MATH 103 Integration Worksheet

We have many methods for integration. There are of course many small tricks in each technique, but these are the big ones. Remember each of them in terms of their strengths and when they have not worked.

1. power rule and trigonometric formulas
2. u-substitution
3. integration by parts
4. trig-subsitution

We have applied integration in the following applications. Take a moment and remember how the application worked. Are there more?

- Area between curves
- volume of rotation
- arc length
- total mass
- center of mass
- displacement or total distance traveled
- solving differential equations
- mean and median

Practice:

1. $\int_0^1 (5x+3)^5 dx = \frac{1}{30}(8^6 - 3^6)$
2. $\int_0^1 x^2(5x^3+3)^5 dx = \frac{1}{90}(8^6 - 3^6)$
3. $\int_{-1}^1 \frac{x+1}{(x^2+2x+2)^3} dx = \frac{6}{25}$
4. $\int_{-1}^1 \frac{x}{x-2} dx = 2 - 2\ln(3)$
5. $\int \frac{2x-5}{\sqrt{x^2-5x+6}} dx = 2\sqrt{x^2-5x+6} + C$
6. $\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2) + C$
7. $\int \frac{1}{x^2\sqrt{1+x^2}} dx = -\frac{\sqrt{x^2+1}}{x} + C$
8. $\int \frac{x^2}{1+x^2} dx = -\arctan(x) + x + C$
9. $\int xe^{x^2} dx = \frac{1}{2}e^{x^2} + C$
10. $\int x^3 e^{x^2} dx = \frac{1}{2}e^{x^2}(x^2-1) + C$
11. $\int e^{-\sqrt{x}} dx = -2e^{-\sqrt{x}}(1+\sqrt{x}) + C$
12. $\int \cos(x) dx = \sin(x) + C$

13. $\int \cos^2(x)dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$

14. $\int \cos^3(x)dx = \sin(x) - \frac{1}{3}\sin^3(x) + C$

15. $\int \cos^4(x)dx = \frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$

16. $\int \cos^5(x)dx = \sin(x) - \frac{2}{3}\sin^3(x) + \frac{\sin^5(x)}{5} + C$

17. The volume of the solid generated by rotating the region bounded by $y = e^x$, $y = 0$, between $x = 0$ and $x = 4$ around the x-axis. $\frac{\pi}{4}(e^8 - 1)$

18. Find the area between the curves $x = 3y - y^2$ and $x + y = 3$. $\frac{4}{3}$

19. What is the length of the $y = x^{3/2}$ over the interval $[0, 1]$? $\ln(1 + \sqrt{2})$

20. Given the velocity $v = 3(t - 3)(t - 1)$ what is the displacement and total distance traveled in the first 2 seconds? displacement $= 2$ and total distance $= 6$

21. Given the acceleration $a(t) = \frac{1}{t^2 + 1}$ and we know the position $s(0) = 0$ and $s(1) = 0$ what is the position function $s(t)$? $s(t) = t\arctan(t) - \frac{1}{2}\ln(1 + t^2) + \left(\frac{1}{2}\ln(2) - \frac{\pi}{4}\right)t$

22. Find the average height of $\cos(x)$ over the intervals $[0, \pi/2]$. $\frac{2}{\pi}$

23. A beam 4 meters long has density $\rho(x) = x^3$ at distance x from the left end of the beam. Find the center of mass $\bar{x}$. Where do we need to cut the bar so that we have two pieces of equal length. Total mass $= 64$, $\bar{x} = 3.2$, and the cut point $= \sqrt[4]{128}$

24. Find $y(t)$ so that $\frac{dy}{dt} + 2yt = 0$ and $y(0) = 1$. $y(t) = e^{-t^2}$

25. Find $y(t)$ so that $\frac{dy}{dx} = \frac{x^2}{y(1 + x^3)}$ and $y(0) = 1$. $y(x) = \sqrt{\frac{2}{3}\ln(1 + x^3) + 1}$

26. What does C have to be so that $p(x) = Cx^2$ is a pdf over $[0, 2]$? Using this C find the cdf, mean, median, variance, and standard deviation. If x represents the hours customers wait before their phone call is answered by a help line interpret the mean and median.

    $C = \frac{3}{8}$, $\bar{x} = 1.5$, $x_{med} = \sqrt[3]{4}$, $V = .15$, $\sigma = \sqrt{.15}$

    $\bar{x} = 1.5$ means the expected wait time is 1.5 hours

    $x_{med} = \sqrt[3]{4}$ means that half the customers wait $\sqrt[3]{4}$ hours or less.

27. An open cylindrical barrel of diameter 7ft and height 21ft is filled to a depth of 10ft by muddy water. The density of the water varies linearly with the depth of the water. If the density at the bottom is 80 pounds per cubic foot and the density at the top is 30 pounds per cubic foot find the total weight of the muddy water in the cylindrical barrel. $\int_0^{10} \pi \left(\frac{7}{2}\right)^2 \left(80 - \frac{50}{21}y\right) dy = 3841.7$