

OSH 2
Math 104 - Section 107

Question 1 (2 points) Use the Intermediate Value Theorem (and your calculator) to show that the equation

$$e^x = 5 - x$$

has a solution in the interval $[1, 2]$. Find the solution's first two decimal digits after the dot (you must justify your answer using the Intermediate Value Theorem).

Question 2 (2 points) Differentiate the following functions:

1. $\frac{x^2}{x^3+1}$
2. $x^2 \cdot \ln x \cdot \cos x$
3. $\sqrt[3]{x} + \sqrt{e^x + 1}$
4. $\sqrt{\ln(e^x + \sin x)}$.

Question 3 (2 points) Find the derivative of the the following functions according to the limit definition of the derivative (no credit will be given for other methods).

- (a) $f(x) = \frac{1}{x^2+1}$
- (b) $f(x) = \sqrt{x^2 + x}$.

Question 4 (2 points) Let

$$f(x) = \begin{cases} x^3 - x & x > -1 \\ 2x + 2 & x \leq -1 \end{cases}$$

- (a) Prove that f is differentiable at $x = -1$ and find $f'(-1)$. (Hint: Compute the left and right limits of $\frac{f(-1+h)-f(-1)}{h}$ as h approaches 0 separately.)
- (b) Find a formula for $f'(x)$ when $x \neq -1$, and draw the graph of $f'(x)$ on the interval $-3 \leq x \leq 1$.

Question 5 (2 points) Let f and g be differentiable functions.

1. Express the derivatives of the following functions using f , g and their derivatives:

$$x^2 f(x) - g(x), \quad f(x^2 - g(x)).$$

2. It is given that

$$\begin{array}{ll} f(2) = 2 & g(2) = 3 \\ f'(2) = -2 & g'(2) = 1. \end{array}$$

Find the equation of the tangent line to the graph of $y = \frac{f(x)+1}{g(x)+1}$ at $x = 2$. (Recall: The equation of a line with slope m passing through a point (a, b) is $y = m(x - a) + b$.)