Assignment 1

Math 104 - Section 107

Question 1 *(3 points)*

The Happy Cookie Bakery is the only producer of cookies in the city of Vancouver. Let p be the price of a cookie (in dollars) and let q be the daily demand for cookies in Genovia. The bakery's owner estimates that the price and demand are related by the following equation:

$$5000p + 2q = 20000 \ .$$

(a) If the bakery sells each cookie for 1$, how many cookies will be sold per day? What will be the factory's daily revenue?

(b) Express the price p in terms of the demand q.

(c) Express the revenue R as a function of q.

(d) Suppose that the daily production cost is 5000$ to keep the bakery running plus 0.2$ for every cookie baked. Express the bakery's daily profit (P) as a function of q.

(e) Continuing (d), how many cookies should the bakery produce in order to maximize its profit? What should be the price of a cookie in this case?

(f) Assume now that the cost of producing q cookies each day is $C(q) = aq + b$ dollars (a dollars for each cookie and b dollars to keep the bakery operating). Find a and b if the bakery's daily profit is maximized when $q = 4000$ and the maximal daily profit is 5000$.

Question 2 *(7 points)* Compute the following limits:

1. $\lim_{x\to -2} \frac{x^2+4x+4}{x^2+3x+2}$
2. $\lim_{x\to -2} \frac{x^2+2x+2}{x^2+x+2}$
3. $\lim_{x\to -2} \frac{x^3+8}{x^2-4}$ (Hint: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$)
4. $\lim_{x\to 1} \frac{\sqrt{x^2+3}-2}{x-1}$
5. $\lim_{x\to 0} \frac{x(x+2)}{\sqrt{x^2+x+1}-\sqrt{x^2+1}}$.
6. **Fact:** $1 + x \le e^x \le 1 + x + x^2$ for all $0 \le x \le 1$.
   Use the fact and the squeeze theorem to prove that $\lim_{x\to 0^+} \frac{e^x-1}{x} = 1$.
7. **Fact:** $\lim_{x\to 0} \frac{\sin x}{x} = 1$.
   Use the fact and limit rules to compute $\lim_{x\to 0} \frac{x^2+x^3}{(\sin x)^2}$.
8. A ball is thrown vertically into the air. It is given that the height of the ball after t seconds is $20t - 5t^2$ meters.
   (a) When will the ball hit the ground? (The ground is height 0.)
   (b) Find the average speed of the ball on the interval $1 \le t \le 2$.
   (c) Compute (according to the definition) the instantaneous speed of the ball at $t = 0$ and $t = 1$.