Announcements

- Course website: https://wiki.math.ubc.ca
- WeBWorK assignments for next week!
- Resources: (Piazza, videos, office hours, textbook)
- Office Hours Today: Math Annex 1110 from 4-5 pm
Last time: **asymptotic thinking**

- Small powers dominate close to $x = 0$; large powers dominate for large $x$.

![Power functions graph](image-url)
Even and odd functions

Even power functions:
- $y = x^2$
- $y = x^4$
- $y = x^6$

Odd power functions:
- $y = x$
- $y = x^3$
- $y = x^5$
Definitions

- An **even function** \( f(x) \) is symmetric about the \( y \) axis:
  \[
  f(x) = f(-x).
  \]

- An **odd function** \( f(x) \) is symmetric about the origin:
  \[
  f(x) = -f(-x).
  \]
Even and odd functions

Example
The function $y = f(x) = C$ for $C$ constant is an even function.

- Analytic argument: Since $f(x) = C$, we know that $f(-x) = C$. This implies that $f(x) = f(-x)$, from which we conclude that $f(x)$ is an even function.

- Geometric argument: a constant function is symmetric about the $y$-axis.
Q1. The product of two odd functions is
   A. an odd function
   B. an even function
   C. both even and odd
   D. neither even nor odd
   E. not enough information to tell
Q2. The function $f(x) = \frac{x^2}{1+x^2}$ (the quotient of two polynomials is called a rational function) is

A. an odd function
B. an even function
C. both even and odd
D. neither even nor odd
E. not enough information to tell
Q3. The function $g(x) = \frac{x^3}{1+x^3}$ is
   A. an odd function
   B. an even function
   C. both even and odd
   D. neither even nor odd
   E. not enough information to tell
Even and odd functions

- If \( f(x) \) and \( g(x) \) are both even functions then \( f + g, f - g, fg, \) and \( f/g \) are all even functions.

- If \( f(x) \) and \( g(x) \) are both odd functions then \( f + g \) and \( f - g \) are odd functions, but \( fg \) and \( f/g \) are even functions.

For you to think about: Why is this true?
Goal: to be able to easily sketch the graph of a simple polynomial function

\[ f(x) = ax^n + bx^m. \]

A polynomial is a sum of power functions multiplied by constants.

Key idea:
- Lower powers dominate near \( x = 0 \).
- Higher powers dominate for \( x \) far from 0.
Power functions and curve sketching

Example

- \( y = x^3 + ax \) is in pre-lecture video and the course notes:

- \( y = 2x^2 - x^3 \) is in the video
- Sketch a graph of the polynomial \( y = x^5 + ax^3 \).
Power functions and curve sketching

Example (Sketch $y = x^5 + ax^3$.)

- Near $x = 0$, $y \approx ax^3$. $a < 0$, $a = 0$, $a > 0$:

- Far from $x = 0$, $y \approx x^5$: 
Power functions and curve sketching

Example (Sketch $y = x^5 + ax^3$.)

$a < 0$:  

$a = 0$:  

$a > 0$:  

\[ \frac{1}{2} \]
Power functions and curve sketching

Example (Sketch $y = x^5 + ax^3$.)

$a < 0$: $a = 0$: $a > 0$: 
Q4. Suppose \( a = -4 \), with \( y = x^5 + ax^3 \). The zeros of \( y = x^5 - 4x^3 \) are:

A. \( x = 2 \)
B. \( x = \pm 4 \)
C. \( x = \pm 2 \)
D. \( x = 0, \pm 2 \)
E. \( x = 0, \pm \sqrt{2} \)
Q5. Which of the functions below has this graph?

A. \(x^3 - x^5\)
B. \(x^5 - x^3\)
C. \(x^4 + x^2\)
D. \(x^4 - x^2\)
E. \(x^2 - x^4\)
Homework Tip: Use a spreadsheet

- Average rate of change calculations from data.
Today...

- Even functions vs. odd functions
- Polynomials: $f(x) = ax^n + bx^m$
- Rational functions: $g(x) = \frac{ax^n + bx^m}{cx^\ell + dx^k}$
- Easily sketching the graph of simple polynomials:
  - Large powers away from $x = 0$
  - Small powers near $x = 0$
- Check the last slides for related exam problems.
Answers

1. B
2. B
3. D
4. D
5. E
1. When $x = 1000$, the function 
\[ g(x) = \frac{6x^4 + 12x^2 + 64x - 87}{2x^3 - 6x^2 + x} \] is closest to
   A. 0.003 
   B. 3000 
   C. 1000000 
   D. 6 
   E. 3 

2. Which of the following graphs is Sketch the graph of 
   \[ f(x) = 8x^2 - x^5. \]