

MATH 103 Conceptual Study Help

1 Chapter 1: Areas, Volumes, and Simple Sums

- Describe the new bounds for $\sum_{i=??}^{??} f(i+3)$ if this sum is equal to $\sum_{i=3}^{10} f(i)$.
- How does the formula for $\sum_{i=0}^n r^i$ relate to $\sum_{i=0}^{\infty} r^i$?

2 Chapter 2: Areas and Riemann Sums

- Describe why $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{b-a}{n} f\left(a + \frac{(b-a)k}{n}\right) = \int_b^a f(x)dx$.
- Using the Riemann sum explain why the integral of a negative function is negative (i.e. why the area below the x -axis is counted negatively by an integral)
- Describe your method to determine $\int_b^a f(x)dx$ from the summation $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{b-a}{n} f\left(a + \frac{(b-a)k}{n}\right)$.

3 Chapter 3: The Fundamental Theorem of Calculus and the Definite Integral

- Why does $\int_0^1 x^2 dx = \int_0^1 t^2 dt$?
- Let $F(x)$ be an antiderivative of $f(x)$ and $G(x) = \int_0^x f(t)dt$. Are $G(x)$ and $F(x)$ necessarily equal?
- Derivatives and integrals are opposites (inverses). State the two fundamental theorems of calculus. How do they reflect this “inverse” idea?
- What is the formula for area between curves? Why does this formula make sense?
- Why is $\int_b^a |f(x)|dx$ equal to the total area trapped between $y = f(x)$ and the x -axis?

4 Chapter 4: Applications of the Definite Integral to Velocities, and Rates

- Why is $\int_a^b v(t)dt$ equal to displacement? Is this the same as total distance traveled? Construct an example where they are different.

- What does $\int_a^b a(t)dt$ calculate?
- Given an acceleration function how do you find the position function?
- What does the area under a velocity graph mean? What does it mean if the area is above the t -axis? What does it mean if the area is below the t -axis?
- What does the area under an acceleration graph mean? What does it mean if the area is above the t -axis? What does it mean if the area is below the t -axis?
- What does the area under a rate-of-change graph mean (like rate-of-change of brain cells per number of calculus problems done)? What does it mean if the area is above the horizontal-axis? What does it mean if the area is below the horizontal-axis?

5 Chapter 5: Applications of the Definite Integral to Mass, Volume and Arc Length

- Why does taking the integral of the density function give you total mass? Why do the units work out?
- The center of mass is $\bar{x} = \frac{1}{M} \int_0^L x\rho(x)dx$ where M is total mass and $\rho(x)$ is the density at x . Why does the x appear in the equation?
- How would you compute center of mass for a two dimensional object? Where is the center of mass for a circle, triangle, and a boomerang. Note that the center of mass of a boomerang may not be on the boomerang.
- Explain how you determine volumes of revolutions? If you have a formula, why does this formula work?

6 Chapter 6: Techniques of integration

- List and describe all integration techniques you know. How do you determine which technique to use and how do you know when a technique isn't working?

7 Chapter 7: Improper Integrals

- What makes an integral proper? What makes an integral improper?
- Why does $\lim_{x \rightarrow \infty} \int_{-x}^x t^3 dt = 0$ but $\int_{-\infty}^{\infty} t^3 dt$ not exist?
- Describe in words the integral tests comparison test. Summarize all methods you have used to form the inequalities used to make comparisons.

8 Chapter 8: Continuous probability distributions

- What is a probability density function (pdf)? What are the units?
- What does the area under a probability density function mean?
- What is a cumulative density function (cdf)? What are the units?
- List all properties of pdfs and cdfs.
- Can a function be both a pdf and cdf? Give an example.
- What is the mean? The median? When are they the same? When they are not equal which one is larger? Why?
- How do you calculate the mean and median given a pdf? How about if you are only given a graph of a pdf.
- How do you calculate the median given a cdf? How about if you are only given a graph of a cdf. Given a graph of cdf how can you estimate the mean?
- How do we understand the concepts of mean and median in terms of mass density?
- What does it mean to normalize a function to a pdf?

9 Chapter 9: Differential equations

- We only learn in this class how to solve differential equations by separating variables. What is this method? Give an example of a differential equation which can not be solved by this method.
- What is a steady state and how do you find it?

10 Chapter 10: Sequences

- What does it mean for a sequence to converge or diverge?
- How do you determine if a sequence converges or diverges?
- Do all monotone sequences converge/diverge? Do all bounded sequences converge/diverge? Provide examples if no and explain to yourself if yes.
- Are all convergent sequences monotone? Are all convergent sequences bounded? Provide examples if no and explain to yourself if yes.
- What are fixed points and what kind of sequence do they apply to?
- What does it mean for a fixed point to be stable and unstable?
- Put the following in order of which grows the fastest as $n \rightarrow \infty$. n , n^2 , n^{10} , n^n , $n!$, 3 , 2^n , e^n , $\ln(n)$, \sqrt{n} , $n^{\frac{3}{2}}$. How can you use this to determine convergence or divergence of a series written as $a_n = \frac{f(n)}{g(n)}$ (like $\frac{n+3}{n^2+3}$, $\frac{\ln(n)+3}{n^{10}+3}$, or $\frac{n^n+n^{10}}{n!+3}$).

11 Chapter 11: Series

- What is a series?
- What does it mean for a series to converge?
- What is the difference between a series converging versus a sequence converging?
- What are the four convergence tests? In what circumstances are they easiest to use? Describe your process of determining if a sequence converges or diverges.
- What is a geometric series, p -series, harmonic series, and telescoping series? How do you tell if each of them converges or diverges?
- Put the following in order of which grows the fastest as $n \rightarrow \infty$. n , n^2 , n^{10} , n^n , $n!$, 3 , $\ln(n)$, \sqrt{n} , 2^n , e^n , $n^{\frac{3}{2}}$. How can you use this to determine convergence or divergence of a series written as $\sum_{n=0}^{\infty} \frac{f(n)}{g(n)}$ (like $\sum_{n=0}^{\infty} \frac{n+3}{n^2+3}$, $\sum_{n=0}^{\infty} \frac{\ln(n)+3}{n^{10}+3}$, or $\sum_{n=0}^{\infty} \frac{n^n+n^{10}}{n!+3}$).

12 Chapter 12: Taylor series

- How do you find a Taylor series of a function $f(x)$?
- How can you use a Taylor series to estimate things like $\int_0^x e^{-x^2} dx$ and $\sin(1)$?
- How do you use Taylor series to estimate a solution for a differential equation?