Today finish ch 4 and start ch 5
- Quiz 2 around 8:50

Announcements
- take best 5 out of 6 quizzes
- Quiz 1 now out of 8 points.

Last time RIG (DEA)

\[ \int_a^b f'(t) \, dt = f(b) - f(a). \]

integral of rate of change = Net change.

Ex

\[ \int_a^b v(t) \, dt = x(b) - x(a) \]

"Area" with sign

"below" velocity graph.

Ex

\[ \int_1^3 P'(t) \, dt \]

= How many moreless lions are there in thousand lions/year. from year 1 to 3.
Clicker #1. A ball is thrown up with speed 2 m/s. Gravity is 10 m/s². How long does it take to return to your hand?

Solution #1

Velocity: \( v(t) = 2 - 10t \)

\[ \text{Why?} \]

\[ v(t) = \text{initial} + \text{change}. \]

\[ = 2 + \int_{0}^{t} a(t) \, dt, \]

\[ a(t) = v'(t). \]

\[ = 2 + \int_{0}^{t} -10 \, dt, \]

\[ = 2 + -10 t \bigg|_{0}^{t}, \]

\[ = 2 - 10t \]

\[ t = \frac{1}{5}. \]

Note: it takes just as long to reach the max height.

Solution #2

\[ v = 2 - 10t \]

\[ x(t) = \text{initial} + \text{change}. \]

\[ = 0 + \int_{0}^{t} v(t) \, dt. \]

\[ = \int_{0}^{t} 2 - 10s \, ds, \]

\[ = 2s - 5s^2 \bigg|_{0}^{t}, \]

\[ = 2t - 5t^2. \]

When return?

\[ x(t) = 0. \]

\[ 2t - 5t^2 = 0. \]

\[ t(2 - 5t) = 0. \]

\[ t = 0, \frac{2}{5}. \]
as it takes to return.

Answer: \( \frac{2}{5} \) s.

Last time,

we start at position 10 at \( t=2 \).

\[
\begin{align*}
\text{displacement } [2,6] & \text{ vs } \\
\int_{2}^{6} v(t) \, dt &= 4+2 - 3 \\
&= 3.
\end{align*}
\]

\[
\int_{2}^{6} |v(t)| \, dt = 4+3+2 = 9.
\]

Where are we at \( t=6 \)?

10 + 3 = 13.

\( \text{(Def) the average value of } f(x) \text{ over } [a,b] \) is

\[
f_{\text{avg}} = \frac{1}{b-a} \int_{a}^{b} f(x) \, dx
\]
Ex Find average velocity over \([0, 4]\) where \(v(t) = \sqrt{t}\).

\[
V_{\text{avg}} = \frac{1}{4-0} \int_0^4 v(t) \, dt
\]

\[
= \frac{1}{4} \int_0^4 \sqrt{t} \, dt.
\]

\[
= \frac{1}{4} \left[ \frac{2}{3} t^{3/2} \right]_0^4
\]

\[
= \frac{1}{4} \left[ \frac{2}{3} \cdot 4^{3/2} - \frac{2}{3} \cdot 0^{3/2} \right]
\]

\[
= \frac{1}{4} \left[ \left( \frac{4}{3} \right)^{3/2} \right] = \frac{1}{4} \cdot 8 = \frac{4}{3} \text{ units/time}
\]

Ch 5 More Applications

- Mass / center of mass
- Volumes of revolution
- Arc length
Total Mass

Discrete Case

\[ 2 \text{kg} \quad 1 \text{kg} \quad 4 \text{kg} \]
\[ 1 \text{m} \quad 2 \text{m} \quad 3 \text{m} \]

\[ M_1 \quad M_2 \quad M_3 \]

Total Mass = 2 + 1 + 4
= \sum_{i=1}^{3} M_i

Continuous Case

A length L rod with density \( \rho(x) \) at \( x \).

Piece at \( x \) length \( dx \) density \( \rho(x) \).

Mass = density \cdot length.

\[ \text{tiny mass} = \rho(x) \cdot dx. \]

Total Mass = \[ \int_0^L \rho(x) \cdot dx. \]

Mass

For a length L rod, which has density \( \rho(0) \) at one end and density 4 kg/m at the other (density increases linearly along the rod).
\[
M = \int_{0}^{L} \rho(x) \, dx.
\]

\[
= \int_{0}^{L} \frac{4}{L} x \, dx.
\]

\[
= \frac{4}{L} \left[ \frac{1}{2} x^2 \right]_{0}^{L}
\]

\[
= \frac{4}{L} \left( \frac{1}{2} L^2 - \frac{1}{2} 0^2 \right)
\]

\[
= \frac{4}{L} \cdot \frac{1}{2} \cdot L^2 = 2L.
\]

\[
\text{Center of Mass}
\]

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**Discrete case**

**Continuous case**
\[ \overline{x} = \frac{1}{M} \int_0^L x \cdot p(x) \, dx \]

Find \( \overline{x} \).

\[ \overline{x} = \frac{1}{M} \int_0^L x \cdot p(x) \, dx = \frac{1}{M} \int_0^L x \cdot \frac{4}{L} x \, dx \]
\[
M = \frac{1}{2} \times \frac{4}{3} \int_0^L x^2 \, dx
\]

\[
= \frac{1}{3} \int_0^L x^3 \, dx
\]

\[
= \left. \frac{1}{3} \frac{x^3}{3} \right|_0^L
\]

\[
= \frac{1}{3} \left( \frac{1}{3} L^3 \right) - 0
\]

\[
M = \frac{1}{2} \frac{4L^3}{6L^2} = \frac{2L}{3L} = \frac{2}{3} L.
\]