Start Quiz set up 9:20-30 min = 8:50

Start Ch 3 Fundamental Theorem of Calculus.

Ch 3 Fundamental Theorem(s) of Calc.

Recall: The Definite Integral of \( f(x) \) over \([a,b]\)

\[
\int_{a}^{b} f(x) \, dx = \text{Area under the curve from } a \text{ to } b. 
\]

\( \int_{a}^{b} f(x) \, dx \) doesn't mean:

\[ \text{Area under the curve.} \]

Area bounded between the curve \( y = f(x) \) and the \( x \)-axis over the interval \([a,b]\).

Note: Term Area is misleading.
Ex. use 4 rec and Right hand Sum of \( f(x) = 6 - 3x \) over \([0, 4]\)

\[
\Delta x = \frac{4}{4} = 1
\]

“Area” = \((1)f(0) + (1)f(1) + (1)f(2) + (1)f(3) + (1)f(4)\)

= \(3 + 0 - 3 - 6\)

= \(-3\).

Note: Area below the \(x\)-axis has a negative weight (counted negatively).

(Def) The definite integral of \( f(x) \) over \([a, b]\)

\[
\int_a^b f(x) \, dx = \text{Area b/w } f(x) \text{ and } x\text{-axis in } [a, b]
\]

\(\int_0^4 (6 - 3x) \, dx \) = Area above/below

\(\int_0^4 (6 - 3x) \, dx = 8 \frac{8}{6}\)
Properties

1. $\int_{a}^{a} f(x) \, dx = 0$

2. $\int_{a}^{b} f(x) \, dx + \int_{b}^{c} f(x) \, dx = \int_{a}^{c} f(x) \, dx$

3. $\int_{a}^{b} c \cdot f(x) \, dx = c \int_{a}^{b} f(x) \, dx$

4. $\int_{a}^{b} f(x) + g(x) \, dx = \int_{a}^{b} f(x) \, dx + \int_{a}^{b} g(x) \, dx$

Note: $\int_{a}^{b} f(x) \cdot g(x) \, dx \neq \int_{a}^{b} f(x) \, dx \cdot \int_{a}^{b} g(x) \, dx$

$\int_{a}^{b} f(t) \, dt$

Consider $A(x) = \int_{a}^{x} f(t) \, dt$
\[ A(x+h) = \int_{a}^{x+h} f(t) \, dt \]

\[ A(x+h) - A(x) = \int_{x}^{x+h} f(x) \, dx \]

So \[ \frac{A(x+h) - A(x)}{h} = f(x) \] as \( h \to 0 \).

Formal Def. of Derivative

\[ A'(x) = f(x) \]
Fundamental Theorem of Calculus part 1

If \( f(x) \) is 1) Bounded and (No \( \infty \) y-values) 2) continuous, (No breaks),
on \([a,b]\) and \( a < x < b \),
then \( A'(x) = f(x) \) has \( A'(x) = f(x) \).

**Example**

\[
A(x) = \int_0^x 6 - 3t \, dt.
\]

\[
A'(x) = 6 - 3x.
\]

\[
B(x) = \int_x^0 6 - 3t \, dt.
\]

\[
B'(x) = - (6 - 3x)
\]

\[
C(x) = \int_x^0 \frac{\sin(t)}{t^2 + 1} + t \, dt
\]

\[
C'(x) = \frac{\sin(x)}{x^2 + 1} + (x)
\]

\[
D(x) = \int_0^{x^2} y \, dt
\]

\[
D'(x) = \frac{dD}{dy} \cdot \frac{dy}{dx}
\]

*Side Note:* \( A(0,4) = 0 \)

*Property 5*

\[
\int_b^a f(x) \, dx = - \int_a^b f(x) \, dx
\]

*Chain rule!*
\[(6-3y) \cdot 2x = [6 - 3(x^2)] \cdot 2x.\]

**General Rule:** \( F(x) = \int_a^{x^2+1} f(t) \, dt \)

\[ F'(x) = f(x^2+1) \cdot (2x) \uparrow \]
\[ \text{insert } x^2+1 \text{ for } t. \]

**Last challenge**
\[ A(x) = \int_x^{2x} 6-3t \, dt \]
\[ = \int_x^0 6-3t \, dt + \int_0^{2x} 6-3t \, dt. \]
\[ A'(x) = -(6-3x) + [6 - 3(2x)] \cdot (2). \]

**Def.** (Given \( y = f(x) \), any function \( F(x) \) such that \( F'(x) = f(x) \) is an anti-derivative of \( f(x) \).)

**Ex.** List some anti-derivatives of \( f(x) = x^2 + 2 \).
$F(x) = \frac{1}{3}x^3 + 2x \quad F'(x) = 3^{\frac{1}{3}}x^2 + 2 = x^2 + 2$.

$F(x) = \frac{1}{3}x^3 + 2x + 2$ has $F'(x) = x^2 + 2$.

Any constant has $\frac{d}{dx}(c) = 0$.

Note: Any two antiderivatives of $f(x)$ are the same up to a constant $+ C$.

**F. T. C. Part 2**

Note: $A(x) = \int_a^x f(t)\,dt$ and $A'(x) = f(x)$. $A(x)$ is an antiderivative of $f(x)$.

Choose any other antiderivative $F(x)$.

$A(x)$ and $F(x)$ differ by a + C (i.e. $A(x) = F(x) + C$).

Note: $A(a) = \int_a^a f(t)\,dt = 0$.

So $A(a) = F(a) + C$.

$0 = F(a) + C - F(a) = C$.

So $A(x) = F(x) - F(a)$.
\[ \int_a^b f(t) \, dt = F(b) - F(a) \]

Let \( x = b \).

\[ \int_a^b f(t) \, dt = F(b) - F(a) \]

**Fundamental Theorem of Calculus Part 2**

If \( f(x) \) is a continuous function on \( [a, b] \),

then \[ \int_a^b f(t) \, dt = F(b) - F(a) \]

Also \( F(x) \) is ANY antiderivative of \( f(x) \).

Then \[ \int_a^b f(t) \, dt = F(b) - F(a) \]

**Example:**

\[ \int_1^2 3 - 6t \, dt = \left[ 3t - 3t^2 \right]_1^2 = \left[ 3(2) - 3(2)^2 \right] - \left[ 3(1) - 3(1)^2 \right] = \left[ 6 - 12 \right] - \left[ 3 - 3 \right] = -6. \]

Antiderivative of \( 3 - 6t \) is \( 3t - 3t^2 \) because \( \frac{d}{dt}(3t - 3t^2) = 3 - 6t \).

**3 things to show to get full credit**