MATH 103 Taylor Series Worksheet

Before your start think about the following.

- What is a Taylor series? What is a MacLaurin series? Recall that the Taylor series centered around $x = 0$ is the MacLaurin series.
- What is the general formula for the Taylor series centered at $x = 0$ of $f(x)$?
- We know the MacLaurin series for $\sin(x)$, $\cos(x)$, $\dfrac{1}{1-x}$, and e^x. What are the first four terms of these series?

Practice:

1. We can get new Taylor series from old ones. For the following write out the first four non-zero terms and also describe the formula for the coefficient of x^n.

   (a) $\sin(2x)$

   (b) $\displaystyle\int_0^t e^{-x^2}\,dx$

   (c) $\dfrac{x^2}{1-2x}$

   (d) $\arctan(x)$

   (e) $\ln(1+x)$

   (f) $(1-x)e^x$

2. Using the formula for a Taylor series centered at $x = 0$ to find the first four terms in the Taylor series of the following functions,

   (a) A function $f(x)$ where $f(0) = 1$, $f'(0) = 2$, $f''(0) = 12$, and $f'''(0) = 3$.

   (b) $\sqrt[3]{1-x}$

3. By using a suitable Maclaurin series find the values of the following infinite sums. Note by ratio test we can confirm all the following sums converge, but ratio test doesn't tell us what they converge to.

   (a) $\pi - \dfrac{\pi^3}{3!} + \dfrac{\pi^5}{5!} - \dfrac{\pi^7}{7!} + \cdots$

   (b) $1 - \dfrac{e^2}{2!} + \dfrac{e^4}{4!} - \dfrac{e^6}{6!} + \cdots$

   (c) $1 + \dfrac{2}{1!} + \dfrac{4}{2!} + \dfrac{8}{3!} + \dfrac{16}{4!} + \cdots$

4. Use the first three non-zero terms of the Maclaurin series to estimate the following values.

   (a) $\sin(1)$

   (b) $\displaystyle\int_0^1 \sin(x^2)\,dx$

   (c) e

   (d) $\displaystyle\int_0^1 e^{-x^2}\,dx$

5. We can use Taylor series to find polynomial solve differential equations. Find either the solution of the differential equation OR the first three non-zero terms of the Taylor series of the solution.

   (a) $\dfrac{dy}{dx} = y$ and $y(0) = 1$

   (b) $\dfrac{dy}{dx} = \sin(x) + y$ and $y(0) = 0$

   (c) $x\dfrac{dy}{dx} = x^2 + y$ and $y(1) = 3$