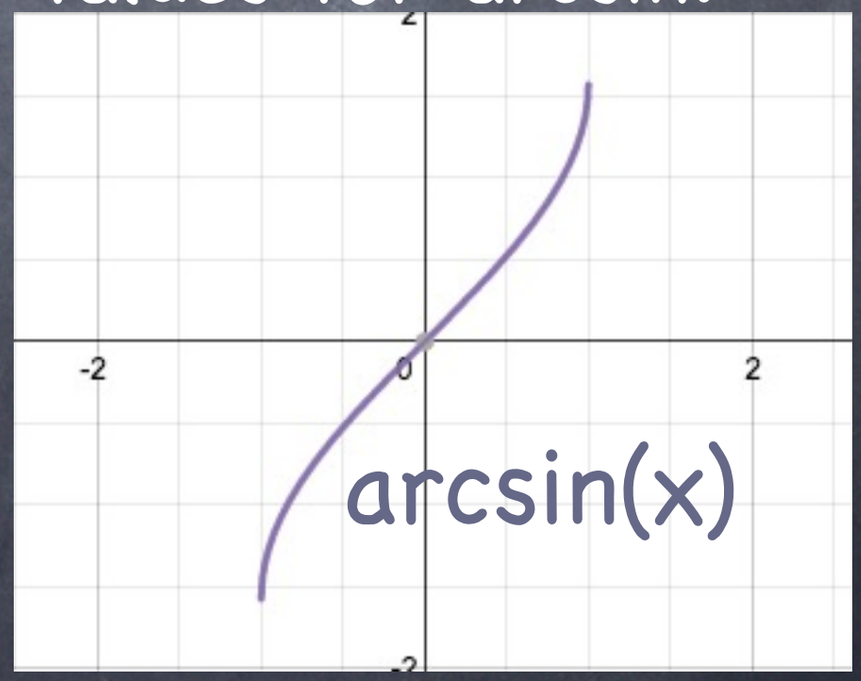

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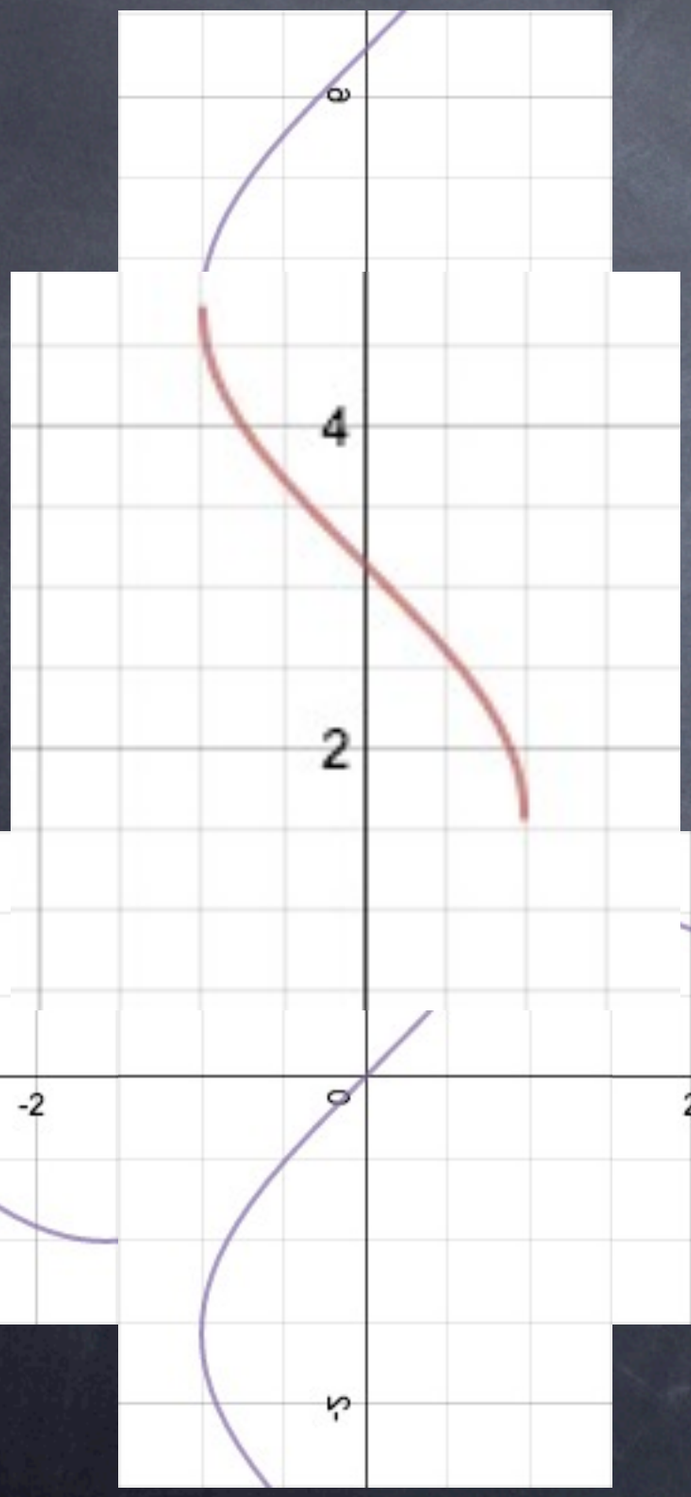

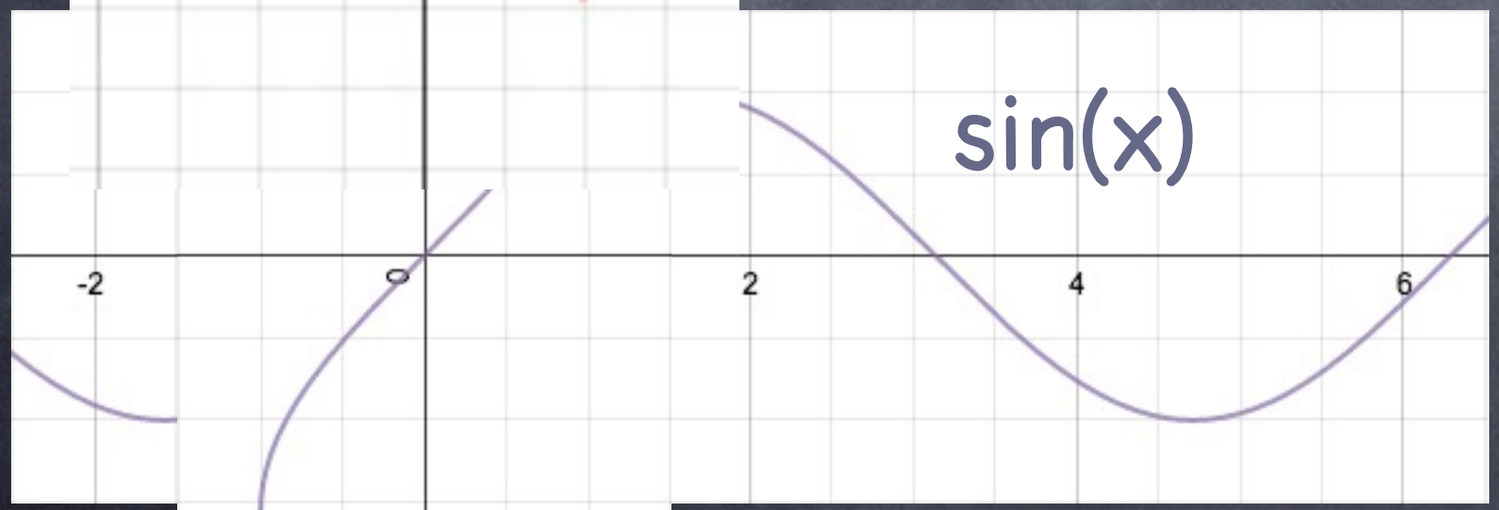

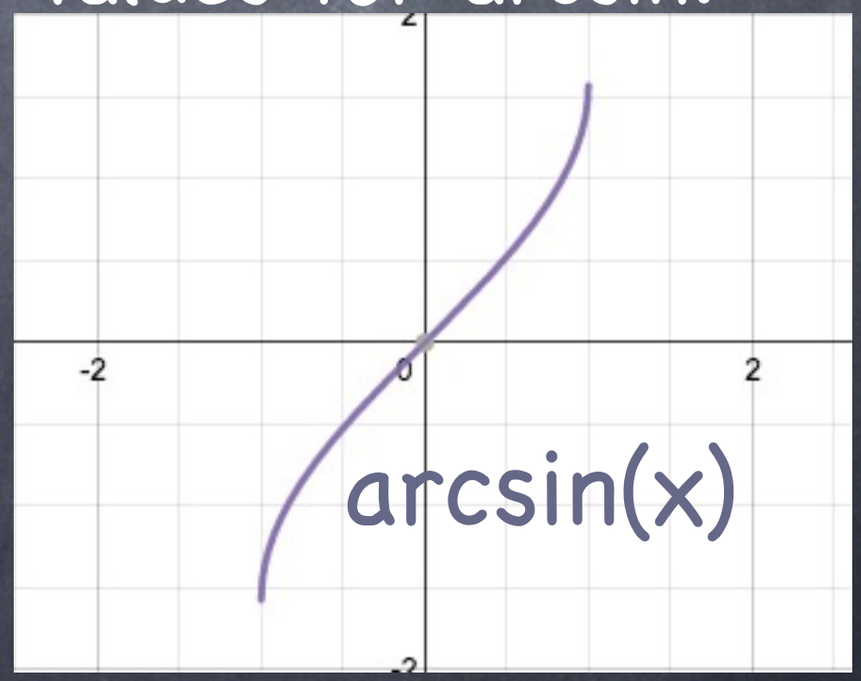

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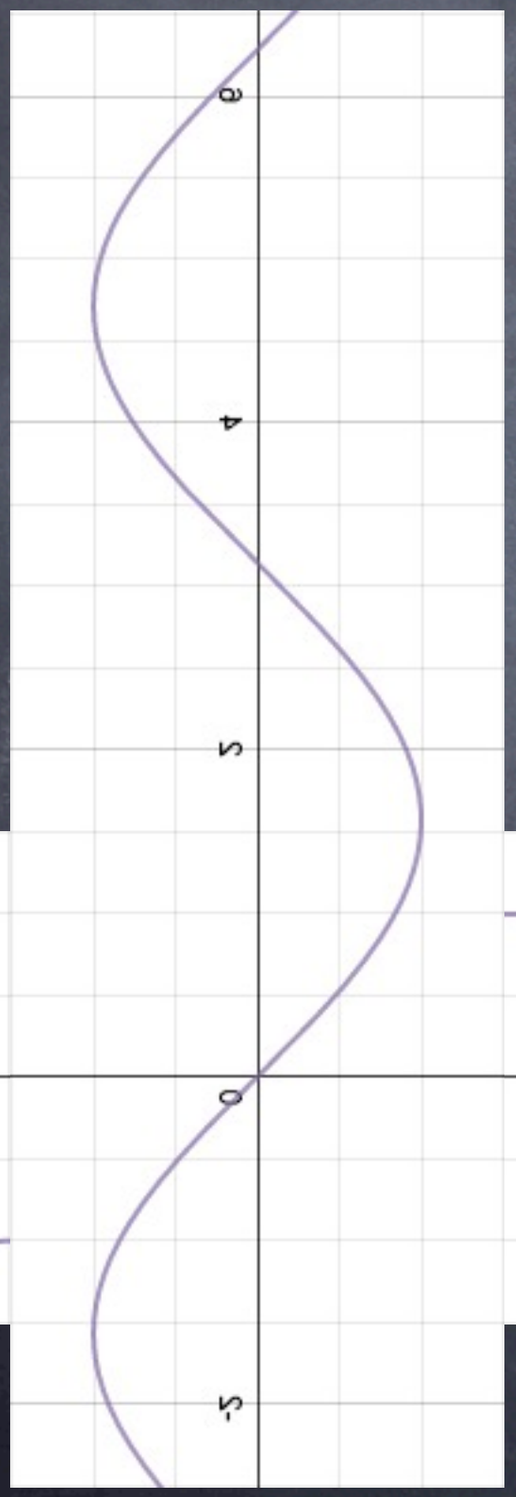

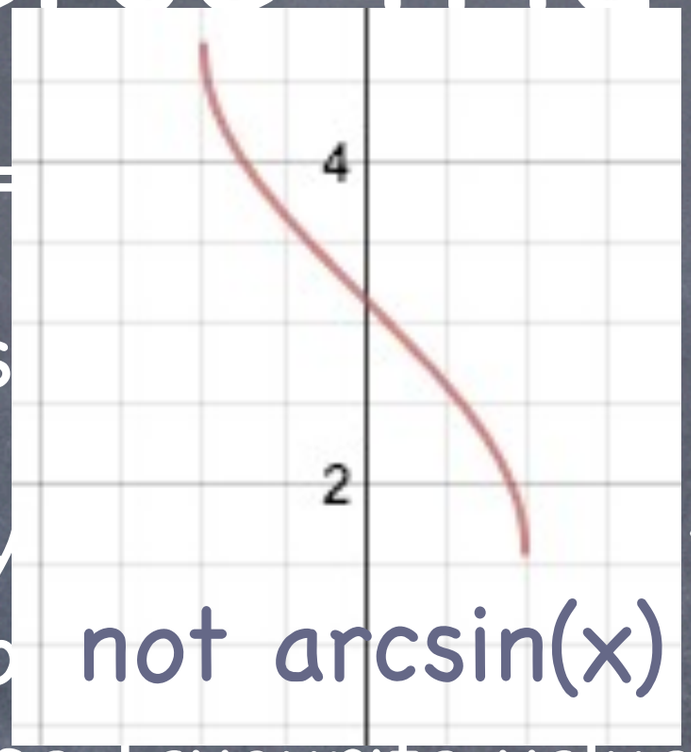

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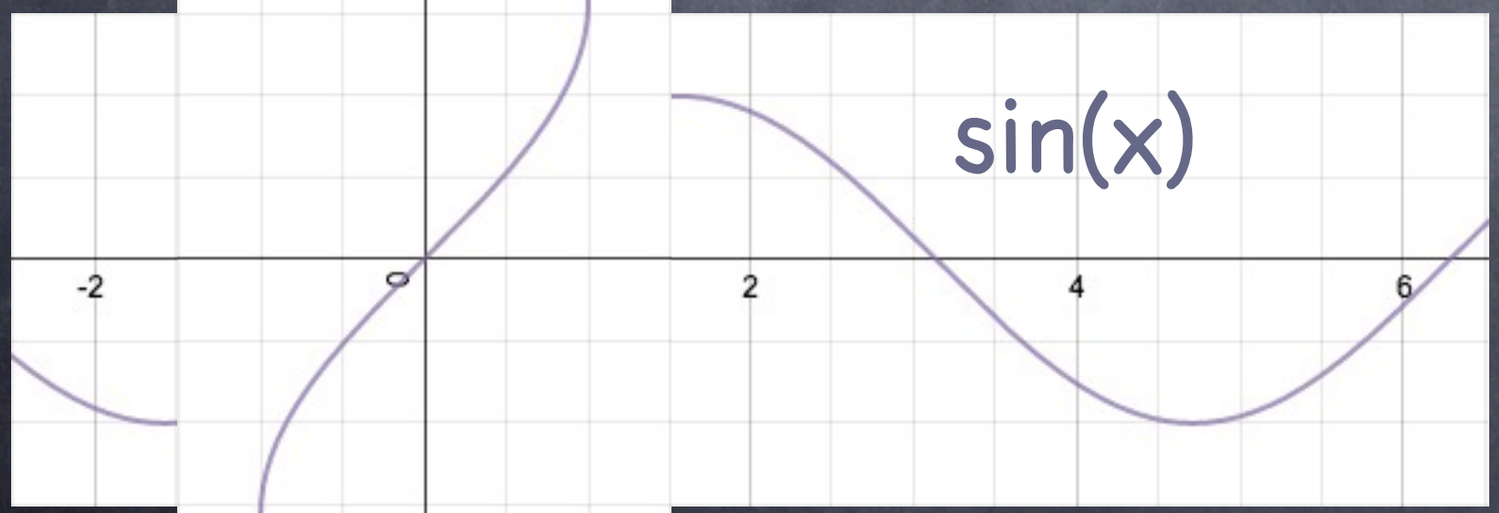

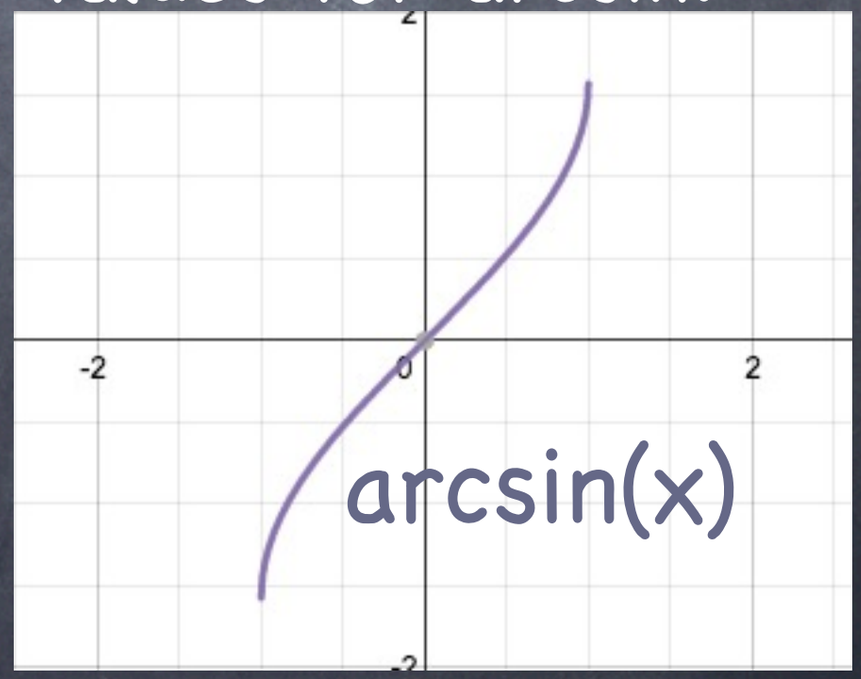

The domain of arcsin is...

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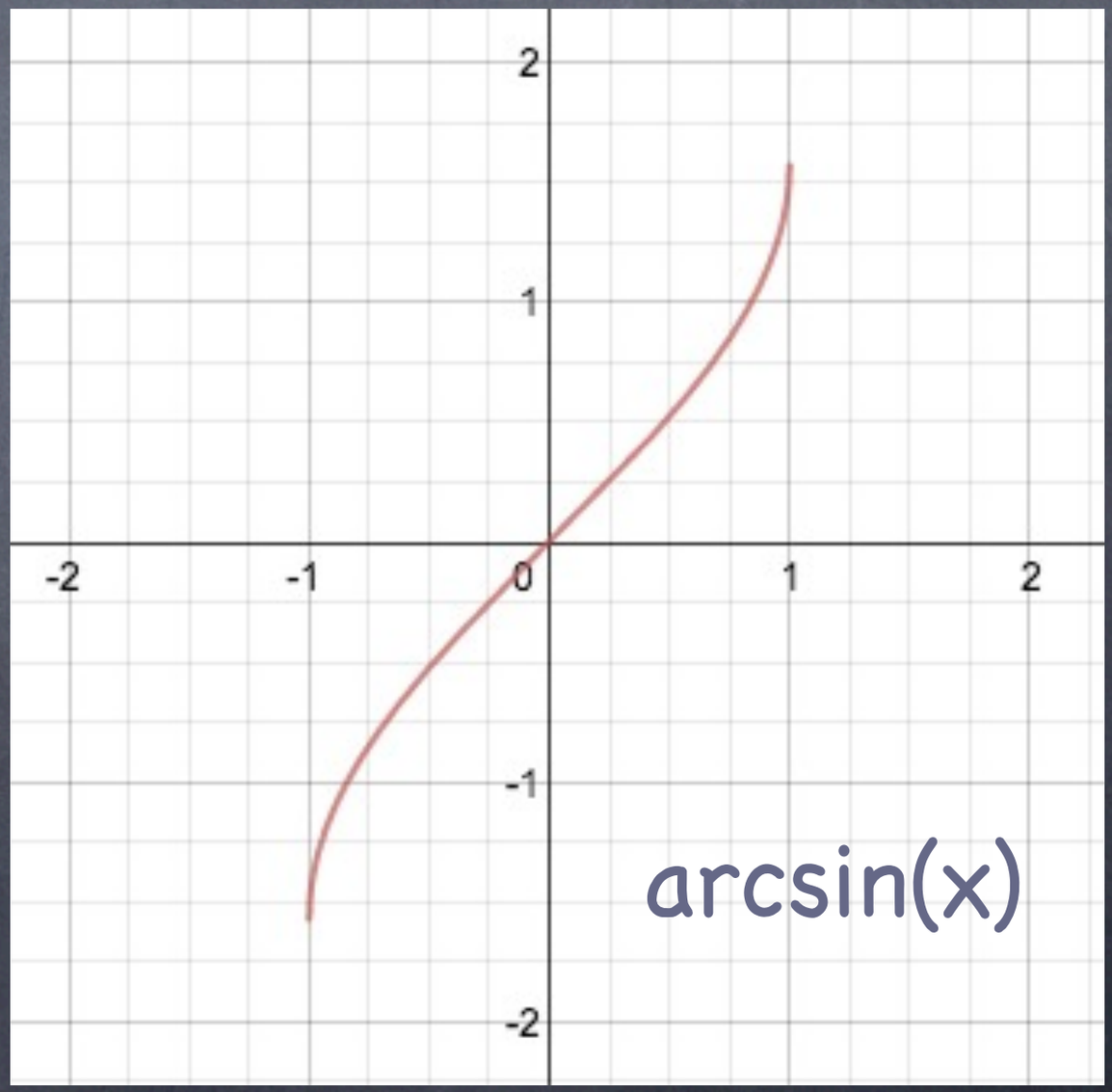

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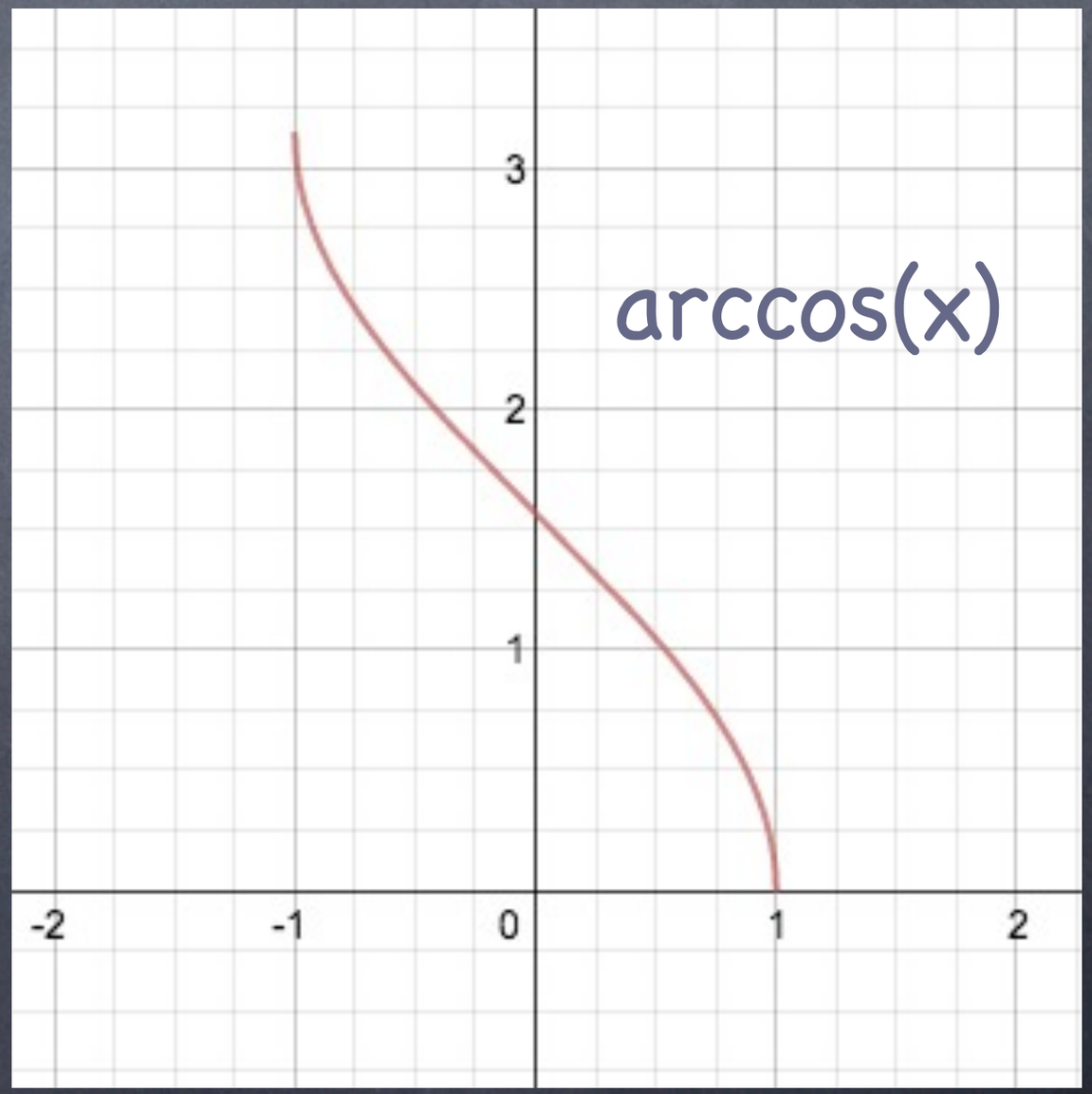

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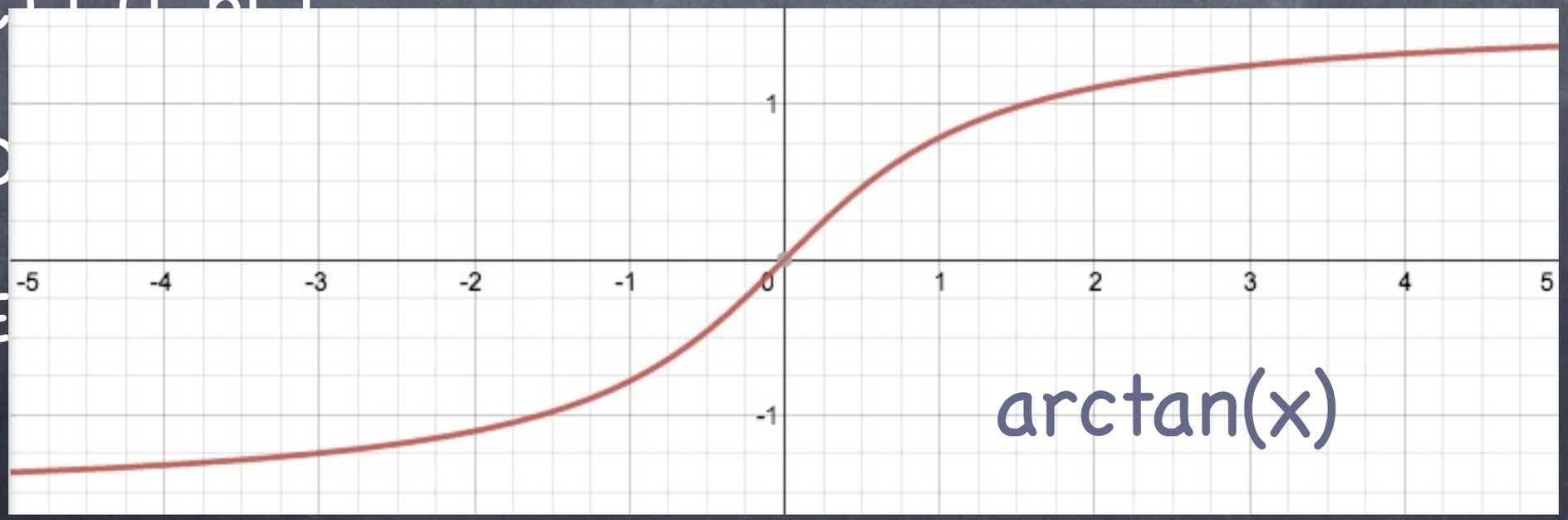

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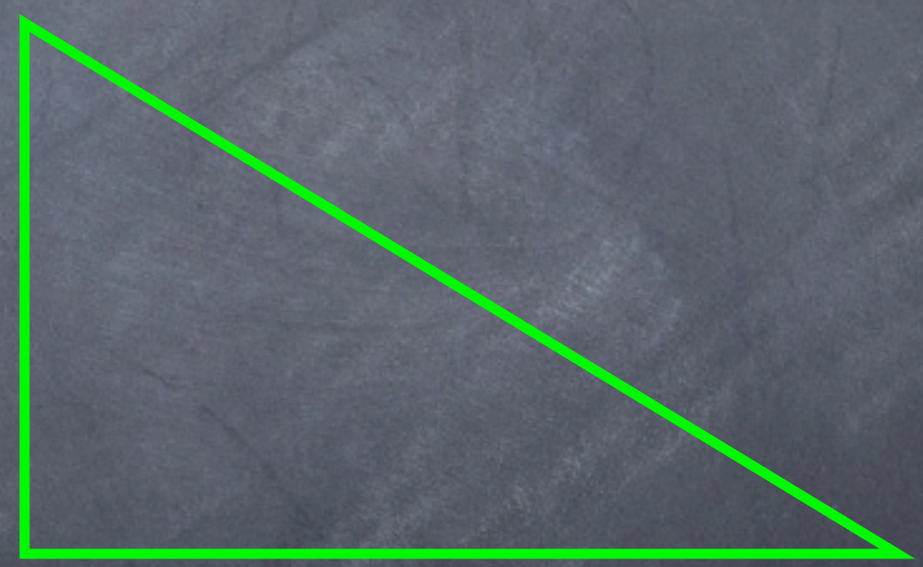

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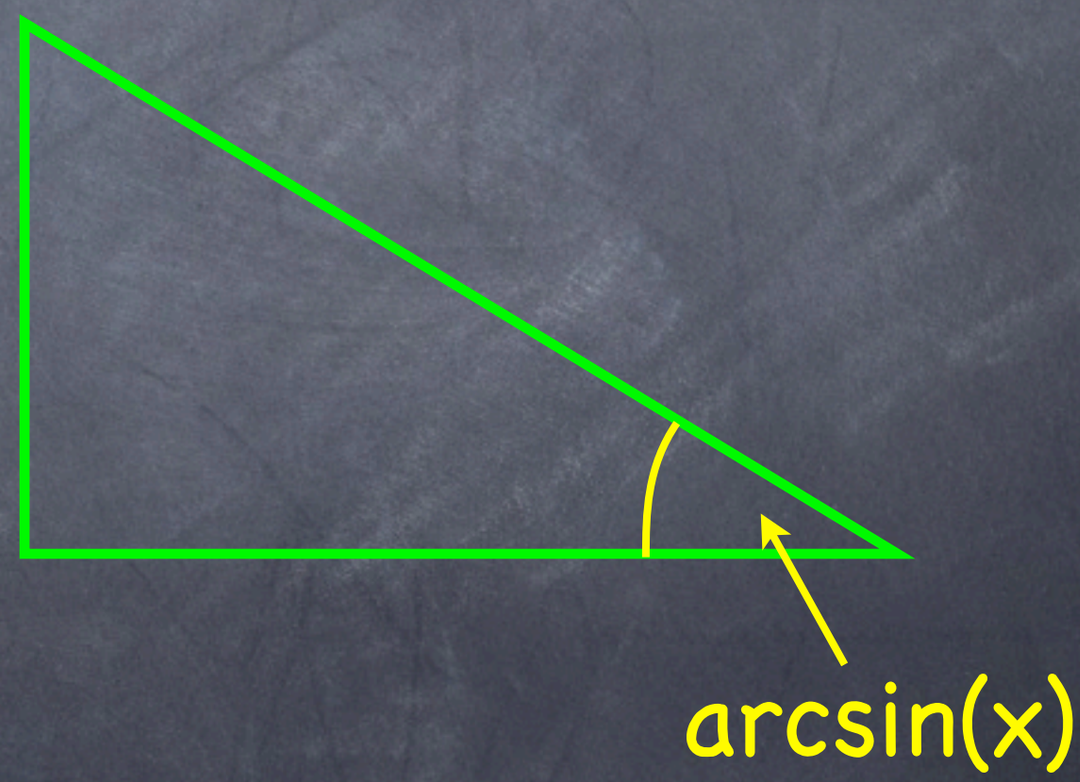

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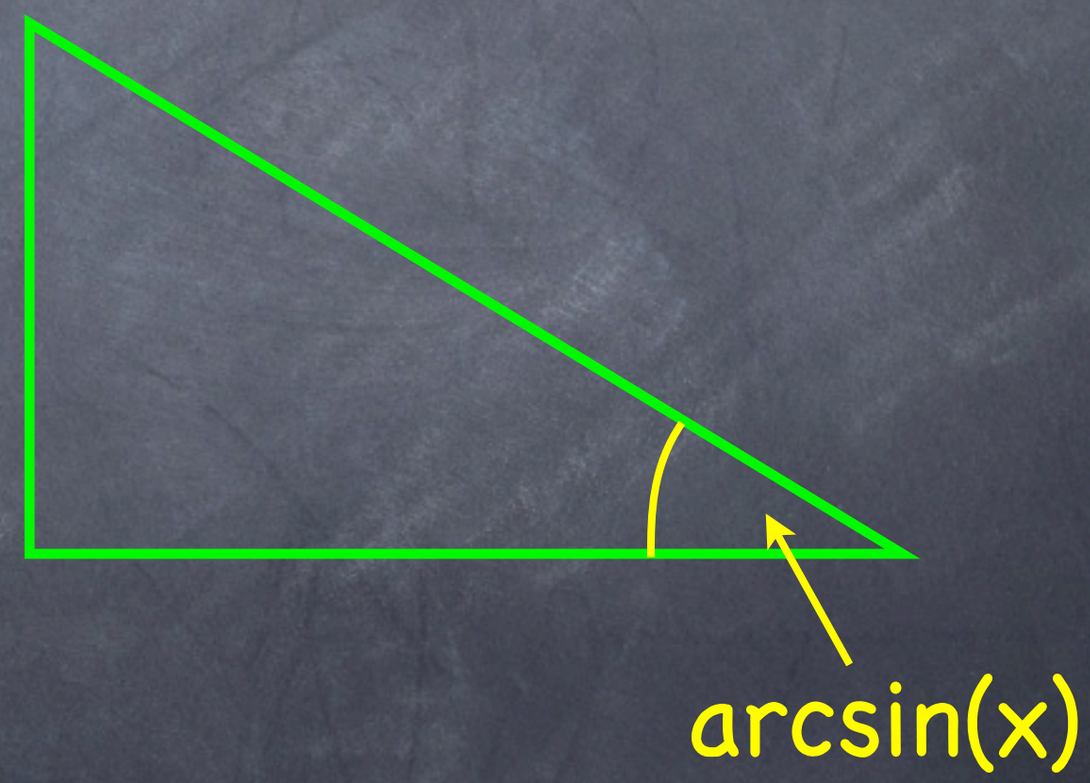

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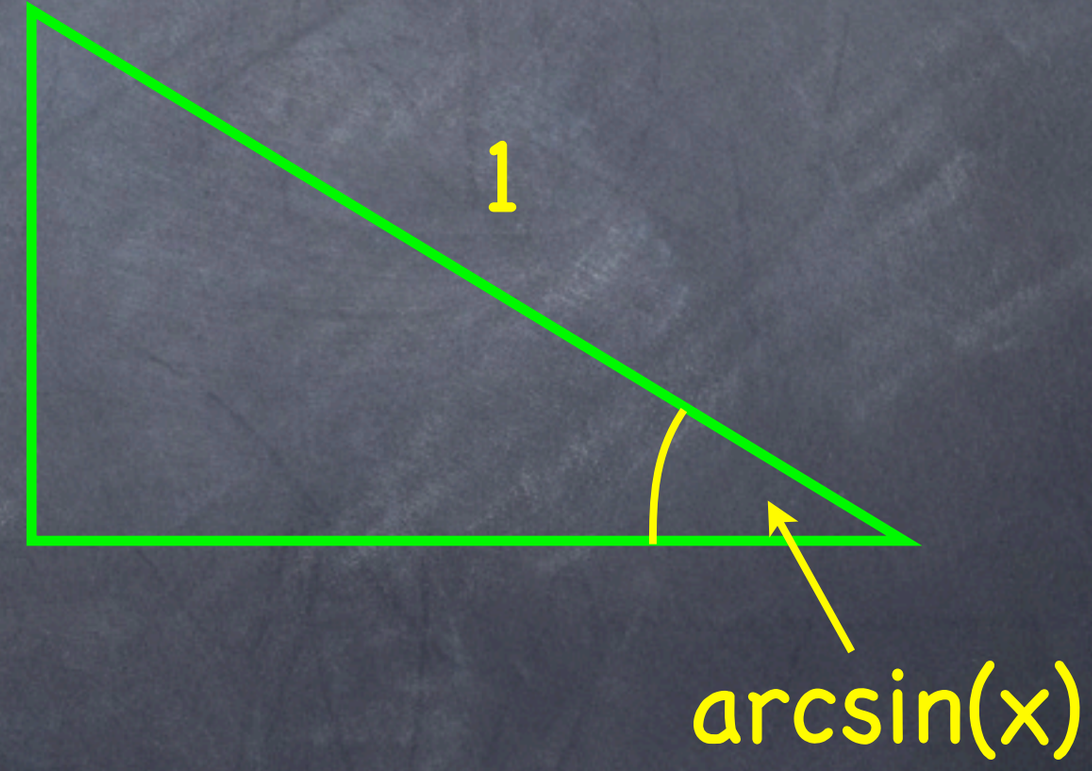

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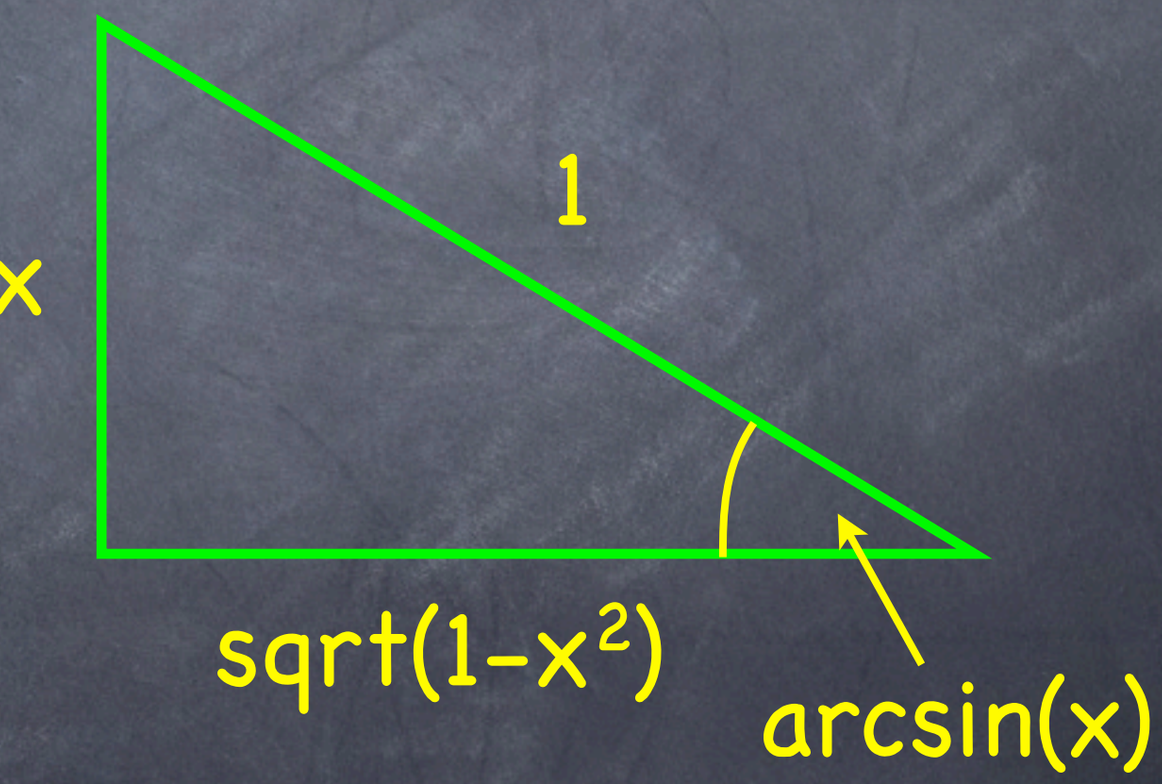

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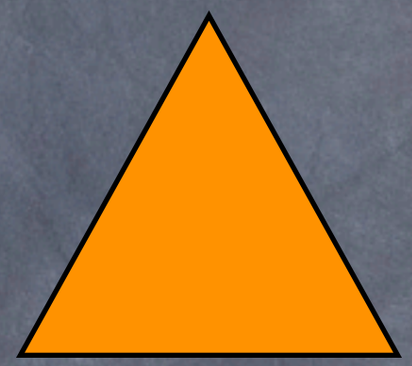
- $y' = 1/\sqrt{1-x^2}$

Trig-related rates

- These usually come down to a triangle that changes in time. For example...



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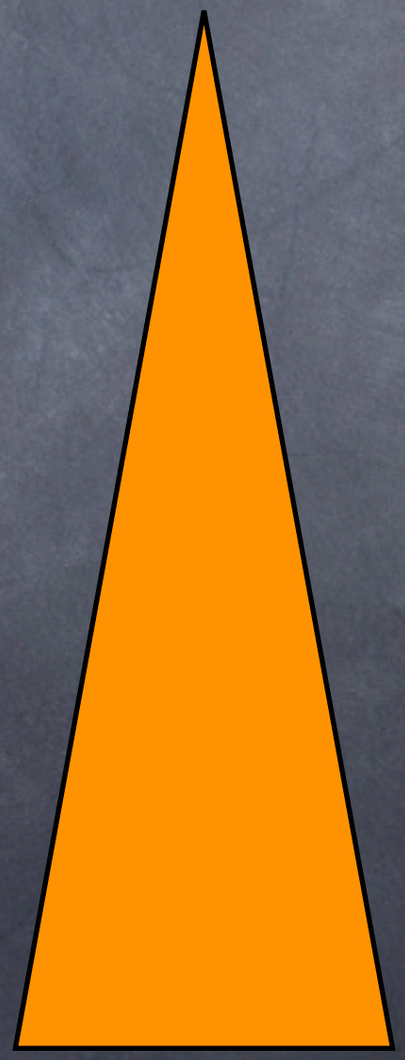

If the height of a triangle with base 2 m changes at a rate $h'=3$ m/s, how quickly is the angle opposite the base changing when $h=\sqrt{3}$ m?

Relate the two changing quantities (h and θ):

(A) $\sin(\theta) = 2/h$

(B) $\sin(\theta/2) = 1/h$

(C) $\sin(\theta/2) = 1/\sqrt{1+h^2}$

(D) $\tan(\theta) = 2/h$

(E) $\tan(\theta/2) = 1/h$