OSH 3
Math 104 - Section 107

**Question 1** *(2 points)* Peter Pan sells flying powder. Denote by $q$ the amount of power produced (in grams) and $p$ the price (in dollars) of one gram of power. It is given that:

(i) $p$ and $q$ are related via $p^2 + q^2 = 5000$.

(ii) The cost of producing $q$ grams of powder is $C(q) = 1000 + 10q$.

Answer the following:

1. Find the revenue ($R$) and profit ($P$). Express them as functions of $q$.
2. Find the marginal cost and marginal revenue. Express them as functions of $q$.
3. Suppose $q = 50$. What is the marginal revenue and marginal cost? Does increasing $q$ increases the profit?
4. For what $q$ is the profit maximal?

**Question 2** *(2 points)* Differentiate the following functions:

1. $2^x + \log_3 x - 2x^\pi$
2. $(5^x - x)^{1.4}$
3. $x^{(e^x)}$
4. $(\ln x)^{\ln x}$

**Question 3** *(2 points)* Use implicit differentiation to express $\frac{dy}{dx}$ as a function of $x$ and $y$ in the following cases:

1. $x^3 + xy + y^3 = 1$
2. $e^x + e^y = xy + 1$

**Question 4** *(2 points)* Find the tangent line to the curve $x + \cos x = y^5 + y^4 - 1$ at the point $(0, 1)$.
**Question 5** *(2 points)* Find all values of $a$ for which the tangent line to the curve $x^2 - axy + y^2 = 1$ at the point $(1, 0)$ passes through the point $(2, 5)$