Syllabus Summary

This is a living list and will be updated throughout the semester.

In this list I summarize the material from the syllabus indicating which material is

• very important,
• of normal importance,
• not required (so can be skipped), and
• not covered in class but you need to know. (I focus on covering the most common, important and non-obvious topics in class. Regretfully, there is not enough time to cover every example type in class. Every good math-diet always needs a good helping of these problem solving opportunities.)

**See syllabus for complete list of topics.

1 Chapter 1: Areas, Volumes, and Simple Sums

• Formulas for area. Besides area of a rectangle and circle you will normally be provided with formulas.
• Writing sums with sigma notation.
  – Arithmetic
  – Increases / decreases by an arithmetic sequence.
  – Geometric
  – Alternating signs
  – Gathering terms together in a useful way / telescoping
  – Manipulations (like index shift)
  – partial sums / convergence (will be covered in more detail in a later chapter)
• Applications (Many of these problems are long, so you wouldn’t find the whole of these problems on examinations, but perhaps parts and ideas from them)

2 Chapter 2: Areas, Riemann Sums

• Approximate definite integrals / area under the curve with a specified number of rectangles. (left end points, right end points, midpoint, trapezoid)
• Approximate definite integrals / area under the curve with \( n \) rectangles. (right end points (used for nearly all problems of this type), left end points, midpoint (very uncommon on exams)).
• Going from Riemann sums to definite integrals and backwards.
• Calculate definite integrals with geometry (i.e. when given regions of squares, rectangles, trapezoids and circles)
• Definite integral properties
3 Chapter 3: The Fundamental Theorem of Calculus, Definite Integral

- Antiderivative of standard functions \( (x^n, \sin x, \cos x, e^x) \)
- Fundamental theorems of calculus parts 1 and 2.
- When can’t you use fundamental theorems of calculus?
- Using symmetry to evaluate definite integrals.
- Area between curves. For examples see [Paul’s Online Notes](#) ← click here
  - Between two functions (where you standardly solve for intercepts)
  - Between two functions from \( x = a \) to \( x = b \) (check for intercepts between \( a \) and \( b \))
    (Similar to example 2, 4 and 5 in Paul’s online notes)
  - Using horizontal strips (Similar to 6 and 7 in Paul’s online notes)
- Reading a graph (many webwork problems on this type)

Circadian rhythm in hormone levels

4 Chapter 4: Applications of the Definite Integral to Velocities, and Rates

- How distance, velocity and acceleration relate with derivatives, antiderivatives and definite integrals.
- The difference between displacement and total distance. (How to calculate both given a function or graph of velocity)
- The general idea: The definite integral of rate of change equals net change.
- The average value formula and how to compute average value.
- Given a graph of a rate of change know how to compute net change. (Also compare two graphs of rate of change like Circadian rhythm in hormone levels in section 4.4 from the course notes)

5 Chapter 5: Applications of the Definite Integral to Mass, Volume and Arc Length

- Total mass formula, center of mass formula, average mass density and how to find the point \( c \) where you cut a rod into two pieces of equal mass. (All of 1D objects)
- Center of mass for 2D objects.
- Volume of revolution about \( x \)-axis, \( y \)-axis, a horizontal line or a vertical line. (Disk method, washer method and shell method)
• Arc length.

• Total mass of other density functions. Examples from class include Actin density (Example 5.2.3 from course notes) and circular bacteria population (Example 5.4.2 from course notes).

6 Chapter 6: Techniques of integration

• Computing differentials.

• Difference between definite and indefinite integrals (and their computations).

• $u$-substitution (simple, using the “$u =$” equation, with trig identities, complete the square, partial fractions (only two linear factors))

• Trigonometric substitution.

• Integration by parts.

• Integrals of all six trig functions.
  – $\sin(x)$ and $\cos(x)$ are standard.
  – $\tan(x)$ and $\cot(x)$ can be done by simple $u$-substitution.
  – When finding $\int \sec(x) \, dx$ you need to multiply by $\frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)}$ and use simple $u$-substitution. There is a typo in the syllabus hint.
  – When finding $\int \csc(x) \, dx$ you need to multiply by $\frac{\csc(x) + \cot(x)}{\csc(x) + \cot(x)}$ and use simple $u$-substitution. There is a typo in the syllabus hint.

• Centroid (center of mass for 2D objects)

7 Improper Integrals

• Definition of improper integrals and the two different types.

• Use limits to evaluate improper integrals.

• Definition of convergence and divergence for improper integrals.

• Use the integral comparison test to show convergence and divergence.

• Compute integrals using techniques and taking limits to show convergence and divergence.

• L’Hopitals rule.
8 Continuous probability distributions

- Definition of PDF and CDF.
- How to normalize a density function to form a PDF (and when you have to do this. Often this is not needed)
- Use a PDF or CDF (given as a function or graph) to compute
  - probability $x$ is in an interval $[x_1, x_2]$
  - average / mean
  - median
- Use a PDF or CDF (as a function) to compute standard deviation and variance.
- The difference between mean and median and how to determine where they are given a graph.

9 Differential equations

- Method of separating variables (This is the only method of solving differential equations you have to know) and solving initial value problems.
- Find steady states and determine whether they are stable or unstable.
- Be able to draw and read the graph of the derivative (i.e. $\frac{dy}{dt}$ verses $y$) and the phase diagram.
- Modeling. (Some examples include population growth, logistic growth, Newton’s law of cooling, draining a container [Click for examples])
- Determine the behavior as $t \to \infty$ given an initial value.

10 Sequences

- The definition of bounded and monotone.
- Determine the closed form of a sequence. This is similar to writing sums in sigma notation which we did in chapter 1.
- Determine convergence or divergence of a sequences with various methods. (Taking a limit, using growth rates, squeeze theorem, L' Hospitals rule, using $a_n = e^{\ln(a_n)}$, facts about iterated maps)
- Know how monotone, bounded and convergence / divergence connect. Which combinations are possible. Which are not possible?
- How to draw a cobweb given an iterated map and an initial value.
- Understand convergence or divergence for any linear iterated map (i.e. of the form $a_{n+1} = ma_n + b$) as well as whether the sequence is monotone or bounded.
• Find steady states / fixed points / equilibria of an iterated map and determine when it is stable or unstable.

• Modeling (Simple growth $a_{n+1} = \alpha a_n$, logistic growth $a_{n+1} = a_n + \alpha a_n \frac{K - a_n}{K}$ and other simple models)

• Compare and contrast the simple growth model for sequences ($a_{n+1} = \alpha a_n$) and differential equations ($\frac{dP}{dt} = \alpha P$) as well as logistic growth ($a_{n+1} = a_n + \alpha a_n \frac{K - a_n}{K}$) versus $\frac{dP}{dt} = \alpha P \frac{K - P}{K}$).