Quiz 3

Math 105 - Section 209

Name: ____________________

Student Nr.: ____________________

Question 1 *(3 points)*

Find the right Riemann sum approximating the area under the curve $f(x) = x^2$ over the interval $[1, 11]$ using 10 subintervals. (Your answer may be left as a short sum of fractions, but no sigmas.)

$$\text{Note:} \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Solution:

The length of the subintervals is $\Delta x = \frac{b-a}{n} = \frac{11-1}{10} = 1$ *(1pt)*.

Then, the right Riemann sum is

$$\begin{aligned}\sum_{i=1}^{10} f(a + i\Delta x)\Delta x &= \sum_{i=1}^{10} f(1+i) = \sum_{i=1}^{10} (1+i)^2 = \sum_{i=1}^{10} 1 + 2i + i^2 \quad (1pt)\\ &= \left(\sum_{i=1}^{10} 1\right) + 2\left(\sum_{i=1}^{10} i\right) + \left(\sum_{i=1}^{10} i^2\right) = 10 + 2\frac{10 \cdot 11}{2} + \frac{10 \cdot 11 \cdot 21}{6} \quad (1pt)\end{aligned}$$

Remark: There was an error in the formula for $\sum_{i=1}^{n} i$ in the original quiz. We accepted both versions.

Question 2 *(2 points)*

Compute $\displaystyle\frac{d}{dx}\int_0^{e^x} \cos(t)dt$.

Solution:

If our area function is $A(x) = \displaystyle\int_0^x \cos(t)dt$, then this question is asking us to compute $\frac{d}{dx}A(e^x)$.

By the chain rule, this is equal

$$\begin{aligned}\frac{d}{dx}A(e^x) &= A'(e^x)\frac{d}{dx}(e^x)\\ &= A'(e^x)e^x \qquad (1pt)\end{aligned}$$

By the Fundamental Theorem of Calculus, part 1, $A'(x) = \cos(x)$, and so $A'(e^x) = \cos(e^x)$. Plugging this all into the above equation gives

$$\frac{d}{dx}A(e^x) = \cos(e^x)e^x \qquad (1pt)$$

Question 3 *(2 points)*

Evaluate $\displaystyle\int_0^1 \frac{y^2}{(y+1)^4}dy$.

Solution:

We will use substitution. Let $u = y + 1$, so $y = u + 1$ and $dy = du$. The lower limit of the integration is then $u_0 = 0 + 1 = 1$ and the upper limit of the integration is $u_1 = 1 + 1 = 2$ (1pt). Then our integral becomes

$$\begin{aligned}\int_0^1 \frac{y^2}{(y+1)^4}dy &= \int_1^2 \frac{(u-1)^2}{u^4}du \\ &= \int_1^2 \frac{u^2 - 2u + 1}{u^4}du \\ &= \int_1^2 u^{-2} - 2u^{-3} + u^{-4}du \\ &= -u^{-1} + u^{-2} - \frac{1}{3}u^{-3}|_1^2 \\ &= -2^{-1} + 2^{-2} - \frac{1}{3}2^{-3} - (1^{-1} + 1^{-2} - \frac{1}{3}1^{-3}) \\ &= \frac{1}{24} \qquad (1pt)\end{aligned}$$

Question 4 *(3 points)*

Evaluate $\displaystyle\int_1^2 x^3 \ln x dx$.

Solution:

We will integrate by parts. The functions $\ln x$ is difficult to integrate, so let us instead differentiate it. We choose $u = \ln x$ and $dv = x^3$ *(1pt)*. This implies that $du = \frac{1}{x}$, and $v = \frac{x^4}{4}$. Hence, for the indefinite integral we get that

$$\begin{aligned}\int x^3 \ln x dx = (\ln x)\frac{x^4}{4} - \int \frac{1}{x}\frac{x^4}{4}dx &= (\ln x)\frac{x^4}{4} - \int \frac{x^3}{4}dx \\ &= (\ln x)\frac{x^4}{4} - \frac{x^4}{16} + C. \quad (1pt)\end{aligned}$$

Applying Fundamental Theorem of Calculus, part 2 we get that

$$\int_1^2 x^3 \ln x dx = (\ln 2)\frac{2^4}{4} - \frac{2^4}{16} - (\ln 1)\frac{1^4}{4} + \frac{1^4}{16} = 4\ln 2 - 1 - 0 + \frac{1}{16}. \quad (1pt)$$